Analysis and Design Principles of Microwave Antennas
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Lecture – 32
Farfield Evaluation of Spherical Wave Radiation by Generalised Antenna

Welcome to this NPTL lecture. We are continuing our discussion on the solution of by Radiated fields when both types of current, Electric currents and Magnetic currents are there.

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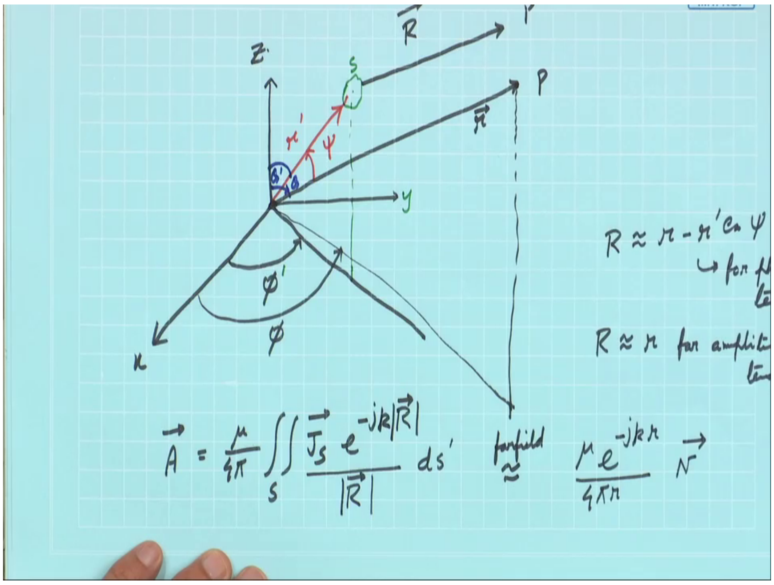

So, I now write the this geometry of the problem again because now, I will make a Farfield approximation. So, for that these geometry you have seen many times; but since it is a generalized thing, I am redrawing it that this is my surface S which we are talking in the field equivalence principle.

So, we have the sources here may be J s, may be M s that I am not showing here. But this point as a in rectangular coordinate this is as x dash y dash z dash potential. So, and I am observing at farfield.

So, I can say that the radius vector of this source is r dashed and the from the source at the observation point, I had the radius vector R and from the center I have to the observation point P. So, P is all these points P. So, r, I have this.

Now, if I if I project this in the sorry this will be my y plane, then this angle that is made that let me call a phi dashed point and similarly, if the this point is projected; let us say here.

So, then this angle will be called phi angle and I can say in terms of solid angle that this angle will be let me call psi that is r dash, this is r and you can say that (I colour that this angle is theta dashed and this angle is theta etcetera. So, this is my geometry.

Now, I can write that what is A? Mu by 4 phi S J s e to the power minus jk R by R ds dashed. Now, I can say that this one, I can also write as.

Now, farfield assumptions all you know that R is equal to r minus r dashed cos psi. This is for Face terms and R is equal to r for amplitude terms.

So, I can write that far field approximation if I put it here, it will become mu e to the power minus jkr by 4 phi r, where what is N?

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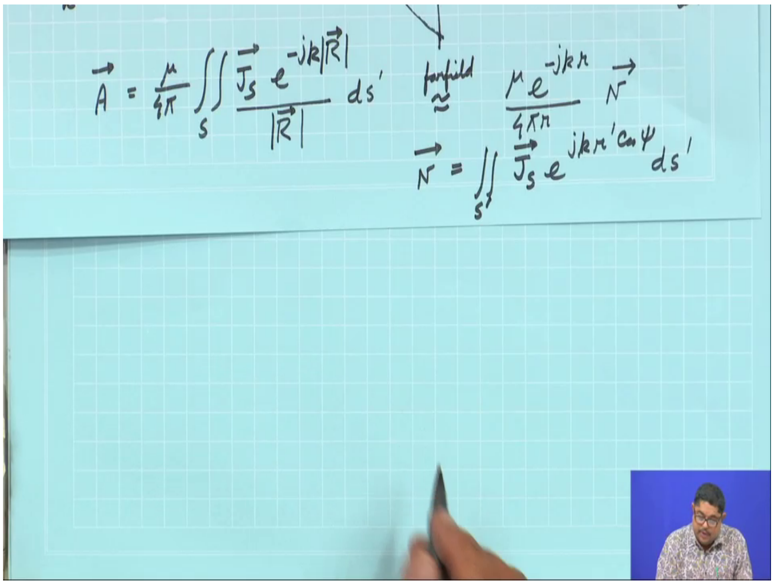

This N is a vector, it is J s e to the power jkr dash cos psi ds dashed. This is just a shorthand that instead of writing that integral I am absorbing that integral.

So, you see this is general, this side there is no source quantity or prime quantities. So, I have separated the prime quantities and unprime quantities. So, this is general thing. This is N. This is all source quantities.

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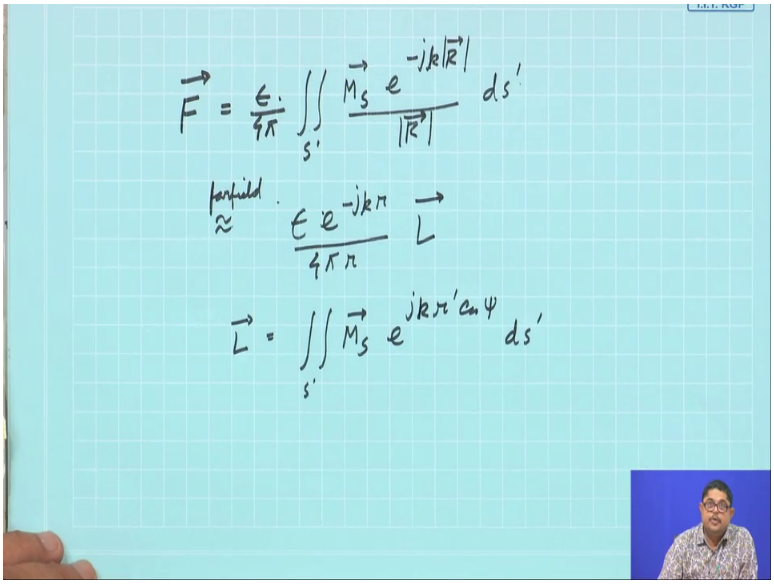

Similarly, I can also write what will be F? F will be as a farfield approximation, this will be epsilon e to the power minus jkr by 4 pi r some L; where, L is Ms.

Now, I leave it here; a typical nature of a magnetic vector potential in the farfield. Actually, this we have not proved; but in current element, we have derived it and that time, we have generalized it for all antennas. But today, we will make certain correction to that. We say that if I have a general antenna of finite dimension, please not this finite dimension.

If I have infinite dimension, then there is something else. Particularly, if you actually when the analysis for a particular class of antenna starts many times we start with infinite dimension in one direction. So that time, there are ways how to tackle those. But if I have a finite dimension antenna, then that radiates spherical waves.

And what I am saying is true for all spherical waves.

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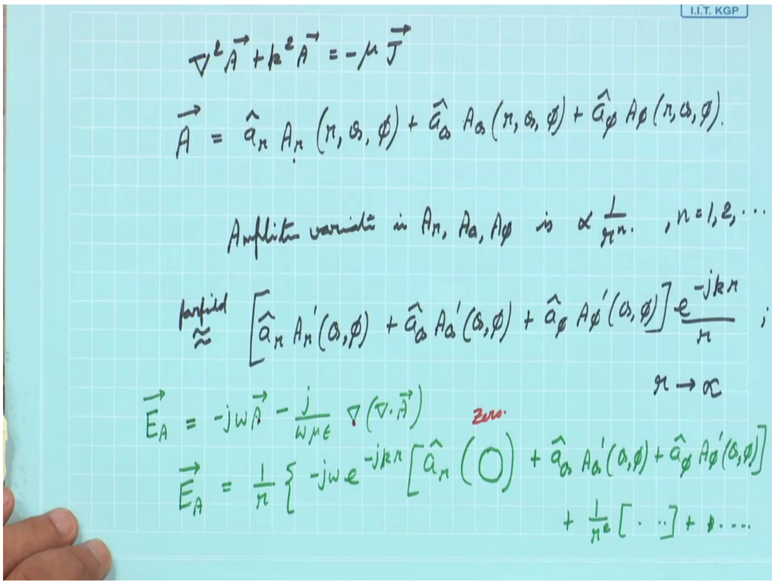

For this spherical waves, a general solution to the vector Helmholtz equation of del square A plus k square A is equal to minus mu J becomes in spherical coordinates this A can be written because already we have seen a.

So, we can write by coordinate transformation that to spherical coordinate. So, it will be a r Ar r theta phi plus a theta A theta r theta phi plus a phi A phi r theta phi.

The amplitude variation of R in each component; that means, in A r, A theta and A phi is of the form Amplitude variation in A r A theta A phi is of the form 1 by r to the power n; where, N may be anything 1, 2 etcetera.

Neglecting the higher order terms of 1 by r N because in the farfield r is very high. So, 1 by r to the power 2, r to the power 3 etcetera that will be high; we can always write this. So, this is a farfield approximation that in the farfield, this A can always be written as.

What I have done? I am claiming that all these components there are variation is of the form e to the power minus jkr by r. This is because they are radiating spherical waves. If they donot radiate spherical waves, this will be something else.

So, in I can separate there r variation. So, what is remaining is there theta phi variation that is why I am writing A r dashed A theta dashed A phi dashed. Now, this can be proved, we are not proving; we are taking that this is the crux of this whole thing that in the farfield we can write like this.

So, if we put this in that equation suppose we have already found this equation that from A how to get E that equation was minus j omega A minus j by omega mu epsilon del del cross A. Actually that time I said that this form is good for farfield approximation.

Now, if this equation what I obtained here, this if we put here, if you do it; just you will have to do this operation because this will be simply multiplication, you do this operation. And you will see that this result will be remarkable. What we get is. So, we perform this operation of divergence once on a and put it here and we separate the 1 by r terms and race terms.

So, there are something and this race terms in the farfield, we can say whatever be their value they will go to 0. But in the 1 by r term, there is a remarkable thing that the radial component that becomes 0. This is 0, this is a r there is not any other function this is actually a, we I can say this is a 0.

So, why this happens? Because this gives operation the gradient of this divergence and this A they actually cancel each other this component.

So, this is a property that if any antenna radiates a spherical wave in the farfield, the radial component goes to 0 for 1 by r type of fields. For others, it may not go. But since, it is in the farfield, these terms are negligible by that and there are no radial components.

So, there are only theta phi component. Now, this thing we claimed in case of current element in case of whatever antennas we have seen. But this is the proof actually, this mathematical operation is behind this. That time also may have said or you have got it, but this is a general feature for all spiracle waves radiated by any finite dimension antenna.

So, similarly we can also see that what will happen to the Magnetic field?

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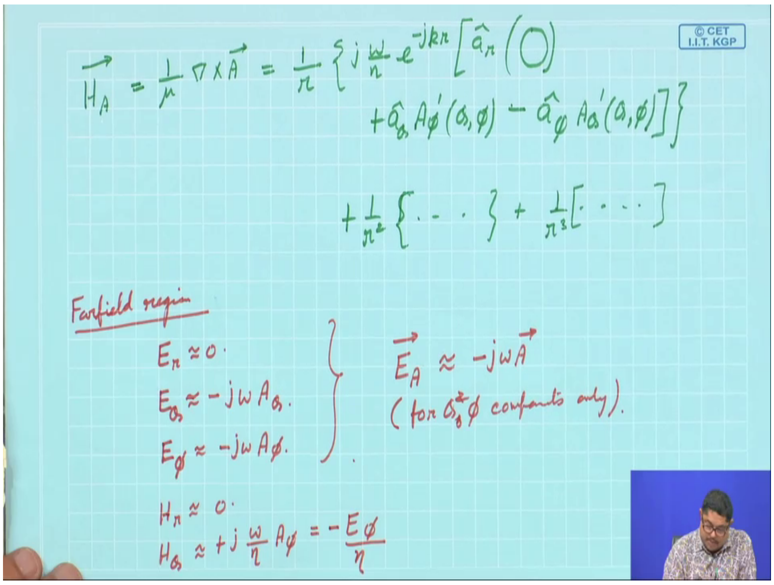

So, magnetic field will be again, we can put because that expression a is there farfield approximation. So, if we put that we will see that 1 by r terms they will be of this form. 1 by r square, then 1 by r cube etcetera.

Now, these terms will go is go. So, here also we see that the magnetic field also does not have any radial component, it may have theta phi component please note there is a this A theta this components is like this in this case.

So, we can say that the radiated electric and magnetic farfields have only theta and phi component. So, now, is the time for summarizing what we have seen in the farfield. So, we can say that in the Farfield region for finite dimension antenna radiated spherical waves, we have E r is going to 0; E theta is can be approximated as you see minus j omega A theta and E phi as minus j omega A phi.

So, from this, I can write that E A will be minus j omega A. But remember that is why we always write here that it is for theta and phi components theta and phi components only. E r is 0 that is why we are writing it like this; but remember that this is not for E r component. E r will be 0, but this relation is for theta phi components only you can write like this.

Similarly, H r is also 0; H theta is plus j omega by eta A phi. So, that is nothing but minus E phi by eta and H phi is minus j omega by eta; eta is the intrinsic impedance of the homogeneous medium A theta is equal to plus E theta by eta.

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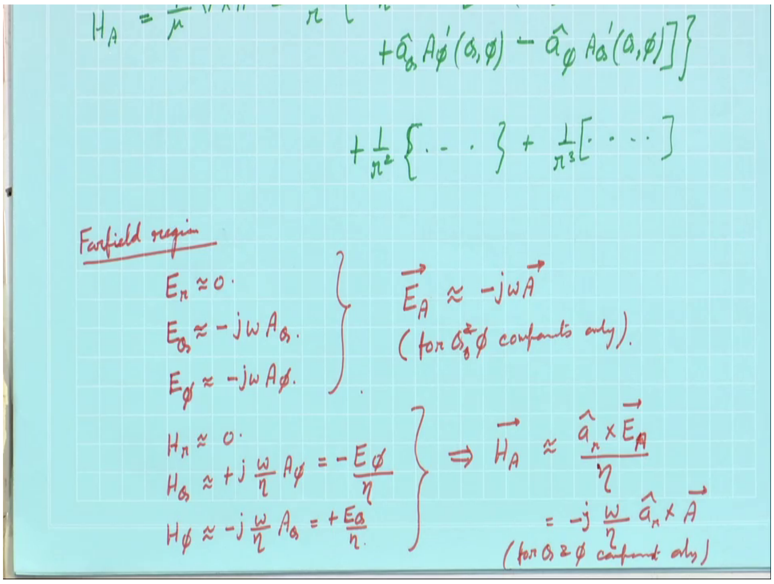

So, again based on these, we can say that H A in the farfield will be like ar cross E a by E capital A by eta o r this is minus j omega by eta ar cross A, if you are in terms of A. Again, for theta and phi components only this is a memory at for remembering.

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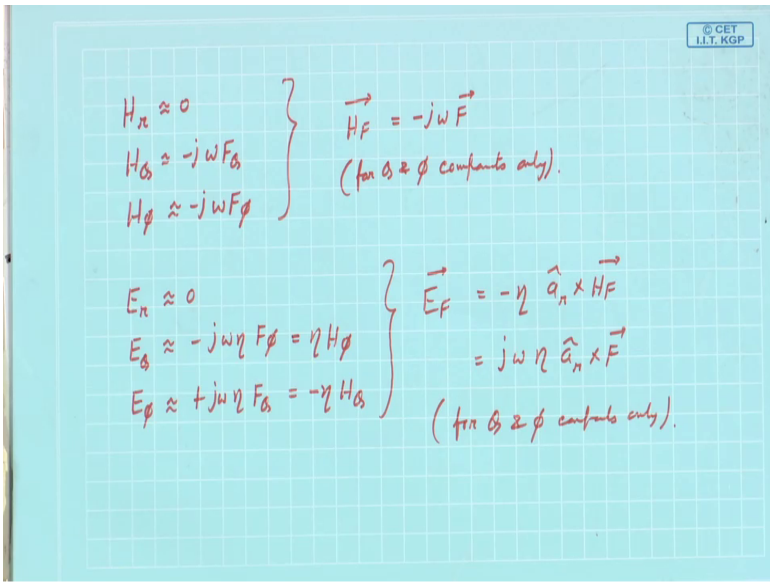

In a similar fashion, the far zone fields due to a magnetic current source M can be written as H R is equal to 0. H theta is minus j omega F theta. H phi is minus j omega epsilon.

So, from here I can say H F is equal to minus j omega F for theta and phi components only. Also E r 0; E theta is minus j omega eta F phi is equal to eta H phi and E phi is plus mu eta F theta and that is equal to minus eta H theta.

So, again I can say that E F is equal to minus eta ar cross H F or if you want to say in terms of F, it is j omega eta ar cross F again I am getting the (Refer Time: 23:11) the theta and phi components only.

So, it turns out that in both the cases the far E and H components are orthogonal to each other and form tm to r type of fields or mods model fields. So that means, in the farfield I have a spectrum of these that we have talked many times. But this is the proof that we have tm 2r; that means, to the r is the observation point also the propagation direction. So, they are tm two that the fields are there.

Now, with this thing we can introduce those N and L vectors that we have introduced from the solutions this L. So, this L and N that from this geometry we have said, we will now introduce them because that integral still needs to be done because A and F contains those integrals.

So, we right now that again we write what are the fields?

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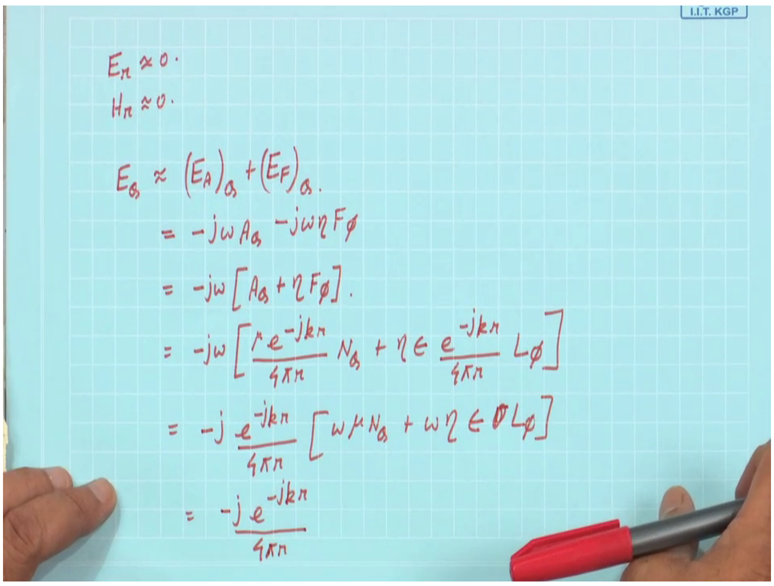

E r is 0; H r is 0; what is E theta? E theta is E A theta plus E F theta. Now, E A theta in the farfield, I can write j omega A theta and E F theta is minus j omega eta F phi is equal to A theta plus eta F phi is equal to minus j omega. I know what is A theta.

You have already seen nu e to the power minus jkr by 4 phi r N theta plus eta epsilon e to the power minus jkr by 4 phi r L phi is equal to minus j e to the power minus jkr by 4 phi r omega mu N theta plus omega eta epsilon sorry L phi is equal to minus j e to the power minus jkr by 4 phi r.

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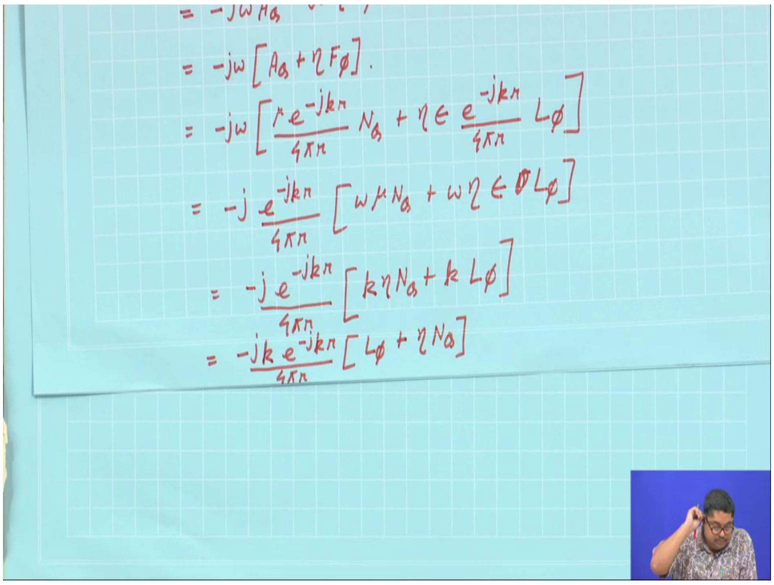

k eta N theta plus k L phi is equal to minus j k e to the power minus jkr by 4 phi r L phi plus eta N theta. So, E theta so; that means, I have this part only I need to find out L phi N theta to evaluate E theta.

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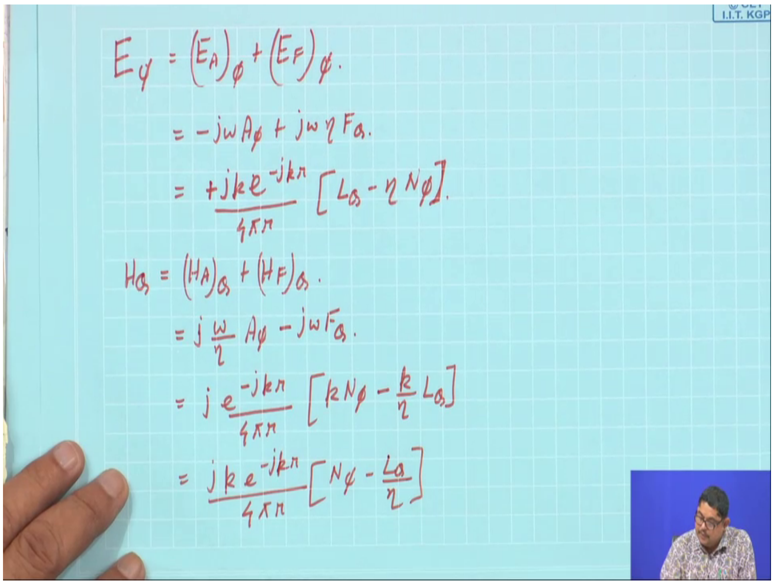

Similarly, let me write E phi is equal to E A phi plus E F phi. So, again this is minus j omega A phi plus j omega eta F theta. So, again you can put this, I am writing the final expression plus j k e to the power minus jkr by 4 phi r L theta minus theta N phi. Also H theta is H A theta plus H F theta is equal to j omega by eta A phi minus j omega F theta. So, if we manipulate like that, it becomes you can be taken out.

So, jk e to the power minus j k r by 4 phi r N phi minus L theta by eta and all.

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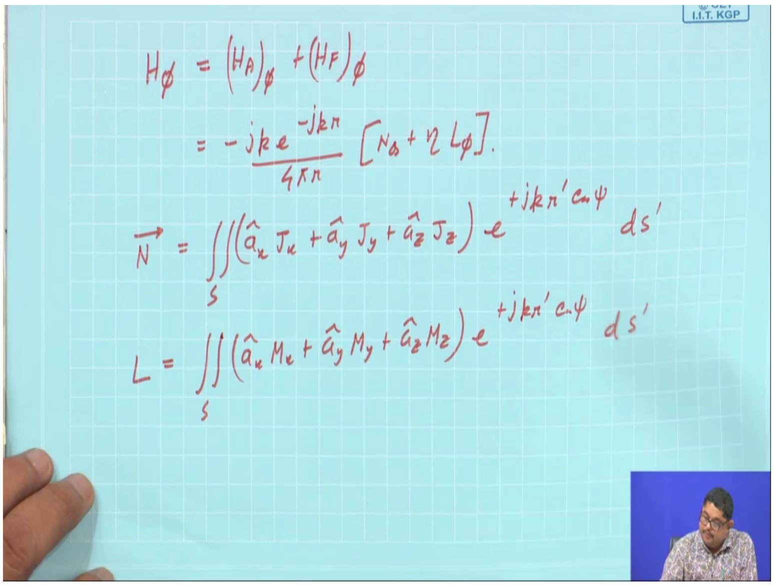

Last one is H phi is equal to H A phi plus H F phi. So, I am not going to the final expression will be minus jk e to the power minus jkr by 4 phi r N theta plus eta L phi.

So, basically it shows that other field things we know, only this N and L these two vectors we need to evaluate their components. Now for that, let us see the simplification; what is N? If you can remember, so, ax J x plus ay J y plus az J z e to the power plus jkr dashed cos phi ds dashed.

This was our N and L was. So, rectangular coordinate, I have expanded that; only thing ax M x ay M y az M z e to the power plus jkr dashed cos phi ds dash.

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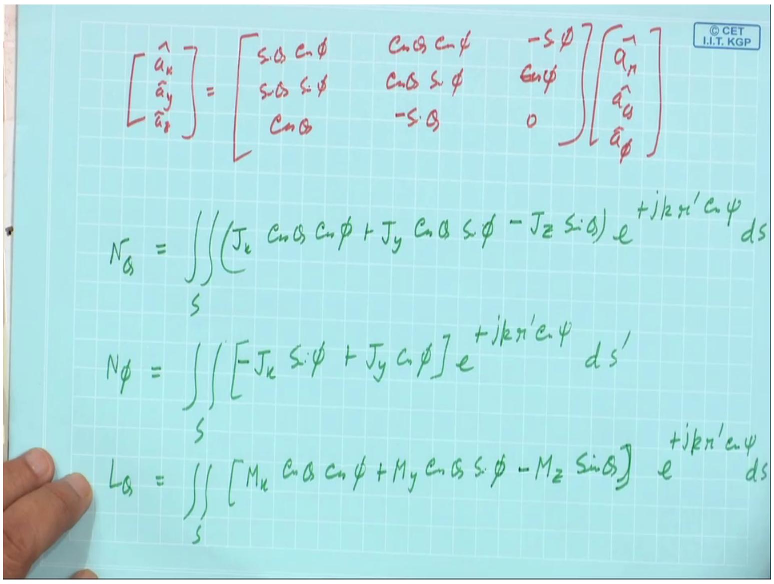

Now, I think all of you know this that ax ay az can be written as sine theta cos phi, sine theta sine phi, cos theta cos theta cos phi, cos theta sine phi, minus sine theta, minus sine phi, plus cos phi, 0 to ar. So, ax can be expanded in terms ar a theta a phi ay can be expanded etcetera.

So, if I put that we get N theta is J x cos theta cos phi plus J y cos theta sine phi minus J z sine theta e to the power plus jkr dashed cos psi ds dash.

N phi is s minus j x sine phi plus J y cos phi e to the power plus jkr dashed cos psi ds dashed, L theta M x cos theta cos phi plus M y cos theta sine phi minus M z sine theta e to the power plus jkr dash cos psi ds dash.

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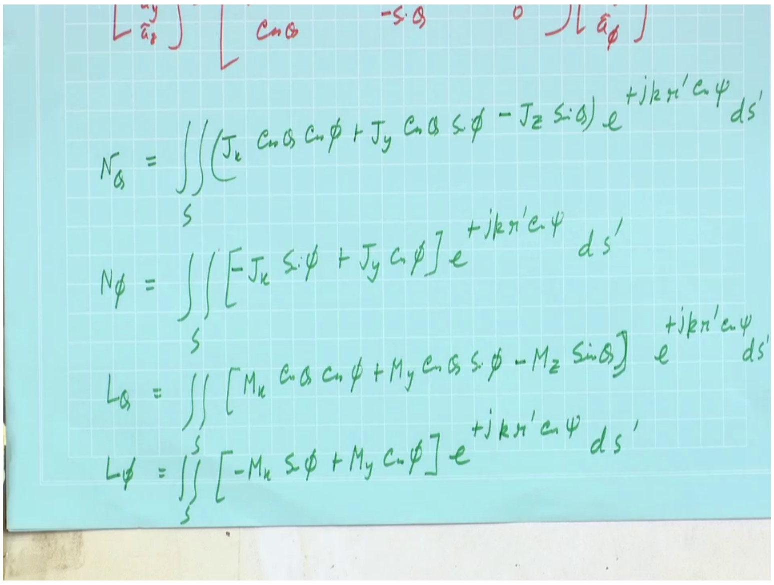

And L psi minus M x sine phi plus M y cos psi e to the power plus jkr dash cos psi ds dash.

Now, this is ok; only thing is I need to simplify this part that this r dashed cos phi, we should write in vector form and in terms of source coordinate. So, for that what I can do?

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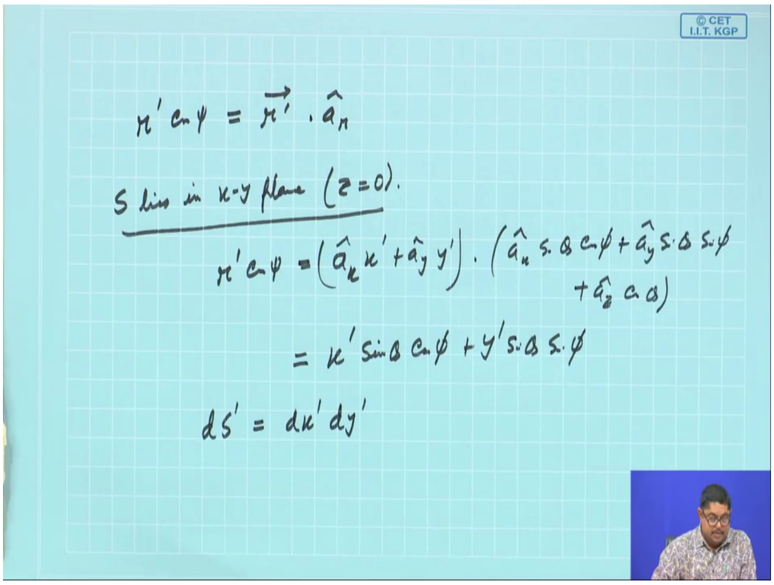

I can write R dashed cos psi is nothing but r dashed dot ar this you can easily see.

So, if the surface s lies in x-y plane; that means, z is equal to 0. Then, r dash cos phi will become r dashed I can write ax x dashed plus ay y dashed because it is at z is equal to 0 plane dot a x sine theta cos phi plus a y sine theta sine phi plus a z cos theta.

So, these becomes x dashed sine theta cos phi plus y dashed sine theta sine phi. Also ds dashed becomes dx dashed dy dashed in the x-y plane ok. So, this now can be solved in particular cases and you can evaluate the farfields easily from these that example we will see in the next lecture.

Thank you.