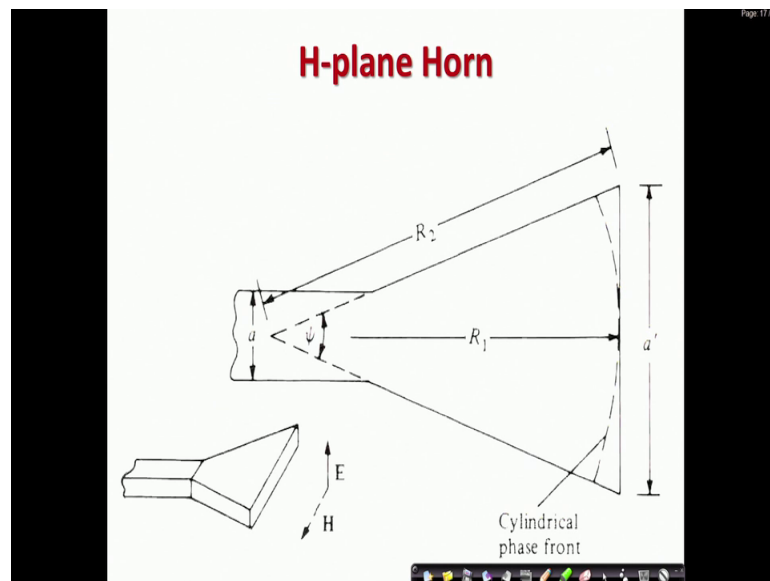


Analysis and Design Principles of Microwave Antennas
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Lecture – 24
Horn Antenna

Welcome to this NPTEL lecture on Horn Antenna.

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We have seen in the previous lecture, the open ended waveguide and found that its directivity is 3.5, which may be a bit better than a dipole, but it is not sufficient. So, we want high directivity. So, for that the idea is that if the aperture is flared, then we can get a much larger aperture opening. So, our area of the aperture in terms of λ will be higher and so, we may be able to get higher gain.

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H-plane Horn

$$k_0(R_2 - R_1) \leq \frac{\pi}{4}$$

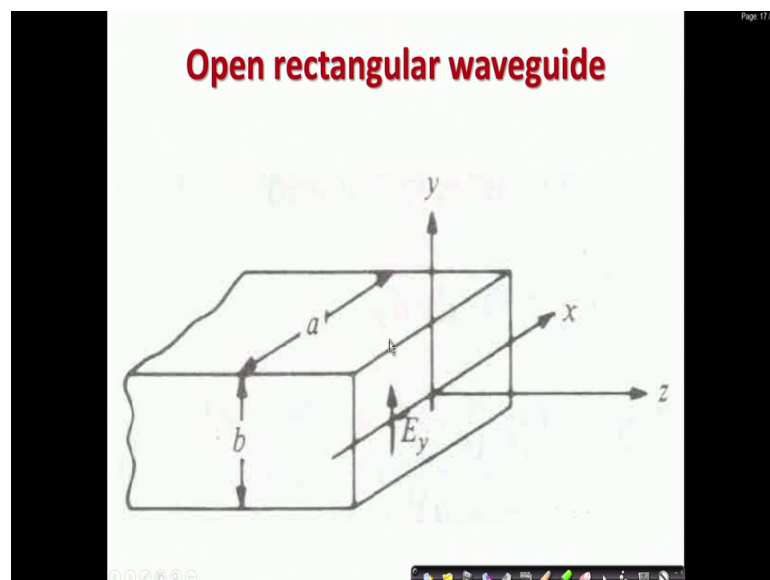
$$R_1 = R_2 \cos \frac{\psi}{2}$$

$$a' = 2R_2 \sin \frac{\psi}{2}$$

$$\tan \frac{\psi}{4} \leq \frac{\lambda_0}{4a'}$$

So, the first of that type is called a horn, which is a H-plane Horn. You can see the view the picture that H-plane Horn, actually the this was the waveguide. Now, we have seen that the wave H field that is in this plane. So, the ff this is the I can say H-plane of the waveguide or if we go back to the rectangular waveguide that will be better.

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So, you know that the electric field is in this direction. So, magnetic field is in this plane this direction. So; that means, we can say that xz plane, that is the H-plane of the rectangular structures. So, if that gets paired so, if you see that this one is getting paired in a horn, this is called H-plane Horn.

So, if you see the side view this is the top view if you see the side view. So, the rectangular wave guide. So, it is broader dimension a now suddenly from here it is getting flared. So, that flaring is done by an angle. And, if you project these 2 lines they meet at a hypothetical point, we will call this apex of the horn this point. So, in this point the flaring is making an angle ψ the flare angle. So, this distance from this apex to the this end that we are calling R_2 and if we go straight from this apex to the center of the horn aperture, that we are calling R_1 .

Now, actually cylindrical waveguide I sorry, the rectangular waveguide was bringing T_{10} mode. Now suddenly when it starts getting flared a line sources is created here. So, a cylindrical wave front is radiated inside this from this flaring start point to the flaring end point. So, here is a cylindrical phase horn with a circular phase distribution. So, the point is; obviously, here you see that at the aperture the phase at this extreme point is different from the phase at the center.

So, there is a phase error here, because the idea of any aperture illumination is that, if we can at illuminate the aperture with an uniform phase, then we get the maximum radiation directivity is maximum at the both side point. So, here this thing will restrict or reduce the directivity, but we are getting much larger aperture. So, these 2 effect actually counter poses, the larger aperture makes the directivity mode and this error tries to minimize that.

Now, actually there is a restriction that you should not flare very much that this phase error that you are committing here; that means, this much phase error, that should not go beyond a point. Typically for H-plane Horns that restriction is the phase error should not exceed plus minus $\pi/4$ at the sides of the aperture. That means, phase error from this point to this point; that means, that should not exceed $\pi/4$, similarly here also it should not exceed $\pi/4$. So, that is the restriction.

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So, here I show you a real H-plane Horn, you see this was the rectangular waveguide gets connected here. So, this side is the rectangular waveguide structure, it is a rectangular waveguide, it is a rectangular waveguide and this you are saying that H-plane is getting flared. The electric field is this directed; electric field is this directed, the magnetic field is going inside. So, the this plane this xz plane is getting flared and you see from here it is getting a flare of this. Apex is something here, if I extend this point and this point I get an apex here. So, with respect to that apex we are now discussing. So, looking into this figure we can say that we can easily calculate, what is the phase error? And, what is the condition that that phase error should be less than $\pi/4$.

So, we can write that the phase error will be $k \sqrt{R_2^2 - R_1^2}$ should be less than equal to $\pi/4$. Now, what is R_1 I will try to introduce the flare angle side. So, R_1 is nothing, but $R_2 \cos \psi/2$ and what is a dashed, a dashed is a dash means a total broad dimension of the on aperture that is $2 R_2 \sin \psi/2$. So, now, you can easily find out that what is the angle that I need? So, if you put that this thing should be less than the by 4 that gives you the $\tan \psi/4$, should be less than equal to $\lambda/4$ a dash.

So, maximum value of ψ is limited, because if you do not do that then you disturb the phase distribution at the aperture. So, it is seen that maximum value of the flare angle, that is inversely proportional to the horns dimension and size. So, actually in order to have a la we said that our main motivation for coming to horn is we want a large aperture.

So, if we want to have a large aperture opening the flare angle will be small resulting in a very long and bulky horn. So, this is the disadvantage of horns, that they become very bulky. So, that is why with the horn we can produce only a modest gain. Actually, if you want to produce higher very higher gains horn is not used, then a reflector antenna is used to have more gain that is why you see satellite people or radio military people ultimately they use a dish antenna for that, this reflector for that, because horn it will be very bulky if you want to have that whole this gain by the horn. Also, it is size etcetera and it is width because this is fully a metallic structure.

So, that is why generally for practical horns we do not have the aperture sizes more than 10 lambda. So, even for 10 lambda thing the horn will be close to 100 wavelength long. Um. Now we can say that, when the case variation is restricted in this manner the um so, that we can neglect the phase variation in aperture field, the aperture field may be assumed to the same as that for dominant T 1 0 mode fitting the horn.

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$$\vec{E}_a = \hat{a}_y E_0 \cos \frac{\pi x}{a'} \quad \left. \begin{array}{l} |x| \leq \frac{a'}{2} \\ |y| \leq \frac{b}{2} \end{array} \right\}$$

$$E_\theta = \frac{j k_0 a' b E_0}{4\pi} e^{-j k_0 r} \sin \phi \operatorname{sinc}(v) \frac{\cos u}{\left(\frac{\pi}{2}\right)^2 - u^2}$$

$$U = \left(\frac{k_0 b}{2}\right) \sin \theta \sin \phi$$

$$u = \left(\frac{k_0 a'}{2}\right) \sin \theta \cos \phi$$

$$\beta = \left(k_0^2 - \frac{\pi^2}{a'^2}\right)^{1/2}$$

$$\beta \approx k_0$$

So, we can write that, we can write the aperture field that will be $a y E_0 \cos \pi x$ by a dash, please remember this is a dash here this is for x less than equal to a dash by 2 and y less than equal to b by 2. So, we can easily find out that is what will be the far field, in this case there would not be any ϕ component, because we do not have any x component in the field that we have already seen in case of open ended waveguide. So, E_θ will be $j k_0 a$ dash $b E_0$ by 4 r e to the power minus $j k_0 r$ sin phi sine v cos u by

π by 2 whole square minus u square. Whereas, before v is equal to $k_0 b$ by 2 $\sin \theta$ $\sin \phi$ and u is equal to $k_0 a$ dashed by 2 $\sin \theta \cos \phi$. And, β where is β is a 0 square minus π square by a dash square whole to the power half. So, you can see from here since a dash is large. So, β is almost equal to k_0 and d the directivity of the horn.

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The image shows handwritten mathematical equations on a light blue grid background. At the top, the equation is $D = \frac{64 a' b}{k_0 \lambda_0^2} = 10.2 \frac{a' b}{\lambda_0^2}$. Below this, it says $0.75\pi = \frac{3}{4}\pi$. At the bottom, the equation is $D' = \frac{10.2}{1.3} \frac{a' b}{\lambda_0^2}$. An arrow points from the $\frac{3}{4}\pi$ term to the denominator 1.3 in the D' equation. A black pen is visible on the right side of the grid.

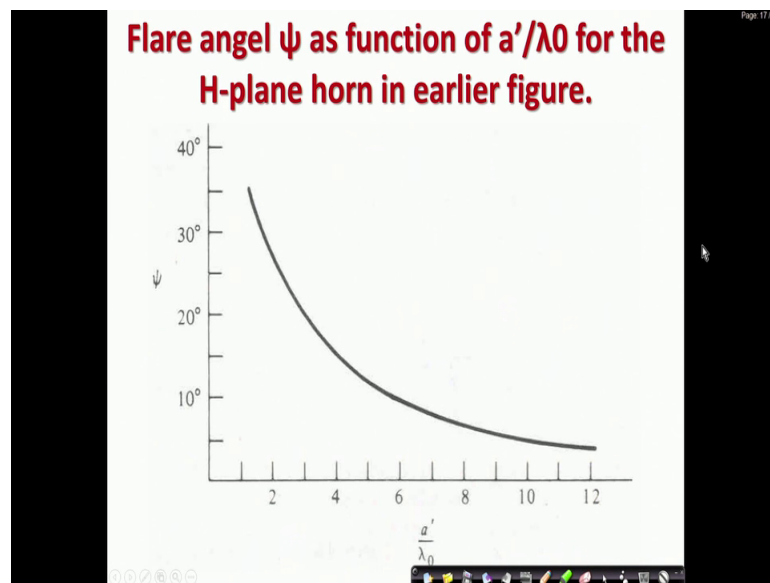
D is 64 this is I am writing from open ended waveguide, only a is replaced by a dash. So, a dash b by $k_0 \lambda_0$ square so, this with putting this k_0 value etcetera, you get 10.2 a dash b by λ_0 square. So, this is your physical area electrical effective electrical physical area. So, actually directivity is 10 times that, you are getting by this one. And so, you remember that in microwave frequency is the gain of horn is essentially equal to the directivity as losses are negligible.

Now, if the horn length is fixed; that means, if this length is fixed, but I want to have more higher gain, than it can be obtained by increasing the flare angle to some extent, that will cause that π by 4 restriction on the phase error, but with the increase in the aperture, because if I increase the flare angle aperture will increase.

So, somewhat the gain that I get more, that somehow compensates the loss in the phase error. So, people have analyzed this and found that, it is possible to get the or the maximum gain is obtained by increasing the aperture with a dashed until a phase error of 0.75 π ; that means, 3 by 4 π , that time we are saying π by 4 now we are saying that 3 by 4 π up to that you can go.

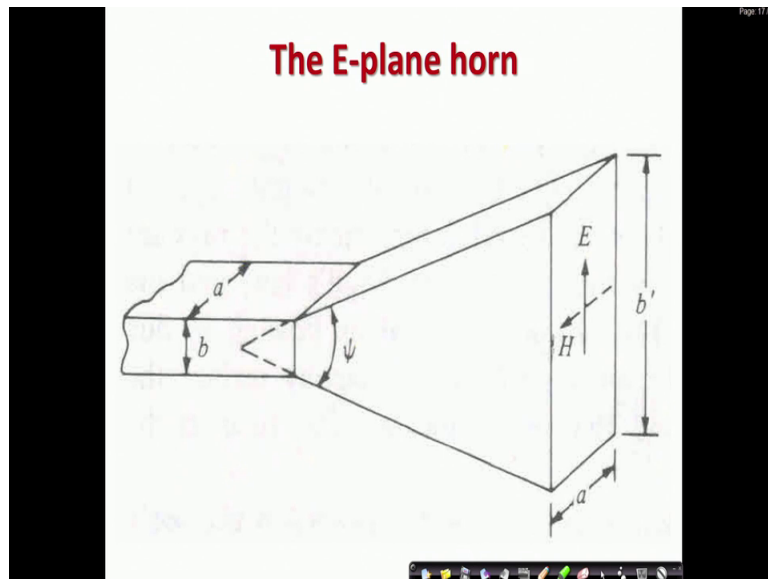
So, the gain that will be from the formula, now gain will be reduced, but that reduction will be the I can say that new gain will be for the H-plane horn will be 10.2 divided by 1.3. Actually, this phase error makes the gain reduced by a factor of 1.3 a dash b by lambda 0 square. Then, you can say why we are choosing, because I am increasing the a dash. So, by that the maximum gain is obtained. So, this is the formula that is generally used. So, if I go up to this 3 point like where 0.7 5 pi ok.

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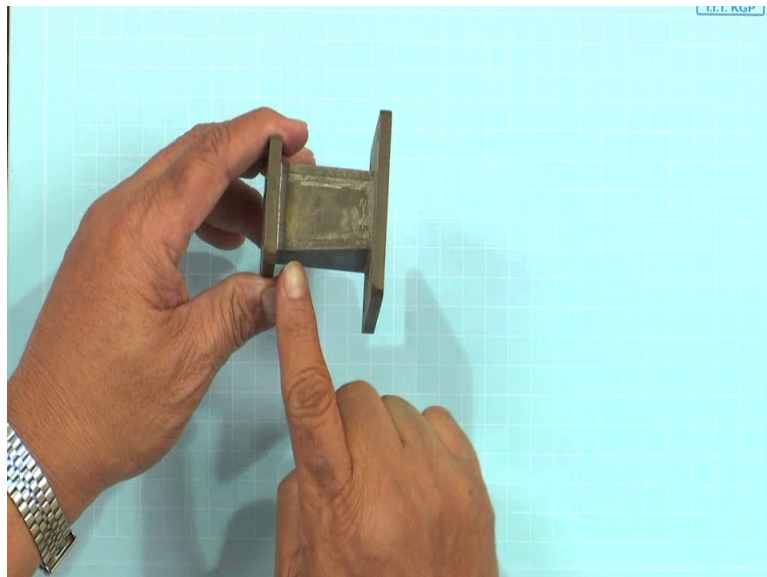
So, this is the H-plane horn, the next one we see is at this is a graph you can see that how flare angle changes the a dash there is a point. So, if I go on increasing the flare angle, then you get that a dashed gets increased, but it is better that you have the larger ones, then flare angle should be kept at a optimum value though, the next one is our E-plane horn.

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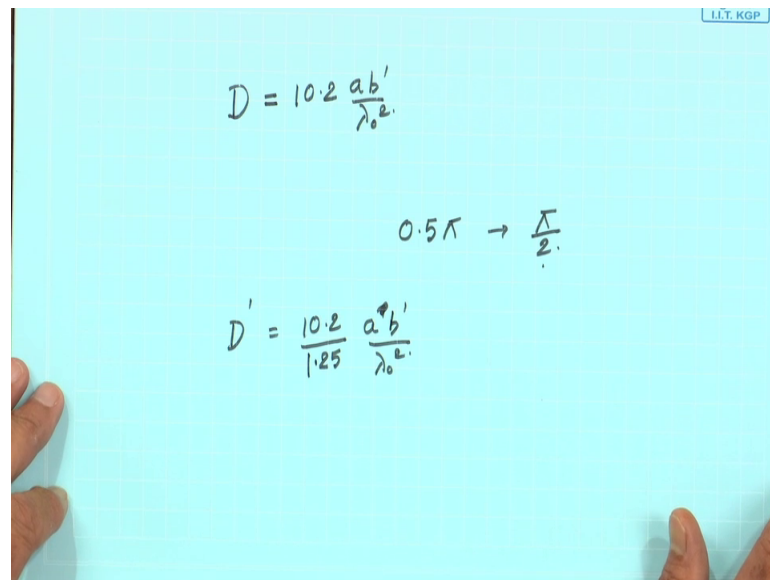
So, let us see the E-plane horn that you see the graph here, the E-plane the electric field is this directed; that means this y direction. So, flaring is done in the y, here it is shown that E and H. So, E is here. So, flaring is here; that means, here a is kept constant the broad dimension the narrow dimension b, that from b it gets there to b dash ok.

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You can see this is an example of a E-plane horn you can see this is the rectangular waveguide side this is the other side, you see from the side if you see that E-plane is getting flare or from the side you see that this is the E-plane. So, E-plane is getting flare the other one is kept constant. So, this is a example of a E-Plane horn. Now, the same consideration as we have done there applies here.

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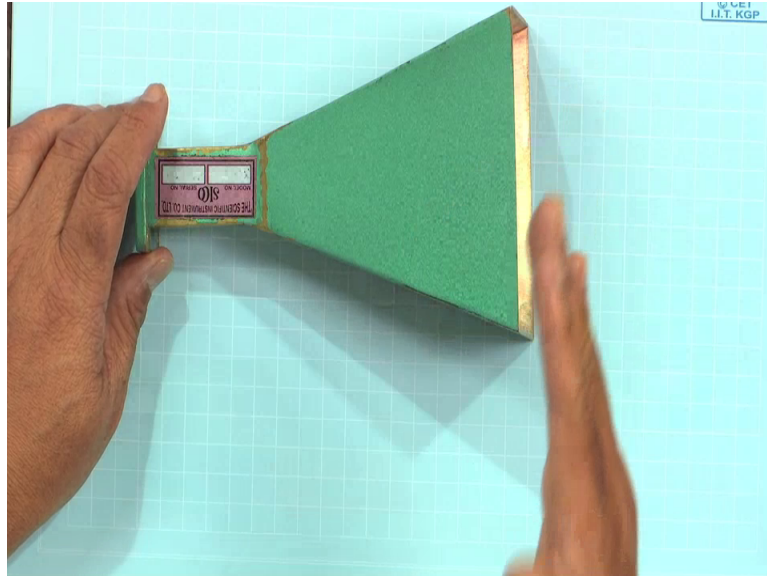


The image shows a hand-drawn slide with a light blue grid background. At the top right, there is a small logo that reads 'I.I.T. KGP'. The main content consists of three mathematical expressions written in black ink. The first expression is $D = 10.2 \frac{ab'}{\lambda_0^2}$. Below it, there is an arrow pointing from 0.5π to $\frac{\pi}{2}$. The second expression is $D' = \frac{10.2}{1.25} \frac{ab'}{\lambda_0^2}$. The expressions are written in a clear, legible hand.

So, here the formula for directivity is $10.2 \frac{ab'}{\lambda_0^2}$. Again people have found that if the horn length is fixed, I want to increase gain somehow then I can violate that, π by 4 thing and bow up to here we I cannot go up to that 3 by 4 π or 0.75 π . Here we can go only up to 0.5 π ; that means, π by 2 only, instead of π by 4 phase error I can go up to π by 4 . So, why it is so? Why in each plane I can go up to 3 by 4 π and in E-Plane, I can go only up to π by 2 , the reason is the in the each aperture field for a rectangular waveguide, that goes to 0 at the sides of the aperture in the H-plane.

Whereas, in the E-Plane the aperture field is constant so, that is why here you cannot go beyond a point, because then you will disturb much, but there you have a chance because at the ends it is going to 0 . So, you have more liberty in the H-plane things ok. And, here also if you do this how much reduction the gain will suffer? That is so, since you are going less you will suffer less. So, 10.2 here instead of 1.3 it is 1.25 a dashed sorry a b dash by λ_0^2 square ok.

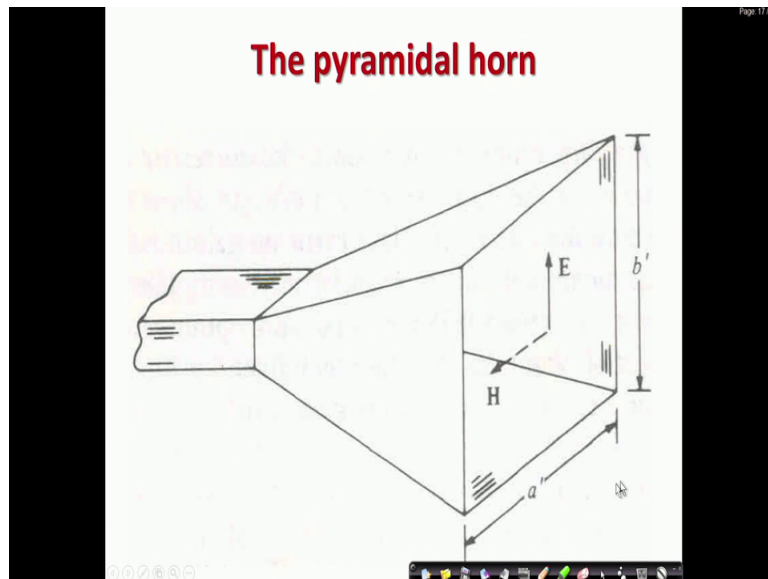
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So, this is the E-Plane horn. Then people thought that to; obviously, a better thing will be that is both E and H-plane things are there like this. This I think all of you have seen many times that this is called pyramidal horn. So, in both direction you see that, this plane also is square this plane also is square this is so, this is the this flaring this bottom one if E sorry this one you will see this flaring is the H-plane flaring this flaring is the E-Plane flaring both are flare.

But remember the very important point, that when it was constructed you see from the top that from the same point when the E-Plane flaring started, the H-plane flaring also started from the same point. This is one of the thing that we will have to keep in mind while designing. So, this is a horn made by sico one of the Ghaziabad companies. So, these are all Indian made things.

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So, you can see that are the you can see the pyramidal horn. So, in both the directions you have flared. So, ultimately your aperture is a dashed by b dash.

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$$\tan \frac{\psi_{E/H}}{4} \leq \frac{\lambda_0}{4a'} \quad \text{or} \quad \frac{\lambda_0}{4b'}$$

$$D = \frac{10.2}{1.3 \times 1.25} \frac{a'b'}{\lambda_0^2}$$

$$= 6.4 \frac{a'b'}{\lambda_0^2}$$

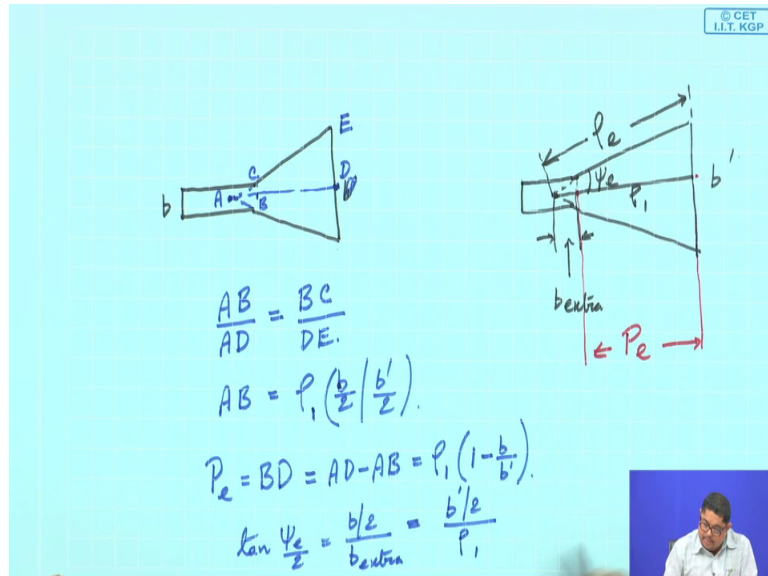
Now, in order to have a negligible phase error in the flare angle in both plane should satisfy the phase error condition, that $\tan \psi_{E/H}$ I am writing to all this to be together satisfied. Obviously, if I satisfy generally a dashed is larger than b dash. So, if I satisfy these this gets automatically satisfied ok. So, the radiated field and directivity, they are also same as E or H-plane horns with a replaced by a dash and b replaced by b dash. So, for a fixed length horn, if I am allowed to do maximum gain thing, then people do that for H-plane they suffer that 0.75 pi phase error and in the E-Plane suffer that. So,

for that what will be the gain of a pyramidal horn, I can write 10.2 , then divided by 1.3 into 1.25 , then a dash b dashed by λ^2 square.

So, these becomes actually this famous formula 6.4 a dash b dash by λ^2 square this is the gain of a horn pyramidal horn. So, pyramidal horns are used for one is the very thing standard gain horn, we will see that in antenna measurement etcetera or various measurements, we want a standard gain. Now, horns horn is one antenna horn dipole etcetera, but horn gives good value of gain. So, horn we know from it is geometry I can tell what is the gain. So, that is why standard gain horns are made always from these pyramidal horns. Because, any gain can be obtained by manipulating the length and so, also for comparison of gain of an unknown antenna, the this gains this pyramidal horns are used, that is why in all antenna laboratories we will see at least the this pyramidal horns, because they are they give standard gains.

Then, small pyramidal horns are used as feed horns, that we will see later that for paraboloidal reflector or the dish antenna. In your homes the dth service there also we will see that there is a dish, but you see that near that there is always sitting a horn antenna. So, that may be a circular horn that may be a rectangular horn, but the there is always a horn. So, that is also another use; that means, in that the paraboloidal reflector system it is called a feed horn. So, as a feed horn also horns are used and also in large array antennas the for feeds etcetera horn is used. Now, since it is so, important particularly for us, we in laboratories this is important. So, I will now spend some time on the design of the horn, because that design is very important.

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So, you see that there will be some geometries, that let us see the horn here you see that first I am taking a flare in the H-plane. So, b oh sorry flare in the E-plane so, b is getting flare to b dashed. So, this is my b dash and let me name it that, let us say the if I the apex let me call it A this point is the apex of the horn A. Now so, the point where the flare is started let me call it B, on this line on this line this point is C and this one is called this point on the aperture let me call D and this point at the top let me call E. So, here you can see easily that I can from geometry I can always write AB by AD is equal to AB by AD you all agree that these are all similar triangles. So, AB by ad is equal to BC by DE I can easily write this.

And so, what is AB? So, AB will be [FL] before that also let me have a another drawing of the same H-plane horn, this is my b dash and this is my apex and we will call this where in rho e and this one from here to here rho 1 ok. Now, actually what I am trying to do I am trying to find from the apex to the flaring point, what is the distance? This one actually I will call these this one is my b extra. So, this is my actually; that means, because these are from outside scene this is an hypothetical point. So, what is this extra point, that I am trying to find. So, I have given the thing the angle let me give this angle is psi E, I am saying because it is an E-Plane horn and I will also say that; that means, the flaring that has taken place for an length P; that means, from here to hear the length is P.

Please remember that rho 1 is from here to apex. So, I can say that rho 1 minus b extra will be P e, because I will have to see that when a horn actually is fabricated the P the physical this length should be equal to in the other plane PH, because that is the

constraint for designing a horn that physically they should be same. That means, this distance in sorry this distance and this distance they should be same otherwise, it cannot be fabricated.

So, I am actually trying to calculate P_e from the given this thing. So, that is why here now I can put what is AB AB is equal to AD means from here you can see ρ_1 into you can see that b by 2 by b dash by 2. So, I can now write that what will be P_e P_e is equal to BD you see BD and; that means, AD minus AB and; that means, ρ_1 1 minus b by b dash. So, now ρ_1 is also generally not given for any horn. So, I will have to relate it with the flare angle. So, now, I can write $\tan \psi$ by 2 is equal to b by 2 by b extra and that is nothing, but b dashed by 2 by ρ_1 .

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$$b_{extra} = \frac{b \rho_1}{b'}$$

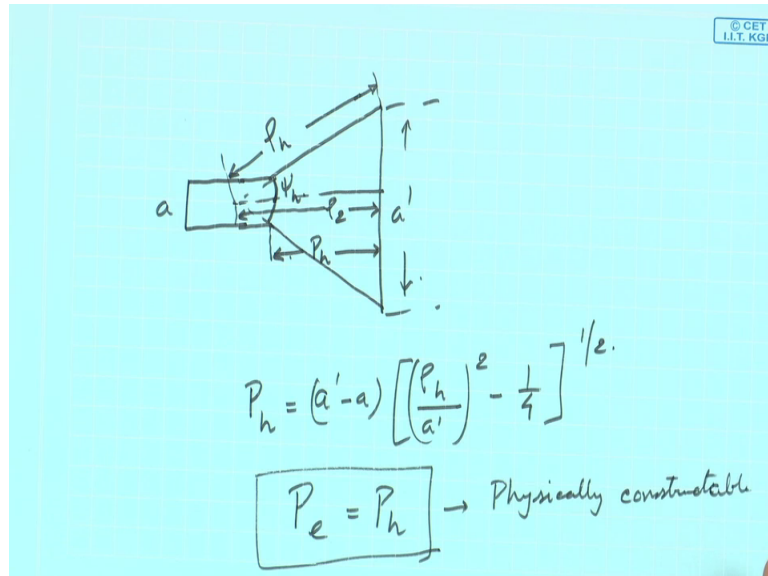
$$P_e = \rho_1 - b_{extra}$$

$$= \frac{(b' - b)}{b'} \sqrt{\rho_e^2 - \frac{(b')^2}{4}}$$

$$P_e = (b' - b) \left[\left(\frac{\rho_e}{b'} \right)^2 - \frac{1}{4} \right]^{1/2}$$

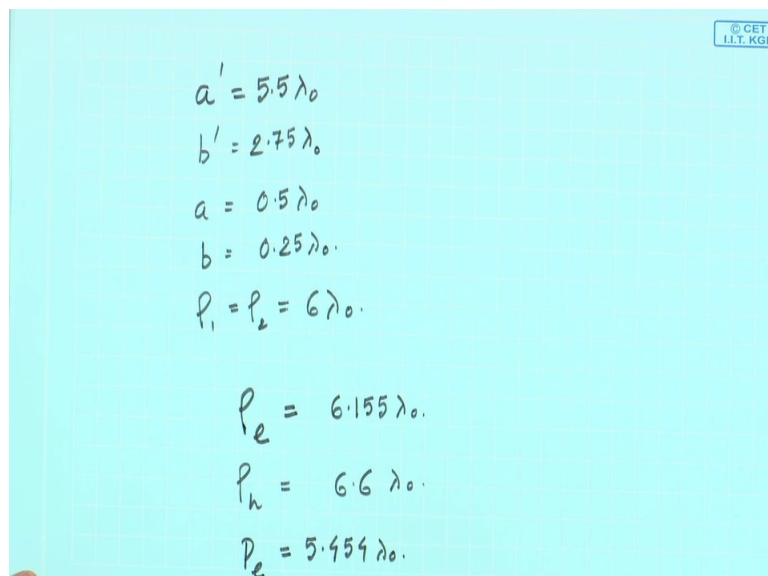
So, from here you can find b extra is b ρ_1 by b dash. So, P_e becomes ρ_1 minus b extra and that, if you do you get b dashed minus b by b dashed over ρ_e square minus b dashed square by 4. So, finally, people write it as b dashed minus b ρ_e by b dash square minus 1 fourth to the power half.

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So, this is ok. Now, I will do what is the same thing for H H-plane horn say there also the diagram is this, I have this. So, this is rho h, this is rho 2 and this is P h and this is this whole thing psi h this is a dashed and this is a. So, again from the similar thing I can say P h will be a dashed minus a rho h by a dashed whole square minus 1 by 4 half. And, to the physically construct a horn we require P e should be equal to P h physically constructible ok.

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So, let us take an example that a pyramidal horn has this dimension a dashed is 5.5 lambda, b dashed is 2.75 lambda, lambda or lambda naught whatever (Refer Time: 31:13) a is 0.5 lambda 0, b is 0.25 lambda 0 rho 1 is equal to rho 2 is equal to 6 lambda

0. Check to see if such a horn can be constructed physically. So, what is the first part? You can directly put the formulas, but it is better that you find out ρ_e and ρ_h first because from the geometry it is easy to understand, what is ρ_h , ρ_h is since ρ_1 ρ_2 is given, which is always it would not be given, but in this case given means. So, I can say that this is the ρ_h is the hypotenuses of this right angle triangle. So, from that I can easily find out ρ_n ρ_e .

So, ρ_e will be we will see that it will be $6.155 \lambda_0$. And, similarly ρ_h will be $6.6 \lambda_0$. So, once you get ρ_e you see the expressions, because since you know b dash ρ_e also you find out, you can easily get P_e from here. Similarly, from the other equation you get 8 check whether they are equal. So, you will see that P_e becomes $5.454 \lambda_0$ and P_h is $5.454 \lambda_0$. So, the horn can be constructed physically. So, this is the first check any designer should make, that suppose this is given these, but I will say that this is a middle part of the design, because no specifier does not all these things. Actual, suppose some other engineer suppose a transmitter system engineer said that you construct a horn of this much gain.

So, in generally unless and until a on a microwave engineer can give you a this type of thing. So, before that actual design is the gain should be specified, you will have to check that what I require? So, that we will take up in the next lecture, that what is the design equation of the horn? Ok.

Thank you.