Analysis and Design Principles of Microwave Antennas
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Lecture – 22
Analysis Procedures of Aperture Antennas

Welcome to this (Refer Time: 00:16) lecture. Now we have seen up till now wire antennas, here types of wire antennas and how to analyze them. And we have seen while seeing the antenna parameters that basically antennas radiation efficiency depends on the effective area of an aperture, effective area of an antenna. Now wire antenna, since they are by definition wire, so they do not have much width, though they have some electrical width that we have shown, but that is not sufficient if we want high gains, high directivity.

So, for that the idea came that instead of wires if we use a two dimensional structure for antenna; that means, aperture to radiate, then it will be better and it turned out. Actually the one of the first wire antenna a aperture antennas were also invented in Calcutta in India by sir J C Bose, because he was the first to use for communication purposes or to prove his experiment, actually he was trying to prove that the laws of electromagnetics also apply (Refer Time: 01:43) optics and for that he first used as a receiver a horn antenna, which is nothing, but an aperture.

He called it collecting funnel, but actually it was an aperture antenna, so horn antenna which is very popular till today and we will see it. So, this aperture antennas are a class of antennas, the examples are, if I have an rectangular or any wave guide rectangular or circular or any cross section wave guide as a transmission line and suddenly I open it; that means, there are no more metals, so it then from the mount of the web guide, it will start radiating, so that is called open ended wave guide antenna.

Then and modification of that to increase the directivity; that means, to increase the effective aperture his horn antenna, then people also cut slots on the conducting walls of web guides, those are slot antennas. Then there are another use that you use a this type of aperture antenna and use reflecting surface which is called dish antenna, so as a reflector that dish antenna that also behaves as an aperture, but actually there should be an antenna some aperture antenna or some wire antenna at its nearby, so that that comes to that length surface and from there it again starts takes this radiation properties.

So, those dish antennas then you can have. Now a days the planar antennas one of that is called patch antenna, microstrip antenna, so you have a aperture, conducting aperture from their put on dielectrics and from there it is radiating. So, all these are examples of aperture antenna

Now, the class the analysis technique for this aperture antennas is a bit different from wire antennas, why because wire antennas we have proved that if we can find the conduction current distribution on the antenna, then we can find the radiated electromagnetic field everywhere in the planet, in the whole universe you can find that (Refer Time: 04:18). Now here in aperture antenna case, it is not always conducting current, because you think of a horn antenna.

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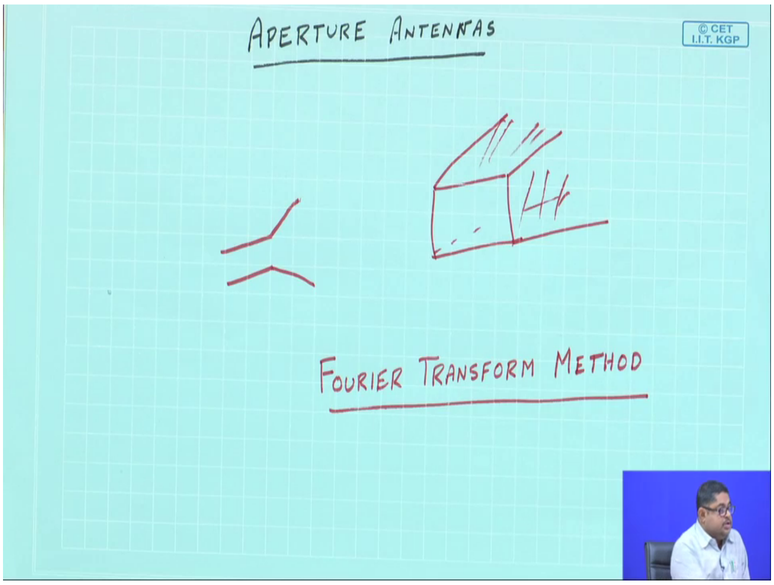

Suppose I have a horn antenna, so this is its radiating mouth or if I see the front view it is something like this, so actually there are. So, these are all metals and suddenly there is nothing there, so there is no conduction current, so we cannot find out the what is a conduction current on an aperture that is why is, but there is an electric field created here, because of whatever source excised that is.

So, aperture antennas are analyzed by the field distribution on the aperture. So, we will see that, so all these planar antennas or aperture antennas, they are analyzed in terms of field with, that is there on the aperture. So, if I know somehow this field, how to know that that we will see there are various ways, but we can, if we can guess what is the field on the aperture, then we can find the field everywhere

So, it is a field, aperture field based analysis technique now. Another thing is this aperture to have meaningful directivity. This we have said earlier also that the both the aperture dimensions. Since it is a two dimension also the it has two dimensions, so its length and width both should be a at least several wavelengths the, so that you can have a meaningful gain; that means, it should have a sizable electrical area the, then only you can have a meaningful thing. Now you think that at low frequency these aperture antennas are not popular, because if you want to have the aperture to be of sizable electrical length.

And since the wavelengths are very large at low frequencies, so you require huge apertures, which is not possible; that is why at high frequencies. Particularly microwave frequencies where typically the wavelength has come down to some centimeters there you can have this type of aperture structures that is why aperture antennas are mostly use at microwave frequencies. So with this background, now, let us see the how to find the radiated field from an aperture whose field distribution is known.

So, there are actually two techniques out of that today we will see the first technique; that is called the radiation from a planar aperture by Fourier transform method. Well I think all of you know Fourier transform and we will use that Fourier transform knowledge the, but there will be a difference here that this Fourier transform is not the time frequency Fourier transform, it is a special Fourier transform. So, to understand this let us look at the this diagram.

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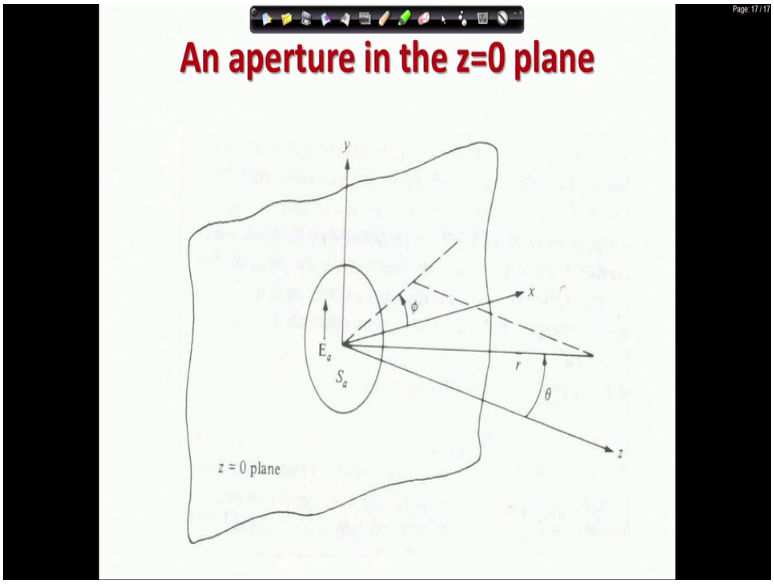

I have an aperture this, this circular thing what is shown in general, it can have any shape, so this is an aperture, so that is called as S a the, this whole aperture is denoted by Sa and we say that we know the tangential field; that means, field on this aperture is known and that field we are calling E a. So this whole thing is a plane, there is a cut and that cut is the aperture, so you can say that this is metal or dielectric or something and there is an aperture there so, this for our coordinate reference we are calling that this whole ah screen that is z is equal to 0 plane.

So, the aperture is radiating in the z is z greater than 0 size, and actually there are some sources definitely at z less than 0, somehow they are there are various ways by which this aperture is excited, we need not go into those details, because all those source gets characterized by this electric field on this, each tangential field. Actually this comes from an theorem called equivalence principle, so if I know the field on this aperture, actually that is sufficient to characterize the excitation ok

So, now our job is to determine the radiated field in z greater than 0. Now let us look at some Fourier transform basic, I think all of you know Fourier transform, but in this case it is a special function So, I do not have a function of time I do not have a function of space. Let us say that I have a function w, it is a function of x, and its Fourier transform I can define by W and the special frequency I will write as kx.

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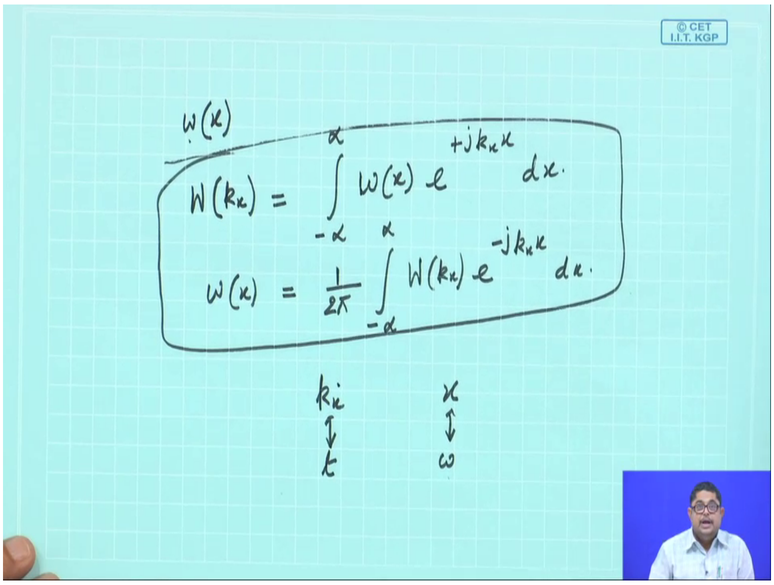

So, W kx I can define as minus infinity to infinity wx, this is small wx e to the power plus j kx x dx, this is my definition of spatial Fourier transform. Please note this plus, generally in time frequency Fourier transform generally, though this is also valid that we generally take this as the Fourier transform we take as minus, but here we are taking plus. Actually it is a general thing, we can take anything in this case specifically for some reason the, that will give us advantage later, this is plus jk. So, we can write that what will be the inverse Fourier transform; in a inverse Fourier transform will be wx is 1 by 2 pi. So, I can say that the variable kx and x in the special transform they follow the same role as t and omega in time signals Fourier analysis.

Please remember that t is not equivalent to x, because of this choice, t is corresponding to kx and omega is corresponding to x, this will have profound implication, but because of our this choice, generally this is the equivalence (Refer Time: 12:25) ok, so this is a one dimensional Fourier transform. Now since I have aperture I need two dimension, so what is a two dimensional Fourier transform? Suppose I have a function of both x and y, u x and y. So, its Fourier transform I will write as and its inverse transform ok.

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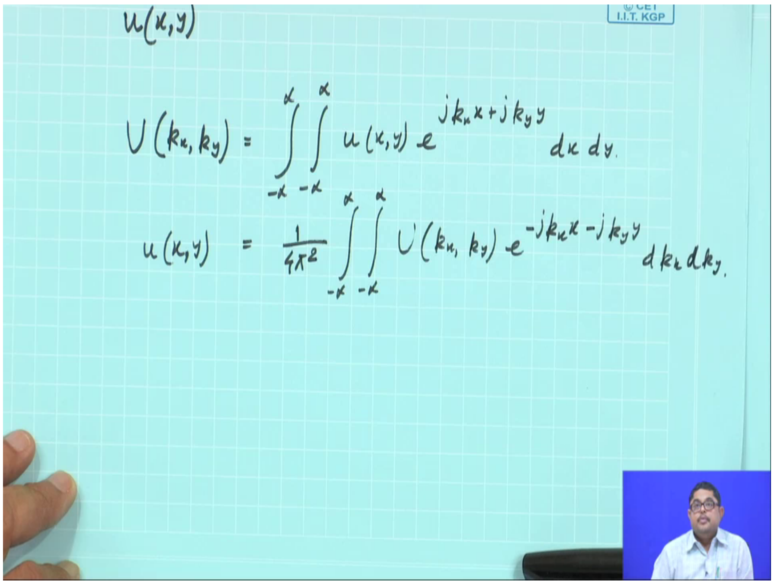

So, with this background let us start. Now we have seen that any antenna, the radiated field satisfies the wave equation.

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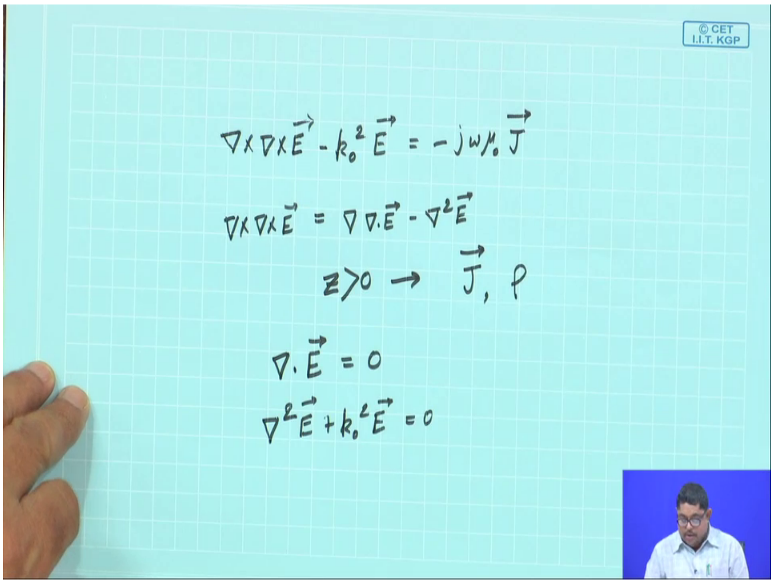

So, what was the wave equation del cross del cross e minus k 0 square e is equal to minus j omega naught j, this was the standard wave equation and we know that this del cross del cross e; that is by vector identity del dot e minus Laplace nop vector Laplace nop ok, and in the region, I am interested in the region z is equal z greater than 0, so it is a radiated zone. So, there j is 0, there is no conduction current and also the there is no charge density, so I can write del dot e is equal to 0 and from these equation del square e plus k naught square e is equal to 0 ok.

Now, these are the two main equations that we will solve to find out e. So I now numbered them, actually here I have written it in the reverse order. Reverse means; that is for my thing, because this is a vector equation.

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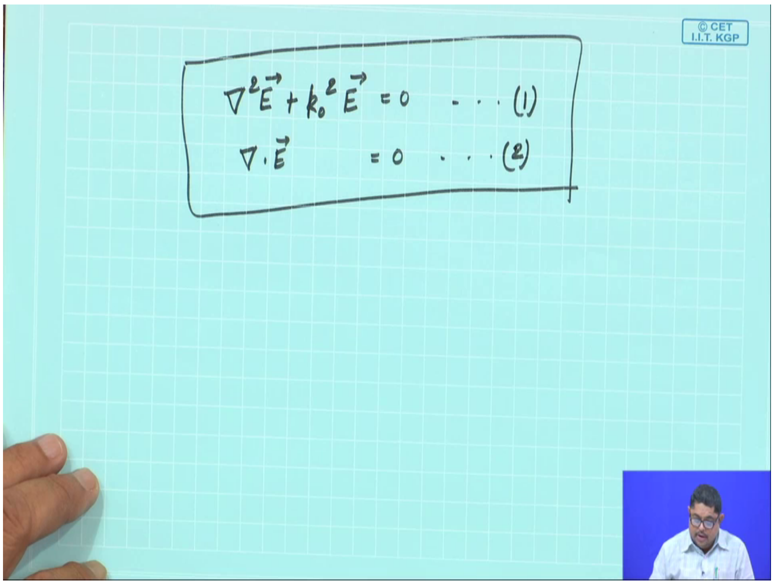

So, let me call this my equation 1 and the divergence equation that is E or it is called 2 later say. So, these are the two main equations that we need to solve. Now, again I recall another thing of Fourier transform.

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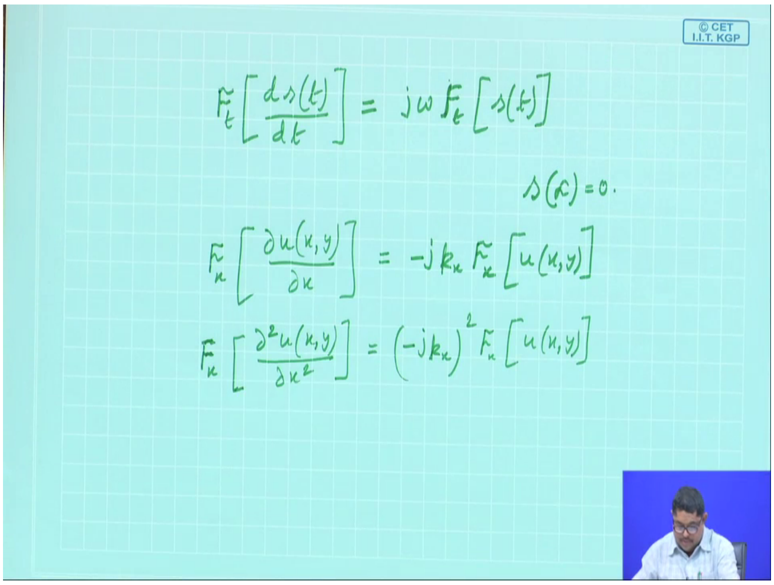

Suppose Fourier transform F t, I am writing is an operator, it is a time Fourier transform operator. So, time, suppose I have a time function s t, what is the Fourier transform of the derivative of s t that we know is j omega this F t s t; that means, it is j omega Fourier transform of the signal s t; obviously, this s t only the to this hold, only restriction is s t should be time limited; that means, s of infinity is 0, so for a time limited signal this is valid. Now let me say that what will be the special part of this.

So, can I say that Fourier transform of x del u x y del x will be; obviously, this is a partial variable because this is two dimensional, so this can I write as minus j k x F x u x y. Why minus? Because of that definition problem in special case the; similarly this F is an operator that is why it is something curly it, I am trying to write. So, for a two dimensional function what will happen to this, del x. So, this will be minus j k x whole square F x u x y etcetera etcetera you can do. Why I am doing this? because I have this persons and this persons, so I have del square here del del x square del del y square del del y square that is why I am doing this. So, let us take Fourier transform of both equation 1 and equation 2.

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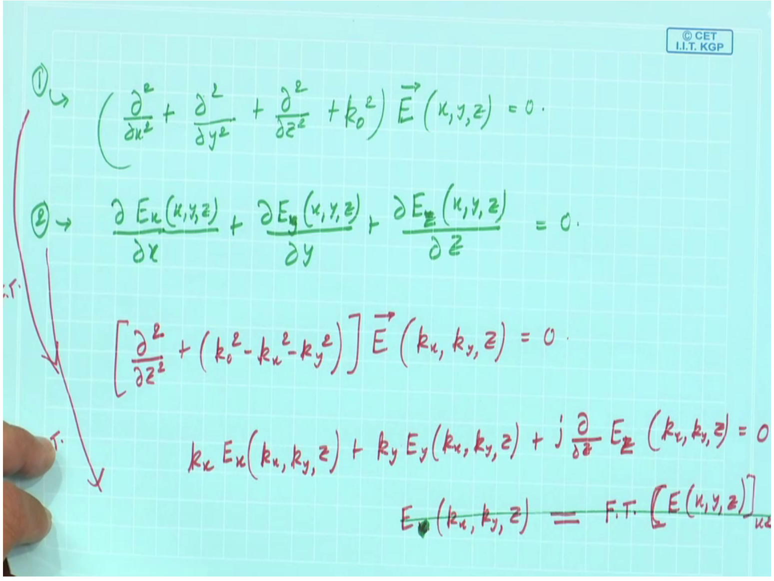

So, I get del square del x square plus del del. This is from the first equation, just I am writing it in Cartesian coordinate. Similarly here the second equation you will give me del E x x y z del x plus del E y x y z del y plus del E e z x y z del z is equal to 0.

Now, let us take Fourier transform of 1 and 2. So, this one if I take Fourier transform, special Fourier transform I get del square. What is this E x, E x k x k y z is Fourier transform of E x y z with respect to x and y.

Here you see I am deliberately making a symbol symbolical mistake. Actually, you see that I have written there, when we take Fourier transform of w x, the Fourier transform is an entirely different function from this original function; that is why we use lower case and upper case, for some other notation we do, because Fourier transform is a completely different function, it is not same as w x, but here I am doing that.

Ah where you will see same notation E x y z and this thing only I am giving a sorry k x (Refer Time: 24; 14) really, so E x yes, no this will be removed. Actually this notation itself I am remaining same only by argument I am changing, you will have to remember this, because it is not simply that E x y z and E they are not same functions, only I am changing argument not that.

But if we use other notations that will get complicated that is why we are keeping the same, but remembering for remember, for understanding or you will have to understand that k x k y by their presence it is the Fourier transform.

So, it is not the same function as E, this is for notational simplicity ok. So, we have got these two things, now I am removing these. So, these are my two things, here you see I have got this variables there is something so I need to simplify, so that I am doing by letting kz square is equal to this constant k naught square minus k x square minus k y square.

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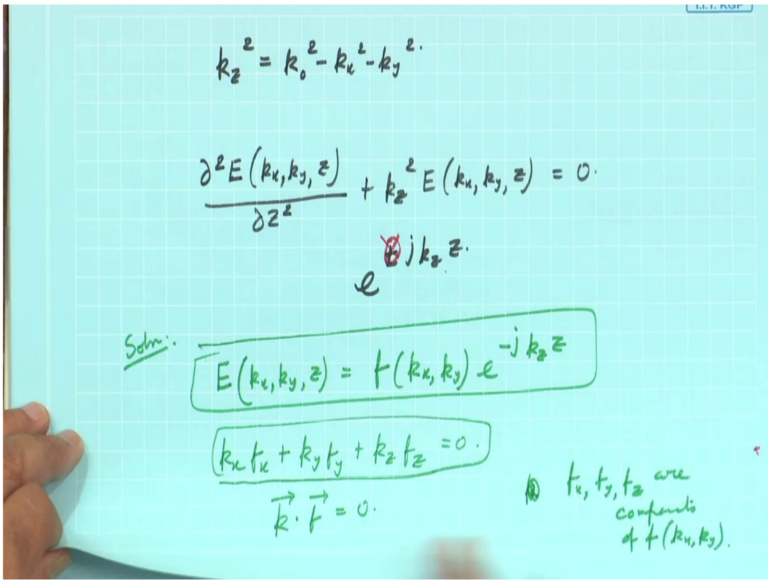

So, then this, this equation if I put that k z square thing it becomes del square E. Now, this I have put in this equation just k z square, the moment I do that you see this looks familiar, this is an wave equation so what is the solution of this? The solution of this is E to the power plus minus j k z z, you can put also and see that this is a solution. So, this wave equation is solved, but remember actually this is the Fourier transforms solution, because this is a Fourier transform expression.

Now for them also, since we are considering on the outwardly propagating wave, so the Fourier transform of an outward wave that cannot have this plus solution, so we will not take the plus solution, this solution we will throw away and we will take only the E to the power minus solution.

So, the what will be the general solution of E. So, solution of E is E k x k y z is f k x k y E to the power minus j. I can write that this is a constant as far as z variable is constant. Now this is to be determined, so the moment this gets determined we will be finding the solution. So, now, this solution is, I have obtained it from this equation, so let it put me in the other equation.

So, if I put it in the other equation this solution you will see that we get k x f x plus k y f y plus k z f z is equal to 0, where what is k, that sorry what is f x? F x f y f z are components of f k x k y x y z components of f x square, or this sometimes this is also written to understand it as k dot f is equal to 0 something. So, what is this equation tells us? This is a very important equation and this tells us that, I am trying to find f x f y f z, it says that they are not independent, 2 are independent, because they are related by this.

So, if I find 2 the other already gets determined ok, so, this is the outcome of these. Now why this is so? Actually if you look back this equation came from where? This equation came from that the restriction on the radiated field; that is divergence should be 0, so that vanishing divergence put this condition that x, three components are not independent actually there is only 2 degrees of freedom there ok.

So, this solution now let me write the inverse from this solution. Now this is the solution, so what is the, this is a Fourier transform what is the inverse transform of this? Inverse transform of this is E x y z is 1 by 4 pi square minus infinity to plus infinity f k x k y E to the power minus j k z z then E to the power minus j k x x minus j k y y d k x d k y ok. This is the solution, this part I have written and then E to the power, this is the inverse k E to the power j k x minus j k minus d to x ok.

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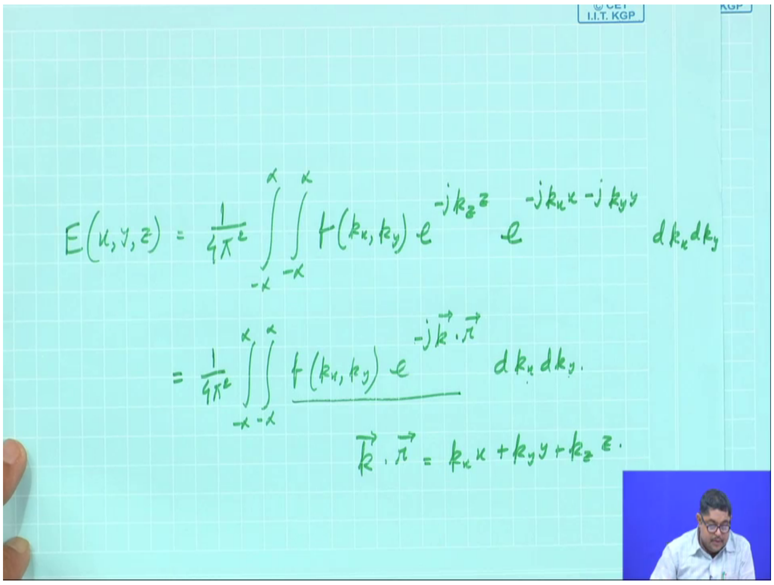

So, this in simplified way I can write as, where what is k dot r k x x plus k y y plus k z z. So, that is all almost that it tells us that what will be the (Refer Time: 32:22) at any point. It is simply the, it can be represented, can I say that this is E to the power j k dot r; that means, various values or x y, so this is a plane wave can I say. This is, these three, this is a plane wave and it is basically superposition of plane waves, they are taking the value k x k y different k x k y. So, this is a spectrum because this k x k y is nothing, but our spatial frequencies.

So, I can say that what is the radiated field that in the z greater than 0 zone, the field can be represented as a spectrum of plane waves, because this one is a plane wave with vector amplitude f propagating in the direction of the propagation vector k ok.

From here we will have to take that what are the different values k z k x and k y can take. Now from the definition of k z, if you see the definition of k z, this was the definition of k z k z square is equal to k 0 square a. So, here you can see that magnitude of the propagation vector that is same as k naught.

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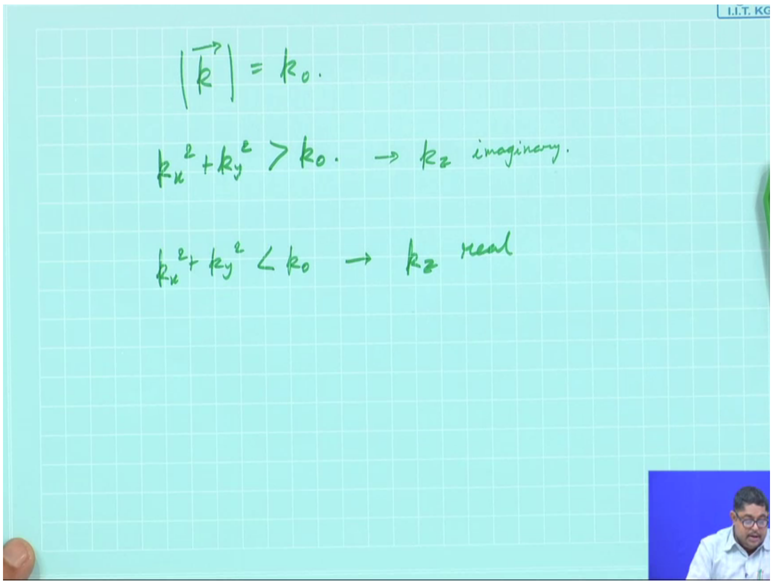

Now, there are two possibilities k x square plus k y square may be greater than k naught or may be less than k naught. If k x square plus k y greater than k naught, then k z, you see k z it becomes imaginary. What is the meaning of a imaginary k z? So these are plane waves which will die down, actually they are attenuating wave, they are called evanescent waves and basically they contribute to the near zone of the radiations. So, near zone radiation is contributed by these evanescent modes.

And for this thing, this k z is real for the other condition. So, they are the contribute to the radiated field, since these are the only plane waves propagating outwards as their k z is real ok. I think the in the next one we will see that the. Till now we have not got the f value, because we have said the solution is this, but this f we have not determined how to find f, so that we will see in the next class.

Thank you.