Analysis and Design Principles of Microwave Antennas
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Lecture – 18
Antenna Array Theory

Welcome to this second lecture of Array Antenna. We were seeing n element array and we have found out what is a total field due to all the radiators.

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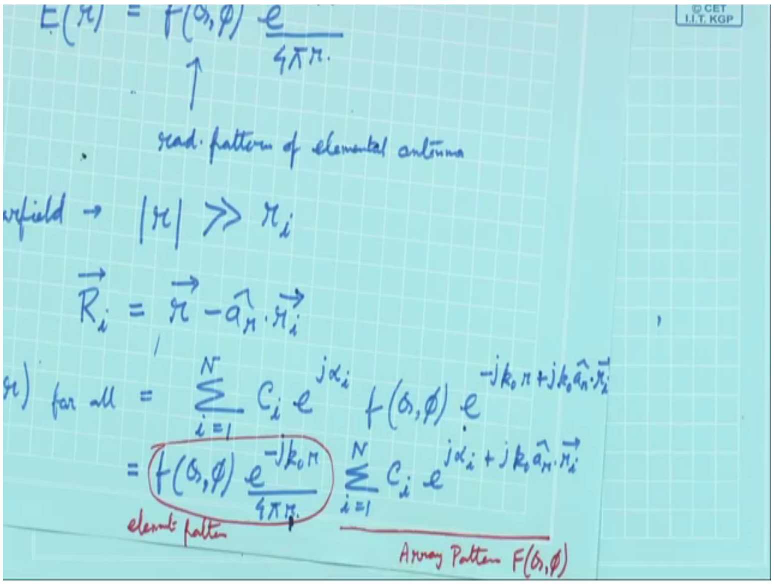

And if you notice in the last expression, we have substituted actually for each one this 4 pi r that actually it is different for different one for r 1 r 2 etcetera it should have been. But since this is the amplitude part we have put for everyone; we have put the far field formula that all r i are same that is why here r i, but here we have not done that here we have put the exact condition.

So, now we can say that if you look at it this whole thing is a element pattern. So, can I claim that this is element pattern, because you see this is the same as this. So, element pattern and this is the array pattern so this is the principle of pattern multiplication.

So, in general also it is valid it is not that only for 2 or only for symmetric thing any n element array you can always break that this thing. And generally this small one we called small f theta and this one array factor. We generally write it as F theta phi because, here also between this angles there are this theta phi is present. So, now study of array antenna is basically this study of this array factor.

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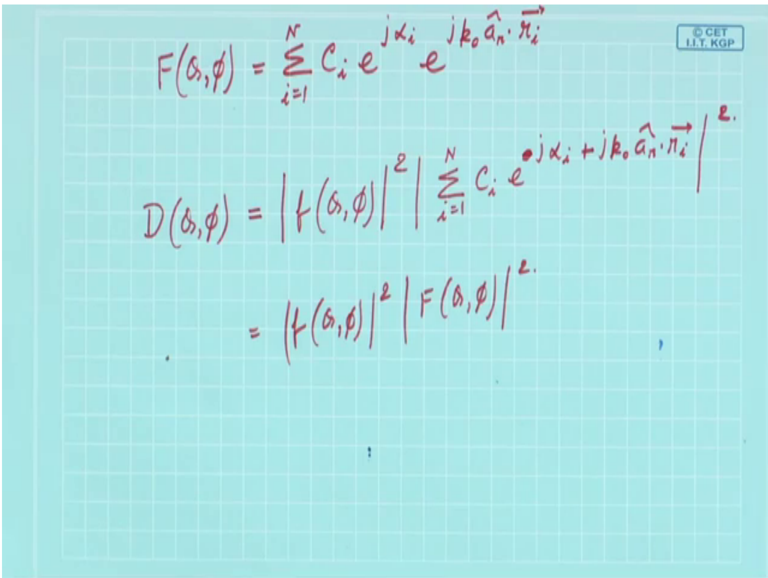

So, I can write the array factor capital F theta phi, that is nothing but i is equal to one to n c i e to the power j alpha i e to the power j k naught a r dot r i. Now, radiation pattern will be square of the magnitude square of these, and directivities can I write here that; directivity of the whole array antenna can be written as f theta phi square ok.

But finding directivity we will see that in case of array factor there will be a directivity and this factor already we know how to calculate the directivity of elemental ones. So, the total directivity will be this thing, but always finding the directivity from this thing array factor is not so easy. We will see some simple cases where we can have the approximately find these.

But the n result is obviously directivity increases much because, of array that is why arrays are used to give directive nature in particular directions and typically 10 15 dB increase in the directivity function can be done with a that is simple practical arrays. Now here, I need to point out one thing that in these all these cases we have assumed that these f theta phi for all elements are same.

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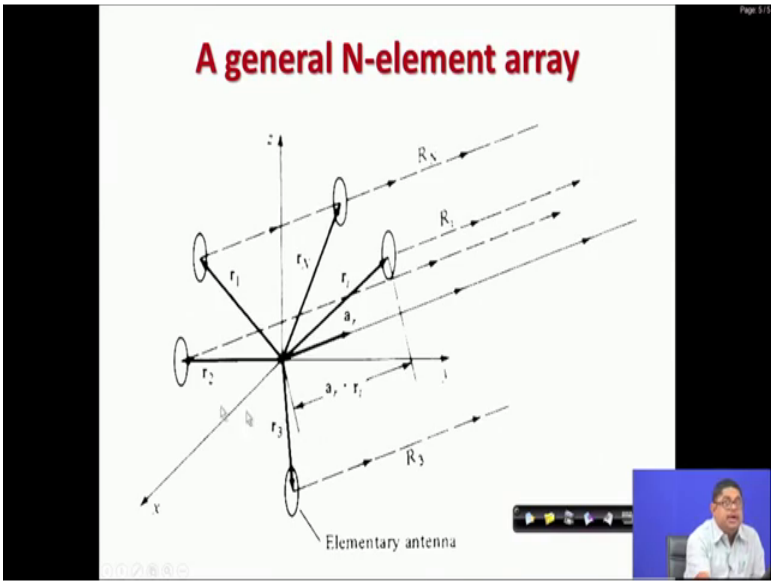

If you look at this wave graph that all these antennas there they are identical antennas so there f theta phi is same. Now pattern multiplication assumes that everyone is having these, but in realistic and practical cases this is not true because, if you have an array antenna now since they are very closely spaced together actually we will prove later that array antennas need to have the spacing need to be less than one wavelength. Now for all array antennas so there will be strong mutual coupling between them. So, if I have an large array, so the central elements and the far away elements or the extreme edges element their pattern is not same because of mutual coupling.

Now, that is neglected in pattern multiplication, so if in an actual evaluation that mutual coupling if it is not taken in to account then the whole calculations goes array. So, pattern multiplication is a simplified one, but at your level under graduate level or graduate level that is, but in the research level or actual evaluation that there are ways to factor in to that mutual coupling factor.

Now here after we will study only the array factor and we will start with the first type of array is uniform linear array. Now uniform there are 2 types of array uniform or non uniform array. Now uniform does not mean that or non uniform does not mean that the array elements are non uniform no, always in array theory all the elements are uniform. But uniform linear array means the spacing between all the elements are same.

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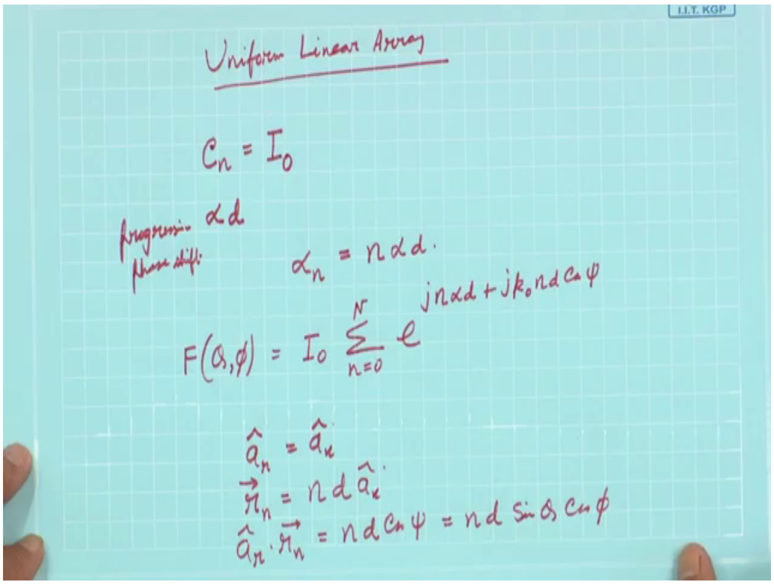

So, spacing is uniform this is the minimum uniform array linear means the all array elements are placed along a line.

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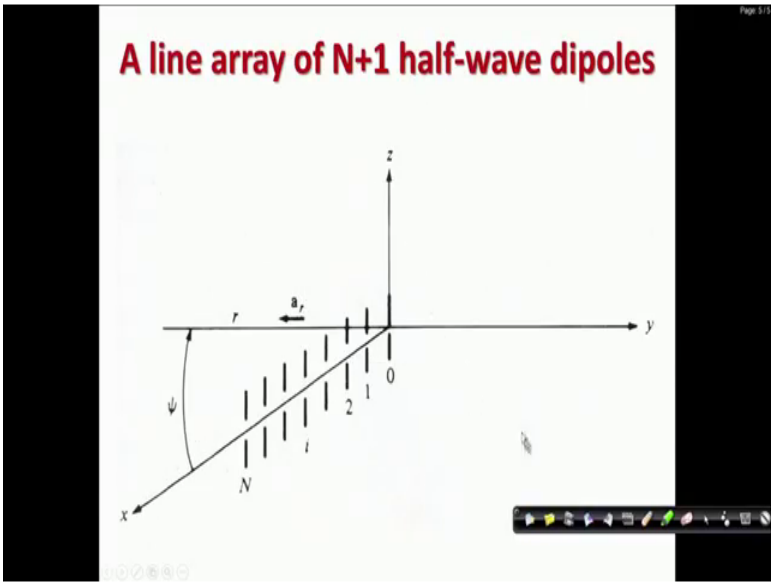

So, this you see that I can say here again, you can see the that this is dipole but it could have been anything, here without loss of generality we are assumed dipoles this could have been monopoles this could have been horn antennas this could have been opponent guide antenna etcetera etcetera. Now to see that the elements are placed along x axis a line and there are N plus 1 such elements because, first one we are naming zero 12 etcetera, I then it goes to capital N. So, this observation point is here p and again this r distance. So this psi is a solid angle this is the solid angle.

So, the observation point or radius vector of the observation point makes an angle psi with the array axis, this is the standard that terminology that will follow ok. So, array axis is x and from that at an angle psi we have the observation point, a r is the unit vector element. And here we say that each antenna is excited with same constant amplitude and since we have assumed dipole.

Now we can say that amplitude excitation is current I naught, but with a progressive phase shift the pro phase shift is alpha d, this is called progressive phase shift. So, I can say that the phase shift of the n-th element small n-th element that will be n alpha d. So, I can write what will be the array factor I naught n is equal to 0 to capital N e to the power j n alpha d plus j k naught n d cos psi.

Why I am writing these? Because, you see my previous expression was this is k naught a r dot r i. So, what is the value of a r dot r i in this case a r is equal to ax and r i or you can say r n will be in n d ax.

So, a r dot r n is equal to n d cos psi and that is equal to n d can I write sin theta cos phi ok. So, I can put these and what is this? This is a geometric series. So, in a geometric series if I have I can say that it is what is that the geometric ratio.

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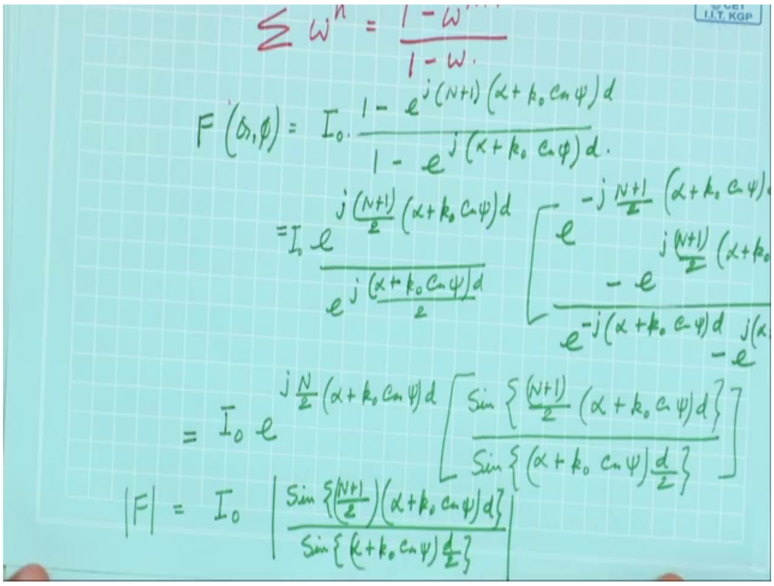

Suppose I have sigma, so what is the value 1 minus omega since I have n minus 1 ok. So similarly here I can write f theta phi is equal to I 0 one minus e to the power j n plus 1 alpha plus k naught cos psi d by 1minus e to the power j alpha plus k naught cos phi d. So, again the trick is you know you take half of these out. So e to the power I naught e to the power and take a common e to the power j n plus 1 by 2 alpha plus k naught cos psi d. And here you take e to the power j alpha plus e to the power j alpha plus k naught cos psi d by 2.

Then you can write it as e to the power minus j N plus 1 by 2 alpha plus k naught cos psi d minus e to the power plus j N plus 1 by 2 alpha plus k naught cos psi d divided by e to the power minus j alpha plus k naught cos psi d minus e to the power plus j alpha plus k naught cos psi d that sum pretty simple. So, this becomes I naught e to the power j N by 2 alpha plus k naught cos psi d sin of N plus 1 by 2 alpha plus k naught cos psi d by sig of alpha plus k naught cos.

So, this is array factor and this is also field expression. So, array field pattern will be a I can easily write what is F is I naught this will go amplitude and this is nothing but sin N plus 1 by 2. Now do not get frightened by looking at big expressions, these are simple expressions may be looking big we will just take steps to simplify that, so that that becomes manageable. What are those step very important steps that we will just assume 2 things we will introduce 2 new variables, one is u is equal to k naught d cos phi and u naught is equal to alpha d.

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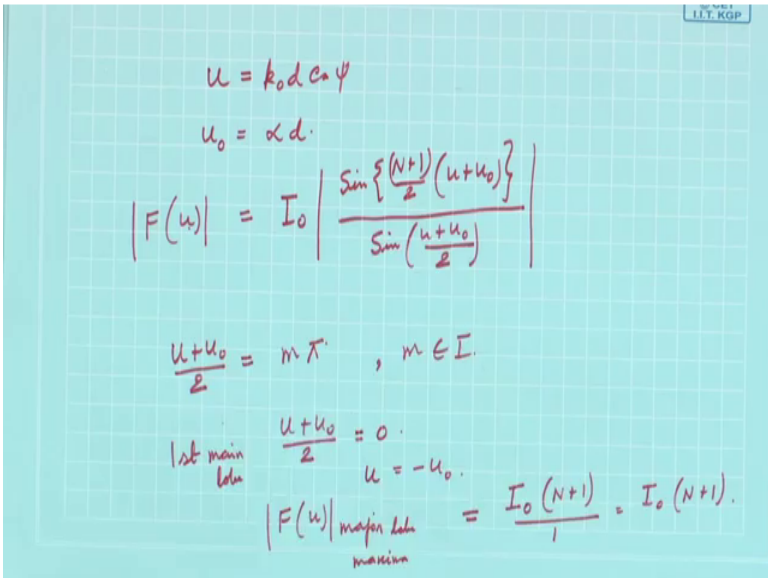

So, the same F where theta phi is now in terms of u, so it is a function of u now and these becomes I naught sin. So, if I we plot these function in the u plane, that means x axis is u and this magnitude is this it becomes something like this.

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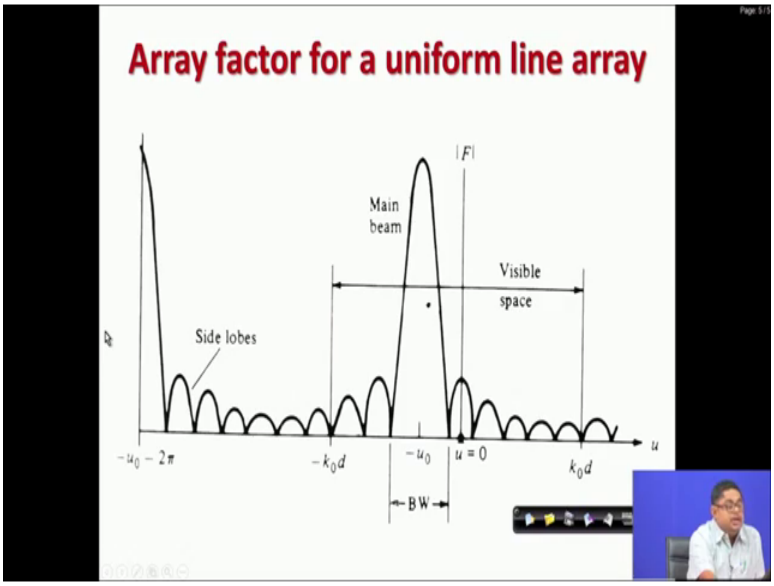

So, you see that there is a maximum value in main lobe then there are side lobes progressively decreasing and seen this is magnitude all are positive. So, this is something like sink function, but only difference that this is a periodic function sink is not a periodic function. So, this is you can say behaves much like the our normal sink function it is sin u by u, but here this is sin N plus 1 by 2 u by sin u.

So, this is a different function this is the heart of array antenna study this is called the array function. Now here you will see that we have plotted versus u, but we will see that this all values of u are not physical values because, by we will see that and the important thing to note from here is that the peak comes at a point where u is equal to minus u naught.

So, the peak is not that u is equal to 0 peak is at u is equal to minus u naught. If you make u naught to be 0 then the peak comes at u is equal to 0. Now what is u naught u naught is dependent on the progressive phase shift. So, if I do not give a progressive phase shift which we have seen we do it in broad side cases, then the main beam will come at this point. But otherwise it is at minus u naught and we can find out what is the; I will explain later why these visible space. And all these things are called because those are important things and you may note that in array things.

We have a beam width definition slightly different from the normal antennas beam width, in antennas beam width we have a generally 3 dB or something beam width here we are taking the beam width definition to be null to null that is the only difference. That time also we have said that null to null can be taken generally in an array antenna we take null to null beam width and what are the main lobes.

So, whenever we get u plus u naught by 2 is equal to m pi you see u plus u naught by 2. If it becomes m pi then I can get a main lobe because, this actually that time this is the singular case that is 0 by 0 case and that is much greater than any of the actual maximum of this point.

So, u plus u as we are m pi where m is any integer, so now you can find what is the first main lobe we will occur at u plus u 0 by 2. So, I can say first main lobe is that means u is equal to minus u 0 and what is the value of that F major lobe maxima value. Here, will have to apply the l hospital rule we will have to differentiate. So, you do that and you will find that it will be I naught N plus 1 by 1 that is I naught N plus 1.

So, basically in the main lobe maxima all the radiated fields from elemental antennas are added in phase. So, at that point all are added in phase. Now let us come to the side lobes when side lobes we will occur what is the conditional side lobe to occur. The numerators sin function should be maximum or 1 and denominator is not 1 then only we will get a side lobe.

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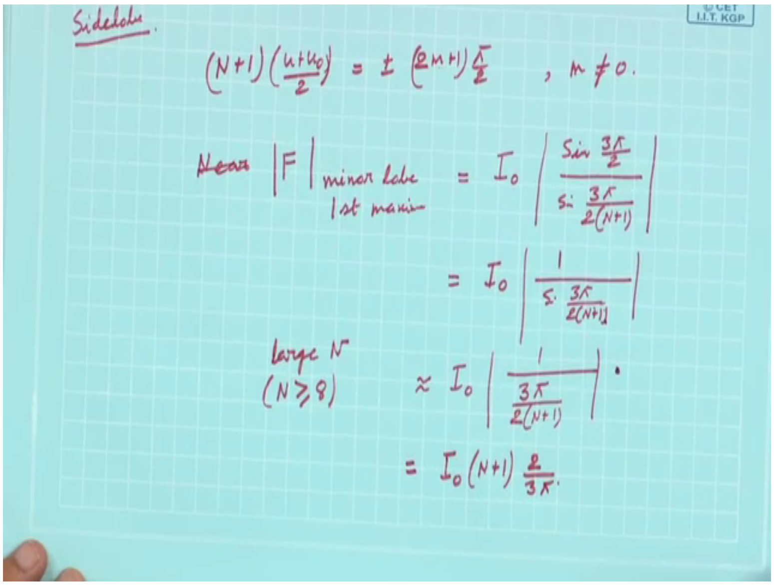

So, put that condition that means what is a side lobe maxima condition that is N plus 1 u plus u 0 by 2 is equal to plus minus 2 m plus 1 pi by 2. And I will have to exclude the main lobe condition m is equal to.

So, now is not it so from these you can see that the amplitude of the side lobe maximum will vary as also can be seen in the graph. You see that amplitude is varies we can find the nearest 1. Nearest 1 will be I can write F minor lobe first maxima that will occur at I naught. So, first m is not 0 so the first value it can take is 3 1 m is 1, so sin 3 pi by 2 by sin 3 pi by 2 N plus 1. Please note that m is equal to 0 this thing becomes what plus minus pi by 2 and that does not give a minima because these also becomes very high. So, this is I naught 1 by sin 3 pi 2 N plus 1.

For large m for how large typically if N is greater than equal to 8, we can say that sin 3 pi 2 N plus 1 that is the argument of sin is very small, so it can be approximated by the angle itself. So, we can put that this is approximately I naught 1 by 3 pi by 2 N plus 1. So, it becomes so that means, please look at the diagram that this is this value and we know this value.

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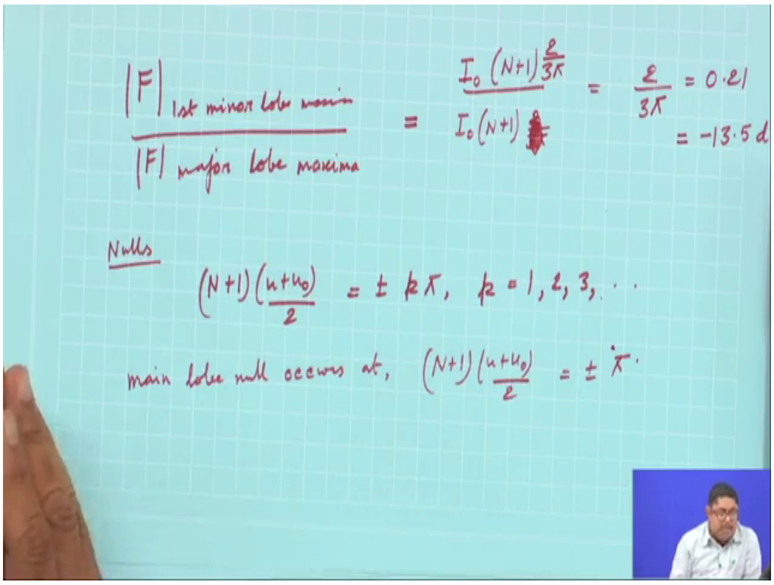

So let us compare F first minor lobe maximum by F major lobe maxima I naught N plus 1 by I naught N plus 1 2 by 3 pi. So, that becomes 2 by tell no they should go up 2 by 3 pi they should be done is equal to 2 by 3 pi and that value is 0.21 and that is minus 13.5 d.

So, this is the general this is a truth, that any uniform linear array any uniform linear array the first side lobe is only minus 13.5 dB the main lobe this is a serious problem in any practical work. By hooker by crook you cannot avoid these, so uniform linear arrays need to be improved, because in practical cases at least minus 30 dB the side lobe should be down. It is better if it is more, but at least minus 30 dB down is necessary for any radar or other type of applications. So, uniform linear array cannot put that beam and that is why people have non uniform arrays and another things.

But this you should always remember that this is a law of nature you cannot break and this is independent of the number of elements in the linear array. So, provided n is at least 8 which is typical. So, if n is greater than 8 then even if you go on pumping more and more arrays you cannot put the side lobe down. That will we will see that if you go on increasing the length of the array, your main beam will be much narrower; that means, you have beam width will be decreasing in main lobe will be nothing but the side lobe you cannot put it down more actually.

And where are the nulls? Nulls are easy to find because if you see the actual function. So, whenever this will give this whole thing is pi sort of thing or multiple will have pi even multiple of pi these won't be even multiple of pi.

So, never 0 by 0 case will come. So, based on that so what are nulls? Nulls of the pattern array N plus 1 u plus u 0 by 2 is plus minus k pi, where k is odd even numbers. So, main lobe nulls occurs at N plus 1 u plus u 0 by 2 is equal to plus minus pi ok.

As a function of u the array pattern repeats you can see the array pattern. So, array pattern repeats every 2 pi units along the u axis. If you put it here u plus 2 pi you will see that the whole thing is repeating and we now note that what is u? U you have introduced to be this.

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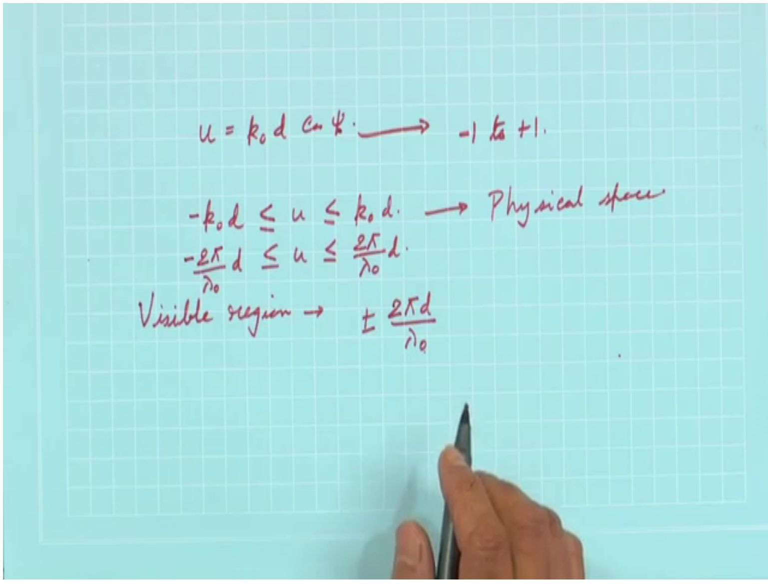

Now cos phi cos psi that can take only cos psi is nothing but sin theta cos phi, so that can take value plus 1 or minus 1 at the most, That means, the range of u corresponding to physical space, because this psi or this theta phi real means this is real means this is physical.

So, visible religion can I say that what will extend from minus k naught u u plus k naught d because, cos phi is from minus 1 to plus 1 cos phi is varying this; so physical space is this that means, u is having a physical value as long as these.

The range of u corresponding to physical space that is called visible region, but actually after every 2 pi we can go on and actually there is techniques by which you can change various antennas array, antennas performance if you go in to the invisible domain hm. Actually that is the way by which the antennas reactive impedance are antennas can be changed, that is an interesting story fortunately after might be take I was introduced to that that was my first professional assignment after passing out and we have a publication from there.

So, ok, but this visible region so you can see that is why here we are making the visible region, is visible space is k 0 d to minus k 0 d in this figure which is this value. So, also we can in practice we require only 1main lobe to occur in physical space if more than 1 comes. That means in this space that means from here to here in instead of this main beam another beam.

That means, you see this is another beam speak now if by some array design this can be changed and come to this space, then there will be more than one main beam in the visible space so that time this is called grating lobe. So, if more than one main lobe comes in to the visible space then they are called grating lobe. To avoid grating lobe we choose the spacing this small enough, so that the region plus minus 2 pi d by also that plus minus this cannot d by 2 means minus 2 pi by lambda naught d 2 pi by lambda naught d. So, the total region is 2 pi d lambda naught.

So, that region plus minus d by lambda naught on either side of u is equal to on either side of u naught does not include another main lobe. So, that is done by the spacing so that means if you make spacing large you can avoid or you can invite grating lobes in to the visible zone. So, we will see that that so what are the requirements on gain the next lecture. And actually if there are 2 special cases of this uniform linear array. I have already mentioned we have shown the photograph for 2 element array 1 will be called broad side array where the radiation on major lobes peaks perpendicular to the axis of the antenna. And another is n fire array where the maximum radiation or a main lobe is along the axis.

So, we will see them one by one broad side array and long side by n fire array, though terms come from radar this those times it was pointed with the that what is that armady

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Thank you.