Analysis and Design Principles of Microwave Antennas
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Lecture – 23
Parasitic Array and Log Periodic Antenna (Contd.)

Welcome to the second lecture. We are still continuing the analysis method of aperture antenna by using Fourier transform.

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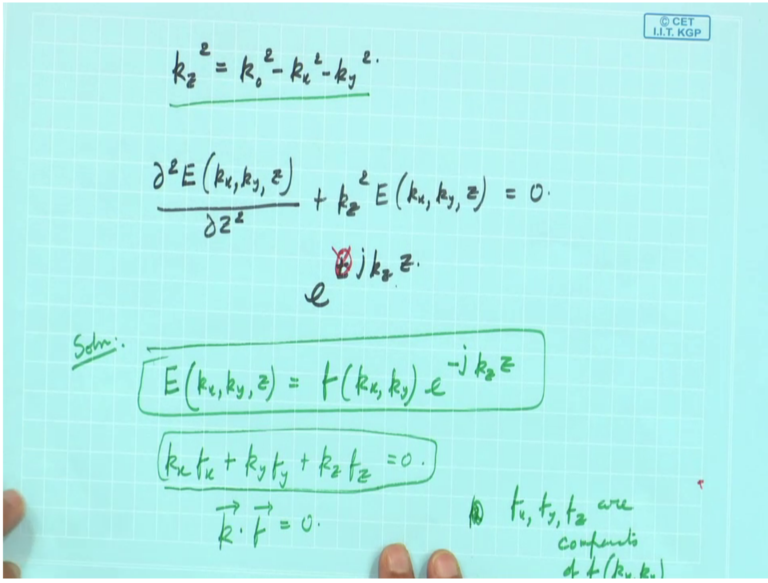

Now, we said in the last concluding part of the last lecture that, till we need to find this f from the solution. So, generally how in a solution, if I get a solution from a differential equation, how do we solve the constant? We put boundary condition. Here, what will be the boundary condition? The boundary condition is, if this is really a solution, let us go back to z is equal to 0 plane and then this solution should give me the aperture field that I have assumed; so, that we will do now, to find the solution here.

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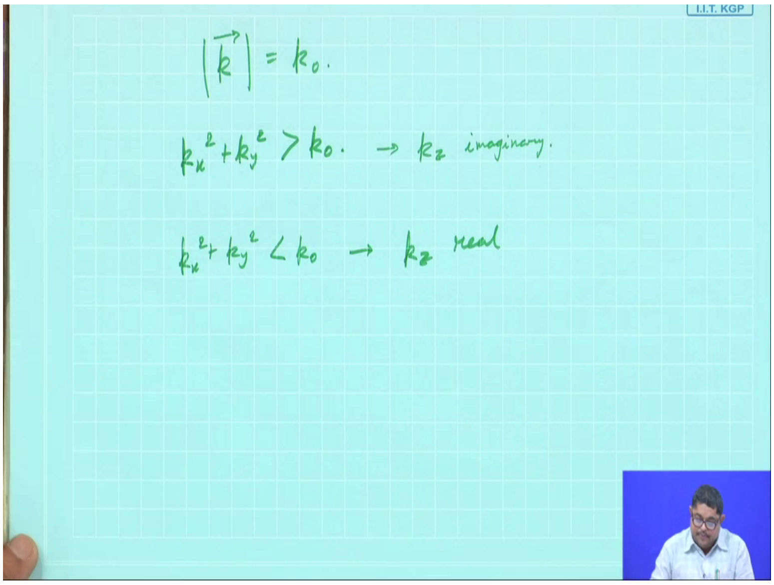

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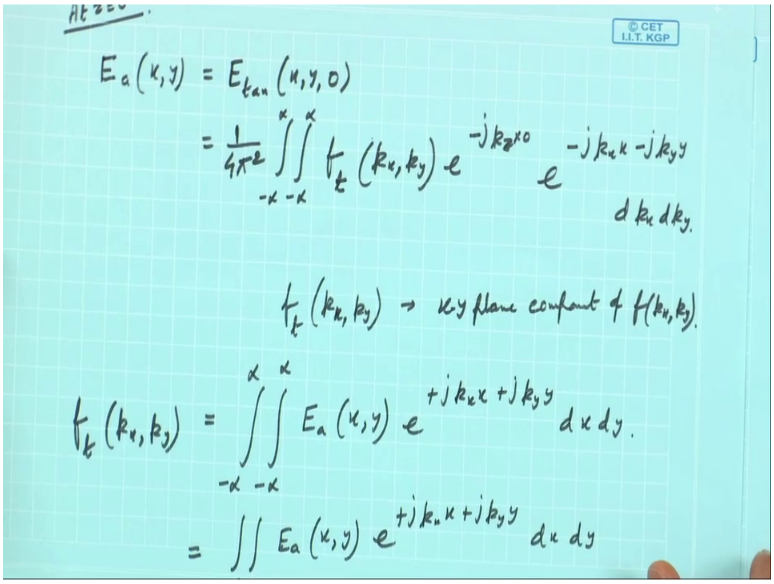

So, what we can say that, we will go to z is equal to 0 and there we know that, we have the tangential field on the aperture as this and this should be equal to; so, I have this solution, I have this solution. Let me put z is equal to 0 here. So, and what is its value? So, I can say that 1 by 4 pi square minus infinity to infinity on, and that a on the aperture, I am calling ft kx ky e to the power minus j kz into 0 e to the power minus j kx x minus j ky y d kx d ky. You say, this was our solution, I have just put z is equal to 0 and only the change is; so, what is ft? ft kx ky is the xy plane component of f kx ky or in, waveguide we will call it transverse component.

Basically, this is nothing, but the transverse component of the f. Now, this relation you see is clearly a 2D Fourier transform relation. So, from our knowledge of Fourier transform, I can now find ft. What is ft kx ky? ft kx ky is equal to E a x y e to the power plus j kx x plus j ky y dx dy and I know that, this aperture field exist only over the aperture, generally, you can say so, but this is the exact expression. From here, I can also write it like this, that it is a say Ea x y e to the power plus j kx x plus j ky y dx dy. This is known to me; so, I can find ft. So, ft is known. Now, it says that f is not known, ft is known ok, but I will say that I have already proved that, they had degree of in freedom is 2.

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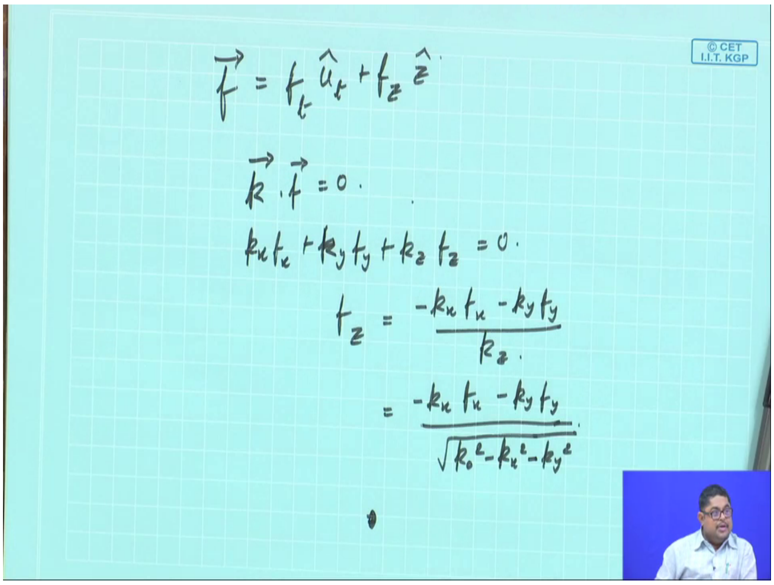

So, I will write f is what; ft ut plus fz z. Already, we have found that k dot f is equal to 0 means, that you can write it in terms of kx fx plus sorry, ky fy plus kz fz is equal to 0. So, from here you can write what is fz; fz is nothing, but minus kx fx minus ky fy by kz is equal to minus kx fx minus ky fy by we have already seen what is kz; k naught square minus kx square minus ky square equal to answer is; so, you see that fz is also known, because we know fx fy we know ft; that means, we know fx fy now, we can find fj. So, we know f. So, we have formally.

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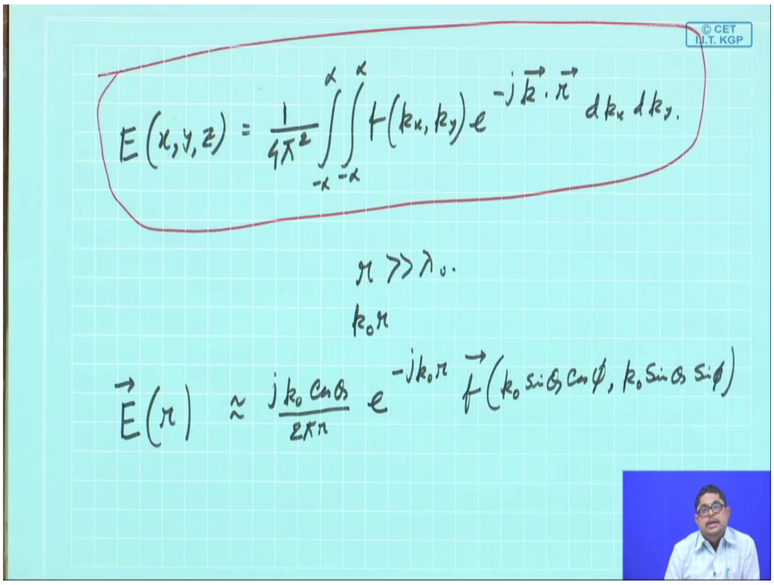

So, now, we can say that what is the field? We will say the field is E x y z anywhere is equal to 1 by 4 pi square ok. So, this is the solution. So, if I know aperture field, I can find f and then I can find the radiated field everywhere. Now, the question is as in case of wire antennas also that can I evaluate this integral? This integral is very difficult to evaluate in general, because this k dot r it is a very rapidly varying an oscillating function. So, generally the, it is not easy to evaluate it, but we are also not interested to know what is the electric field everywhere.

We want to know what is the electric field in the far zone and in the far zone; that means, when r is much-much greater than lambda naught; that means, k naught r is large, there actually, there is a method, which is called stationary phase method. Actually, in that method what happens that if you have an oscillating integral, you can see that generally, the oscilla, the effect of this oscillation, cancels out, but near the stationary points. Stationary points means where the, that function as a maxima or minima those stationary points, there it is not an oscillating thing. So, what it says that you evaluate the integral on those stationary points. Generally, that will be the, asymptotic value of the integral.

So, by applying that people have found that at very large or far fields, the field is given by this actually classes. We derive this where stationary phase method that how this comes, but here r naught if any of you are interested, you can contact me, I will tell, share the notes that how can this be evaluated or these are available in books particularly

R. E. Collins book. It has been shown that how to do that. So, this is ultimately the useful expression the far field of the antenna is given by this, you see in terms of this f, it can be expanded. So, the far field is simply related to the Fourier transform of the aperture field.

Now, in the evaluation of this ft the integrals over x and y are taken over all portions of the z is equal to 0 plane on which non-zero values of the tangential electric field exists, if S a is an opening cut in a perfectly conducting spring then everywhere outside S a will be 0 tangential electric field. So, then, also it has been seen that for an aperture that is large in terms of wave length, which is our case I said that any practical antenna, aperture antenna should be sizable dimension, electrical dimension. So, there it has been seen that, ft is highly peaked in the forward direction means along the z axis.

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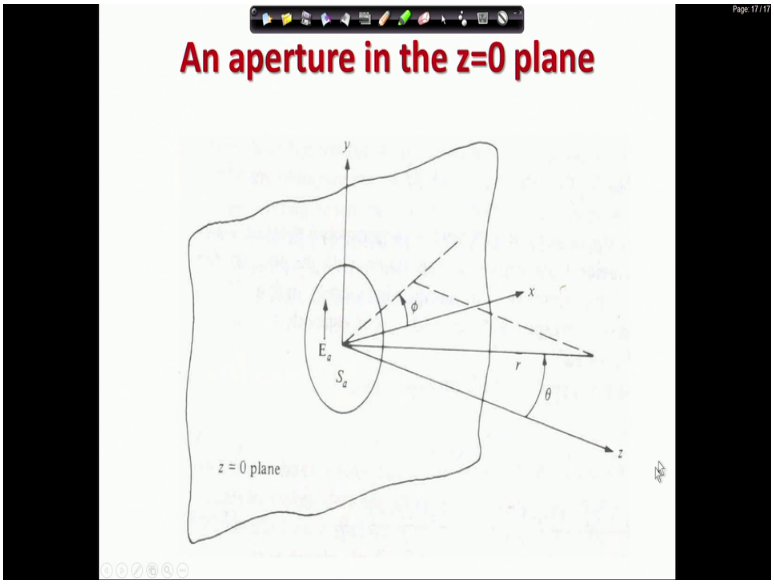

If you look at this diagram, so along this z axis, the ft is highly peaked and you know that since the fx kx fx that formula, this kx fx is ft is highly peaked then fz, because of this is very small. So, in this zones where we are interested, in the far zone and also in the both side direction, we can say that generally, is also here in that zone. This cos theta that will be, their cos theta becomes 1, because theta is 0.

So, here you see the expression, this is 1 and ft is, sizable, fz is that is why very small. So, radiated field can be given nearly by ft in this region and this so; that means, in most of our zone of interest if we are looking both side to the aperture, then this f, we can replace by ft also if not then we will have to do.

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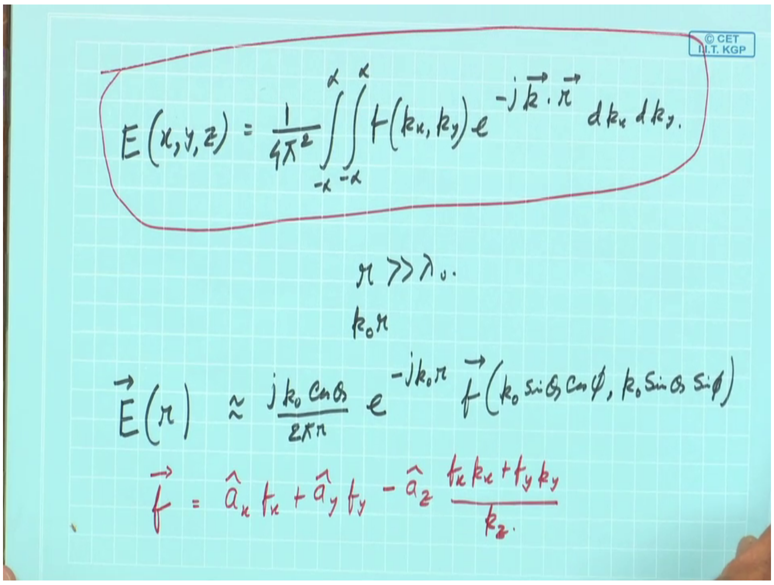

So, I will write better that in general that f can be written as you see ax fx plus ay fy minus az fx kx plus fy ky by kz. This I have already shown that how a fz comes. So, I am putting the value this. So, you see you have all these values. Now, generally in the observation zone, we want the whole thing in terms of spherical coordinates.

So, your job is to now, convert this ax ay and az vector to spherical coordinates; that means, I am giving you the clue that ax ay, this you know spherical to rectangular coordinate transformation just.

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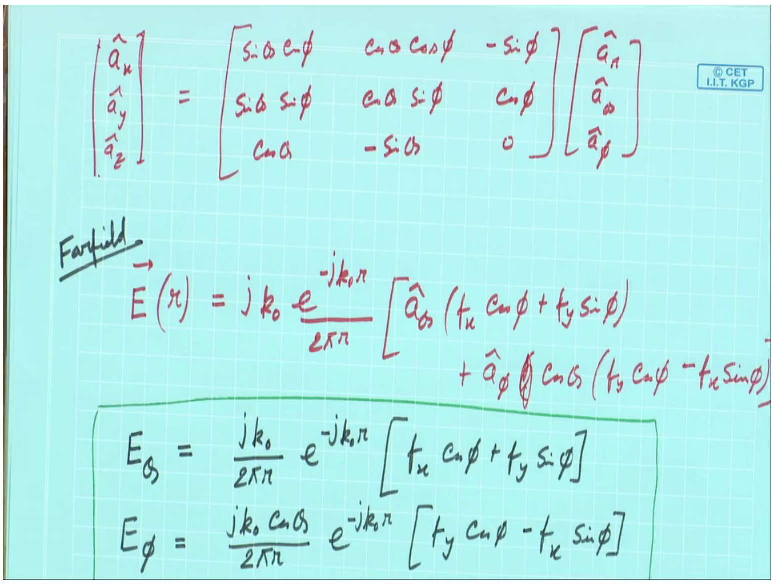

So, with the help of this you can see that the far field can be written as j k o and then we can write in our usual way, what is E theta far; obviously, these are all far field. So, what is E theta for later reference, I am writing it, very-very important. Let us see that E theta and E phi, we got in the far field I can easily get h theta h phi from this. So, here you see that I have this components, another point, I want to emphasize that since, we have this k dot f is equal to 0.

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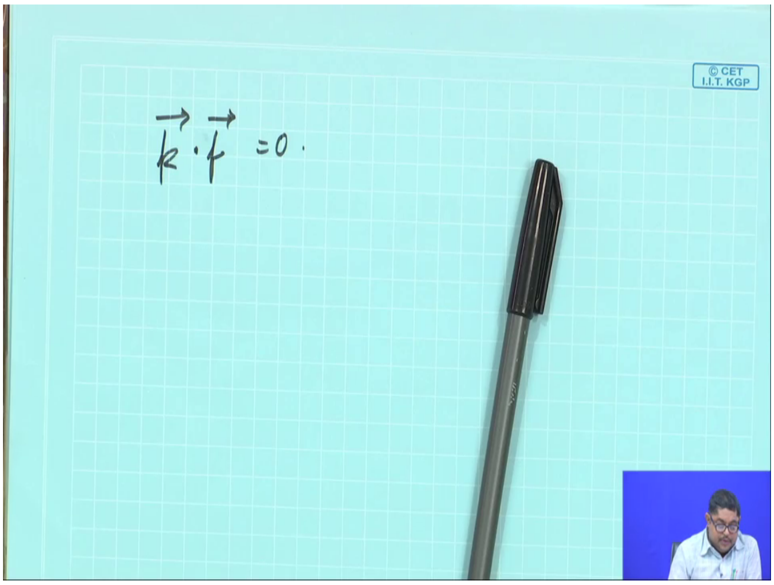

So, I can say that f does not have any component in the direction of propagation vector k. So, and also we have seen that the electric field is directly related to f. So, can I say that the electric field also does not have any component in the direction of the propagation vector.

What is the meaning; that means, in the radiation zone the fields are tmof s. So, basically we have superposition of tmof s in the um, far zone. So, from an aperture when the field is radiated basically, we will get in the far field the superposition of various tmof s that we can see in the spectrum, the special frequency spectrum shows that what tmof s are there ok. So, this concludes the analysis part. So, by Fourier transform then if you can guess the, a think where aperture field, then you can find the radiated field, we will see one by one application of that. The first application will be open ended waveguide.

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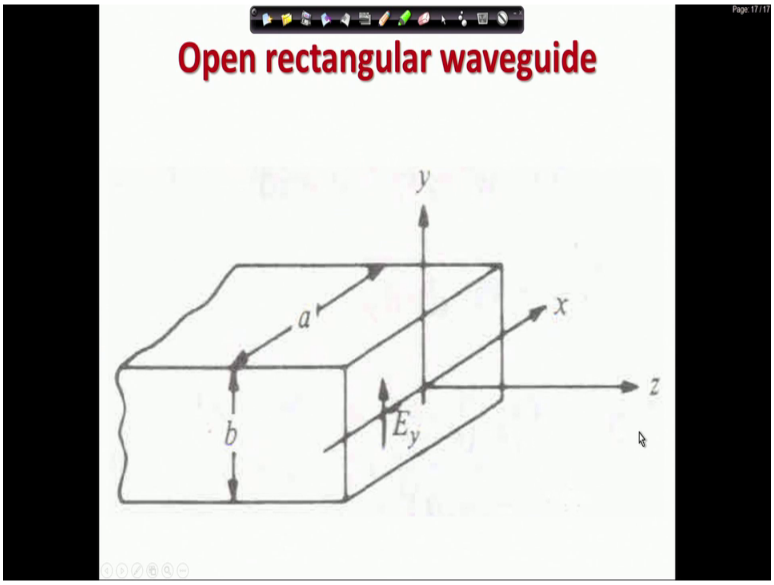

You see this shows an open rectangular waveguide. It is the wave actually, this backside it was a waveguide suddenly, after this at z is equal to 0 nothing is there. So, inside there was E 1 0 mode, dominant mode was propagating and you see the dimensions etcetera shown. So, electric field is y directed electric field does not have any variation in the y direction; electric field has variation in the x direction. Typically, the variation is sinusoidal variation a is the broader dimension of the waveguide, b is the narrower dimension of the waveguide etcetera, etcetera. The aperture is in the z is equal to 0 plane.

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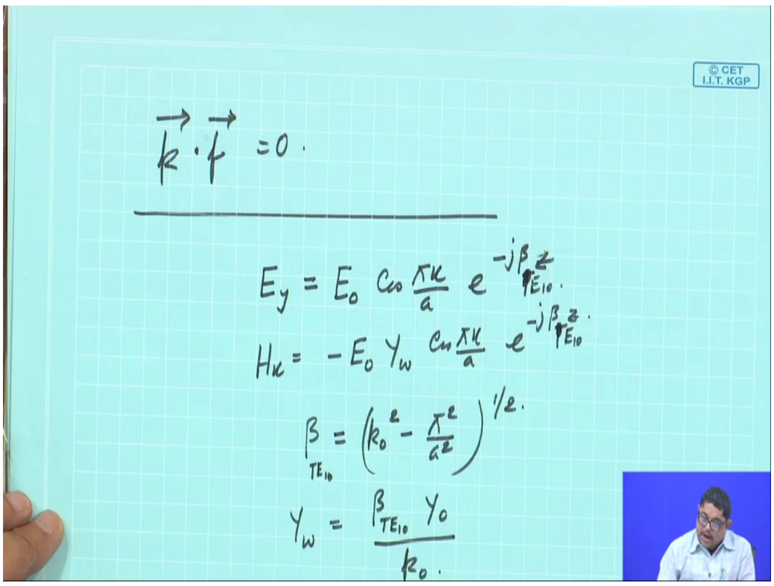

So, I can write the transverse component of the fields as Ey, all of you know this from your waveguide knowledge. I can write according to my uc coordinate, my coordinate is a center of the guide. So, field will be cos pi x by a e to the power j beta naught z and Hx is minus E naught Y w cos pi x by a e to the power minus j beta naught z not beta naught sorry, sorry, sorry. I think it is actually beta TE 1 0.

It should be that I should not call it a actually, this is I will later and. So, what is beta the propagation constant that is k naught square minus pi square by a square whole to the power half ok. This is sometimes where i theta is T 1 0 also. So, you can write here as TE 1 0, TE 1 0 ok. And what is this? This is the wave admittance and wave admittance for this mode is beta TE 1 0 Y naught Y k naught ok.

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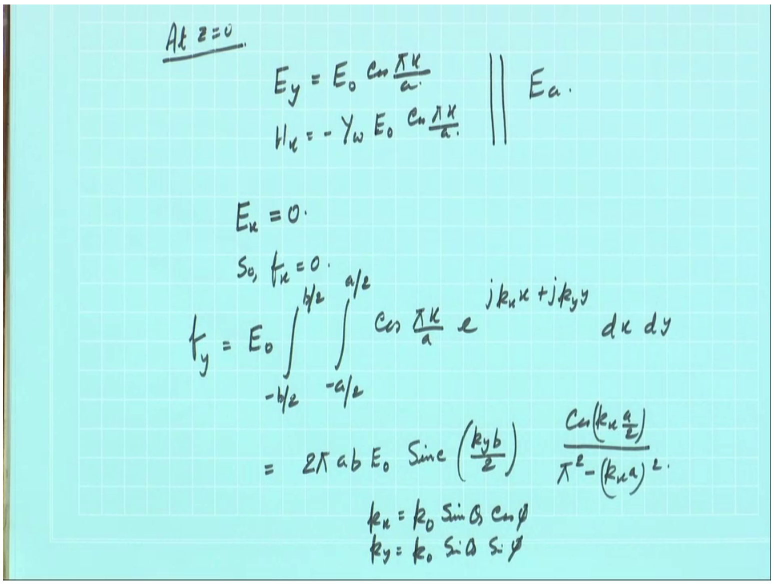

So, at z is equal to 0 what are the fields; because those are my aperture fields. So, at z is equal to 0 the fields are Ey is E naught cos pi x by a and Hx is minus. Now, at this aperture suppose, wave was coming with dominant TE 1 0 mode; so, it was seeing an wave impedance of 1 by y omega. So, y w 1 by that wave admittance suddenly, it sees free space impedance that is 377 ohm.

So, what will happen? There will be immediately a reflection, there will be a reflected TE 1 0 mode wave also, because of this discontinuity there will be some higher order modes will get generated. So, a small amplitude of that are excited near the open end. So, that can be taken into the analysis, but as a very, first cut analysis we can neglect both of these, if we neglect them then we can say that the aperture field is also given by the dominant TE 1 0 field.

I am again repeating that actually aperture field will be equal to the reflected, but the actually the what has incident field plus the reflected field plus the higher order modes, but I can neglect the reflected another part as a first analysis. So, it has been found that this assumption if I made, that the aperture field is as if the same as the dominant mode field, then in the main lobe of the radiation there is no discrepancy, but in the side lobes, there are some discrepancy ok.

So, this is at z is equal to 0, this is the field. So, I say that this is my e a. So, you see that that means my Ex is 0. So, can I say fx will be also 0, fx is the Fourier transform. So, I will have only fy and what is fy? E naught minus b by 2 to plus b by 2 minus a by 2 to plus a by 2 cos pi x by a e to the power j k x plus j ky y dx dy is equal to 2 pi a b E naught sin ky b by 2 cos kx a by 2 by pi square minus kx a whole square, where kx is k naught sin theta cos phi ky is k naught sin theta sin phi ok.

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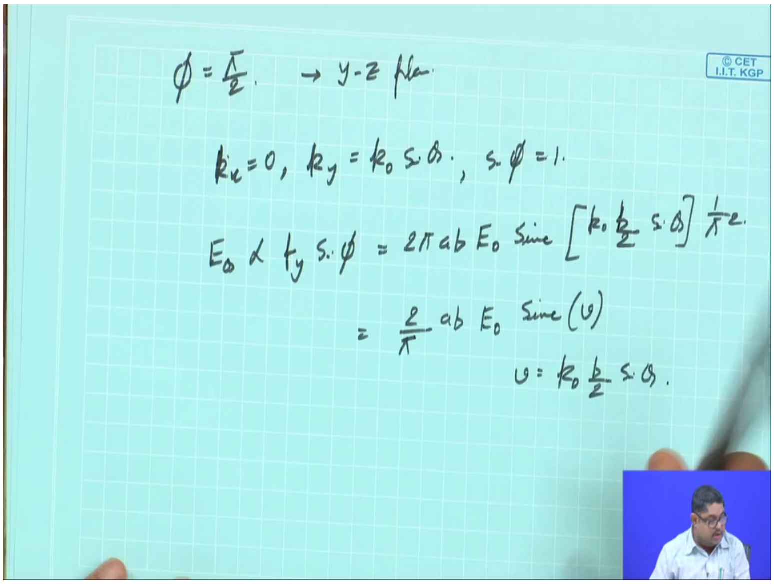

So, in the pi is equal to pi by 2 plane; that means, if you look at the diagram yz plane; that means, in this plane that is and you show that this plane, can you show my hand (Refer Time: 25:36) in the yz plane, the radiated field is given by E theta there will be what will be kx kx is 0 in this for pi is equal to pi by 2 and ky is k naught sin theta and sin phi is equal to 1. So, E theta is proportional to fy sin phi is equal to 2 pi ab E naught sin k naught d by 2 sin theta 1 by pi square is equal to 2 by pi ab E naught sin v where v is equal to k naught b by 2 sin theta.

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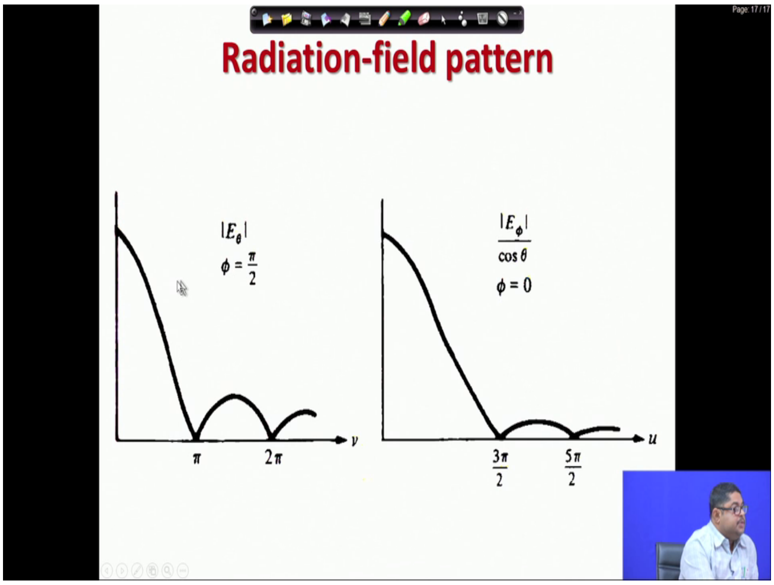

It has been shown here in the, this slept diagram you will see that this is the radiation pattern e theta as a function of v. So, due to actually why it is like this seen you see an interesting thing again you show that it is an interesting thing actually along y direction; that means, yz plane they are the along y direction the aperture elimination is uniform.

So, Fourier transform of that will be what; radiation pattern is sin function that is what it came and in the pi is equal to 0 plane; that means, x z plane you can write kx is k naught sin theta ky is 0 cos phi is 1. So, you can put in our all those solution actually, I am putting all those here, you see these are my solutions here, I am putting and getting if you (Refer Time: 27:59) with that.

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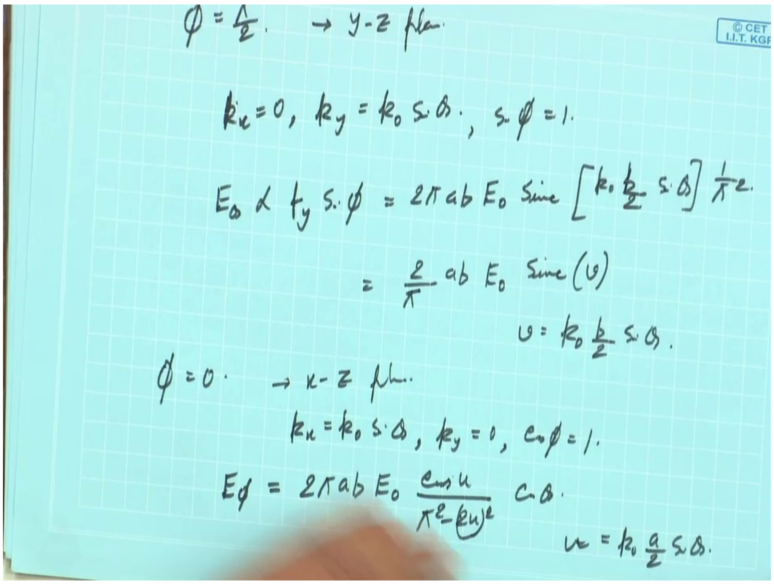

So, here I will get the e phi value to be 2 pi a b E naught cos u by pi square minus 2 u whole square cos theta, where u is k naught a by 2 sin theta. So, that has been put plot here, E phi by cos theta and it shows like this. Here, it was not uniform illumination and so, it is not a pure sin function, but Fourier transforming.

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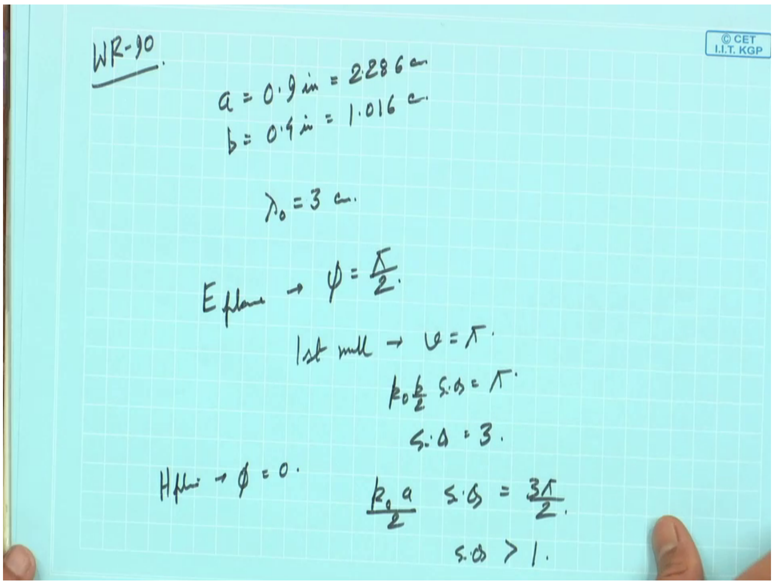

Now, if, if you have at x band WR 90 wave guide it is a x band waveguide, it is dimensions, we know a is 0.09 inch that is 2.286 centimeter and it is b is 0.4 inch that is 1.016 centimeter. Let lambda naught is 3 centimeter; that means, 10 gigahertz. Now, beam width you can find, what the beam width is in the E plane. E plane means phi is equal to pi by 2 E plane, where the E vector lies with the propagation direction.

So, there the first low local set v is equal pi bs value you put k naught b by 2 sin theta is equal to pi. So, sin theta value if you put you know all these values, it will come to the 3 what does that means, that theta is in the invisible zone. So, sin theta is greater than 1; that means, in the theta plane we do not have a null.

You will have to go beyond that, but that is physically not possible. So, you will have to say that null is not there. So, it is a very, when a v is too small for a null to occur in visible space to beam width, it is undefined here, similarly in the H plane means the other plane phi is equal to 0 null occurs at k naught a by 2 sin theta is equal to 3 pi by 2 please note that at pi by 2. It does not occur, because that pi by 2 we get f y is equal to 0 by 0

So, here if you solve for sin theta, this sin theta also is greater than 1. This null here also at invisible space. So, very flat beam to a directed nature so, for directive calculation, we have to do actual integration that can be done, we know the fields now radiated zone.

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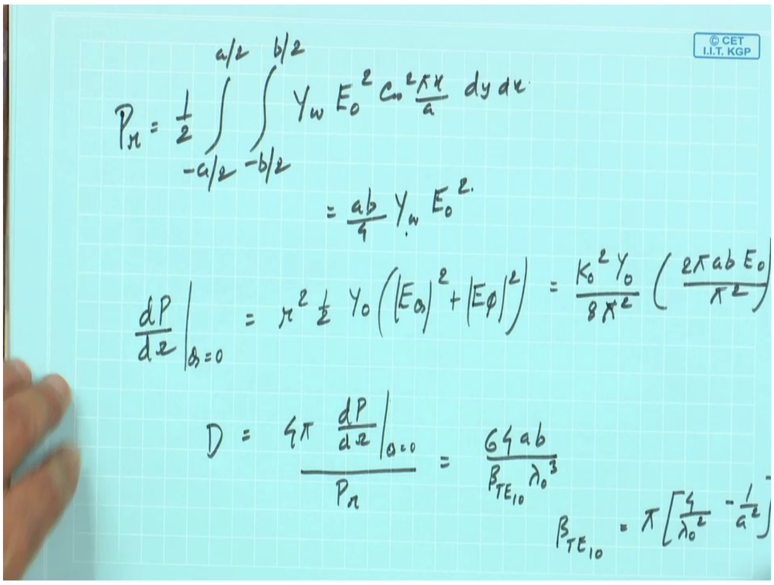

So, total power radiated we know. So, Pr we can calculate half minus a by 2 to a by 2 minus b by 2 to b by 2 and w E naught square cos square pi x by a dy dx, this will give me ab by 4 and we know from the this, these two principle patterns from here, we know where is the maximum radiation happening it is along z axis.

So, we can find the radiation intensity at z axis; that means, dP d omega at theta is equal to 0 that will r square half y naught E theta square plus E phi square this, if you evaluate will come to be. So, directivity d is 4 pi dP d omega f theta is equal to 0 by Pr. So, you do this you will get finally, you will have to put those things values of this y w, this wave impedance that I have already told. So, if you do that you will get 64 ab by beta TE 1 0 lambda naught cube and beta TE 1 0 is pi by 4 by lambda naught square minus 1 by a square whole to the power half.

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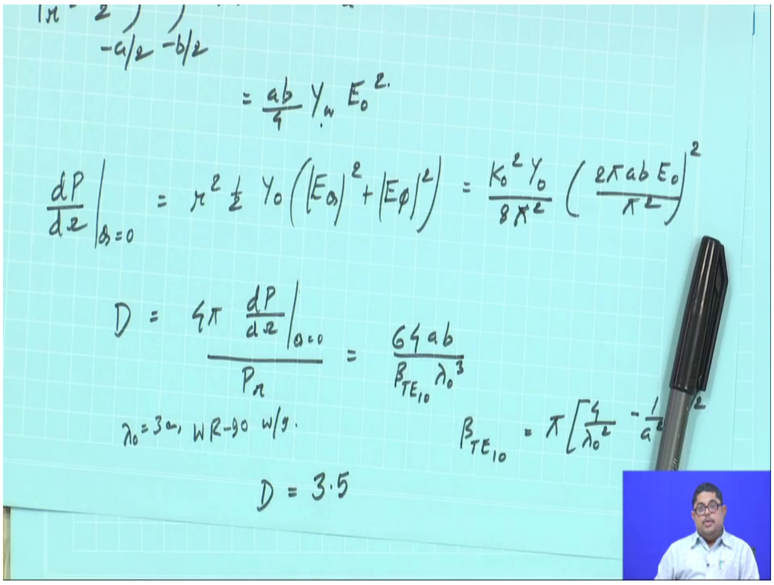

Again, if we do that thing that lambda naught 3 centimeter WR 90 waveguide then you can find this D becomes if you put these values 3.5. So, directivity of a open ended the waveguide in x band that WR 90 the directivity is 3.5. So, it is definitely better than a dipole, you say it is definitely better than a dipole, but in aperture thing, it is a such large thing 3.5 is not good, but open ended waveguide is a very popular antenna, because in arrays it can be used in array, it is a very good element. It can, sustain very very high power, compared to all those wire antennas.

So, in various radar applications you have open ended waveguide then it is a very good structure, if the you see feed is coming suddenly that is becoming an antenna. So, there is no need to have a separate antenna and all those things and on a ground plane everyone is put; so, the front part is very planar. So, that is why in missiles etcetera guidance systems, they are, when there are guidance systems, missile guidance system. It is heavily used this open ended waveguide and it is antenna properties are very well known.

So, now we will see, to today we are concluding this, but we will do the same thing in case of other things, the horn antennas. Because now, the question is if I flared the aperture that becomes a horn and the moment I flare my motivation is, I should get better directivity, which actually we will get, by flaring; So, that we will see in the next lectures.

Thank you.