Analysis and Design Principles of Microwave Antennas
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Lecture - 02
Radiation From a Current Element (Hertzian Dipole)

Welcome to the second lecture. We are still continuing the theme microwave radiation fundamentals. So, we have seen the inhomogeneous Helmholtz equation in magnetic vector potential in the first class. Now, we will find a solution to that with a simplified case. First we will consider a current distribution which is very fundamental, but which is very fictitious in nature that is called Hertzian Dipole or electric Hertzian Dipole. It is an electric dipole. Also this is called a current element; interchangeably there are many names to it.

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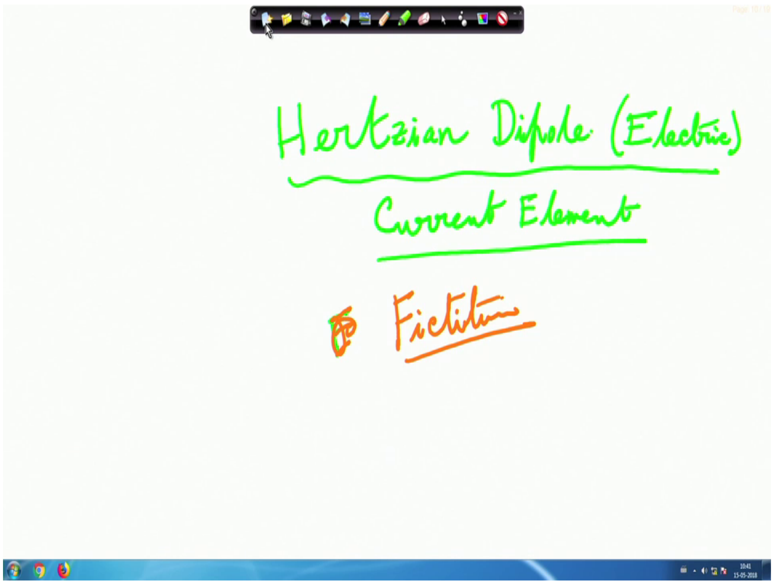

Now, this is actually a fictitious. Why? Because first let me draw that the current element.

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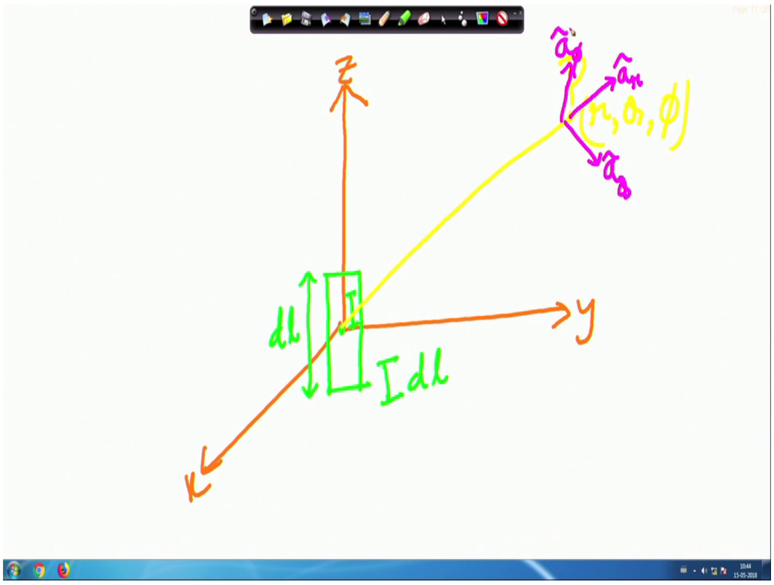

As I have already shown that in Cartesian coordinates the magnetic vector potential and the current density vector they are co linear. So, it is easier to find out magnetic vector potential in Cartesian coordinate. So, I have drawn a Cartesian coordinate. Let us say that this is my z direction this is my left handed system. Now, there the let the current element is centered here this is the very thin its length is infinite decimal, it is a very thin current, but it is carrying a current I. So, this is generally always written as Idl that means, I is the length and d l. So, I is uniform throughout in magnitude and phase, but after reaching here there is no I, after reaching here there is no I.

You know that these type of thing cannot exist in practice that how I will put these current element, but we will see that to understand radiation this is a vital concept, because the radiation that takes place from this type of current element that is present in all types of antenna radiations.

And also we will be actually breaking a distributed radiator because all radiators are distributed antennas are distributed structure and they are they can be broken into these current elements. So, that is why this is fundamental. So, we consider that this I is a time varying current of frequency omega etcetera and let us say that we are observing it at a point field point r so that means, in generally field points are always specified in terms of the spherical coordinates r theta phi.

So, at a point P we are interested to know what is the electric field, what is the magnetic field at that point. So, I again remind you that this is the direction of the radial vector at point P the direction of the a theta vector will be like this and a phi vector is orthogonal outside this plane is this, ok. So, let us consider this radiation. So, a short thin filament of current located at the origin oriented along the z axis. So, now, we can say what will be then current density, that will be J z into a z, a z is a and we have found that these value is Idl a z.

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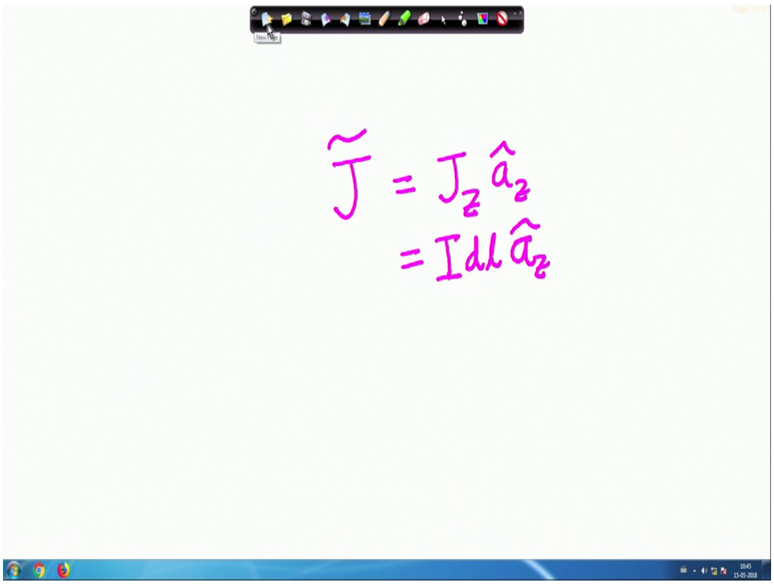

Now, So, this source distribution if we you see this source distribution is definitely spherically symmetry because this source does not have any theta or phi variation in all theta and phi it will be looking same definitely it has some r variation. So, we can say the and we have already found that the source that you can let us know that the current, current density vector is z directed, so we can say that a also will be z directed magnetic vector potential a that will be also, so I can say that magnetic vector potential will be a z.

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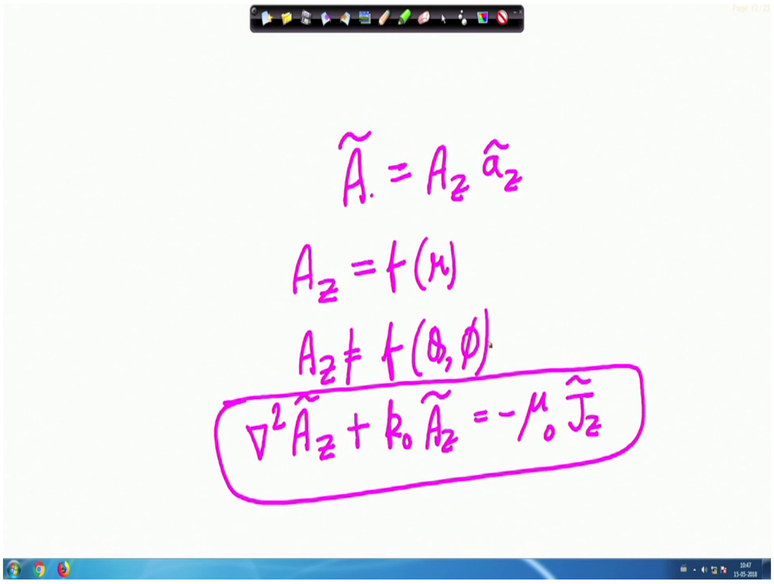

And from the symmetry I can say that A z will be a function of r and A z is not a function of theta or phi because the source is spherically symmetric. Once this is there I can write, so what will be my equation that Laplace equation del square A z plus k naught A z is equal to minus mu naught J z. I will have to solve this equation. So, I can immediately write the del square equation in the spherical coordinate.

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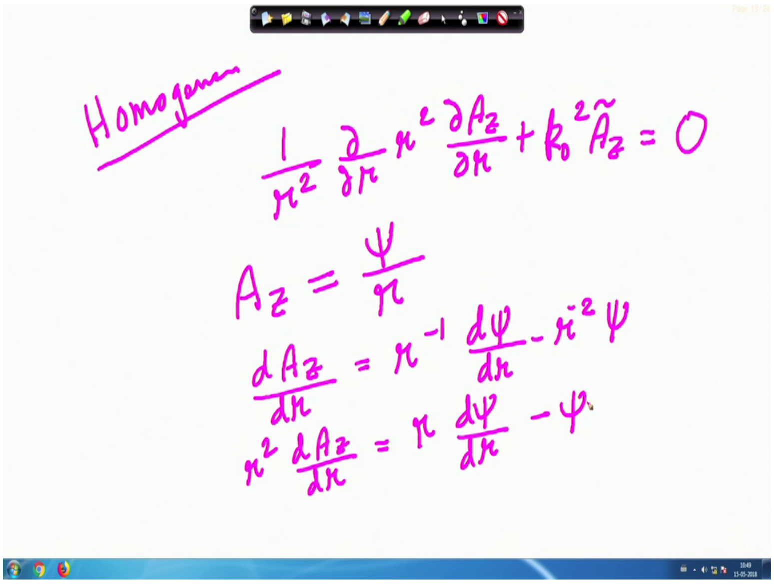

So, I will write 1 by r square del, del r, r square, del A z del r plus k naught square A z ah. Here you know that solution of any in homogeneous differential equation first we solve the homogeneous equation these homogeneous differential equation that give us the general solution and then we add to that the particular solutions. So, that will do by that time putting the source value that means, j j value.

So, first we will doing the homogeneous part of the solution. This you know from your knowledge of differential equation solution. Now, the solution as you know that, in second order differential equation we first find the solution, so solution will be obviously, have. So, A z is a function of r let us put that the function is that functional dependency is 1 by r and psi is a constant.

So, if we do that then we can find what is instead of del z, I can write d A z because it is only a function of r. So, d z d r will be this is simple mathematics you will be able to do on your own similarly you can find what is r square d A z d r that will be r d phi d r minus phi.

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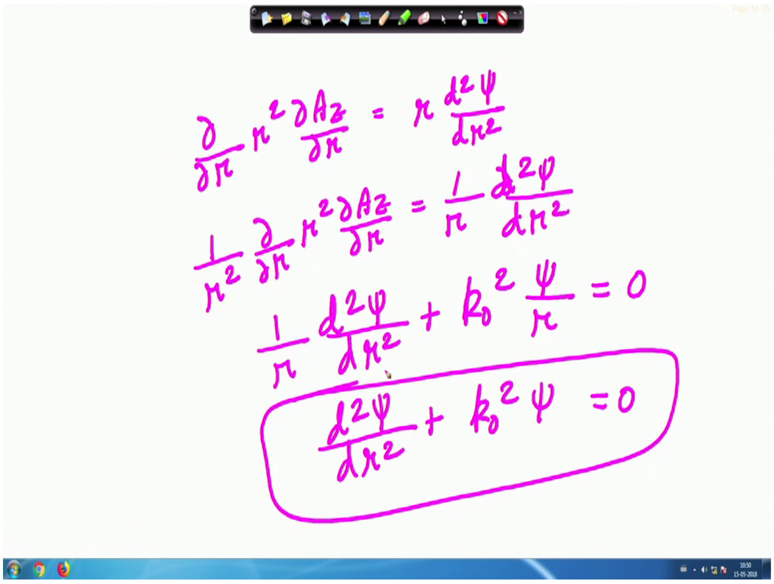

Then del r r square d r that will after some substitution I am not doing it is simple you do because it is a one only one variable dependence. So, we can say that this 1 by r square del r r square del A z del r that will be 1 by r del square phi sorry it is d square by d r square. So, these we have got one part.

Now, let us put it. So, we can say that 1 by r d 2 phi d r 2 plus or we can write it as d 2 phi d r 2 plus k naught square phi is equals to 0. So, this is a simple scalar second ordered equation, so differential equation. So, all of us from our college days knows these solution this is simple harmonic motions solution.

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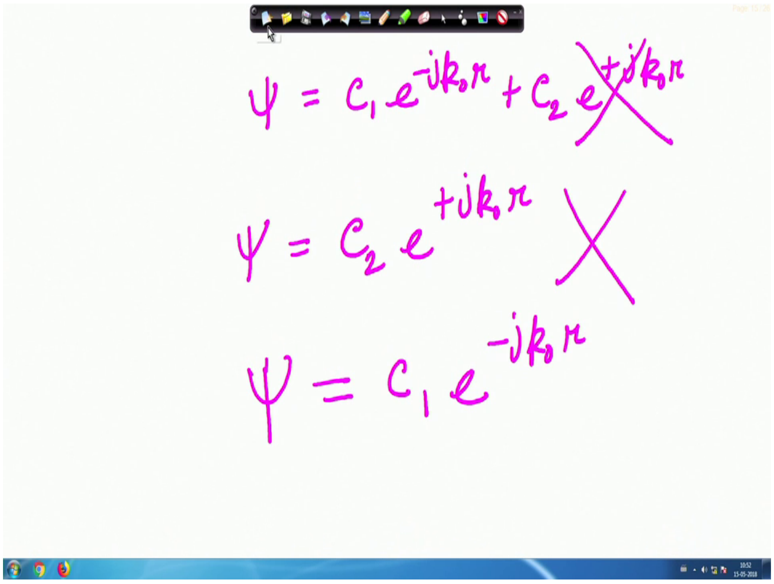

So, what will be the solution? We know that phi will be equal to c 1 e to the power minus j k naught r plus c 2 e to the power plus j k naught r. Both this solutions are possible that is why we are writing it as super position of them c 1 and c 2 are unknown that is value to be determined.

This is mathematics but physics says that out of this one solution is not acceptable. Why? Because it is a radiation taking place, so radiation means transport of energy. So, at a infinite distance from the source the value of the field should go to 0. So, out of these actually what is if I choose phi is equal to c 2 e to the power plus j k naught r you know this is an wave, which is coming in side and if I put r is infinity then this value becomes infinite. So, that radiation condition says that this is not possible, this wave any wave coming here at infinite it should have high infinite value. So, this is not possible. So, this solution is not possible. So, acceptable solution is, ok.

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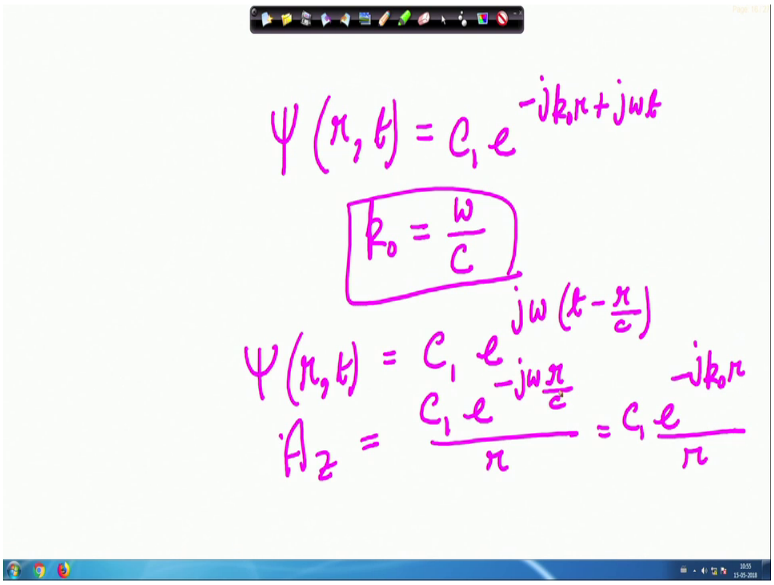

Now, let us this was a Phasor solution. So, if we now write the time variation then it will be c 1 e to the power minus j k naught r plus j omega t.

And already if you recall that when we define the wave number k 1 that time we have said k naught is equal to omega into square root of mu naught mu epsilon may for free space. So, that is actually c, c is the speed of light in free space. So, in that terms we can write that phi r t that will be c 1 e to the power j omega t minus r by c. So, this is definitely an equation of a wave. A wave which is travelling outward and reaching the point P, in my drawing that the observation point P at a delay time delay of r by c. So, this phi which is called retired potential because due to this delta the potential is getting retired in time.

So, this is an wave solution and here still we have to find the value of c 1 that will do. So, again if we suppress the time variation we comes back. So, what is our A z? We have assumed the solution as A z is equal to phi by r. So, now we will have to write that A z will be c 1 e to the power minus j omega r by c by r or in our previous nomenclature that means, in terms of k minus j k naught r by r.

Here we will see that in time domain generally we all bring this k naught as omega by c but in antenna parlance we will see that this wave number domain has some advantages that is why we prefer when we are in phasor domain we have considering the radiation things we do it. But if we come to time domain generally we bring back to this relation omega, ok.

So, this is a solution A z is this phi 1 is yet to be found out so that means, this is the solution for the homogeneous part or the general solution of the differential equation. Now, we will have to solve the in inhomogeneous part to evaluate this c 1 ok. So, what we have done? From the current element we found out that this is the direction of the current. So, according to we found that vector potential also will be z directed we assume that vector potential is some constant by r by that we found that ok, it is satisfying the wave equation.

So, we have found the solutions out of two solutions, one we have physical consideration radiation condition from that we have discarded, we have kept the valid solution and we have found the general solution. Now, we will have to put the value of c 1 from the inhomogeneous part and that will be our thing. So, next we are finding the inhomogeneous solution part.

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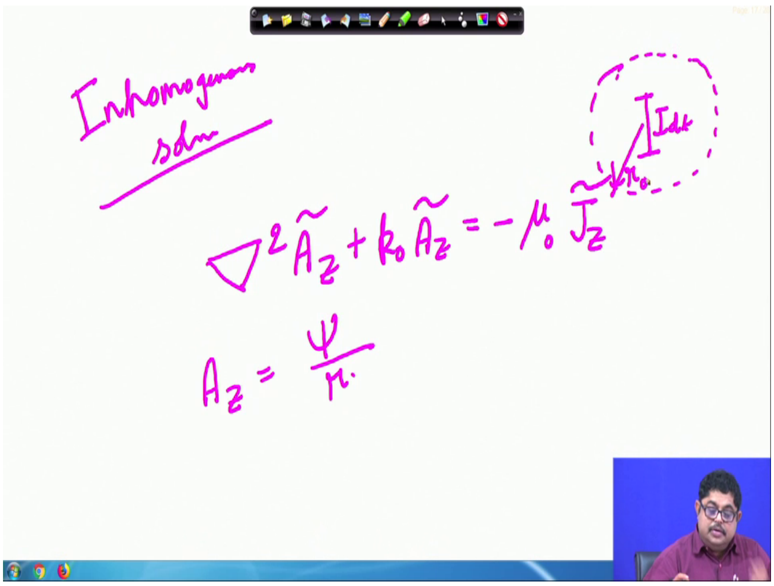

So, what is the equation write the inhomogeneous equation this was our you know inhomogeneous equation. Now, what we do, you see what was our solution A z was here is a problem that if we want to determine what is the value of A z at r is equal to 0, it is a division by 0 case because phi is a nonzero and fine. So, I function is nonzero and r is 0.

So, this happens in all the electric fields magnetic fields you have seen these that at r is equal to 0 we cannot define it.

But the source or the actual we call it singularity that is present there the strength of the source that will miss out if we do not consider the r 0 point. So, what is the wave out? We look at a very small volume around the source we are not exactly r, but we know that very near to r if we can enclose the source we will get its radiation effects that means, I effect of the source.

So, that is why we actually to find the source strength, we do this that we integrate the both sides of this scalar equation, scalar Helmholtz equation over a small spherical volume of radius r. So that means, if we have this current element source this is our source Idl, over this we put a spherical volume of radius r naught we find out this and then we force that r naught to go to 0. So, that will be the effect of this source.

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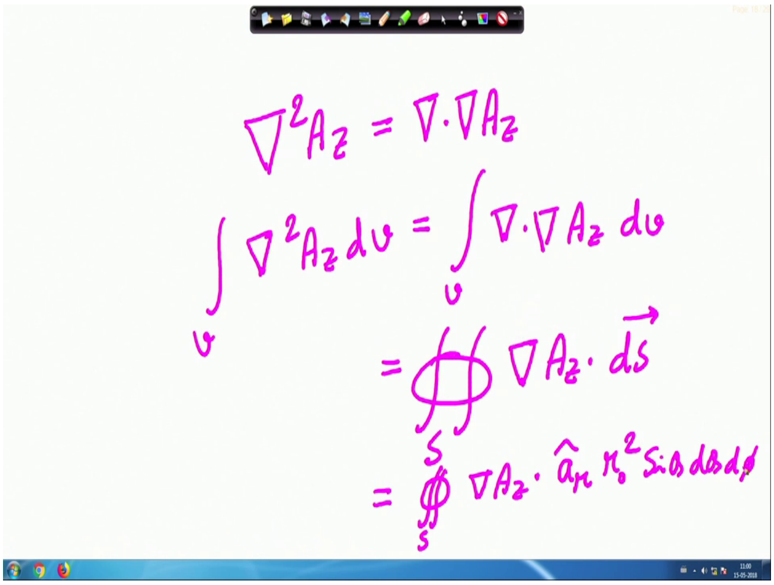

So, this is the idea. So, we know we can use divergence theorem. And what is del square A z? Del square A z is nothing, but this the lap lesbian. So, that is A z is a scalar, so gradient of that divergence of that. So, now, using divergence theorem can I write this del square A z d v is equal to del dot del A z d v and that now if I apply divergence theorem that will be a surface closed surface del A z dot d s.

Now, what is d s vector? In spherical coordinate we know the d s vector is a r r naught square sin theta d theta d phi. So, putting these in the scalar Helmholtz equation which we are doing, so in scalar Helmholtz equation while integrating.

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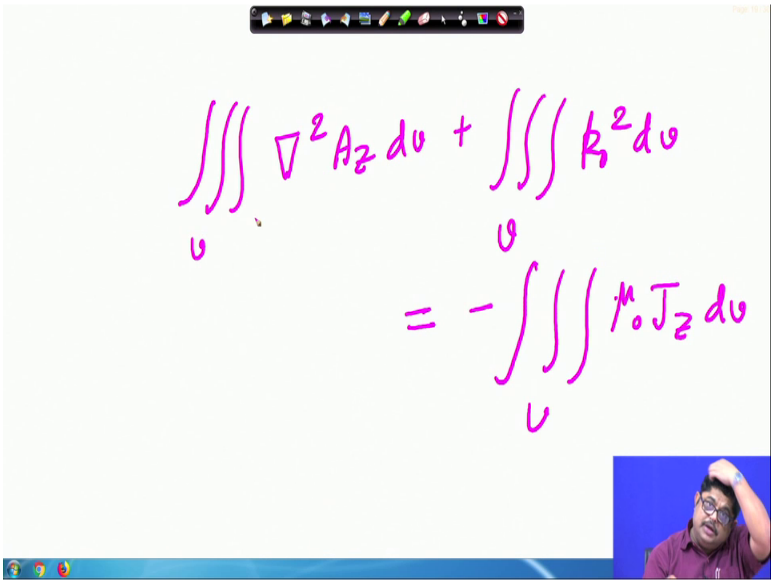

We actually our actual equation will be this is a volume integration del square A z d v plus k naught square d v is equal to minus 1 integration mu naught J z d v. This is the integration of in homogeneous Helmholtz equation.

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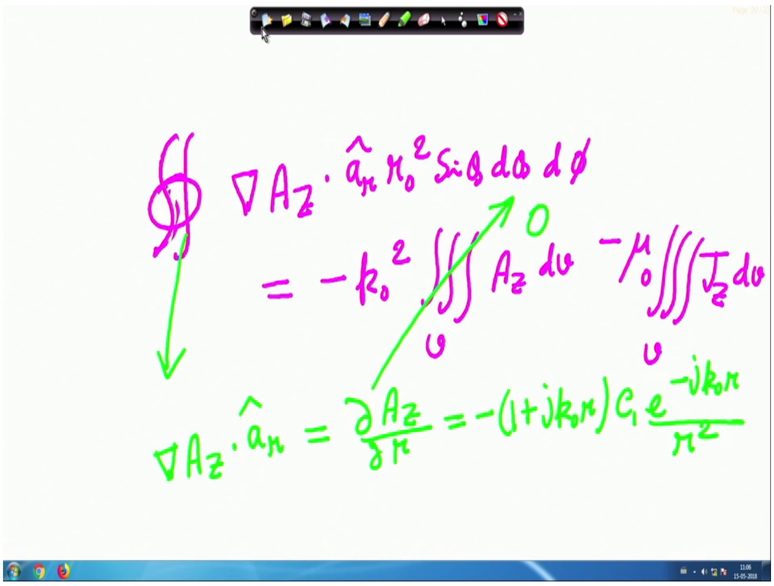

Now, there we have already simplified these are del square A z d v will be actually here surface integration. So, I can write that oh sorry.

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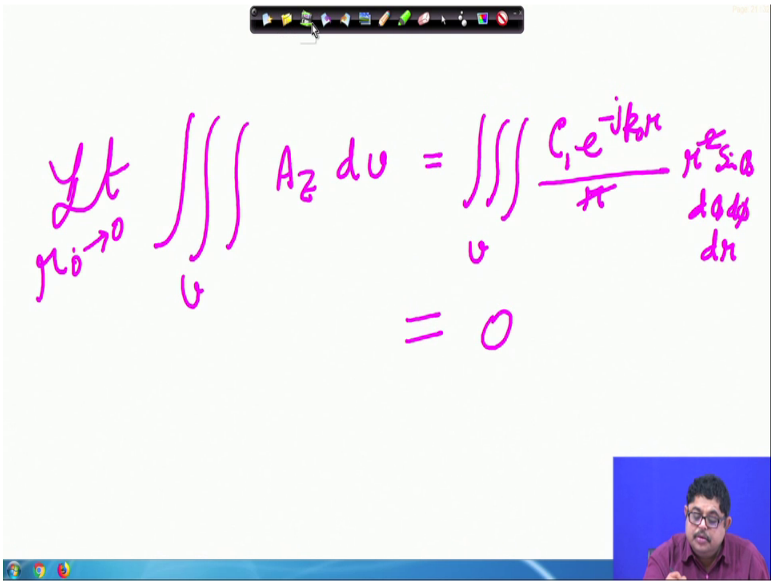

Now, let us look this term A z d v. A z d v, A z I know what is it? C 1 e to the power minus j k naught r by r r square sin theta d theta d phi d r, this is our d v in spherical coordinate r square sin theta d theta d phi d r. So, this term you see that this is proportional to r. So, finally, when I will put that limit r naught tends to 0. So, this is proportional to r, so this will become 0. So, this term I can forget. Also (Refer Time: 24:16). So, this term I have put that this is 0. I am left with this term.

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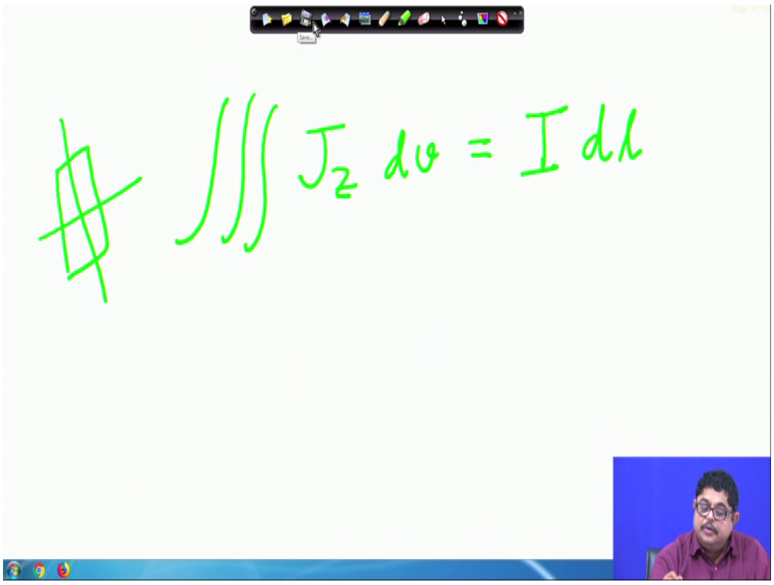

Let us see this term. This term is we have seen that this term is nothing, but because I have only a current element Idl. So, volume wise if I integrate I will get Idl and so, this term already I got. So, this term also I have solved. Now, this term let me further manipulate this term what will be this term if you see what is actually del A z dot a r others will be coming later.

So, this means what? This is nothing, but del A z del r I know the value of A z, c 1 e to the power minus j k naught r by r. So, if I do this I will get minus 1 plus j k naught r c 1 e to the power minus j k naught r phi r square you will see this that if I do this it will be this.

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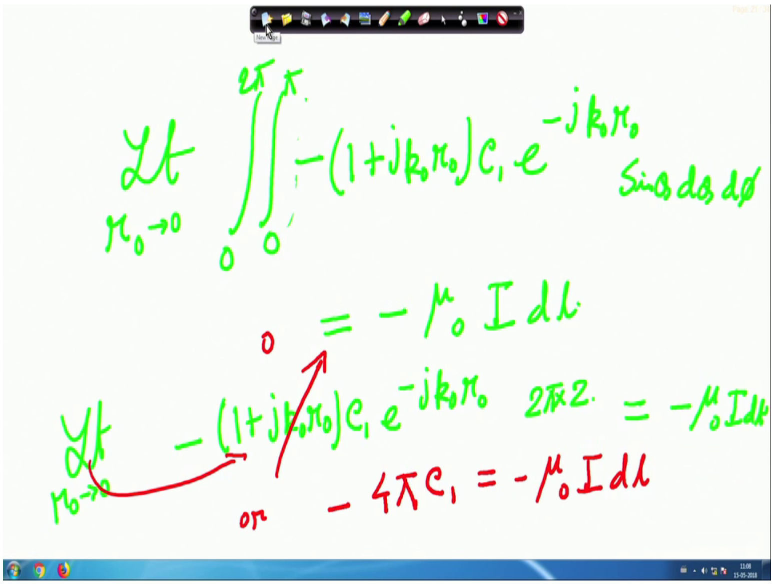

So, everything now is ready I am now putting it limit r 0 tends to 0. Already this has surface integral. So, I will have. So, this is minus 1 plus j k naught r naught c 1 e to the power minus j k naught r naught sin theta d theta d phi, is equal to minus mu naught Idl. Now, these are all constant things. So, sin theta d theta d phi and I will have to put these value things. So, phi will be 0 to phi this will be 0 to 2 phi. So, this one then will become minus 1 plus j k naught r naught c 1 e to the power minus j k naught r naught and due to integration there will be 1 2 phi and 0 to phi sin theta d theta is another 2. So, 4 phi into this.

Now, if I put this is equal to minus mu naught Idl. But here is a limit everything r naught goes to 0. So, if this goes to 0 this term cancels. So, this term goes to 0 with the thing. So, I get the final thing minus 4 phi c 1 this term also 1 is equal to minus mu naught Idl.

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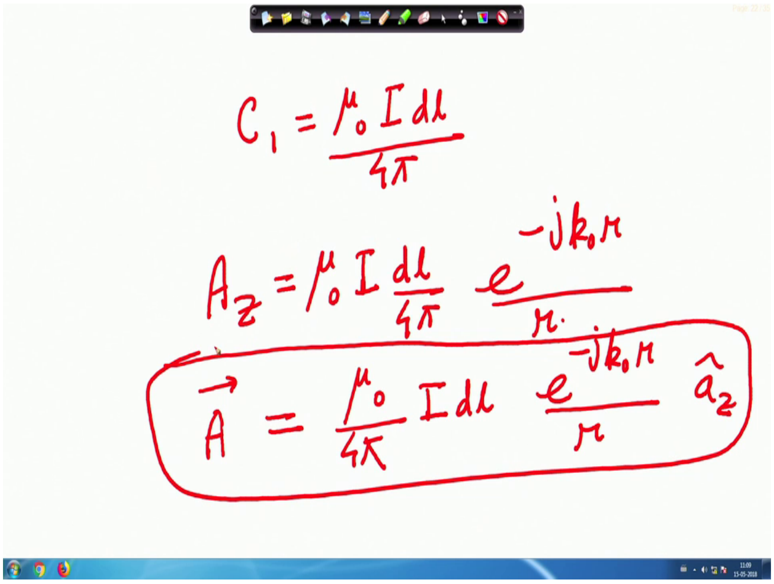

So, this helps me to find the value of c 1 is nothing, but mu naught Idl by 4 phi. So, we can now find that actual solution is A z is what? A z is equal to mu naught Idl by 4 phi e to the power minus j k naught r by r.

So, you see this c 1 actually represents the source strength Idl is the strength of the source these are the constants etcetera. So, A z is this. So, we can say that finally, we found the A vector for the current element that is mu naught by 4 phi Idl e to the power minus j k naught r by r A z.

So, solving the Helmholtz equation inhomogeneous Helmholtz equation we could obtain the magnetic vector potential. So, this is a remarkable achievement that we can find these. So, now, I can say that if I have a current the current element was y directed then this A z instead of A z that time it will be a y and this will be a y directed. If I have a current element in x direction then there will be a x that will be there that directed. So, we can get now this from here it is very simple to find out the first the defining equation was H. So, what is H? H is 1 by mu naught del cross A.

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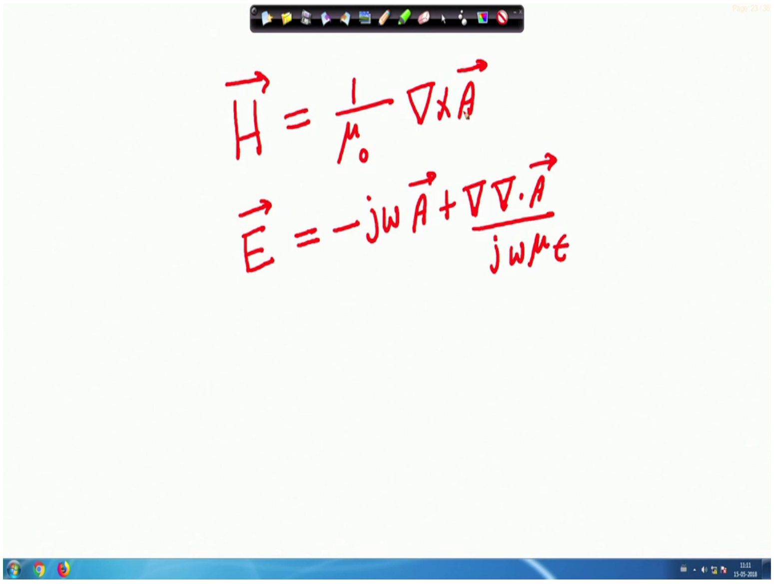

So, if we do that we know A. So, if we do this we will be able to find H and we know what is E? E is if we put this solution into the Maxwell's equation you get minus j omega A plus del del cross A by j omega mu epsilon. So, I know A, so E can we obtain or you can this first find out H then put the H into Maxwell's equation you can do or directly from here also you can find out.

So, this is the whole journey that we can solve for the all radiation, but we have not evaluated these. So, we will have to evaluate that and we will have to express it in terms of field coordinates. So, field coordinates are generally spherical. So, that we generally express in spherical coordinates and we will convert that we will find the fields and we will by that we can analyze the structure. So, we have done an useful thing.

In the next class we will see what is the importance of these derivation. Coming through this potential route we have seen we can simplify because knowing the current given source currents direction we can easily find the vector potential magnetic vector potential, and from magnetic vector potential easily we can find the magnetic field and electric field that is radiated by the antenna at the points ok. So, some more steps are there to actual writing the expressions of this that we will take in the next class.