Analysis and Design Principles of Microwave Antennas
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Lecture - 26
Reflector Antennas

Welcome to this NPTEL lecture on Reflector Antenna systems, but before going there actually in the last lecture we have discussed about corrugated horn that time I forgot to show you some diagrams.

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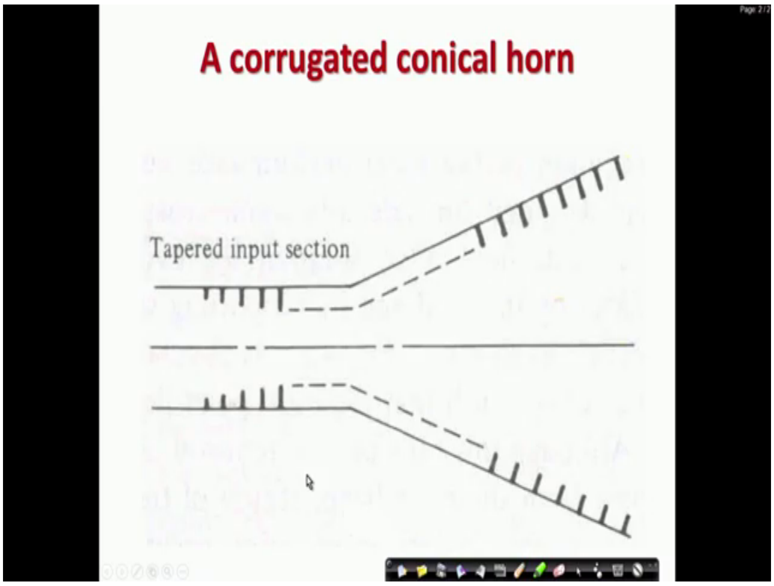

So, a corrugated conical horn this is conical, but corrugated section looks like these in the input feeding section there is a tapered thing. And, then in the radiating zone or near the aperture there is this corrugation.

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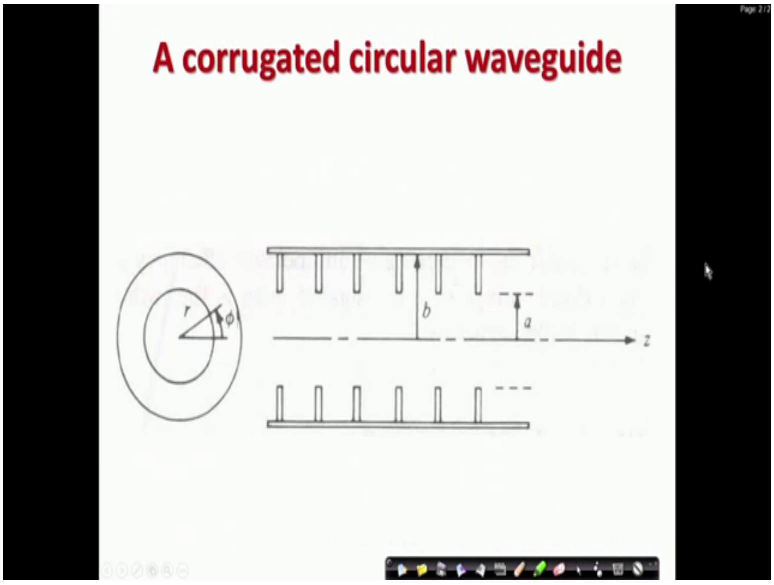

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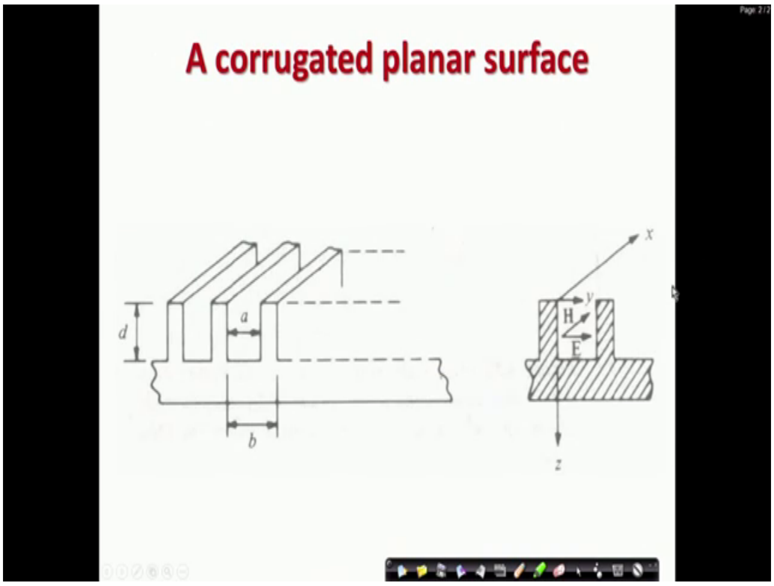

Now, this is the you can say side view, but if we look at the things actually corrugation means this that some portion you are removing the metals. So, this is for a planar surface corrugation looks like this. So, on those two surfaces on this surface these are the corrugations; that means this portions are metallic, but between portions are non-metallic similarly here. So, that is extending for the surface. So, this I you please add this portion to the previous lecture. Now, today we start with the reflector antenna.

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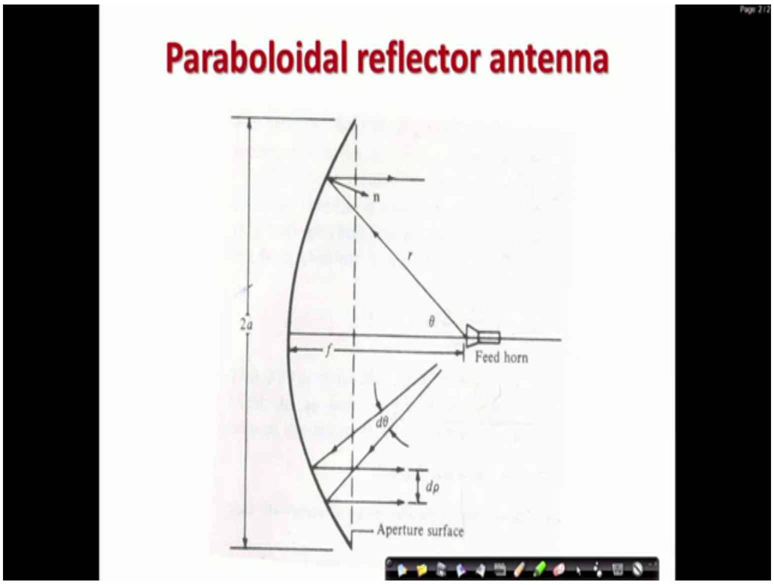

So, at microwave frequencies this is mostly used antenna and majority of satellite links use this reflector antenna systems. The geometry of the reflector system you can see here we are seeing it as a paraboloidal, because this is actually the conic section paraboloid which is obtained by revolving this paraboloid about this axis. If this is the axis of the parabola, if you revolve it you get the paraboloid. So, the reflector is the paraboloidal we have a feed horn at the focus of the paraboloid.

So, now here please understand this dashed line that we that is called a plane because, actually if you think of a dish antenna all of you have seen. Now, think that if I take from one edge to other edge, if I take a plane that plane for the paraboloid it will be circular disk. So, this is a plane thing this is called aperture, aperture surface. So, actually it is a circular disk planar disk for this paraboloid thing.

So, rays that are going from the speed if they go here after getting reflected from the paraboloid, that becomes parallel to the axis similarly here. Now, the this diameter of this aperture is 2 a or radius is a. Now, this is an important ratio as we will see that these f by 2 a or f by d that is a specification for a parabola, because it determines all its radiation properties f by d ratio people generally call it.

Now, we first find out because, we will have to put our previously derived result that if I know the aperture field distribution; that means, if I know the field distribution on this aperture I will be able to say what is the field, that we have seen the by Fourier transformer method etcetera. So, we will have to know what is this apertures equation etcetera. Now, parabola equation we know. So, from the basic geometry now we will derive the geometry of this thing. So, the other portions of the drawing I will come later.

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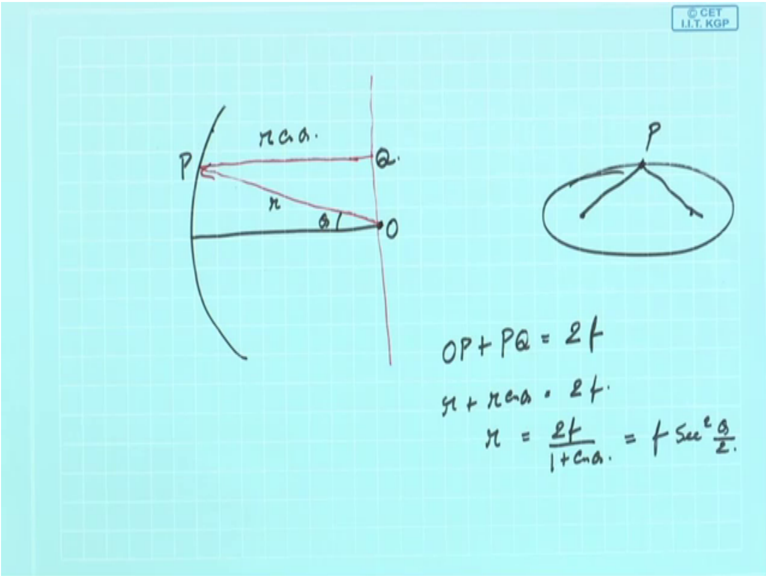

So, I can say that let me draw a parabola and this is its point, let us say the focus I am calling O and the suppose I have an incident ray from here that ray is coming to the focal plane. And, the point where the parallel ray meets the focal plane let me call that as point Q and this point let me call it P, where the ray has heat. Now, from the parabolas basic any conic section if you remember that if from the distance from the focus to any point on the conic section that is always a constant.

If you remember the conic sections which has two focuses so, from a point on the section if you add those two points distance focus; suppose I have a ellipse and it has you know two focus. So, this is the point so, sum of these two distances is always equal to twice of the focus. So, here the same thing we can say that OP plus PQ that is equal to twice f a constant. Now, you can see from the geometry that this angle is theta. So, if this rays or this distance is r then this will be r cos theta. So, I can put that r plus r cos theta is equal to 2 f or r is equal to 2 f by 1 plus cos theta or I can say this is f sec square theta by 2. So, this is the this r; that means, if I take the r here then the corresponding theta I will have to say and this equation will be satisfied.

So, basically the paraboloid is a locus of this point r ok. Generally, the horn is placed at the primary focus as a feed. Now, here actually the analysis for that we will assume that since, this is a microwave thing we assume that ray optics which has a strong foundation from Maxwell's equation. Actually, Sir Jagadish Bose proved that optics also waves Maxwell's equation for the first time he proved it because Maxwell did not say that explicitly. He just conjuncture probably these laws which I have found they also are valid for optics, but Sir J. C. Bose proved it. So, we will take help of ray optics at this fields and it had been seen that those laws would not put here because, the results obtained after these fully matches with the experimental observations.

So, the relevant results that I will take that is called that locally all the waves are clean waves. Actually, in ray optics this holds good because they are wavelength is very small, but here also we will assume these that locally all the rays. So, first thing is the waves travel in a ray that we will assume here also; actually in a tube of ray because we have a beam. So, that is why tube of ray and locally they behave as a plane wave and propagation straight line along state rays in homogeneous medium. At a boundary between medium they are reflected or refracted at the tangent plane according to Snell's law and the fennel reflection coefficient holds good.

The power is viewed as flowing in flux tube and if a flux tube is reflected from a perfectly conducting surface, the power in the reflected flux tube equals that in the incident flux tube. These are all these results you know; just I am saying it that the electric and magnetic fields are transported along the flux tubes. The electric and magnetic fields are mutually perpendicular and orthogonal to the rays because, rays are the direction of propagation. Since, it is plane waves so, electric and magnetic fields are orthogonal to these. And, for the paraboloid surface all rays from focus are reflected to become parallel to axis. At the edges the rays are diffracted, but we will neglect the effect of diffraction in this analysis.

I already said that the aperture is a plane in this case which is circular disk, just in front of the surface. Now, after saying all these my actual intention is that we have already seen the horn antenna. So, this we know the radius suppose, we know the radiation pattern of the feed horn, but to calculate what is happening to the radiated field by the reflector we need to find this field distribution on this aperture. So, we now relate that if I know this power radiation pattern of the feed, what will be the aperture field distribution. This will be our first task, but before that I will want to show you two slides.

What about I said that reflection of a flux tube if you have a plane reflector then this is the incident flux tube and then this is the reflected flux tube. What I said this is the central ray there are other rays, these are the two edge rays. So, you see that E and H filed they are orthogonal. So, E H and the ray they form a triplet etcetera. So, this is plane reflector, but we do not have a plane reflector so, I will have to see some next slide. So, our case will be like this we are having an incident tube, then that incident tube will go here on the one ray is shown, but actually here also there will be tube. And, incident tube thing and the reflected tube total power that should be conserved because, this we are assuming to be a conducting surface ok. So, with that I again go back here.

Now, you see that actually you see that there are two ways by which this paraboloid reflector can be analysed. One is the way I am saying that aperture field I will calculate and from aperture field we know what is the far field etcetera that study within the last lectures we have derived. But, there are also people you can say that I can also find out at this paraboloid reflector what is the current because, this is not an aperture problem. Because, this is a real paraboloid conducting surface is there so, I can find the conduction current here.

And, from that also I can find the thing both methods are there, this is called induced current method by which where antennas are analysed. So, that can be done also this aperture is a aperture so, that is why here from the field we go. So, we will take the aperture field method because it will help us to illustrate what were we have derived for planar antennas etcetera. So, for that let me introduce that, suppose g theta phi is the power angular radiation pattern of the feed located at the focus. This is actually horn antenna, we are not saying horn because there can be other things also, but mostly in all the cases it is on. So, g theta phi is the power angular because I am saying it is expressed in terms of theta phi.

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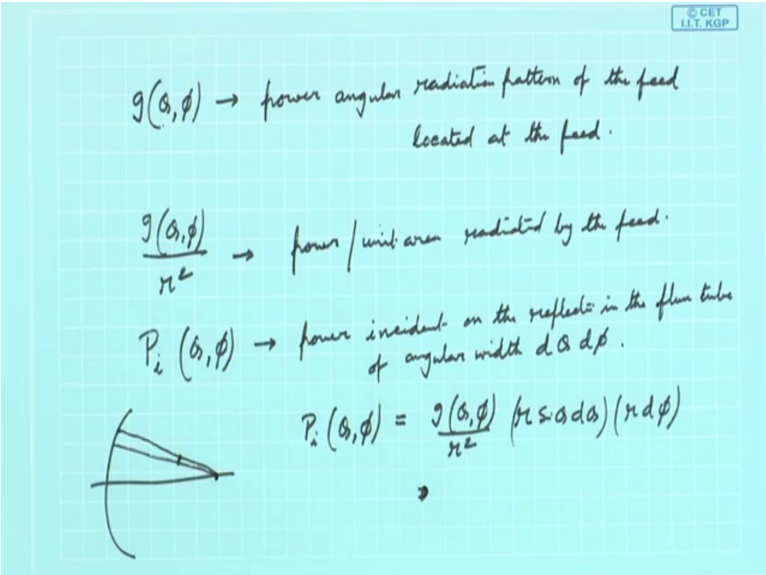

So, can I say that g theta phi by r square is the power per unit area radiated by the feed. Now, the power incident on the reflector, power incident on the reflector in the flux tube of angular width d theta d phi, is what. You see that I am saying that we have these now, this is d theta and in the azimuthal plane there will be d phi. So, in that tube how much power is there that is called P i theta phi. So, that I can easily calculate because I know these.

So, P i theta phi is g theta phi by r square into the area in that angular steep and can I say that what will be that area: one will be r sin theta d theta, another will be r d phi ok. So, this is then got actually you will see that this r will get cancelled. Now, this same power from the log of optics the same power must appear in the reflected flux tube. And, for the reflected flux tube you see that so, suppose this d theta d phi this power has gone here, this is that tube incident tube. Now, while it is reflected you see I have something like it will occupy the portion, if I call that the distance from the centre these variable is rho this becomes, a cylindrical type of thing.

So, can I say that d rho d phi that will tell me a because, now no more theta is there because this has now become a straight rays. So, in d rho d phi direction in d rho in an steep consisting of d rho d phi its variation of these two parameters, whatever power is there that should be equal. So, I can say that the same power must appear in the reflected flux tube of width d rho on the aperture surface. So, I can say that this should be equal to and for that suppose it produces a power density per unit area in the aperture surface.

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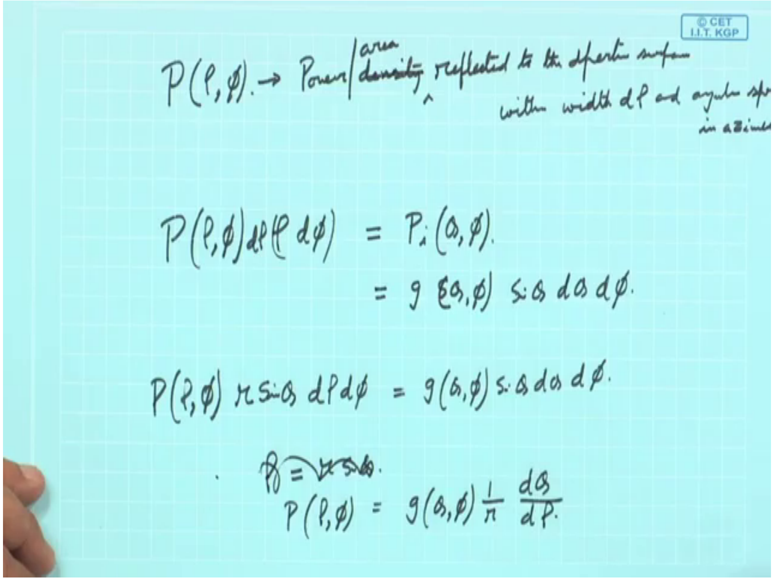

It will produce a power density per aperture area in the surface and let me call that or let me change it that let me call this P rho phi, is the power density reflected to power density or power per unit area. Let me say power per area reflected to the aperture surface within width d rho and angular spread in azimuth d phi. So, I can say that P rho phi then rho d phi and d rho this is the total power, that should be equal to that P i theta phi.

So, I can say that this P i theta phi already I have shown that it is g theta phi sin theta d theta d phi. So, let me write P rho phi and in place of this rho to cancel out I write rho is what, from the figure you can see it is nothing, but r sin theta. So, r sin theta d rho d phi is g theta phi sin theta d theta d phi that solve my problem. So, I can say that also since rho is equal to r sin theta [FL] so, let me simplify this first. So, this is P rho phi will be g theta phi 1 by r d theta d rho ok. Now, this d rho d theta needs to be evaluated, I know what is the relation between rho and theta.

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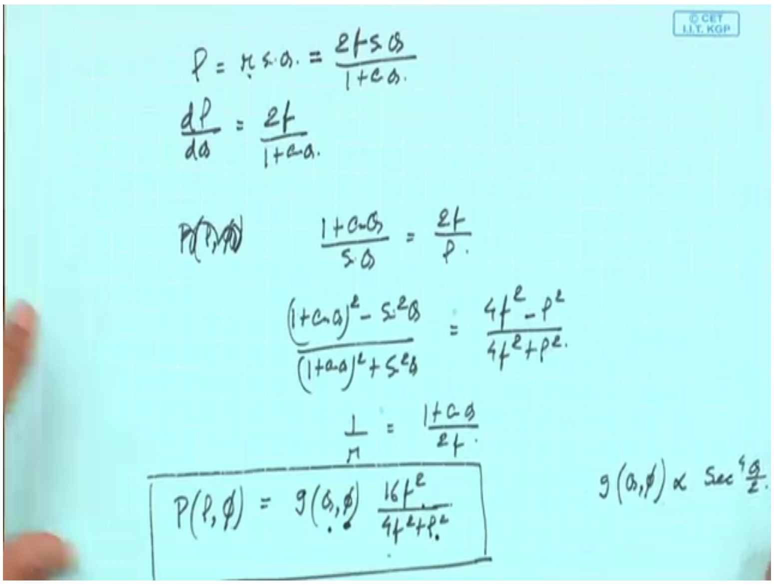

So, I have rho is equal to r sin theta. So, d rho d theta will be 2 f also, r sin theta means I can put r value I can put 2 f. So, that it become constant otherwise r is a variable, but 2 f sin theta by 1 plus cos theta. Now, you differentiate with respect to these that becomes 2 f by 1 plus cos theta. So, now you put it they are so, get P rho phi [FL] ok, you need to have something 2 f 1 plus cos theta. Now, some more algebraic manipulation so, I can say 1 plus cos theta by sin theta is 2 f by rho. So, you can write 1 plus cos theta whole square minus sin square theta by 1 plus cos theta whole square plus sin square theta will be 4 f square minus rho square by 4 f square plus rho square also, we have that 1 by r is 1 plus cos theta plus 2 f so all that. If we put finally, I can write P rho phi is g theta phi 16 f square by 4 f square plus rho square.

So, this is the expression I was finding out that what is the power density at the aperture, I know g theta phi everything else you see that is constant. So, at a particular rho if you ask me that what is the power density putting that value I can tell you; that means, at all points on the power aperture, which is a circular disc I now know the power density. So, my rho varies from 0 to a or 0 to or you can say plus a to minus a. So, I put and I get so, now I know the power distribution on the antenna.

Now, from here we will take certain things that you have seen that it is always better or we have not seen that, but please note actually due to lack of time we did not do that. If an aperture is illuminated uniformly that gives us the maximum radiation; that means, the radiation is in the broad side direction and peaks. Actually, from array theory concept you can see that when we uniformly illuminate the array elements then also you get a peak; in other cases you do not get because all are consecutively added. The same thing in a distributed fashion that if you uniformly illuminate with same phase distribution, same amplitude then you get very peaky thing along the broad side.

So, that is always desirable because that gives you maximum directivity thing so, always that is a dream. So, if I want to make P theta rho phi uniform then what is my requirement on g theta phi, from here you can see that g theta phi should be proportional to this sec 4 theta by 2. How just you put it bring this angle theta because, this things actually this relation that 1 plus cos theta by 2 f. Actually, by putting angle or you can say that see the other way that g theta phi suppose a thing the feed is uniform. The feed giving you uniform radiation then P rho phi will be proportional to this 16 f square by 4 phi square f square which is nothing, but 1 plus cos theta whole square, we will see that.

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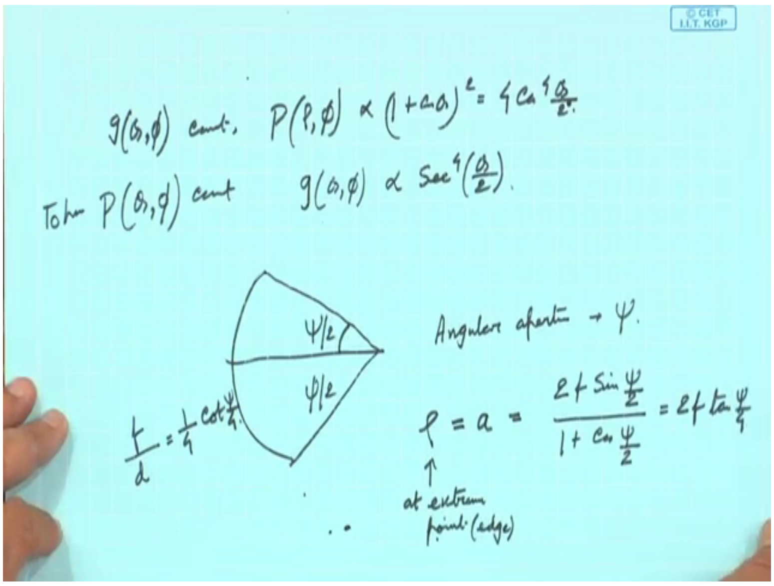

So, that is so, I will rather write otherwise you that if g theta phi constant then P rho phi from this expression becomes, 1 plus cos theta whole square which is 4 cos 4 theta by 2. So, that says that the aperture is not getting uniformly illuminated. So, there is a distribution and where is the maximum aperture field that is obvious a power that is; obviously, at the theta is equal to 0.

So, compared to the centre the edges of the aperture; that means, here I get maximum radiation and here I get minimum radiation. So, this in reflected term people call it there is a taper in the amplitude, if we have these. Now, this is not desirable rather we the other thing is desirable we want to have P theta phi constant. So, your g theta phi; that means, the feed pattern that should be proportional to sec 4 theta by 2.

Now, what do you mean by that g theta phi sec 4 theta by 2; that means, for theta is equal to 0; that means, at the central location the sec theta is 1. And, for other places that means, at the edge that is very high. Now, that is what will happen that actually; that means, what we are saying that it is having very high power in this 90 degree directions. So, and very small power in this that means, minimum here radiating and so, we will see that that we will give that many power will be lost, the reflector would not be able to intercept that power that is called spill over loss; we will come later to spill over loss. So, it is not very desirable.

So, that mean this is optimum, but this is not desirable and practically we cannot do that because, that will give rise to high spill over loss. So, it will definitely give us very high directivity. This choice if I make I will get very high directivity, but the efficiencies they will kill that directly. So, that is why we will see that this is not done and also here I will say that what is the angle because, you see that though I can make this paraboloid reflector come to this phi by 2, but we do not do that because then the our aperture becomes very small. So, up to certain angle it is there.

So, that is called angular aperture that means, if I have a paraboloid so, this angle is called psi or psi by 2. So, this angle is also psi by 2 so; that means, angular aperture of the paraboloid is psi. So, at that point what is the value of rho? Rho is a so, rho at extreme point or edge; at extreme point or edge of the paraboloid rho is definitely equal to their a. And, that is 2 f sin psi by 2 by 1 plus cos psi by 2 the paraboloidal equation. So, this is nothing, but 2 f tan phi by 4.

So, I said that basically you see this psi is governing the whole thing. So, this thing people also sometimes this whole thing is specified by specifying the f by d, d means 2 f. So, f by 2 a is what? 1 by 4 cot psi by 4. So, sometimes if the angular aperture is specified f by d ratio is specified or if f by d ratio is specified, I can find what is the angular aperture of the paraboloid. Now, here I will say that ultimately we will see that the amplitude taper what is the this that amplitude taper in the aperture field; these reduces the directivity because, this increases the beam width. So, reduces directivity.

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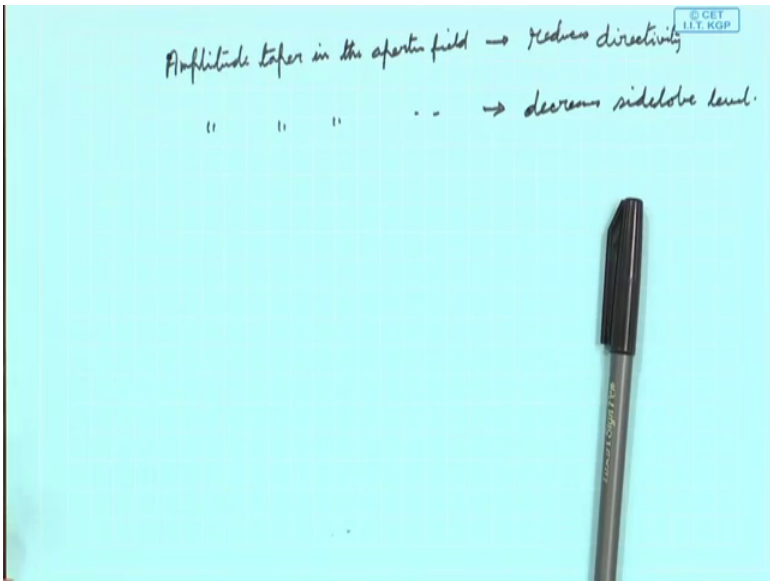

So, that is not desirable, but amplitude taper in the aperture field also decreases the side lobe level this we will show later. So, now this is the problem, this is the designers problem that how much amplitude taper I will have to give. Because one side it reduces directivity, another side is puts the sidelobe level down. Now, in a communication problem generally this side lobe level reduction is more important; that means, if you have an interferer nearby then you want to put a side lobe to that place that is more important. Because, you are not so, concern particularly in communication the directivity or SNR is sufficient.

So, directivity is not a concern you can suffer a low gain also. So, that is proven, but in radar type of applications there the directivity is prime thing and generally they do not consider, they do not bother about the interference from others. So, there actually this is the thing that what is the optimum value of the taper so, that I can get maximum thing. But this thing qualitatively remember always because, this is the dilemma in which a designer always sees so, this is important.

I now that compromises actually our whole study of engineering. Actually, the next lectures will be actually how an engineer makes that compromise between these. So, I stop it here. In the next lecture we will see now, we have got the aperture field we can immediately find out what is the directivity of this antenna.

Thank you.