

Analysis and Design Principles of Microwave Antennas
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Lecture - 28
Paraboloid Reflector Antenna (Contd.)

Welcome to this NPTEL lecture on Reflector Antennas. In the last lecture we were discussing about the various efficiencies. So, we have found out the directivity.

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$$G = D \eta_A \eta_s$$

$$\eta_s = D_f \int_0^{2\pi} \int_0^{\psi/2} \frac{g(\theta, \phi)}{4\pi g(\theta, \phi)} \sin\theta \, d\theta \, d\phi$$

$$\eta_A = \eta_i \eta_p \eta_u \left| \int_0^{2\pi} \int_0^{\psi/2} [g(\theta, \phi)]^{1/2} \tan\frac{\theta}{2} \, d\theta \, d\phi \right|^2$$

$$\eta_i = \frac{4L^2}{\pi a^2} \frac{\int_0^{2\pi} \int_0^{\psi/2} g(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\psi/2} g(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

And we have written in the last class the directivity of the reflector system into various efficiencies. So, out of that we have written up to this there will be other efficiencies later. Now, we have also found out the expression for spillover efficiency. So, I think if you remember you have written the spillover efficiency expression as the directivity of the feed and about the aperture efficiency we have said that it is the product of three efficiencies; the illumination efficiency, the polarisation efficiency and the uniform field efficiency. So, when we have assumed these two to be same the illumination efficiency expression we have written as up to this we have done.

Now, if in many cases, the feed that the radiation pattern for that power radiation pattern for that does not have a dependence on the azimuthal angle phi. So, if we say that g theta phi is independent of phi then this equation readily gets simplified because this d phi that comes out this integration we can perform. So, these 2 pi comes out readily both in the

numerator and denominator.

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if $g(\theta, \phi)$ is independent of ϕ

$$\eta_i = \frac{8f^2}{a^2} \left[\frac{\int_0^{\psi/2} \sqrt{g(\theta)} \tan \frac{\theta}{2} d\theta}{\int_0^{\psi/2} g(\theta) \sin \theta d\theta} \right]^2$$

$$= 32 \left(\frac{f}{d}\right)^2 [\quad]$$

$$= 2 \cot^2 \frac{\psi}{4} [\quad]$$

aperture eff. can be maximised:

- small angular aperture.
- $g(\theta) \propto \sec^4 \frac{\theta}{2}$.

So, I write that; $g(\theta, \phi)$ is if this is independent of ϕ then you can further simplify this aperture illumination efficiency that will be $8f^2$ by a^2 . Now, $g(\theta, \phi)$ becomes $g(\theta)$ only and these already if you see when we found the f by d ratio. So, this expression can be related to f by d ratio. So, putting that we can write that this 32 because f^2 by a^2 from here I can write f by d^2 into this third bracket as it is and that f by d ratio we have seen that that can be written as 2 in terms of the angular aperture $2 \cot^2 \frac{\psi}{4}$ into this.

So, these shows that the when we say that for here you see that G is equal to these so, if we want to maximise gain we want to maximise $\eta_A \eta_S$, aperture efficiency into spillover efficiency product. Now, aperture efficiency is basically this illumination efficiency into these. So, for the time being these are put to 1 . So, this efficiency is now shown to these. So, from here we can see that so, the maximise the maxi optimization that we can do. So, aperture efficiency can be maximised.

Now, I can write that aperture efficiency can be maximised if we do two things; one is that we use small angular aperture because you see that \cot function ψ . So, smaller ψ will give you more aperture efficiency. So, one thing is small angular aperture and previously we have seen that if we take the g , the feed power pattern that is proportional to $\sec^4 \frac{\theta}{2}$, then also we can maximise.

But the problem for this is if we take this choice then the spillover efficiency gets increased because that becomes a pattern which has much more radiation in the edges. So, that is why separately this illumination efficiency cannot be a thing. So, we can now we have to find a function that which can maximise this. So, in instead of taking sec function, now people have found out that we can have a better function that.

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Handwritten notes on a light blue background. On the left, a diagram shows a radiation pattern with a peak at $\theta = 0$. To the right, a piecewise function is defined: $g(\theta) = \begin{cases} 2(n+1)\cos^n \theta & , 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & , \text{elsewhere} \end{cases}$. Below this, the total power radiated by the feed is given as $\int_0^{2\pi} \int_0^{\pi/2} g(\theta) \sin \theta \, d\theta \, d\phi$. At the bottom, the directivity D_f is calculated as $D_f = 4\pi \frac{\text{max of } g(\theta)}{\text{Total power radiated}} = 4\pi \frac{2(n+1)}{4\pi} = 2(n+1)$.

Suppose, the feed radiation pattern is something like $2n + 1 \cos^n \theta$; that means, instead of $\sec^4 \theta$ we take $\cos^n \theta$ some power for $0 \leq \theta \leq \pi/2$, so, in this zone and 0 elsewhere. Now, this factor $2n + 1$ that is chosen for a reason that we can see that if a feed has this power pattern, what is the total power radiated? So, the total power radiated by the feed that will be $\int_0^{2\pi} \int_0^{\pi/2} 2(n+1)\cos^n \theta \sin \theta \, d\theta \, d\phi$.

Now, so, if you do this already it is $d\theta$. So, this ϕ integration will come out and if you evaluate this because this expression is easy to evaluate, you will find that it will turn out to be 4π . So, this is the reason the total radiated power for this choice becomes this. So, with this choice we can now find out what is the feed directivity, D_f will be 4π then maximum of $g(\theta)$ by total power radiated.

So, maximum of $g(\theta) \cos^n \theta$ can have maximum value 1; so that means, it will be $2n + 1$ and total power radiated just now we have calculated 4π . So, that cancels this and the directivity becomes $2n + 1$. So, that was the idea that; so, this now if you

remember if you see this spillover efficiency expression derived earlier that spillover efficiency is in terms of D f into these. So, we can put this value of D f there.

So, spillover efficiency becomes with this value spillover efficiency will you can now you know the pattern; so that means, this g theta expression you know. So, you can put it, it will be $2n + 1 \int_0^{2\pi} \int_0^{\psi/2} \sqrt{2(n+1)} \cos^n \theta \tan \theta d\theta d\phi$ by $2 \int_0^{2\pi} \int_0^{\psi/2} 2(n+1) \cos^n \theta \sin \theta d\theta d\phi$ into $2 \int_0^{2\pi} \int_0^{\psi/2} (1 - \cos^2 \theta)^{n/2} \tan \theta d\theta d\phi$ by $2 \int_0^{2\pi} \int_0^{\psi/2} \cos^n \theta \sin \theta d\theta d\phi$; this can be easily integrated and it will become $1 - \cos^2 \theta$ by 2 whole to the power n sorry $n + 1$; this will be the this will be the spillover efficiency. And now also with this choice of g theta phi what will be the illumination efficiency?

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$$\eta_i = \frac{4f^2}{a^2} \frac{\left[\int_0^{2\pi} \int_0^{\psi/2} \sqrt{2(n+1)} \cos^n \theta \tan \theta d\theta d\phi \right]^2}{\int_0^{2\pi} \int_0^{\psi/2} 2(n+1) \cos^n \theta \sin \theta d\theta d\phi}$$

$$= \frac{4f^2}{a^2} \frac{2(n+1) 4\pi^2 \left[\int_0^{\psi/2} (\cos \theta)^{n/2} \tan \theta d\theta \right]^2}{2(n+1) 2\pi \int_0^{\psi/2} \cos^n \theta \sin \theta d\theta}$$

$\frac{\eta_s}{(n+1)}$

Eta i is $4f^2/a^2 \int_0^{2\pi} \int_0^{\psi/2} \sqrt{2(n+1)} \cos^n \theta \tan \theta d\theta d\phi$ this thing square divided by $\int_0^{2\pi} \int_0^{\psi/2} 2(n+1) \cos^n \theta \sin \theta d\theta d\phi$. Now, this you can easily integrate but in the denominator actually we will do something. So, let us see the numerator $2n + 1 \int_0^{2\pi} \int_0^{\psi/2} 4 \pi^2 \cos^n \theta \tan \theta d\theta d\phi$ whole square and in the denominator we write $2n + 1$ into 2π , but in place of this the. So, what will remain is $\int_0^{\psi/2} 2 \cos^n \theta \sin \theta d\theta$.

Now, here we will put the expression of spillover efficiency. So, this instead of this we will write eta s by $n + 1$. Now that gives us you see eta s eta i product which will be easier for maximisation.

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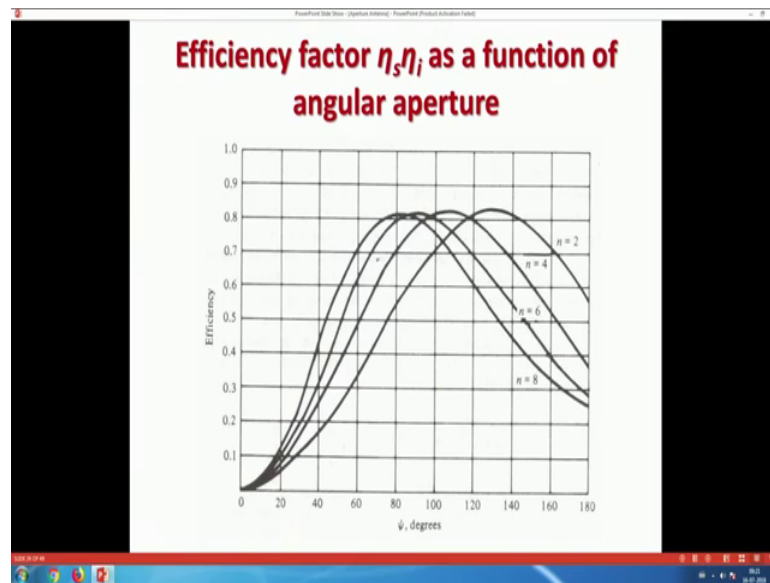
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$$\begin{aligned} \eta_i \eta_s &= \frac{8f^2}{a^2} (n+1) \left[\int_0^{\psi/2} (\cos \theta)^{n/2} \tan \theta d\theta \right]^2 \\ &= \frac{8f^2}{a^2} (n+1) \left[\int_0^{\psi/2} (2 \cos^2 \frac{\theta}{2} - 1)^{n/2} \frac{\sin \theta/2}{\cos \theta/2} d\theta \right]^2 \\ \text{even } n &= \cot^2 \frac{\psi}{4} \begin{cases} 24 \left(\sin^2 \frac{\psi}{4} + \ln \cos \frac{\psi}{4} \right)^2 ; n=2. \\ 40 \left(\sin^4 \frac{\psi}{4} + \ln \cos \frac{\psi}{4} \right)^2 ; n=4. \\ 14 \left[\frac{1}{2} \sin^2 \frac{\psi}{2} + \frac{1}{3} (1 - \cos \frac{\psi}{2})^3 + 2 \ln \cos \frac{\psi}{4} \right]^2 ; n=6 \\ 18 \left[\frac{1}{2} \sin^2 \frac{\psi}{2} + \frac{1}{3} (1 - \cos \frac{\psi}{2})^3 + 2 \ln \cos \frac{\psi}{4} - \frac{1}{4} (1 - \cos^4 \frac{\psi}{2}) \right]^2 ; n=8 \end{cases} \end{aligned}$$

So, we can write that from here I can write eta i eta s is 8 f square by a square n plus 1 0 to psi by 2 cos theta n by 2 tan theta by 2 d theta whole square. So, this needs to be maximised. So, this people have further make it. Now, this integral is 0 for odd n and non zero for even m. So, this can be written as cot square pi by 4; then its value people have evaluated that. So, only for n even this is non zero.

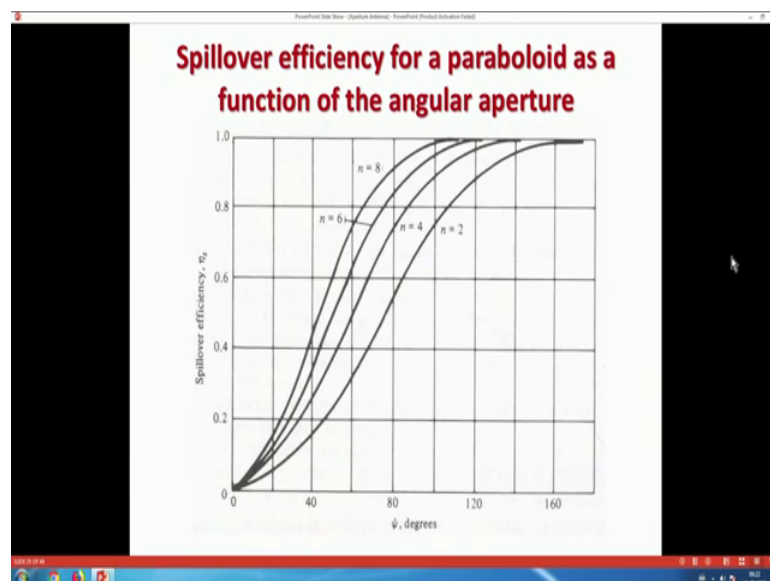
So, those values are 24 sin square psi by 4 plus 1 n cos psi by 4 whole square; for n is equal to 2, 40 sin 4 phi by 4 plus 1 n cos psi by 4 whole square; for n is equal to 4, 14 half sin square phi by 2 plus one-third 1 minus cos psi by 2 whole cube plus 2 l n cos psi by 4 whole square; for n is equal to 6 and 18 half sin square phi by 2 plus one-third 1 minus cos phi by 2 whole cube plus 2 l n cos psi by 4 minus one-fourth 1 minus cos 4 phi by 2, when this thing whole square for n is equal to 8 etcetera. So, these curves are available, you can easily put these curves as a function of phi.

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So, if you look at the diagram, this as a function of angular aperture this shows this. So, you see that every curve has a maximum thing. So, for n is equal to 2 you are getting a maximum I can say something around 130 degree aperture. So, if we take the angular aperture to be 130 degree then we get further n is equal to 2 it is gives the maximum efficiency. Similarly, for n is equal to 4 it is something like 110 degree, for n is equal to 6 it is 90 no n is equal to 6 will be something like 90 degree and for n is equal to 2 something like 80 degree etcetera.

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Also we can see that if we separately see the spillover efficiency then it shows that spillover efficiency goes to almost 10 percent, the more we give the n value and that is obvious because what is n? Physically, if you look at this expression what is sorry, what is n? N is n come from here that more directed feed, cos function you are making suppose you have a cos function is generally these, but if you square it will be more picky that will the value will be this sorry.

Let us look here that if you have these the more picky thing will be something like this make another factor that will be something like this. So, that means, if I make it more directed then the obviously, spillover will be less. So, that is shown by this graph that up to 8 if you take then it is very (Refer Time: 20:24) very small angular aperture it goes to high. So, main thing is from there the joint one you can easily find out. So, from these 2 figure, the designer chooses the angular aperture, so and decides.

Now, then while deriving all these we have assumed that uniform field illumination in the aperture but obviously, with the cos to the power n theta type of excitation there will be taper. So, what will be the taper that also we can calculate that what will be the amplitude taper in the field. So, let us say that we have chosen from this figure, some optimum psi value. So, let us call at psi opt.

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$$\psi = \psi_{opt}$$

$$g\left(\frac{\psi_{opt}}{2}\right) = 2(n+1) \cos^n \frac{\psi_{opt}}{2}$$

$$P(a) = g\left(\frac{\psi_{opt}}{2}\right) \frac{(1 + \cos \frac{\psi_{opt}}{2})^2}{4f^2}$$

$$= 2(n+1) \cos^n \frac{\psi_{opt}}{2} \frac{(1 + \cos \frac{\psi_{opt}}{2})^2}{4f^2}$$

Aperture field amplitude at the edge $\rightarrow \propto \sqrt{P(a)}$
 at the centre $\rightarrow \sqrt{\frac{2(n+1)}{f^2}}$

So, we know that $g(\psi_{opt}/2)$ that will be $2(n+1) \cos^n(\psi_{opt}/2)$. So, we have already found out power density in aperture power density P a in terms of the feed

pattern. So, we can from that expression we can write. Now, I am putting the value of this feed power pattern. So, once I know these it is obvious that what will be the at the edge the aperture field please remember this is amplitude pattern; field amplitude, no this is power pattern from that I am writing the field amplitude at the edge; this is the power pattern aperture pattern.

So, field amplitude at the edge; that means, there my angle is this optimum. So, that will be proportional to root over P a and field amplitude at the centre that will be when this angle become 0. So, that you know you can put the in the cos 0 angle. So, root over 2 n plus 1 by inside bracket f square or outside f whatever. So, we can write what will be the amplitude taper?

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$$\frac{A(a)_{\text{edge}}}{A(a)_{\text{centre}}} = \frac{1}{2} \cos \frac{\psi}{2} \left(\frac{\psi_{\text{opt}}}{2} \right) \left(1 + \cos \frac{\psi_{\text{opt}}}{2} \right)$$

$n=2, \psi_{\text{opt}} \approx 130^\circ$
 Aperture taper = -10.44 dB.

$n=4, \psi_{\text{opt}} \approx 110^\circ$
 taper \rightarrow -11.74 dB.

$n=6, \psi_{\text{opt}} \approx 90^\circ$
 amp. taper = -16.42 dB.

A a edge versus centre will be half cos n by 2. So, for n is equal to 2, from the graph we have seen psi opt is something like 130 degree. So, we can say this one this is called aperture taper that if you calculate that will become minus 10.44 dB. Remember that this aperture taper to calculate this dB we have taken a 20 log because this is the field. For n is equal to 4, the optimum angular aperture is from that graph 110 degree.

So, taper is minus 11.74 dB, for n is equal to 6 this from the graph is almost 90 degree. So, amplitude taper this is minus 16.42 dB etcetera. Also you can find the corresponding taper at the feed radiation pattern.

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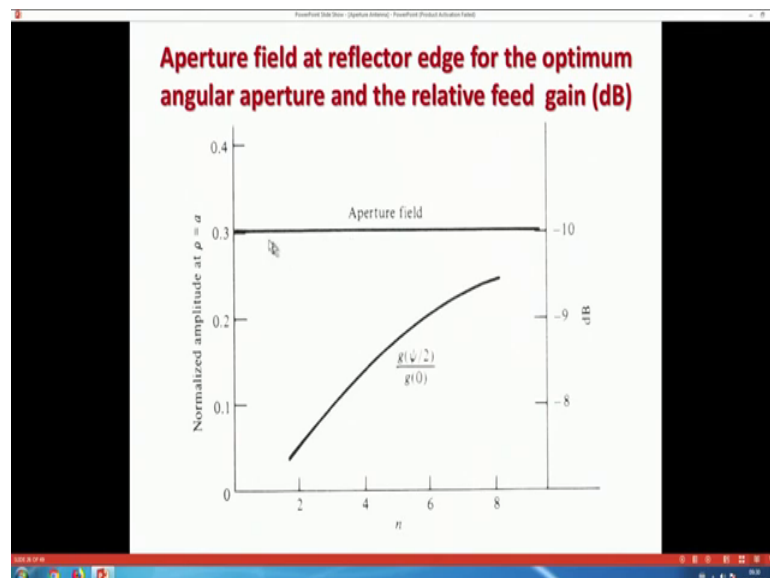
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$$\text{feed power taper} = \frac{g\left(\frac{\psi_{opt}}{2}\right)}{g(0)}$$

$n=2,$	$g_{\text{taper}} = -7.48 \text{ dB.}$
$n=4,$	$g_{\text{taper}} = -9.66 \text{ dB.}$
$n=6,$	$g_{\text{taper}} = -9.03 \text{ dB.}$

So, feed power taper because the designer of the feed is this value that will be given by g sorry ψ_{opt} by 2 by $g(0)$ and. So, this for n is equal to 2 we can write g_{taper} will be. So, this value was 130 degree. So, g of 65 you can put it and please remember here it will be $10 \log d$ because it is a power pattern. So, these becomes minus 7.48 dB, n is equal to 4, g_{taper} becomes minus 9.66 dB, n is equal to 6, g_{taper} becomes minus 9.03 dB ok. So, that shows you the thing.

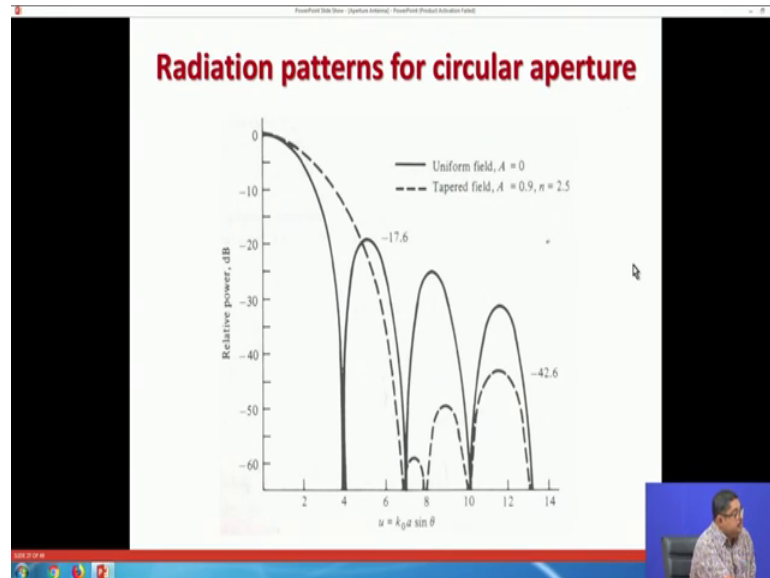
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So, we can; so, from this people have found out that various graphs this this whatever I

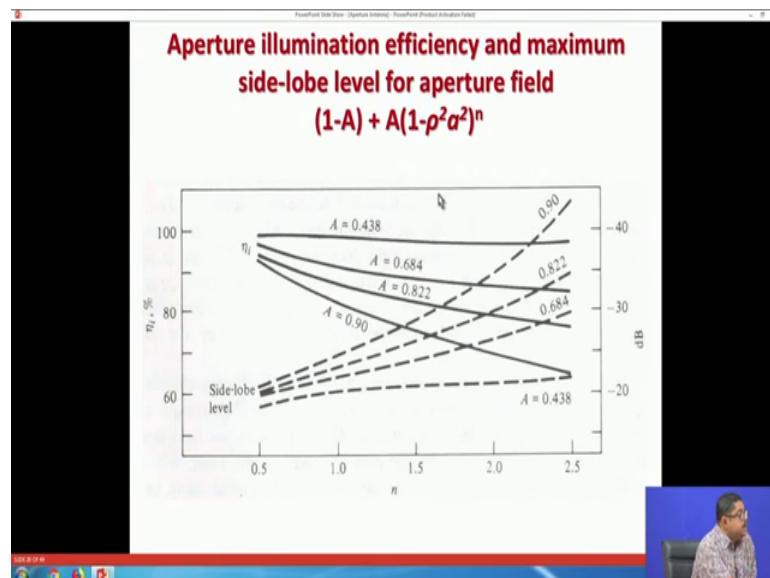
have said that aperture field in terms of actual values if you give it in dB then there will be difference and this is the g taper, this is the aperture field taper for various choice of n ok.

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Then this is circular.

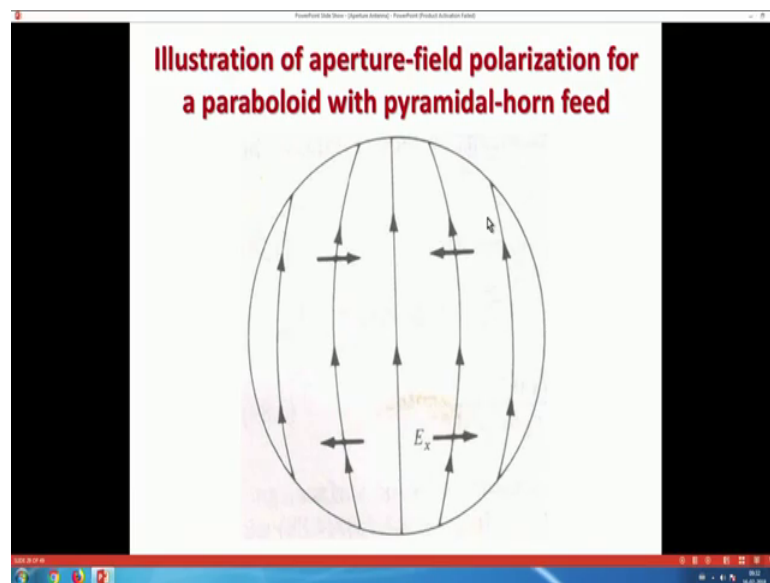
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Aperture illumination efficiency; that means, η_i and maximum side lobe level for aperture field, so, for a η_i how much you get? So, you see that if we take the this aperture taper to be more and more for then you see that illumination efficiency that

actually falls and side lobe level you see that if we increase the aperture taper the side lobe level actually goes down because this is minus 40. So, that is good; that means, if you give more taper the side lobe falls down which qualitatively we said earlier; so, compared to uniform distribution of the aperture field taper gives 25 dB reduction in side lobe typically. So, in communication I already pointed out that the this side lobe reduction is more important and so, people have.

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So, that thing that you should consider that what is a efficiency you are getting; so, based on that you will have to modify the aperture illumination efficiency; that means, where from we have started. Then there will be; [FL] so, we can I think from whatever we have also this thing let me discuss that. Actually I have a large surface and there we are assuming that we are having a suppose polarisation like these, but there will be also because this is an aperture surface.

So, there is no point of preventing this cross polarized field. So, some cross polarised field will come because there is the horn is also giving that cross polarised thing then there are scatterings there are various reflections on the feed etcetera, so, all that gives this cross polarised thing. So, that also need to be taken that what is that that can be estimated we are not going there. So, let us see a problem that suppose we have a 10 meter diameter reflector; a let us do a problem.

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10m dia reflector.
 $f/d \rightarrow 0.5$.
 $f_{\text{req}} \rightarrow 3 \text{ GHz}$.
 feed pattern is symmetric: $g(\theta) = 6 \cos^2 \theta$.
 $\eta_i \eta_s = \cot^2 \frac{\psi}{4} \left[24 \left(\sin^2 \frac{\psi}{4} + \ln \cos \frac{\psi}{4} \right)^2 \right] = 0.75$.
 $\frac{f}{d} = \frac{1}{4} \cot \frac{\psi}{4}$
 $0.5 \quad \psi = 106.26^\circ$

A 10 meter diameter reflector with f by d ratio of 0.5 because generally, these reflectors are specified like this and frequency operating, frequency is 3 gigahertz. The feed is having a pattern which is feed pattern is symmetrical and given by g theta is equal to 6 cos square theta. So that means, you see it actually satisfied our that feed optimum feed distribution that people have found. So, here n is equal to 2. So, $2n + 1$ is 6 and this is cos square theta.

So, first find out what is the overall efficiency; that means, let us see other efficiency. So, basically we will have to find $\eta_i \eta_s$. So, first thing is we know f by d ratio, the f by d ratio is given that actually gives us the angular aperture because we know f by d is nothing but one-fourth cot psi by 4 and this is specified as 0.5. So, from there I can find out the psi value will be 106.26 degree.

So, now we can find out $\eta_i \eta_s$ because $\eta_i \eta_s$ is cot square psi by 4 24 sin square psi by 4 plus ln cos psi by 4 whole square; this expression I have already given earlier. So, if you calculate you will see that this value comes to be 0.75. So obviously, this is not an optimum because f by d ratio is given. So, we could not choose that. If we had our thing then we could have chosen from the graph to maximise $\eta_i \eta_s$, we can find for n is equal to 2 that value would have been 130 degree, but here it is actually 106 degree. So, that is why it is coming as 0.75. Now, also I can since the a is given, what else that diameter of the reflector is given. So, immediately I know what is the diameter is given

and also the frequency is given. What will be the directivity? Directivity can be easily calculated.

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$$\text{Aperture directivity} = \frac{4\pi}{\lambda_0^2} (\pi a^2)$$

$$= 49.99 \text{ dB.}$$

$$\text{gain} = \eta D = 0.75 \times 98636 = 48.69 \text{ dB.}$$

$$\eta_s = 1 - \left(\frac{c}{\lambda_0}\right)^{n+1}$$

$$= 0.784$$

$$\eta_a = \frac{\eta_i \eta_s}{\eta_s} = \frac{0.75}{0.784} = 95.66\%$$

$$\eta_b \rightarrow \text{Blockage eff}$$

Aperture directivity; please remember this is aperture directivity.

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Here, you can see this is actually a paraboloid dish, this is the horn feed; you can see the horn feed here. This is the horn this is the circular horn instead of a rectangular horn, it is at the focus and aperture means this circular zone. We see this is the circle here; hypothetical circle that is the aperture; that is diameter is being said. So, f by d ratio

means this diameter by this f , the focal (Refer Time: 34:46). So, aperture directivity will be 4π by λ naught square into πa square. So, frequency is said frequency is 3 gigahertz; that means, you have 10 centimeter and a is 10 meter. So, if you put all those values we will see it comes to be 49.94 dB and then what will be the gain? Gain of this reflector system; here we assume that other efficiencies are all 1; η_i and η_s we have calculated. So, gain will be η into D . So, η is 0.75 and this gain is absolute value is 98696. So, that becomes 48.69 dB.

And now what is the spillover efficiency? Spillover efficiency, since we know the aperture we can find spillover efficiency; $1 - \cos \psi$ by $2n + 1$. So, that will be 0.784. So, 78.4 percent and then what is the illumination efficiency? Since, I know that $\eta_i \eta_s$ by $\eta_s \eta_i$ value I have found 0.75 by 0.784. So, that is 95.66 percent, illumination efficiency ok. Now, there will be other efficiencies also which we have not said, one of that is the surface roughness.

Actually, the surface you see here the rays are coming. So, if the surface is having a rough surface then there will be some error because here we are assuming that the rays are regularly reflected but due to the surface imperfection there will be a. So, that is specified in terms of surface error. So, any dish antenna for fabrication actually the very critical thing in the fabrication is the surface roughness, actually the random error is specified that how much surface error is there. So, in terms of that people have found that is some distribution for a random Gaussian distribution they assume and from that there are formulas for finding the surface roughness. So, that efficiency is called η_r .

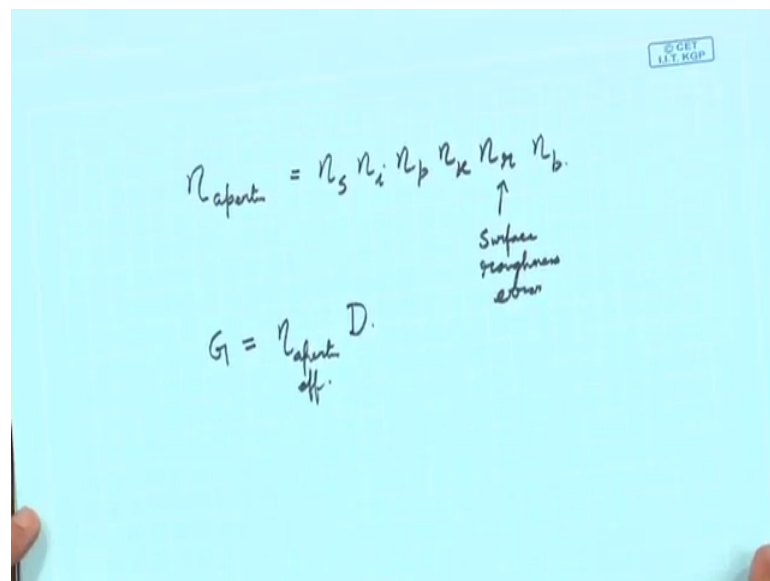
Also you see that this problem actually we will later discuss that the rays are look at here the speed is getting it here, but there are some arrangement here for maintaining the feed. You see this whole arrangement is to maintain the feed. So, this is feed and its supporting thing also there will be cables from we may be rf cables may be waveguide here. Now, the problem is after this thing gets reflected, the reflected ray this feed is blocking that.

So, some portion of the power is not going to the way thing that is one thing also another thing is suppose this thing the paraboloid reflector is looking at top supposed to a satellite, but the feed is looking to the ground. You see so, feed has below this paraboloidal surface also it is receiving signals from here. So, lot of ground reflected noise it catch. So, that we will reduce the sensitivity, but before that this efficiency that

will be mattered because this feed is blocking this that is called blockage efficiency.

So, one was spillover that all the power was not captured. Now whatever is being reflected the feed and its accessories are blocking that. So, there will be another thing which is called eta b this is blockage efficiency. So finally, I write that what are the efficiencies that is involved in the aperture, aperture efficiency.

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$$\eta_{\text{aperture}} = \eta_s \eta_i \eta_p \eta_u \eta_m \eta_b$$

↑
Surface
roughness
error

$$G = \eta_{\text{aperture}} D$$

So, that will be one is spillover efficiency, one is illumination efficiency, one is polarisation efficiency, one is uniform field efficiency, then this surface roughness, this is surface error; surface I will say roughness error efficiency and then the blocking efficiency. So, all this effects the aperture efficiency. So, finally, gain is nothing but eta aperture efficiency into the directivity of the aperture ok.

With this, I conclude. In the later, we will see that some of the techniques for by which this blockage thing can be tackled.

Thank you.