Analysis and Design Principles of Microwave Antennas
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Lecture - 27
Paraboloid Reflector Antenna (Contd.)

Welcome to this lecture in NPTEL course, we are continuing the discussion on reflect Paraboloid Reflector system. We have seen the aperture field distribution from the feeds radiation pattern, now we will see the directivity of the this whole system; that means, aperture directivity.

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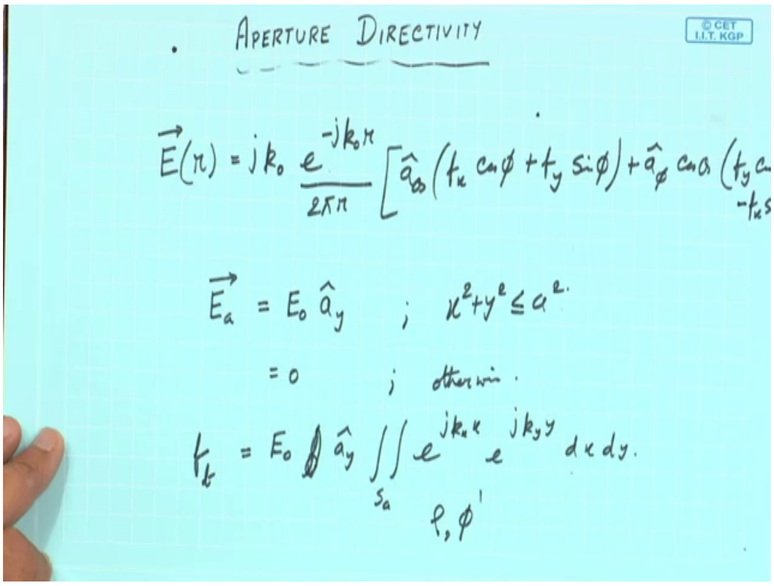

So, we know actually in few lectures back we have proved by Fourier transform method that for a uniform constant phase aperture field with linear polarisation, the radiated field is given by the expression E r j k naught e to the power minus k j naught r by 2 pi r; a theta f x cos phi plus f y sin phi plus a phi cos theta f y cos phi minus f x sin phi, this we have already proved.

Now, our aperture is circular with radius a in z is equal to 0 plane. So, we can write our aperture field is E naught a y linearly polarised uniform electric field, we are assuming it. Now, we have seen that it is it has got problems, but first let us see the directivity; that means, what maximum we can achieve, if we can create this uniform field let us arbitrarily I have taken y polarised field in any polarised single polarisation. So, this is valid for x square plus y square less than equal to a square is the circle, circular disc and 0 otherwise.

So, with this aim what will be my Fourier transform f t, f t will be E naught sorry E naught that a y then on the aperture e to the power so, this is 1, 1 into e to the power j k xx e to the power j k yy dx dy. Now, to evaluate this integral this integral evaluation is not so easy. So, I will help you. So, since this is a circular disc we introduce the polar coordinates, but since it is the aperture is here source we say the polar coordinate is rho instead of phi I call it phi dash. I should have call the rho dash also, but it would not get confused because the other feeds thing is theta. So, since it does not have any theta so, there is no confusion. But, since there is also phi so, the observation field phi and source field phi that is why I am showing phi dash.

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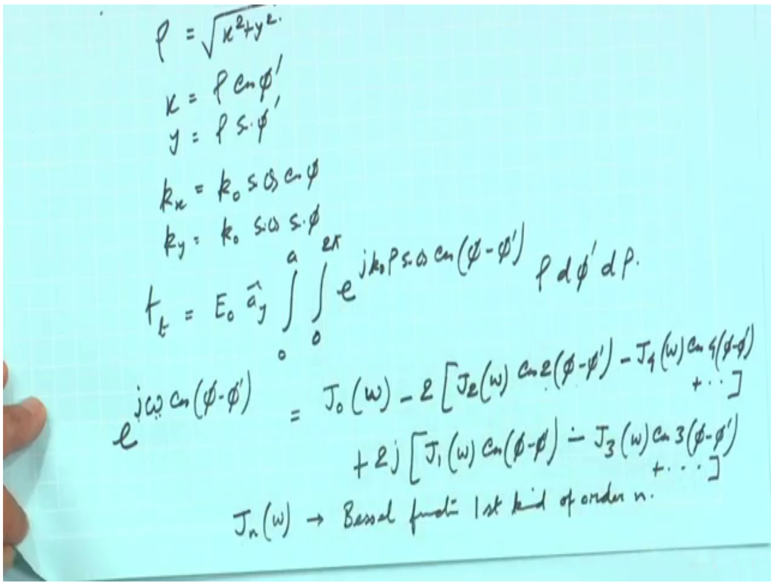

Now, what is my rho? Rho is nothing, but root over x square plus y square. So, x is equal to rho cos phi dash, y is rho sin phi dash and also we know k x is k naught sin theta cos phi and k y is k naught sin theta sin phi. So, I can put it on that f t expression so, it becomes E naught a y, then 0 to a 0 to 2 pi e to the power j k naught rho sin theta cos phi minus phi dash rho d phi dash d rho ok. Now, this thing e to the power j something cos that can be, that has a series expansion, anyone knows that this expansion is in terms of Bessel functions. So, we can write if I have e j some constant into cos phi minus phi dash, the Bessel expansion is J naught that w you can say or omega whatever this is a constant; that means, j into these.

So, J naught omega minus 2 J 2 omega cos 2 phi minus phi dash minus J 4 omega cos 4 phi minus phi dashed etcetera etcetera plus 2 j J 1 omega cos phi minus phi dash minus J 3 omega cos 3 phi minus phi dash plus etcetera. So, big but so, what is the, if I write J n omega for various subscript. So, this is the Bessel function which comes in all engineering science problems. So, Bessel function this is J means it is the first kind and n means of order n ok. Now, the if I put it here fortunately the integration of this cos 2 phi minus phi dash where, phi dash this varying from 0 to 2 pi cos 4. So, everything from 0 to 2 pi integration that becomes 0; only the thing is who does not become 0 is J 0 omega.

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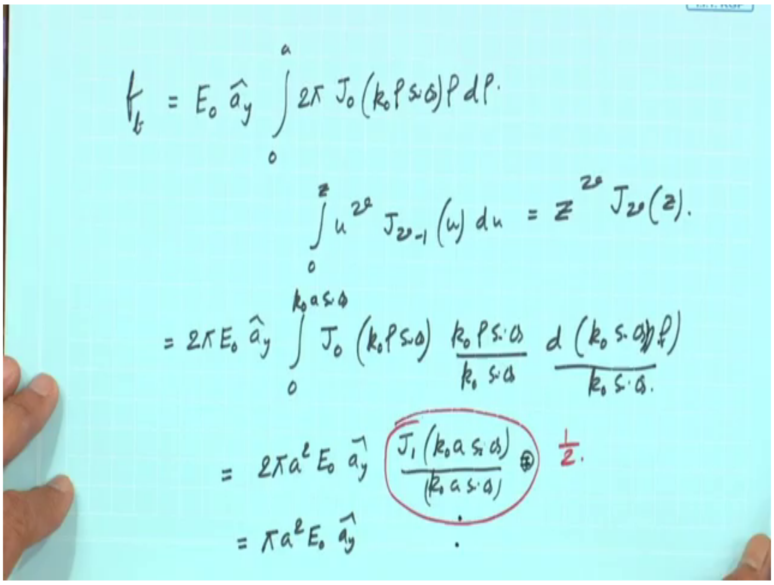

So, f t comes out as E naught a y 0 to a 2 pi J naught, we will see that k naught rho sin theta rho d rho. So, it is an integration of Bessel function of 0 order. So, there the there is another formula which will help you. So, with this formula you can find out that this will become 2 pi E naught a y 0 to k 0 a sin theta J naught k naught rho sin theta k naught rho sin theta by k naught sin theta d of k naught sin theta sorry k naught sin theta rho by k naught sin theta.

So, if you do that it finally, becomes 2 pi a square E naught a y J 1 k naught a sin theta by k naught a sin theta. Now, on the z axis on the z axis theta is 0 so, k naught sin theta fully 0. So, this J naught 0 by 0 this term will give you J naught 0 by 0. Now so, that is a (Refer Time: 11:09) rule so, we will have to differentiate these and then put the limit. So, this part gives you half so, it becomes pi a square E naught a y.

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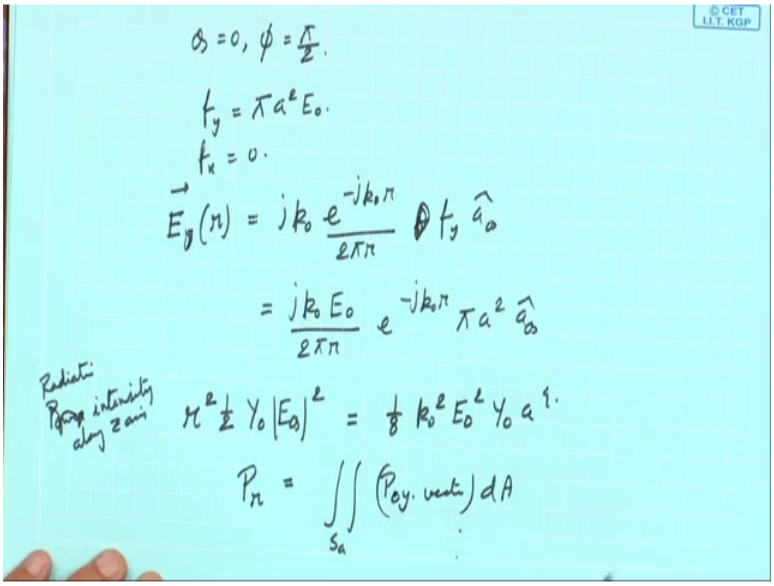

On the z axis the e plane pattern is theta is equal to 0 is equal to pi by 2. So, we get f y will be pi a square E naught and f x is equal to 0. So, we have f y f x I have got so, we can say that E radiated field that will be j k naught to the power minus j k naught r by 2 pi r, f y you can put or I will write f y a theta. So, that will be j k naught E naught by 2 pi r e to the power minus j k naught r pi a square a theta. So, this will be the power field.

Now, the power density per unit solid angular radiation intensity in this direction will be r square half Y naught E theta square. So, I will say power radiation intensity let us say radiation intensity along z axis that will be this. So, if you do it will become 1 by 8 k naught square E naught square Y naught a to the power 4. Another thing we need the total radiated power P r so, for that will be over the aperture, the pointing vector into d A.

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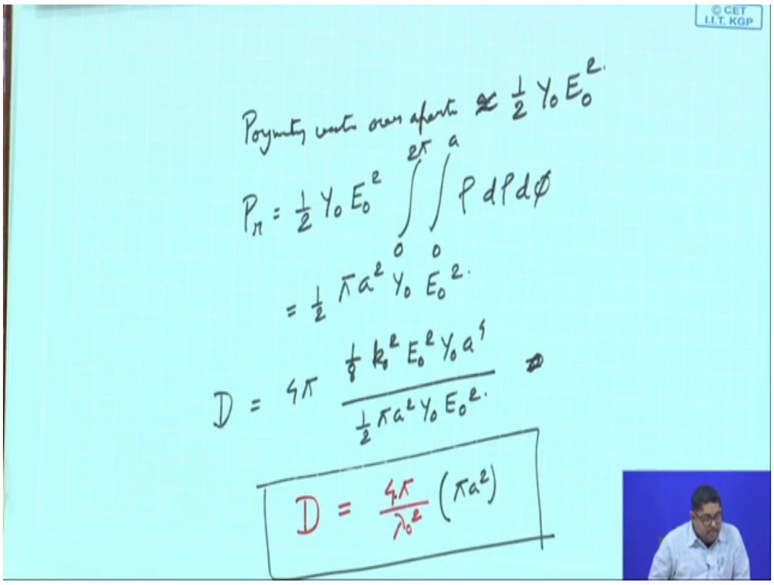

Now, pointing vector over aperture is what? Pointing vector over aperture that actually here we can say that though aperture is not very far away, but we are having the already a parallel beams. And, it has been it can be proved that they are the magnetic field is also like the per field; that means, that n cross that thing.

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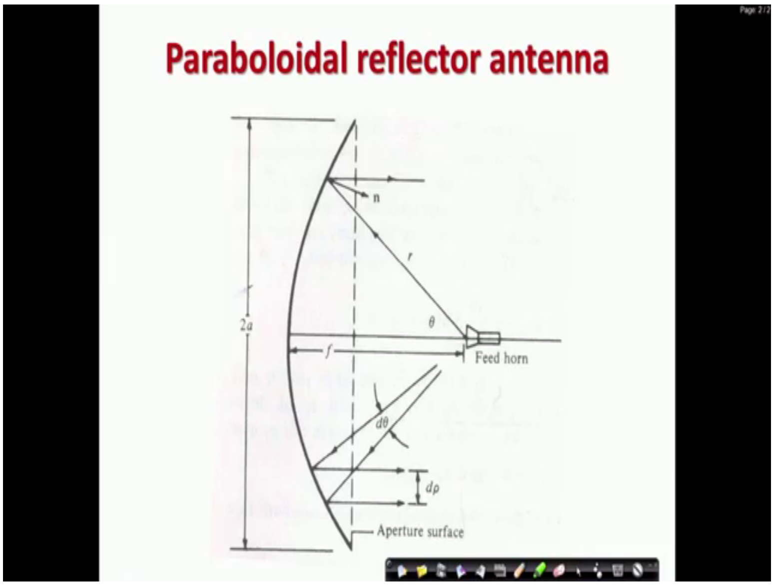

So, we can almost take it that it will be half Y naught E square E, E naught square; so, just like far field it becomes there. So, if this is there the total radiated power P r will be half Y naught E naught square then the area 0 to 2 pi 0 to a rho d rho d phi. If you do it becomes half pi a square Y naught E 0 square. So, now you are in a position to do directivity 4 pi into that radiation intensity 1 by 8 k naught square E naught square Y naught a to the power 4 by total radiated power half pi a square Y naught E naught square. These gives you and I will write it like this, directivity is 4 pi by lambda naught square such a simple formula, such a useful formula.

Now, what is it pi a square means what? The aperture physical area. So, aperture physical area is getting multiplied by 4 pi by lambda naught square. So, if you have a d shop 1 meter radius then pi. So, 4 pi square gives you 40 roughly and if you are at 10 gigahertz 3 centimetre means 0.03 into 0.03 so, 0.009. So, I can say 0.01 so that means, 4000 is the directivity; 1 meter radius can give you directivity of 4000, quite remarkable.

But obviously, we cannot when this is directivity; that means, there are no losses. Now, actually there will be losses etcetera. So, the maximum directivity obtainable from an aperture antenna with a constant phase uniform field is physical area multiplied by 4 pi by lambda naught square; very simple formula easy to remember. But, what will be the gain of the parboiled antenna reflector antenna system that will be G is equal to 4 pi by lambda naught square into pi a square into efficiencies.

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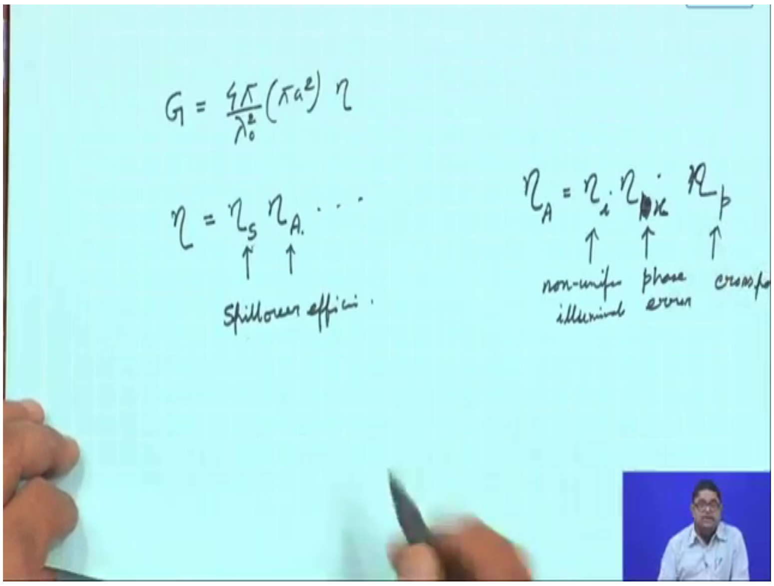

Now, these efficiencies the first one actually we will see a long list of efficiencies at the end. Initially I am writing eta, that eta to start with I make that there is a two things: eta S eta A later others we will come so, I will say these. Now, this I will see first this then, this, then others. Now, this eta S this is called spillover efficiency, eta A is aperture efficiency. Now, aperture efficiency I need to talk a bit because whatever we have done that if I have an uniform illumination, if I have a constant phase illumination, if I have a single polarisation aperture field then I get the directivity I have found.

Now, aperture efficiency is violation of all these. So, aperture efficiency will comprise of illumination efficiency because, we have seen that there is a taper and it is desirable from the application point of view. So, it will be multiplied by eta i then it will be that phase. So, uniform phase we have we know that there will be a phase error. So, that is phase and I think that third parties now sorry this is polarisation. So, the next one non-uniform thing let me call it taper t eta x.

So, this is due to this is due to non-uniform illumination, this is due to phase error, due to phase non-uniform phase and then eta p it is the cross pole presence of non-linear thing. So, spillover efficiency what is it? Spillover efficiency is that some radiation from the feed is not intercepted by the reflector because, reflector is finite it has an angular aperture. So, it is not intercepting all the rays. So, power is lost there and definitely then we are assuming total power radiated, but if that power is not reflected so, ultimately aperture is not getting that power. So, gain will definitely reduce so, that is called spillover loss and efficiency factor due to that is called spillover efficiency.

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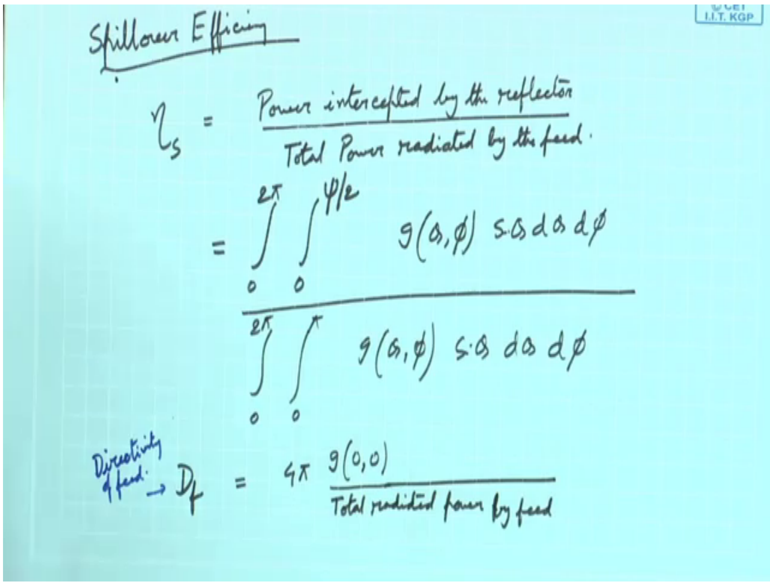

So, what is spillover efficiency now? So, I am now talking of spillover efficiency, eta S is power intercepted by the reflector by total power radiated by the feed. So, what is it? Power intercepted means I have g theta phi is the power radiation pattern, sin theta d theta d phi. And so, phi 0 to 2 pi and theta is from 0 to psi by 2 half of the angular aperture because, 0 to 2 pi I am giving. So, here 0 to psi by 2 this is the power intercepted by the reflector and here 0 to 2 pi 0 to pi g theta phi sin theta d theta d phi that is all, that is spillover efficiency ok.

Now, ultimately we will see that I am not interested to find simply this spillover efficiency or it is we will later see that jointly what happens here. Actually, the others can be separately done, but these two are as physically I explained that if I want to have an uniform illumination then spillover increases, that I said earlier that the of best possible illumination was that sec to the power 4 theta by 2, but that gives lot of a. So, these two are coupled and one effects the other. So, that is why we will jointly maximise this instead of trying to do that. Now, to that in instead of evaluating it further what I do, I want to put here the directivity of the feed; that means, this is radiation pattern here I want to put the directivity of the feed so, D f.

Now, what is directivity of the feed? D f let me write directivity of feed D f. What will be its expression? 4 pi g 0 0 by total radiated power by feed. So, you see that this is nothing, but total radiated power by feed this denominator. So, can I write that eta S is D f then 0 to 2 pi 0 to psi by 2 g theta phi by actually, this will come out 4 pi g 0 0 sin theta d theta d phi ok, I leave it here ok.

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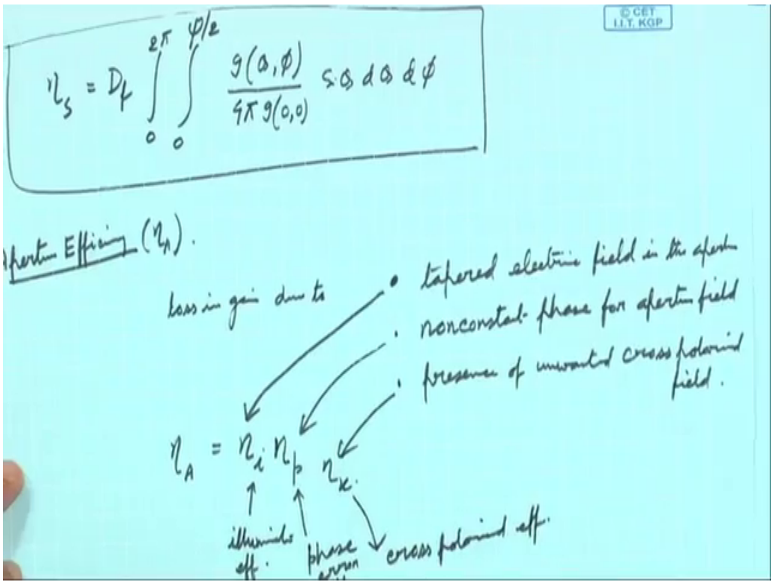

Now, next I start I will use that as I said that since I will jointly do. So, I will take this after see finding an expression for aperture efficiency eta A. So, who gives this aperture efficiency? The loss in gain due to first I have a tapered electric field in the aperture, then non-constant phase for aperture field and then presence of unwanted cross polarised field ok. So, actually this expression is available or it can be derived, but I would not do that because that will be more mathematical in nature. So, I can write eta is eta i this is called illumination efficiency, then non-constant phase is phase error efficiency and cross polarised thing is eta x.

So, this is illumination efficiency, this is phase error efficiency, this is cross polarised efficiency. So, I am not going there are some long list of things. So, we assume while deriving the, this illumination efficiency that eta p and eta x they are one, later we will put their values. So, under that assumption it can be derived, if anyone interested we see in the may be in the course page we can upload this the derivation. But finally, what we need is that eta i the illumination efficiency that becomes an expression something like this. This is the expression that is obtained for the illumination efficiency.

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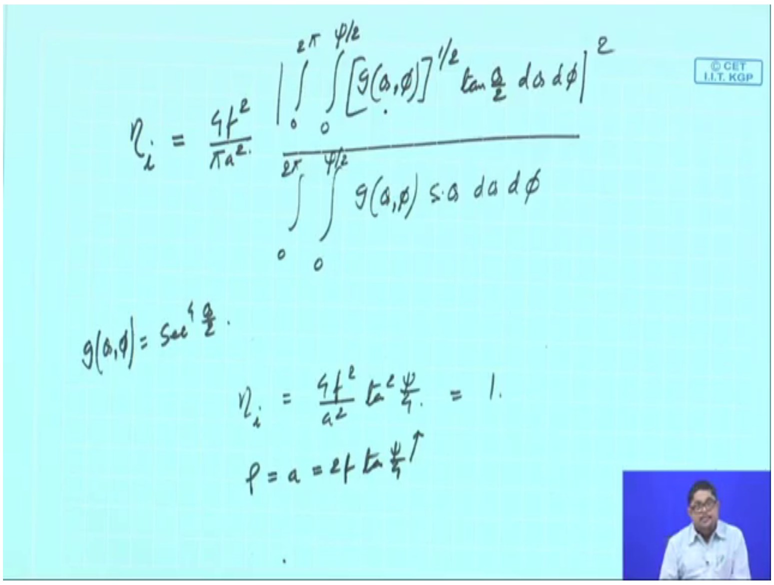

Now, if you do not believe this expression, I can give you a check. For uniform aperture field we require that g theta phi should be proportional to sec 4 theta by 2. Put that in this expression g theta phi values and eta i should turn out to be 1, then that will be indirect way of saying this expression. So, you can do that, that if you put it there you will see this will become sec square theta by 2 tan theta by 2 d theta. This whole thing will come out because, g theta phi is independent of phi. So, the phi integration will give you 2 pi and here also you can integrate. So, you will see that under that assumption that g theta phi is sec 4 theta by 2, this eta expression becomes 4 f square by a square tan square pi by 4.

Already, we have found the value of a when we derived that rho is equal to a is 2 f tan pi by 4. So, put that value of a so, eta i becomes 1 putting it here it becomes 1. So, this expression is correct. So, now we will see that how to manipulate these, that we will see that this jointly if we put this two expressions eta A and eta S how to manipulate that, but that we will see in the next class. So, we have now found out the illumination efficiency, found out means this is I have given this formula and the spillover efficiency we have derived. So, now we will see how to play with that as an engineer in the next class.

Thank you.