Analysis and Design Principles of Microwave Antennas
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Lecture - 33
Slot Antenna

Welcome to this NPTEL lecture on generalized analysis. We were continuing that generalized analysis of Antennas. Now in the last lecture, we have found all the formulas required to analyze the antenna by the potential concepts. So, now, today we will first see a application of that; actually there will be 3 examples we will show by which we will see that various types of antenna which are commonly used. They can be analyzed by the same techniques that we have found.

The first one is the rectangular aperture in an infinite ground plane.

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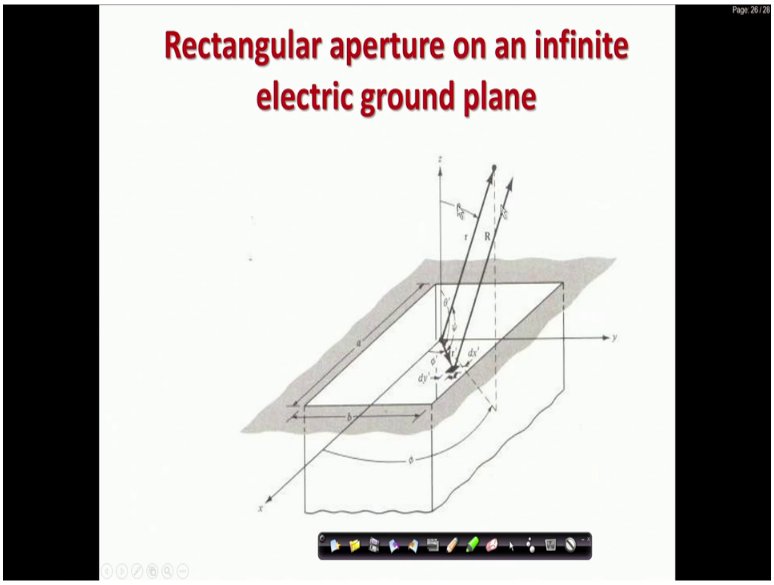

You can see this, this is there is a infinite conductor. Though here it is shown as finite, but assume it is an infinite. So, for infinite generally we say that in each side, the conductor should be at least 5 lambda long. So, if we do that; that means, if we are the lowest frequency let us say that we are having x band so 8 gigahertz lowest frequency; so, roughly 2.5 centimeter. So, 5 times that lambda so, 11 12 centimeter inside if we keep we can call it call infinite electric ground plane and we can have an aperture.

Now, aperture may be of any shape. Here, we are taking rectangular aperture and this aperture actually you see this is the way you can analyze actually slot antennas by these. So, let us say that we have a rectangular slot basically. So, it is an aperture in an infinite ground plane and I can write here the basic aperture diagram.

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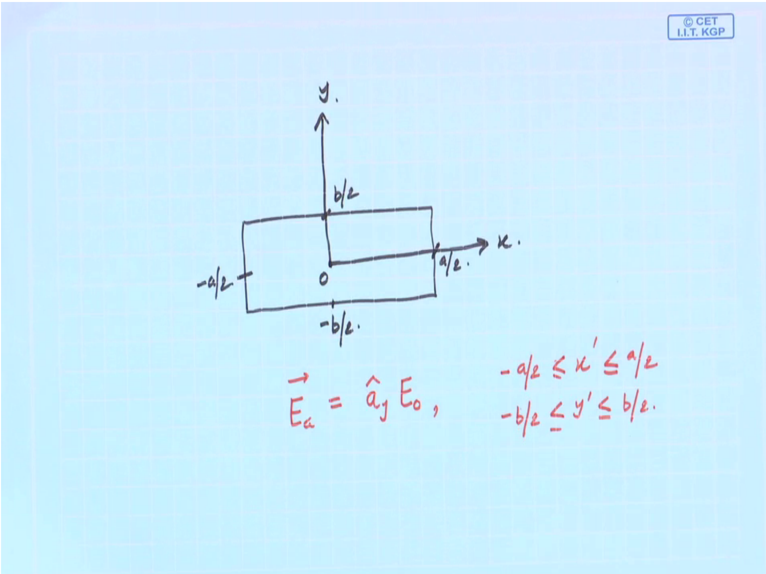

So, let us say that in the x y plane, the aperture is there; that means, that a sorry x y. So, that is z is equal to 0 plane and let us say that this point is b by 2 and this point is a by 2. So that means, this point will be minus a by 2 and this point will be minus b by 2.

So, the center of the aperture that there we take the origin of the coordinate system and we assume that aperture field is uniform. Now that thing is valid, if we have this width of this; that means, that b is very small compared to a. So, in that case slots are generally that width are very small.

So, you can assume that the field is uniform there or if you can in a more advanced thing you can assume some other field also based on where the slot is cut etcetera. But to start with you will see that this gives a good result. So, we can say that the aperture field, we assume that it is uniform and obviously, here it will be y directed as the thing and let us say that it is uniform. So, there is x y variation so, E naught is a constant.

So, I can say that this is valid for minus a by 2 x dashed. Again, I am writing x dashed though I have shown axis as x y, but since this is my source. So, I am using prime coordinates and ok. So, the moment we do this, our next step is straightforward we will have to find what is the magnetic current for this surface magnetic current.

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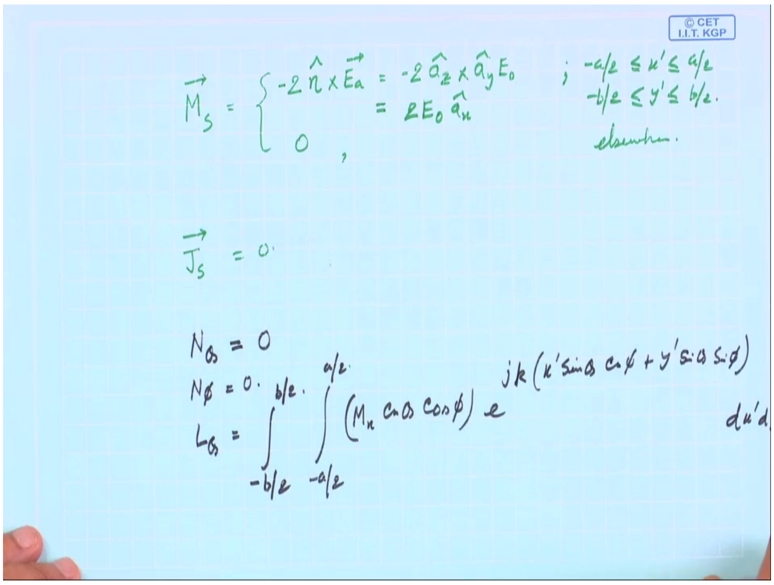

So, M s is as we know that M s will be minus 2. So, n in this case is z directed. So, a z cross a y E 0, you can write the region where this is valid and 0 elsewhere. And also, we know that since we have an infinite ground plane; so, z s is 0.

So, the source we have found the surface with a equivalence principle the surface sources; so, currents. So, now, if you see your old notes that you can easily find out that since J s is 0, N theta will be 0; N phi will be 0 and there will be L theta L phi.

Now, L theta you see M s is only having a z y a y means that will have I can say also here that it will only have an x component and that is nothing but if you do 2 E naught a x. So, L theta will be minus b by 2 to plus b by 2 you can see it that this is.

So, here you can cos theta cos phi etcetera will come out. So, only because this integration is x dashed thing. So, and M x already we know a M x value is 2 E 0.

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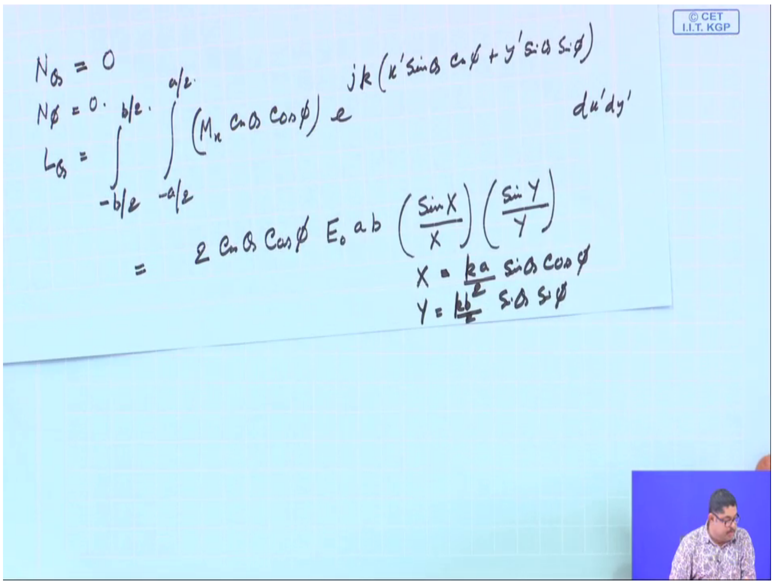

So, we can easily do this integration because it is E to the power j k x dashed sin theta cos phi all this constant x dashed. So, you can easily do and you will find that this becomes I can write 2 cos theta cos phi E naught a b then sin capital X by X sin capital Y by Y; where, X X is k a by 2 sin theta cos phi and Y is k b by 2 sin theta sin phi so, multiplication of 2 sin functions.

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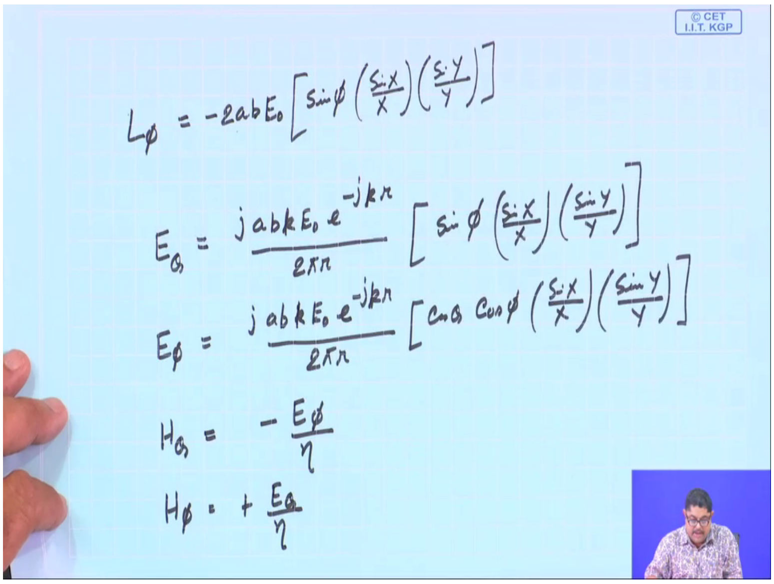

And similarly we can find L phi L phi will be minus 2 a b E naught sin phi sin X by X sin Y by Y. So, once you know N theta N phi L theta L phi, you can find out by E theta;

we have already done that before. So, now, it will be j a b k E naught e to the power minus jkr by 2 pi r sin phi sin X by X sin Y by Y.

And E phi will be j ab so, these are the far fields. E naught e to the power minus jkr by 2 pi r cos theta cos phi sin X by X sin Y by Y. Also H theta will be minus E phi by eta and H phi will be plus E theta by eta ok.

So, we have found easily you see from the analysis that we have done. You can find the far fields. Far field has E theta E phi components and H theta H phi components and we can now find the radiation patterns. So, since our actual aperture field is y directed; so, we can define the principal planes.

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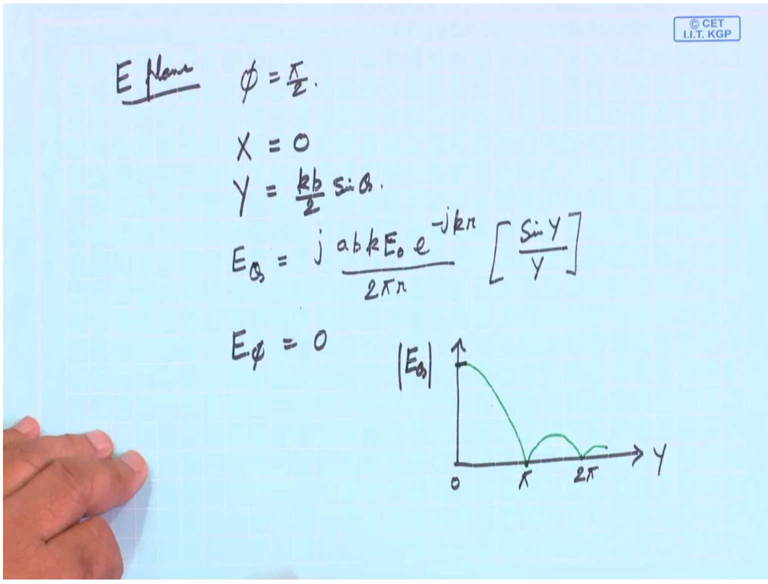

Principal planes will be; so, one is E plane we call where the y z plane. So, that is nothing but phi is equal to pi by 2. So, in if we put pi by 2, let us look at X and Y definitions. So, immediately X becomes 0 and Y non zero. So, X becomes 0; capital X and Y becomes k b by 2 sin theta.

So, immediately if you see the E theta, if I expressions. So, you can see that E theta is j ab k E naught E to the power minus jkr by 2 pi r sin Y by Y and E phi will become 0.

So, in the principal plane you do not have E phi. So, you can put this normalize. This is actually constant and this part if I take the magnitude of this field, this part goes. So, basically it is a sin Y. So, I can put the radiation pattern in the E plane that will, it is a field radiation pattern; power radiation pattern will be square of this.

So, if I normalize this; that means, I divide by these. So, basically it is a sin functions. So, here it is Y. So, now, you can take that this value will be 1 at 0 and then, it will go you know the sin pattern and since I am taking the magnitude. So, it will only positive things like that.

So, where are those points? This point is pi; this point is 2 pi etcetera. And I will later just let me draw the H plane pattern; then, we will analyze various beam width, we will have to find from here ok.

So, this is E plane. Now, let me see, what is the H plane pattern?

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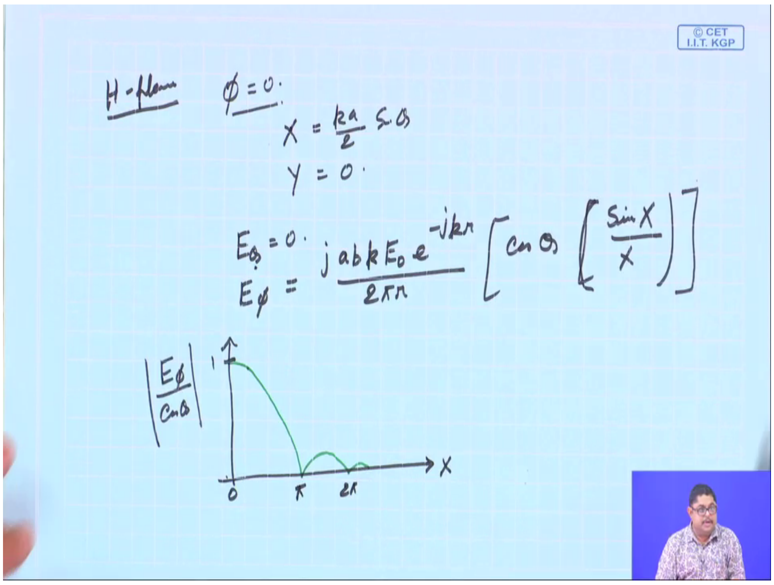

So, H plane is definitely the other one; that means, phi is equal to 0 that principal plane. So, they are immediately my X becomes k a by 2 sin theta and Y becomes 0. So, my E theta here goes to 0; but E phi exists and E phi is again that same constraint thing j a b k E naught e to the power minus jkr by 2 pi r. But here not only a sinc function it is a cos theta into that sin X by X, it will like this.

Now obviously, this sinc function; where, this x is given by this k a by 2 sin theta. So, these sincs variation is much faster than this cos theta variation, if you plot. That is why generally people plot it like this that the E phi by cos theta. So, near about theta is equal to 0, this gives you 2 things.

But actually, you can always put it also. So, it will be same that sinc type of function. But obviously, you will have to give the theta. So, this is with respect to X and it is again this is one and this is pi this is 2 pi etcetera right.

Now, let us now look more closely this E plane pattern. So, where are the nulls occurring because we know the main beam the maximized at Y is equal to 0; that means, in the broadside direction z is equal to 0; z direction. So, let us see in the E plane pattern again.

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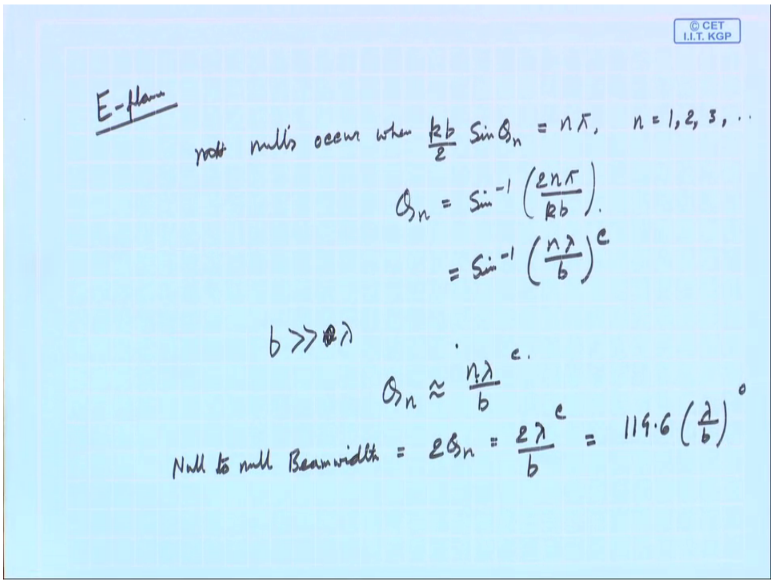

E plane; so, nulls sorry occur when we have kb by 2 sin theta n is n pi; where, n is equal to 1, 2, 3 etcetera.

So, what is the value of theta n? That means, I can say that E plane. So, here corresponding to this Y, this theta n is coming. So, what is the value of this theta n? It is sin inverse 2 n pi by kb. So, if you put the value of k which is 2 pi by lambda, it becomes sin inverse n lambda by b and remember that this is in the radian.

So, generally the bs are taken much larger than lambda. Actually for any practical antenna, you always said that the dimensions that should be several lambdas long; otherwise you do not get the interference pattern. So, we can assume that b is much

much larger than or you can say lambda. So, I can say that theta n is approximately n lambda by b in radian ok.

So, what will be the null to null beam width; null to null beam width? So, that will be 2 theta n and that will be 2 lambda by b put actually the first null so that n is equal to 1. So, this is in a thing and you can convert it to degree that will give you 114.6 lambda by b in degree.

Similarly, you can find half power beam width. So, first we will have to find where is the half power occurring? That means, obviously, half power is occurring somewhere here. Let me corresponding theta, let me call theta 3 dB.

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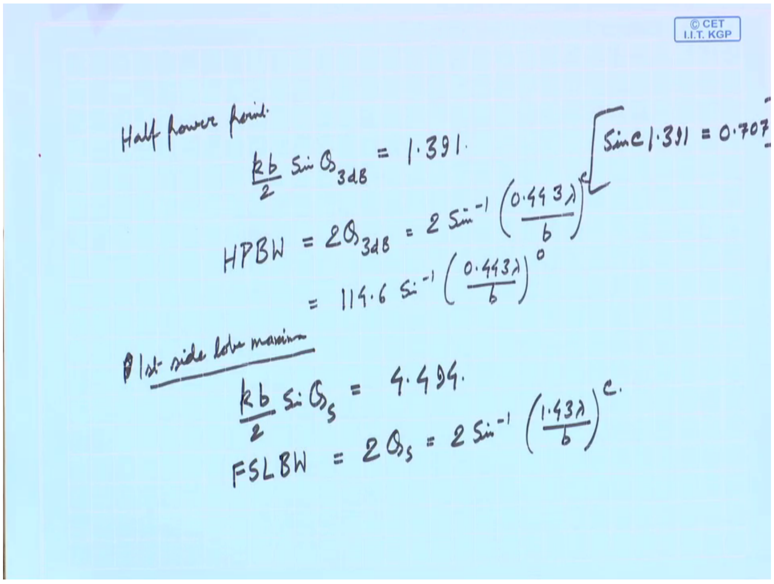

So, Half power point, point occurs when kb by 2 sin theta 3 dB; is anyone knows this value? This is from a sinc function. So, since we know the sinc functions, actually please note this you can check these at sinc of 1.391 is equal to 0.707 which is our half power point.

So, I knew this that is why I wrote that 1.391. So, from there, you can now write half power beam width that will be 2 into theta three dB and that is 2 sin inverse 0.443 lambda by b in radian and that will turn out to be 114.6 sin inverse 0.443 lambda by b in degrees. So, if you know b in terms of lambda, you can easily find out this.

Similarly, 1st side lobe maxima; that means, 1st side lobe maxima where occurs? That occurs, again a known thing that. So, I will write first side lobe maxima. So, for a sinc function, this value is known. So, that is if we say sin let us say side lobe maxima.

So, theta s value that occurs when this thing x is 4.494. So, from here you can say that first side lobe beam width that will be 2 theta s and that is 2 sin inverse 1.43 lambda by b in radian.

So, important thing is what is a 1st side lobe level, you see this is beam width. But we want to know that this maximum value that means, if I have here, what is this divided by this; that is called 1st side lobe level. So, obviously, this is 1. So, if I know this value I can find and that value is known.

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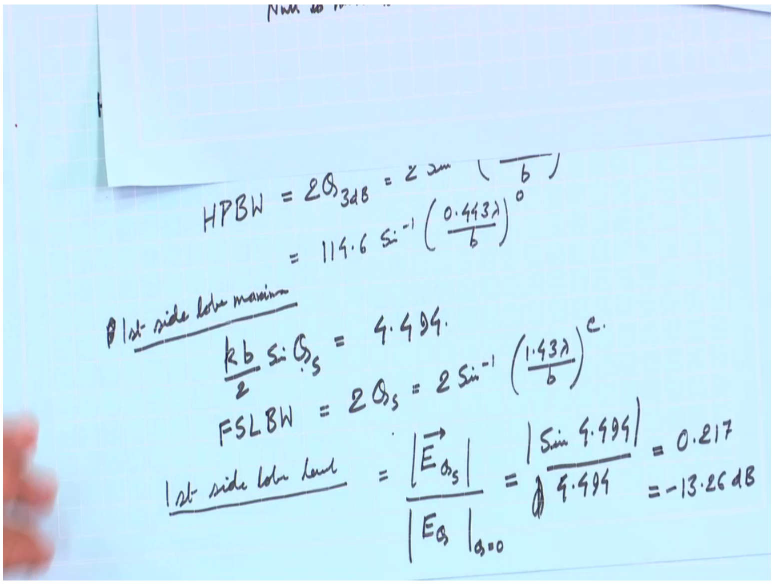

So, I can say that 1st side lobe level, let me adjust the page; 1st side lobe level that will be E theta s magnitude by you can say theta s by E theta at theta is equal to 0. Magnitude is equal to sin of 4.494 divide magnitude because this can be a by 4.494. This is positive, but this may be here. So, that is 0.217 and that becomes minus 13.26 dB.

Instead of this, this is an exact value of side lobe level, but sometimes approximately people say that side lobe delta theta is instead of putting this value, sometimes people say that numerator of sinc function is maximum. That means numerator will be so approximately you can see.

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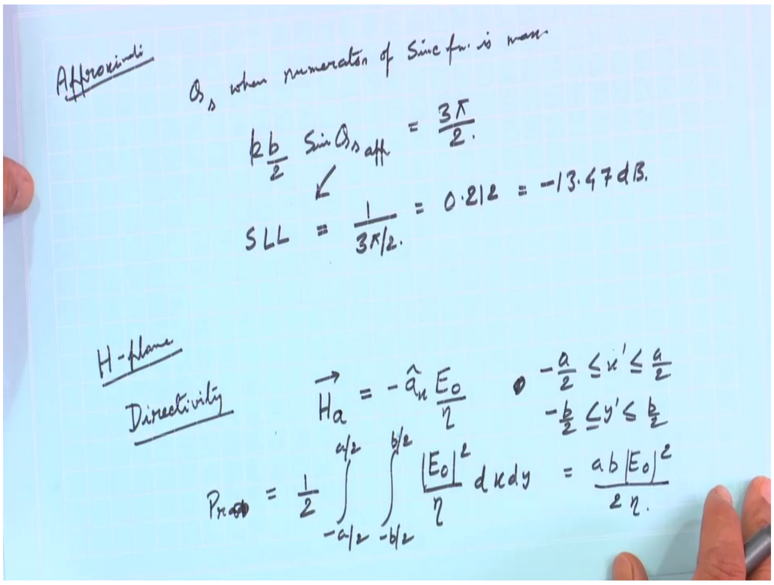

This is much better expression, but approximately people say that theta s happens when numerator of sinc function is maximum. This is not true that we have seen that at this value of sinc only you get, but so that time you can put kb by 2 sin theta s, I am writing approximate and that will occur at 3 by 2 maximum.

So, from here with this you can say SLL turns out to be 1 by 3 pi by 2 and that is 0.212 and that is minus 13.47 dB. So, you see that ultimately, there is not much difference one is 0.217, another is 212. So, that is why minus 13 dB etcetera.

Now, you can here you can assume how this 13 dB; do you remember this minus 13 dB thing? Actually, in an uniform array, we have seen the 1st side lobe level. Level is always minus 13 dB down, if we have number of elements something like 8 or more.

So, the same thing here because actually this aperture, I am having an uniform illumination and I said that actually this is something like any theory that error sources distributed sources, but since their uniform; they are interfering and that interference is giving this pattern. So, this pattern and the uniform array pattern, almost they are similar ok. Expression also wise we could have proved that.

So, this is about E plane. Now, let us see the H plane. So, H plane you can easily find out because same things that the all these terms will be similar expressions because the function is same only the x and y are interchanged that is why in all this expression, where b is coming; you replace it by a, you get the these things. But actually I will say that the 2 patterns we have drawn, E plane is this and H plane pattern here we have made a cos theta thing. So, that cos theta if you bring, then this one we simply that a and b replacement.

So, error from both side cos theta term also needs to be incorporated so, you can always calculate that because you know the basic principles. So, you know the filled this pattern. So, you can always put the value and numerically solve it. There are may not many analytic expression; but you can solve it with trial and error etcetera you can do.

So, I am not writing H plane things, you can approximately you can put a and if you want more sophisticated thing, do the actual thing. Next thing that I want to say is what about the directivity. That is an interesting thing because now we know the fields, E theta and H phi. So, we can find directivity.

But, if we want to do that; that means you find the P r from those E theta H phi expressions, you will see that there will be an integration coming which is not so easy to solve which a cumbersome thing. So, what people do that the maximum power radiation direction we already know that is z is equal to 0.

So, directivities numerator, we can easily find out. The thing is how to find, what is the total power radiated. So, we can say that total power radiated instead of calculating from the farfields, we can also calculate from the aperture fields. Because whatever aperture is sending; so that is an easier way because it is an uniform field. So, that is why for P r calculation, total power radiated will take the help of the aperture field.

So, we know that what is the aperture field. So, already x, E y we have seen and there will be also the magnetic field in the aperture. So, that I can easily write that that will be ax into E naught by eta because E a already I have said it is ay E naught. So, this is a over aperture.

So, those all those things minus a by 2 less than x dashed minus a by 2 and minus b by 2 less than y dashed less than b by 2 with that 0 elsewhere. So, you can find P rad. It will be P r or P r let us say because we have that initially P r.

So, half you integrate over the aperture a by 2 minus b by 2 b by 2 E naught square by eta and dx dy because your surface area of the waveguide that will be z directed so, dot product that will cancel. So, ultimately it will be a b E naught square by 2 eta.

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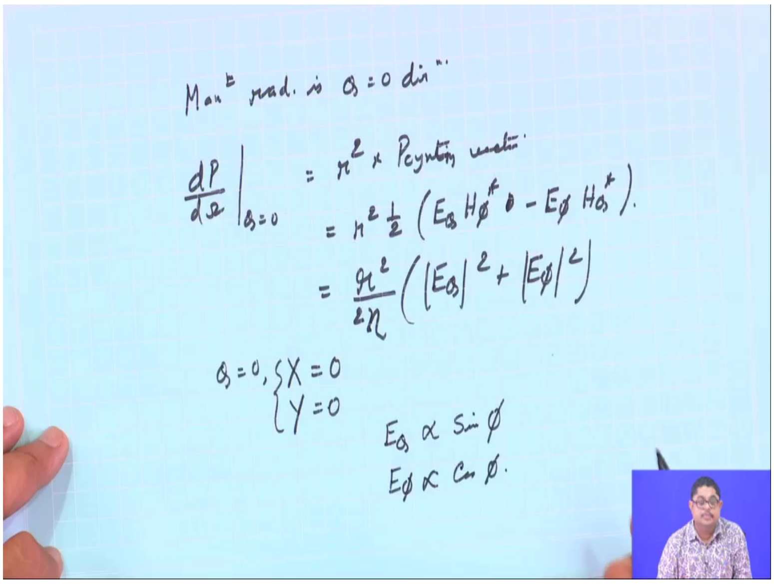

And maximum radiation, we know at z is equal maximum radiation is a theta is equal to 0 direction. So, dP d omega theta is equal to 0 will be r square into pointing vector pointing vector. So, that is r square half E theta H phi star minus E phi H theta star and if you put this values you know. So, r square by 2 eta 2 E theta square plus E phi square ok.

So, this E theta if I we know now put theta is equal to 0. So, X capital X becomes 0; capital Y also becomes 0. So, E theta expression, if you see it is proportional to sin phi and E phi expression is proportional to cos phi.

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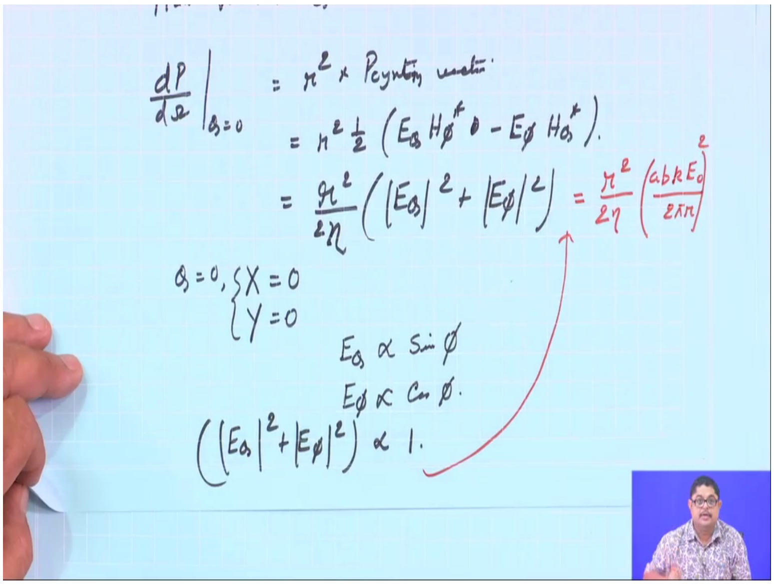

So, now you can find out what will be this E theta square plus E phi square. This thing will be proportional to 1; that means, no phi variation ok.

So, then you can put it here. I am again putting in that this value if I put it so, it becomes r square by 2 eta into E theta E phi have a constant part that part will be there; other things are not there. So, that will be a b k E naught by 2 pi r whole square. So, dP d omega you got; P r I have got. So, now, I can find directivity.

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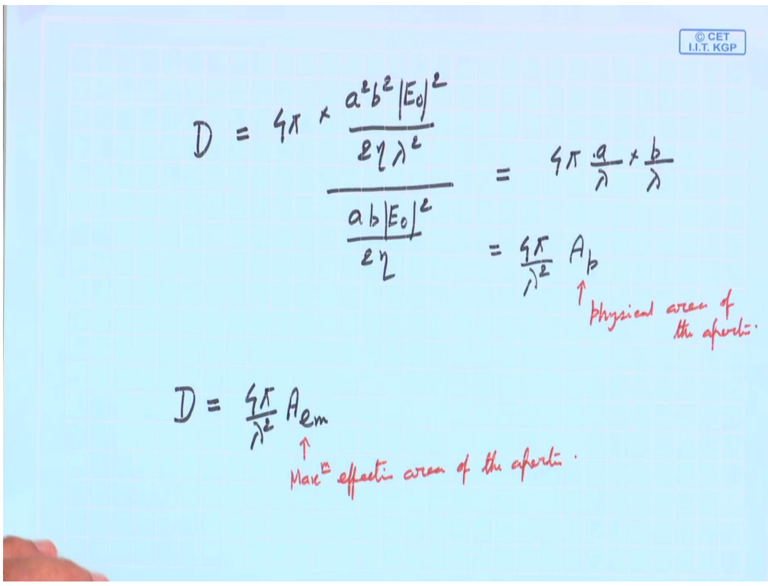

So, D will be 4 pi into a square b square E 0 square by 2 eta lambda square divided by a b E naught square by 2 eta. So, that becomes 4 pi a by lambda into b by lambda.

Now, I can say that two things; one is I can write it as a square into a b is I am writing the physical area of the aperture. Also I know that D is equal to we are proved that for any antenna, D can be written as 4 pi by lambda square into effective maximum effective area.

So, you see this is physical area of the aperture. This is maximum effective area of the aperture. So, what is turns out that in this case both A p and A em are same. Now, we have that time defined a term called aperture efficiency. What is aperture efficiency?

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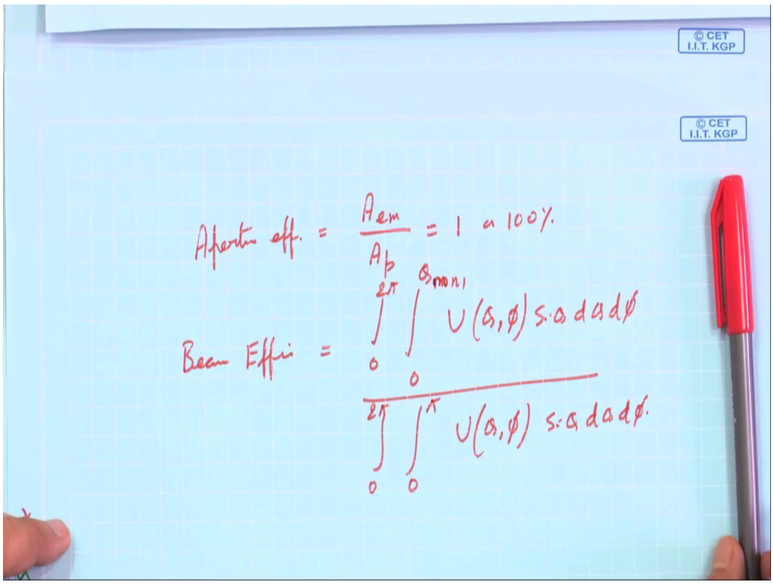

Aperture efficiency is A em by A p because any antenna designers wishes I want to make A em as large as possible. So, how much we are getting in this case, this is 1 or 100 percent.

So, you see that physical area and the aperture the maximum effective or electrical area, they are same; why? Because this happens for uniform illumination we will see other examples, if I not illuminate the aperture uniformly, I will not get these; that time while introducing array antennas in the first lecture I said that we try to make the antenna should be large in terms of lambda. So, some factor of that physical area will be the directivity.

Now, we will see that. You can get the full area as the full effective area that efficiency to be 100 percent if you have uniform illumination; if not, then you will have to have a reduced efficiency. And from here, we can also calculate other things like beam efficiency that we have discussed in case of a that what is the power in the main beam? So, I can also write beam efficiency that will be power in the main beam.

So, 0 to 2 pi 0 to if I say theta 1 up to theta 1 is a main beam or theta n, you can theta 1 n let me say or theta n 1 that first. So U theta phi sin theta d theta d phi by 0 to 2 pi 0 to pi U theta phi sin theta d theta d phi

So, for radiometry, radio astronomy, radar etcetera requires very high beam efficiency; near about 100 percent so that noise in the side lobes etcetera get thing.

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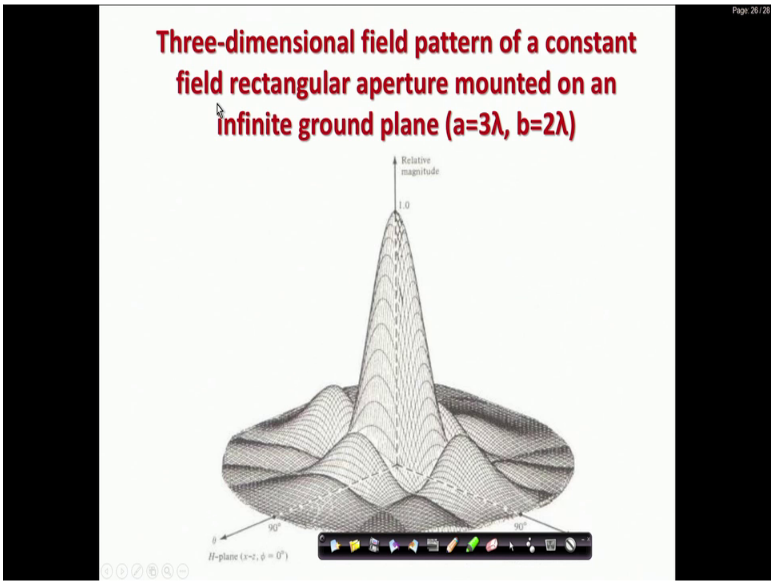

So, you can see that this is actually the field pattern of an rectangular aperture mounted on an infinite ground plane. Here, to plot this, this is from Balinese book that he has taken a is equal to 3 lambda, b is equal to 2 lambda. It is a rectangular aperture. So, you see that the peak is at theta is equal to 0 direction. But, how many side lobes are there? You can always calculate I think 4-5 side lobes are there etcetera.

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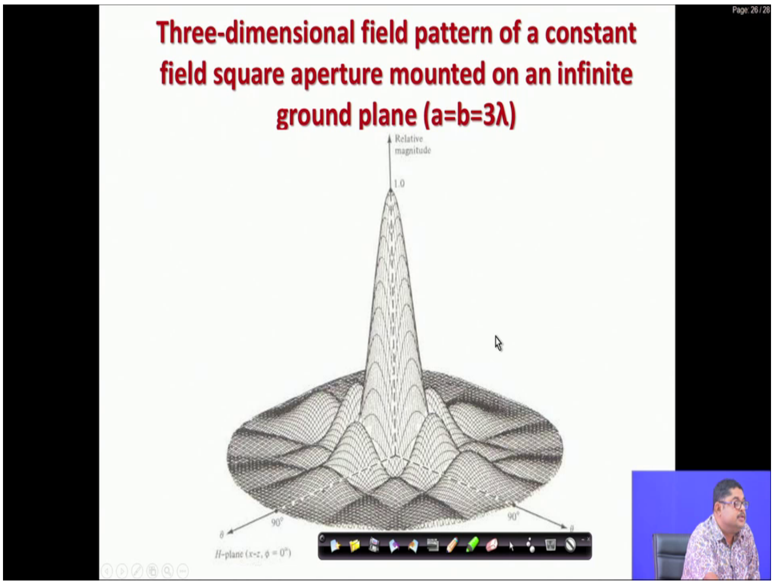

So, this is a square instead of a rectangular aperture, this is a square aperture a b is equal to 3 lambda is having this.

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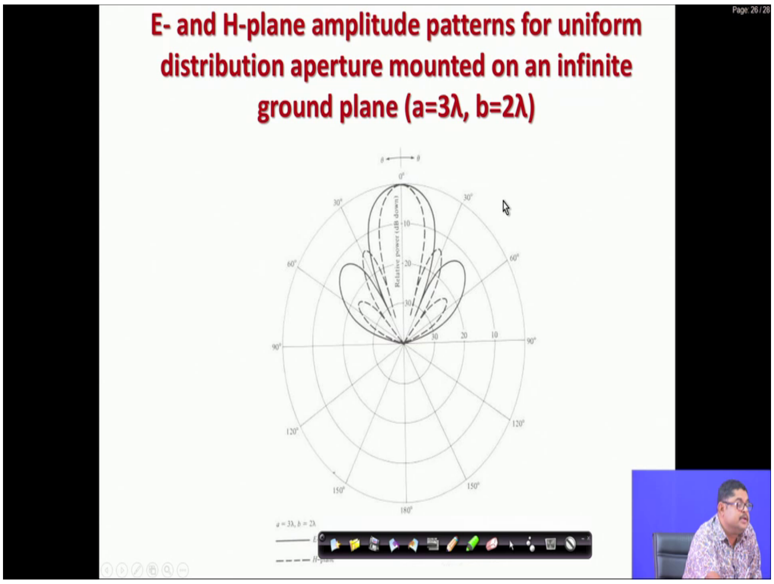

Then, this is the patterns you see black one is the black one is the E plane and the dashed one is the H plane pattern. Now, since a is larger that is why you see H plane is narrower. This is a thing that if you have the broad broader dimension large, then in the H plane you have more constricted thing and this is again, rectangular aperture and this will come later.

So, with that we will conclude this lecture. In the next, we will see that instead of uniform illumination if we have some other illumination, how this thing behaves.

Thank you.