Analysis and Design Principles of Microwave Antennas
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Lecture - 30
Generalised Analysis of Antenna

Welcome to this lecture on NPTEL, we will today discuss a new concept the Analysis of Antennas by Generalised currents. So, actually I recall all of you have heard of Huygens principle, actually in optics Huygens gave this principle actually this is true for electromagnetics also. The I just state the principle Huygens principle is each point on a primary wave front, can be considered to be a new source of a secondary spherical wave. And that a secondary wave front can be constructed as the envelope of the secondary waves.

So, what he says that when a wave propagates, so every point on that propagated wave that waves has a secondary wave front secondary source and that you can consider that that is radiating. So, effectively all the secondary sources their radiation is same as the main waveform that was Huygens principle. From that actually (Refer Time: 01:39) a very famous electromagnetic theory man. He formulated a principle which is called equivalence principle in 1930s. He made this contribution basically there is a theorem called uniqueness theorem by which he found out this equivalence principle.

So, what he said is the field in a lossy region is uniquely specified by the sources within the region, plus the tangential components of the electric field over the boundary or the tangential component of the magnetic fields over the boundary or the former over part of the boundary and the latter over part of the latter over rest of the boundary.

So, it is just I have stated it for your knowledge. We will discuss about this so it is not that you will have to think what. Here he has made a term lossless medium actually, sorry lossy medium. So, lossy medium is nothing but a special case of lossless medium, where loss is equal to 0 actually our pre space actually it is lossy, but we consider it that loss is very small. So, it is can be considered as our usual lossless medium also.

So, from equivalence principle we can say; that for a source free region you see (Refer Time: 03:45) said that you require the knowledge of sources as well as the tangential fields at the surface. Now if I do not have a source at some region, so for a source free region we can say that if the tangential electric and magnetic fields are completely known over a close surface. The fields in the source free region can be determined and actually by uniqueness theorem there is a guarantee that, what we determined that is the correct solution.

So, what is the practical application of this equivalence principle is, field outside an imaginary close surfaces, imaginary close surfaces are obtained by placing over the close surface suitable electric and magnetic currents, densities that satisfy boundary conditions.

Now, here there is a thing that for the first time I think in this lecture, I am telling about magnetic current densities actually this is a again I am hypothetical thing. Actually mankind could separate the electric charges so we can separate the positive and negative charges. We cannot do that for North Pole and South Pole. So, magnetic poles they cannot be separated, that is why we cannot have a flow of magnetic poles flow of north poles or flow of south poles etcetera.

But in the field wise we can equivalently say that if there is an electric current density which is flow of charges. So, equivalently there will be also magnetic current densities, and we will see just after this how to analytically make that assumption. So, that because if we make the, this thing in the Maxwell's equation, then the whole Maxwell's equation becomes symmetrical that we will see; that Maxwell's equation suffer from one thing. Suppose if you look at just Maxwell's equation I have del dot D, divergence of the electric flux density that is rho, but del dot B that is 0.

So, you see that there is a thing, but if we have separate magnetic charges I can put this to be rho m.

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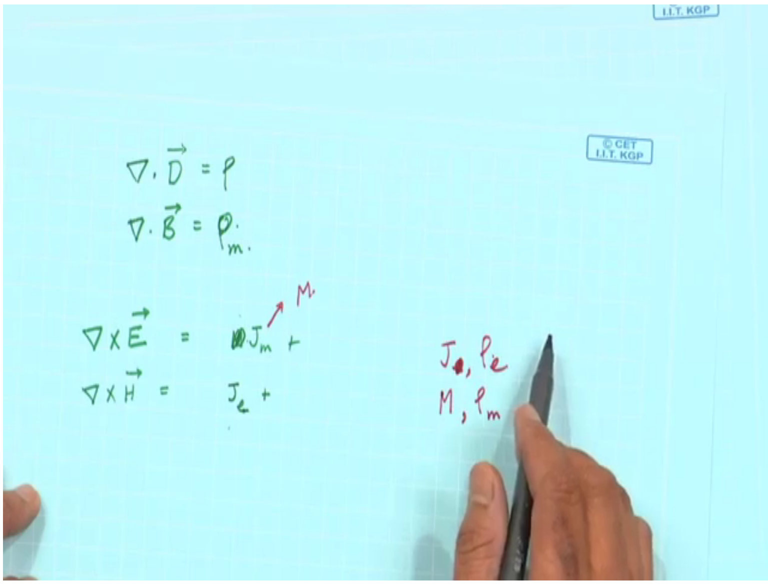

Similarly, what is del cross E, and what is del cross H? You can see that in del cross H, I have a J and there is something else, but here you see that in del cross E I do not have this counterpart of g. So, here if we put a sorry, now if I call this J e and if I call this Jm then these 2 also becomes symmetrical. So, that is the idea and finally, we will see that what is the actual expression. So, this whole thing gets very symmetric and that is why in electromagnetics we always assume these that there is a magnetic current also. and this satisfies the introduction of these does not change the Maxwell's equation solutions.

So, also before going to this actual discussion of this equivalence principle also what is the close surface? It a close surface means for this equivalence principle that, a surface which encloses the sources and they are taken to coincide with the conduct and part of the structure or antenna because then our actually the thing is our integration domain gets very much reduced, if we can include in the surface the conduction conducting boundary, because in the we know that in conducting zones various spill things gets cancelled.

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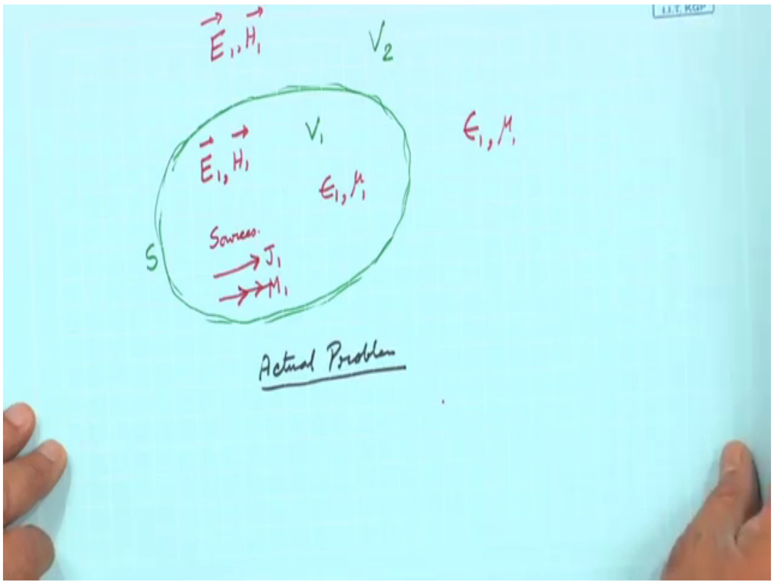

Now, let us come to the actually actual problem; is let us say that I have a surface like this hypothetical surface. Actually there are let us say that there are sources. So, how do characterise sources? A source just as we said that in actual life in Maxwell's equation you see, who are the sources the sources are J e and rho.

But we say that why there can be also the J M which actually we do not call J M we call M actually this Jm is given the name M. So, this is generally the conducting current that generally call J. So, J rho so M and rho M I can say. Now already these J and rho are related by continuity equation. So, actually I need not consider these, similarly I need not consider this. So, actual sources I can say are for electromagnetics are J and M.

So, sources means I can say that there is a electric source, electric type of source to be precise because actually and this is called magnetic type of source. Generally, for magnetic this is not a correct notation. Generally, 2 arrows are placed on a magnetic sources to distinguish it from J and let the medium constitutive thing is epsilon 1 mu 1.

So, this sources has produced fields everywhere let us say E 1 and H 1, because electromagnetic field they will sources mean these are all time varying sources. So, they are producing E 1 and H 1. So, here also they will be producing E 1 H 1, because this is actually this surface is an arbitrary surface. So, everywhere there will be E 1 H 1s. And here also I write that this is 1 epsilon 1, I am not saying about the conductivity that lost thing we have discussed that more or less that is we can say that lossless medium type of thing.

Now, to distinguish that, this zone you see that this is a close surface, why? Because this is completely enclosing the sources. So, this is hypothetical I have taken this. So, this and let me name this is the surfaces close surfaces enclosing this volume V 1. So, everywhere there are the same thing and this outside volume, outside of the surface let me call V 2. So, this is my actual problem.

Now, my actual intension is to find E 1 H 1, you see think of an antenna problem; that I do not want to know what sources is producing this field etcetera. I just want to find out at a radiated zone what is E 1 H 1? So now, this thing can be. So, these J 1 M 1 they are radiating fields. So, here inside the surface also it is producing that field outside also producing that field etcetera.

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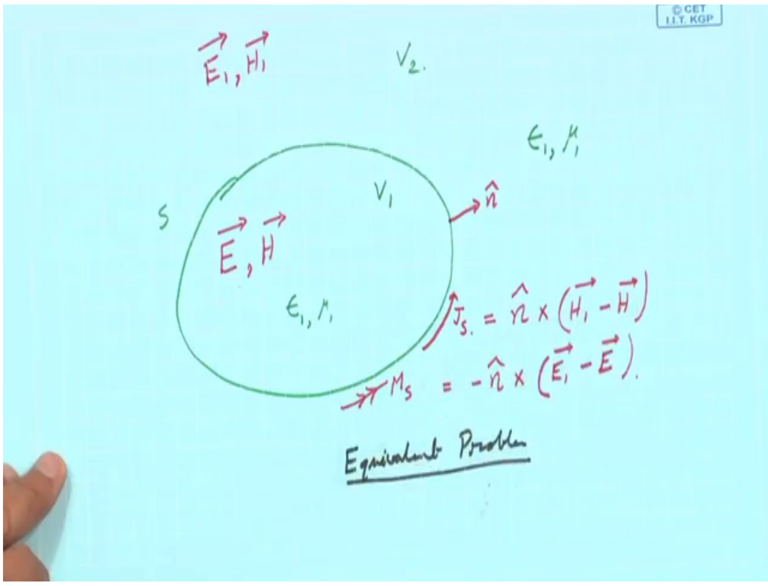

Now, this actual problem I will replace with the help of equivalence problem, fill equivalence principle as this same surface S, I have the volume V 1, and I have these epsilon 1 mu 1, outside also I have epsilon 1 mu 1 that has not change anything.

Now, this one I am calling V 1 outside, I am calling V 2. What I now let me say that this is my the surface has a outward normal. So, this is my direction of that, now I say that this is my equivalent problem I have written it where is the black, this is my equivalent problem. Here I say that inside I do not consider any sources. So, these V 1 which was earlier containing sources, now I do not consider any sources, but I satisfies the boundary condition here. So, I put on this imaginary surface to current densities, this is actually equivalent currents one is J S another is M S.

What is J S? It is the tangential component of the discontinuity in the magnetic field. And M S is the negative of the tangential component of the discontinuity in the electric field. What is there I will have to show that actually ; still previous case this outside field was E 1 H 1 now also outside field is E 1 H 1. And obviously, since I have removed these so the fields this sources which was earlier J 1 M 1 actual sources I have represent them with some surface sources J S M S here. So, they will be radiating inside a different field than the previous 1 so that I am writing E and H. A different from E 1 H 1, but these are taken to be such that they will radiate the same thing in the outside, outside I am not disturbing and inside field is getting disturbed.

But why this outside is not is getting disturbed because I have taken the sources so that it becomes the discontinuity that is created by placing these that is tackled here. So, the boundary condition on the surface is unaltered by this choice. And that is why the outside fields are not getting. So, J S and M S radiate field inside the surface S that is different from actual, but in the volume V 2 they produce the correct fields E 1 H 1.

Now, this is the equivalence principles followed; that I can do this and then if I am interested only about this I can go on doing this. So, please understand that this is the equivalent problem. And you see that if we do not take M S, we cannot get because in the if you look at the boundary condition of the electromagnetic field that we have learned in our M theory days; that there can be discontinuity in the tangential component of magnetic field, there can be discontinuity in electric field also.

As happens suppose in a horn antenna, suppose in a horn antenna there is no discontinuity in the magnetic field, but there is a discontinuity in the electric field. Now that we were taking furrier transform and all these, but aperture there is some think of the open (Refer Time: 18:23) or horn antenna that there is an aperture. So, there is certain field was going suddenly the electric field is getting change. So, that discontinuity we will have to take and that is why this introduction of magnetic current.

So, magnetic current it is hypothetical we are not saying there is a flow, but discontinuity we are calling, tangential component discontinuity of the electric field that we are calling magnetic current and tangential component discontinuity in the magnetic field that we are calling. So, I think I got confused that magnetic field discontinuity in the tangential component of the magnetic field is electric current, and the discontinuity in the tangential component of the electric field is magnetic current.

So, both should be present because in some cases this may be present in some cases, this may be present or in a generalized case both can be present. So, sources should be of these 2 type because you see we generally considered in the electromagnetic cases; that if you have a discontinuity in the tangential component of the magnetic field there should be a surface current, which is also tangential electric current, but it is in a another direction orthogonal to the in direction so that is here.

So, similar thing should be here and that correct thing is here please remember that there is a sign change here, these are all for Maxwell's equation thing

Now, this is equivalence principle followed. So now, I need not consider this so over the surface if I put based on the actual problem; then I can find out because if I can find out already we have seen for wire antennas that if I know this current. Current on the wire is nothing but current on that surface. So, from there we have seen we will go to potential and then we will find out the field.

Similar way we will show that for M S also this is the current you can find out, we will from Maxwell's equation we will prove that this you can do. So, if there are 2 present it will be some of the field due to J S. Some plus some of these super position holds if only one this is a classical wire antenna, where M S is not 0; the other thing is this is J S is 0 in case of a classical aperture antenna in general everything.

Now, there are some engineering shortcuts to this problems that you see this E and H can be any field. So, without loss of generality we can say E H can be 0 also. So, if I take E H 0 then actually law of one scientist proposed, that is why it is called loves equivalent.

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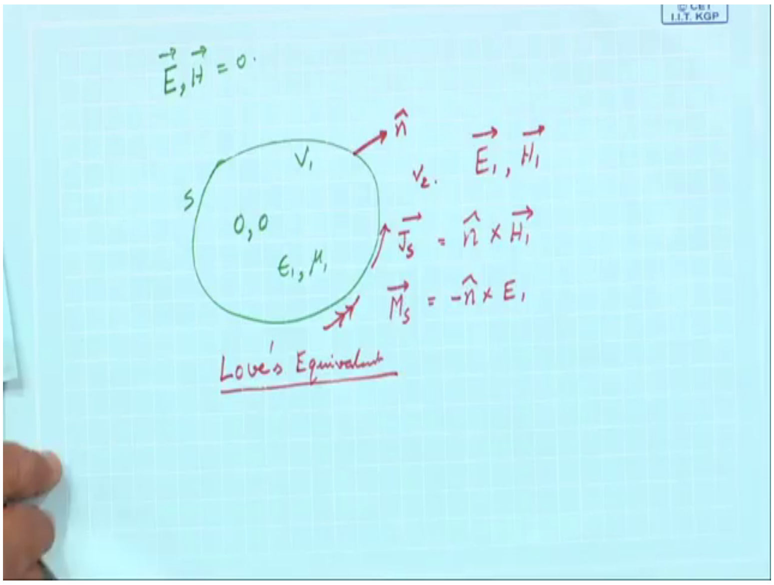

So, let me draw that E and H these fields if we take to be 0. So, this is my V 1 so I say fields are now 0 0 these are same epsilon 1 mu 1 etcetera. So, this is my S and so this is my n outward, this is my V 2, in V 2 the fields are same E 1 H 1. And I have a J S is equal to n cross H 1, and I have M S into minus n cross E 1.

So, you see the surface current densities J S and M S radiate in an unbounded medium epsilon and omega, epsilon 1 omega mu 1. So, our formulas for E H that we derived for wire antennas can be used here, but this requires knowledge of both E 1 and H 1 in tangential component over S.

So, you see that in wire antennas we have had the knowledge of only J S, but here we require both this knowledge; that means E field and H field both an. Now to simplify these you see that, if we now put so for let me write this is called loves equivalent.

Now, to simplify you see that problem that; why I will deal with the 2 things.

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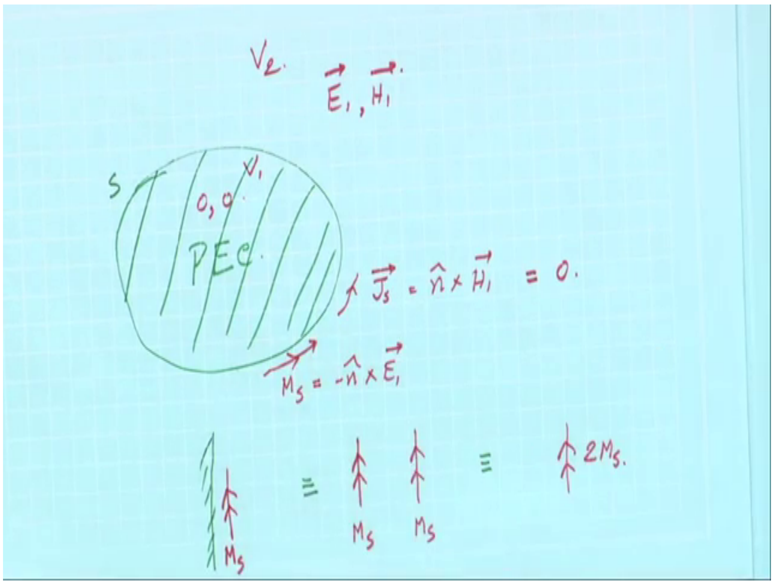

Let me put that same surfaces, but now this is totally I put a perfect electric conductor PEC. I think you know PEC inside that the fields are 0. So, already here the fields were 0, but I have put a PEC here, you have this is my V 2 and this is my E 1 H 1 so PEC I said 0 0.

So now there will be I will have to put over this surface and M S is equal to minus n cross E 1. And a J S is equal to n cross H 1. So, what is the difference between these; we will show that we will show something, but now you see that point is we cannot use the same formulas as we were having in case of unbounded medium; that means, up to here those if on the J S present I can use the formulas I developed, but this case and this case is different. Because this case was just a J S and M S present they are radiating in an unbounded medium not this is bounded that is bounded.

But now this is radiating and that there is a PEC. So, this is not the same problem. So, this is a in presence of PEC how they are radiating. So, that is a different problem, but I will now say that in this case these J S value become 0. Why? That means, it says that no field gets radiated by an electric current impressed along the surface of a PEC, how?

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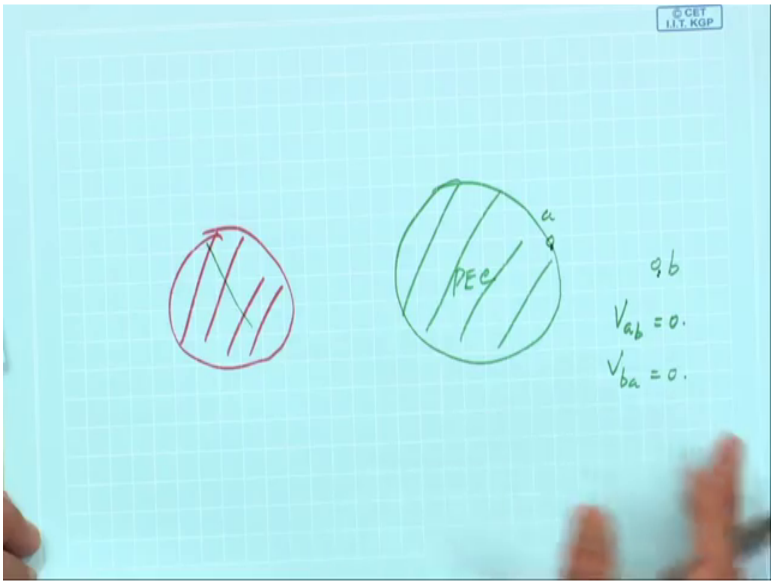

So, what you do to understand these understand that; I have this p sorry, I should have used this colour you have a PEC. Suppose I have a port here let me call it a and at a distant point, I have another port b. Visualize the set of port or terminals a on the this PEC conductor and another set in b in space a bit away from the conductor.

Now, if I have a current element here at b, then it produce what tangential electric field it will produce here, since it is PEC it will be 0. So, I can say that even if there is a current element here this voltage is 0 because no fields here. Now I (Refer Time: 28:10) so if I have a current element here, what will be the field here that is 0. So, Vba is also 0, now terminal b is arbitrary it can be taken anywhere.

So, I can say that current element on or the any conduction current on a PEC does not produce anywhere any field. So, that is why I can put that J S is 0. Now so; that means, we have only M S present here so MS but the problem is it is having a PEC. So, now we can invoke image theory so I can say that if this surface is an infinite plane, the surface is an infinite plane and in front that I have a name S.

So, image theory will give me that I do not have a PEC, I have sorry we should write MS. So, I have M S I do not have anything; I have the original M S like this so this is the image. So, this equivalently I will have 2 M S, radiating in unbounded space. So, I can use the formulas so; that means, this is done actually you see that an open ended waveguide, if we think that the waveguide the antenna always we make a ground plane along the open ended waveguide.

If we assume that that open that ground plane is infinite how you assume though in practice, we have something. If you take generally 5 lambda sort of thing for a conductor that ground plane link in all the directions, then you can say that it is an infinite ground plane. So, ultimately you find the current so this is a magnetic current, how that magnetic current comes; it is that minus n cross E 1. So, twice of that now that is radiating so you can easily find this formula, so you can find the field.

Actually we will show if time permits that the aperture Fourier field thing etcetera etcetera, inside this also gives you the same result.

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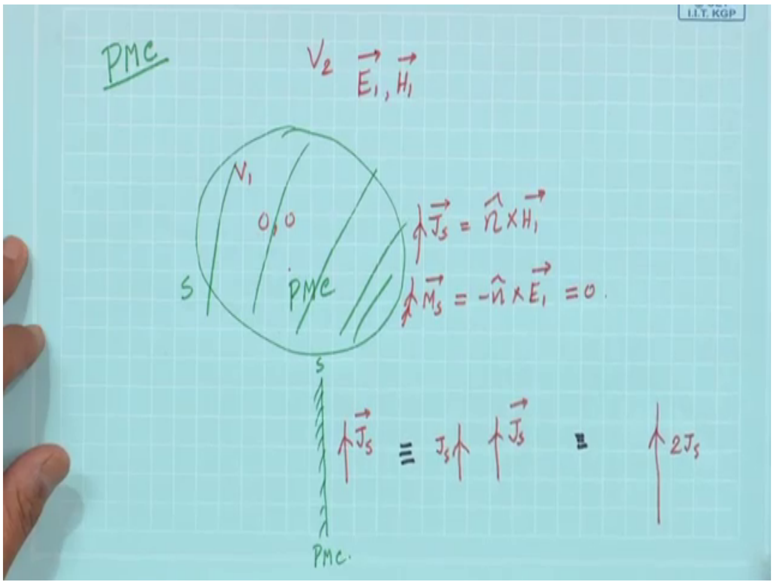

Similarly, we have a we can also put instead of PEC a perfect magnetic conductor. So, if we do that this is our surfaces the inside is V 1. So, the fields are 0 0 and this whole thing is a this is the PMC behind that surfaces. And the outside V 2 get the original fields E 1 H 1 exist.

So, for equivalence what will have to put, we will have to put a J S here and then you M S here. Now this J S is will be nothing but n cross H 1 and this M S will be minus n cross E 1. So, just same as PEC analogy, that if I have an M S in front of a PMC actually it will be 0. The same way you can consider another point here and then you invoke in reciprocity, this can be proved so I do not have an M S so I have simply a J S.

So now these J S radiates in presence of an PMC. So, that is difficult to evaluate. So, what we can do again like PEC we can invoke the image theory. So, let us consider that this surface S is an infinite plane. So, we can take that basically the problem is like this, I have an infinite PMC case this is S and in front of that I have a J S, tangential J S. So, the by invoking image theory I can write that equivalent to this is; basically this J S and the image of that will be another J S.

So, basically I can say that I have nothing but a 2 J S, and the medium is unbounded. So, 2 J S is radiating into an unbounded space. So, this expression we know already if I have a current density electric, current density in an unbounded homogeneous medium how it radiates, so we can easily find the field from here. So, this is the from loves equivalence principle, you can get these 2 cases where it becomes easier only will have to have either this J S in PMC case or M S in case of PEC ok.

Now, if this S is not only infinite plane, but it is curvature is much smaller than the lambda then we can have these same we can infinite plane approximation holds. Depending on what out of this whether loves equivalent, we will use or any other equivalent we will use, or PMC PEC etcetera we will proceed and do that.

So, based on that we will now discuss that; the thing it will be very simplified, but one thing we will have to proceed because this is today we introduce the concept of magnetic current. So, we do not know if a magnetic current is present in an unbounded medium how fields radiate. So, we will have to go to the Maxwell's equation and first we will derive that if an magnetic surface current is present on a surface, how the fields, what is the expression of the electric field, and magnetic field then we will make the power field approximation go to the actual cases ok.

Thank you.