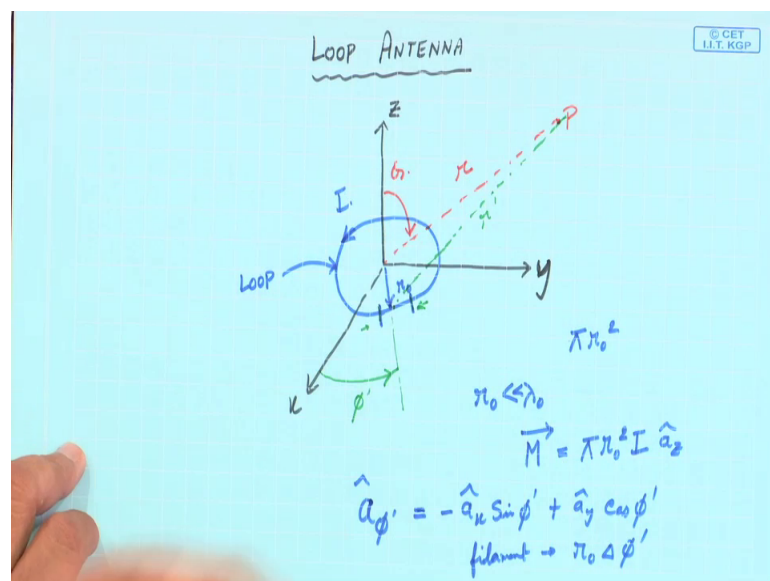


Analysis and Design Principles of Microwave Antennas
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Lecture – 15
Loop Antenna

Welcome to today's lecture, today will see another basic antenna which is a Loop Antenna.

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So, we have seen current element which is a wire antenna; a type of wire antenna which is constant current throughout that the dual to that is a loop where we have constant current, but current is in a loop. Let us say the loop is placed in the xy plane here is it is the xy xy plane.

So, the loop is blue circle that is a loop placed in the xy plane. So, the loop axis is in the z axis. Now here you see that from your knowledge from class intermediate classes that the magnetic field that will be in the axial direction. Because of the symmetry of the structure each opposite pair they will cancel and the magnetic field will in the z direction

So, this is from duality we can easily claim that whatever we have seen for current element it is a dual structure. So, we can say that if we change the electrical magnetic field; we get the answer, but we will not do that from duality it can be easily said what it

will be its field, but we own think of duality we will try to derive the from the basic first principle.

So, what is the field at a point P? So, let the radius of the loop is r naught. So, as we have seen that this loop radius is r naught and so, the area of the loop is πr naught square and a current I is flowing in the loop. So, axis of the loop I only you said in the z direction; now we assume that this is a small loop that actually helps in solving some mathematical complexity.

So, we assume that r naught the radius of the loop is much less than the much less than the operating wave length. So, if we say this then we can say that the loop may be treated as a point source. So, it is a magnetic dipole it is magnetic dipole moment you know that magnetic dipole moment M that we can say will be the area of the loop into the current. And dipole moment is z directed I said this you already know because one side of it will be south pole, another side is north pole will be created and the dipole moment will be in the Z direction.

Now, this one will radiate because you see always the current is accelerating because the direction of the current is changing. Though the current magnitude is same, but vectorially the current is always changing. So, that is why it is an always accelerating current; so, there will be radiation from the loop. And we know I already said that the field radiated by a small magnetic dipole is dual to the field radiated by a Hertzian dipole are current element. So, electrical magnetic field can be interchange etcetera.

So, let us see that consider; so how to analyze this from first principle? We will take a small portion infinite definite portion of the loop that is for circular cases; we call it a filament. So, this small filament at an angle ϕ dash azimuthal angle ϕ dash; we are taking. So, again the prime quantities are source quantity; so we are taking a small filament here and what is the vector for that? The unit vector for this ϕ we can write; obviously, it will be directed in this the increasing direction increasing of the angle ϕ dash.

So, it will be in that direction; that means, in this case it will be in this point. So, at here it will be increasing in this direction. And we can write it in our Cartesian coordinate; so, that we can write as minus $a \sin \phi$ dash plus $a \cos \phi$ dash.

So, here the length of this filament is clearly $r \sin \theta \, d\phi$. So, the filament's length I can write the filament that we are considering its length is $r \sin \theta \, d\phi$. So, this is the counterpart of the dl in the elemental a . So, the contribution of this we can find out the vector potential of this filament of current. So, if we know that then for all the filaments we can just sum that will be sum over all this contour.

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Handwritten derivation on a blue board:

$$\vec{A} = \mu_0 (I dl) \frac{e^{-jk_0 r}}{4\pi r} \hat{a}_z \rightarrow \text{Current Element}$$

$$\text{Loop} \rightarrow d\vec{A} = \mu_0 (I r_0 d\phi') \frac{e^{-jk_0 r'}}{4\pi r'} \hat{a}_{\phi'}$$

Magnetic vector potential due to filament $r_0 d\phi'$

$$= \frac{\mu_0 (I r_0) d\phi'}{4\pi r'} (-\hat{a}_x \sin \phi' + \hat{a}_y \cos \phi') e^{-jk_0 r'}$$

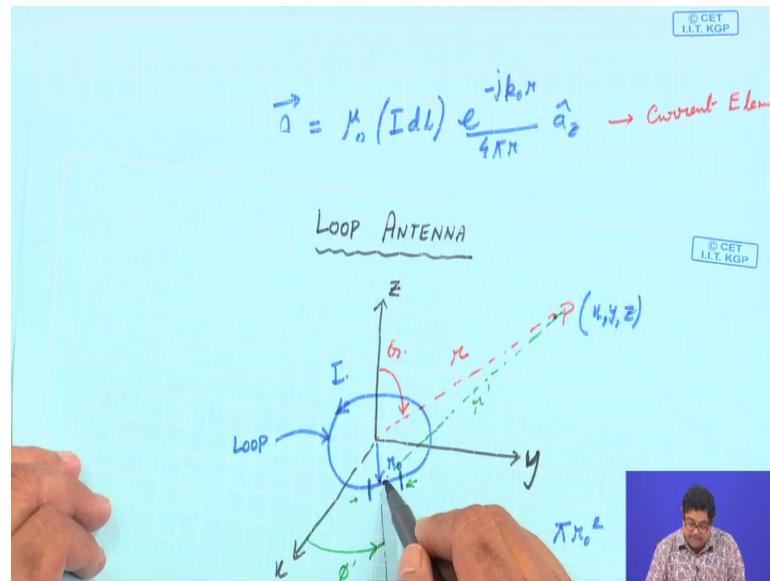
$$r' = \left[(x - r_0 \cos \phi')^2 + (y - r_0 \sin \phi')^2 + z^2 \right]^{1/2}$$

$$\vec{A} = \frac{\mu_0 I r_0}{4\pi} \int_0^{2\pi} \frac{e^{-jk_0 r'}}{r'} (-\hat{a}_x \sin \phi' + \hat{a}_y \cos \phi') d\phi'$$

So, let us recall that for a current element Idl what was our magnetic vector potential, magnetic vector potential was μ_0 naught; this is for current a current element Hertzian dipole. So, I will write this is current element case; so, here we can write that for loop what will be magnetic vector potential? For this current; so, it will be μ_0 naught in place of Idl I will write $I r_0 \sin \theta \, d\phi$ or $d\phi$ to the power minus $j k_0 r$ by $4\pi r$ this one. So, let me write to the power minus $j k_0 r$ dash by $4\pi r$ dash a_{ϕ} dash.

So, now this a_{ϕ} dash I have this unit vector I have expressed. So, I can write it in terms of that and this r dash also. Now I can write it in terms of Cartesian coordinate, what is it?

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Because this point P; so it is a; if I say it is at x y z; then easily what is r dash? x minus the x coordinate of this x dash point. So; that means, can I write x minus r dashed cos phi dashed square plus y minus r dashed sin phi dashed square plus z square to the power half.

So this is the loop I can say this is the magnetic vector potential due to filament at due to filament delta phi dashed. So, total vector potential or these also I can write it as dA. So, the total vector potential for all the filaments; that means, for the whole loop that I can write as mu naught I r naught by 4 pi 0 to 2 pi e to the power minus j k naught r dashed by r dashed minus ax sin phi dashed plus ay cos phi dashed d phi dashed.

So, this integral if can be integrated, we get the vector potential for a loop of current. But this integral is not so easy to evaluate, on this we make certain approximation; we will make those approximations now. And please remember that we are primarily interested in the far field of the antennas.

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$$r \gg r_0.$$

$$r_0 \ll \lambda_0.$$

$$k_0 r_0 \ll 1.$$

replace r' by r

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$r^2 = x^2 + y^2 + z^2.$$

$$n' = \left[(n \sin \theta \cos \phi - n_0 \cos \phi)^2 + (n \sin \theta \sin \phi - n_0)^2 + n^2 \cos^2 \theta \right]^{1/2}.$$

So, for the that case; that means, we are mainly interested for r much greater than r_0 naught the far field. So, already we have defined that r_0 naught is much less than λ_0 naught because that is our small loop. So, can we say that $k_0 r_0$ naught that will be much less than 1. So, by these 2 we can replace that r' dashed by r ; so, we can replace r' dash by r because you can see from here that if this point is far away so, the difference between r and r' dash is this.

So, that can be neglected in the amplitude part so; that means, a here this r' dashed that we can replace by r not here because that will disturb the whole thing. So, we can now write what is r' this; now what is x can we write is now as x is equal to $r \sin \theta \cos \phi$, y is $r \sin \theta \sin \phi$ and r^2 square is x^2 square plus y^2 square plus z^2 square. So, we can write r' dashed as $r \sin \theta \cos \phi$ minus $r_0 \cos \phi$ square plus $r \sin \theta \sin \phi$ minus r_0 plus r^2 square whole square θ to the power half.

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$$\begin{aligned}
 r' &= \left[r^2 + r_0^2 - 2r r_0 \sin\theta \cos\phi \cos\phi' + r r_0 \sin\theta \sin\phi \sin\phi' \right] \\
 &\approx r^2 \left\{ 1 - \frac{2r r_0}{r} \sin\theta \cos\phi \cos\phi' + \frac{r r_0}{r} \sin\theta \sin\phi \sin\phi' \right\} \\
 &\quad [1+u]^{1/2} \approx 1 + \frac{u}{2} \rightarrow u \ll 1. \\
 &\approx r - r_0 \sin\theta \cos\phi \cos\phi' + r_0 \sin\theta \sin\phi \sin\phi' \\
 e^{-jk_0 r'} &\approx e^{-jk_0 r} e^{+jk_0 r_0 \sin\theta \cos\phi \cos\phi' - jk_0 r_0 \sin\theta \sin\phi \sin\phi'} \\
 &\approx e^{-jk_0 r} \left[1 + jk_0 r_0 \sin\theta \cos\phi \cos\phi' - jk_0 r_0 \sin\theta \sin\phi \sin\phi' \right]
 \end{aligned}$$

So, this; that means, r' can be written as $r^2 + r_0^2 - 2r r_0 \sin\theta \cos\phi \cos\phi' + r r_0 \sin\theta \sin\phi \sin\phi'$ to the power half. So, we now put the far field condition; so, we can neglect this r_0^2 in presence of this r^2 . So, we can write this after neglecting this term that; this will be r^2 along there will be another one.

Now, please remember that $1 + u$ to the power half can be approximated as $1 + \frac{u}{2}$ when u is much much less than 1. So, here also can I write that this term because r_0 you see already we have said that r is much larger than r_0 ; that means, r_0 is much smaller than r and these are all terms which are less than 1. So, we can say that this is very small compared to 1 and then we can write it as $r - r_0 \sin\theta \cos\phi \cos\phi' + r_0 \sin\theta \sin\phi \sin\phi'$.

So, this we can write here in the exponential function; that means, in the phase term we will have a term involving $k_0 r_0$; when we substitute the already derived this expression, but $k_0 r_0$ will be less than 1. So, we may again use this thing or e to the power u becomes $1 + u$ when u is much much less than 1. So, by that we can write e to the power $j k_0 r'$ that will be to the power minus $j k_0 r$; then e to the power plus $j k_0 r_0 \sin\theta \cos\phi \cos\phi' - j k_0 r_0 \sin\theta \sin\phi \sin\phi'$ because this is the r' is this.

So, this term into this e to the power minus j k naught to the power plus jk r naught sin theta cos phi cos phi dash plus sin phi sin phi dashed. And this can be written as 1 plus j k naught; r naught sin theta cos phi cos phi dash plus sin phi sin phi dashed.

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$$\vec{A} = \frac{\mu_0 I N_0}{4\pi r} e^{-jk_0 r} \int_0^{2\pi} (-\hat{a}_x \sin \phi' + \hat{a}_y \cos \phi') [1 + jk_0 r_0 \sin \theta (\cos \phi \cos \phi' + \sin \phi \sin \phi')] d\phi'$$

$$= \frac{j k_0 \mu_0 (N I)}{4\pi r} e^{-jk_0 r} \hat{a}_\phi$$

$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A}$$

$$= -\frac{1}{\mu_0 r} \frac{\partial}{\partial \phi} (N I a_\phi) \hat{a}_\theta$$

$$= -\frac{N I}{4\pi r} e^{-jk_0 r} \hat{a}_\theta$$

So, this term we have got now. So, we will put this into the vector potential equation and that will give us mu naught I r naught by 4 pi r e to the power j k naught r 0 to 2 pi minus ax sin phi dashed plus ay cos phi dashed into this term; 1 plus j k naught r naught sin theta cos phi cos phi dashed plus sin phi sin phi dashed d phi dashed ok.

So, from here we will see that only terms that do not integrate to 0 are the cos square and sin square terms all other terms since we are having a 0 to 2 pi integration. So, a single terms all will integrate to 0 and cos square; whatever angle from 0 to 2 pi or sin square phi, they will give rise 2 each integration 0 to 2 pi a cos square term gives you a factor of pi sin square will get that.

So, we can write this as j k naught, mu naught, pi r naught square I by 4 pi r sin theta e to the power minus j k naught r a phi; where a phi is the actually this term. So, here we getting a phi, but this is a filed quantity. So, a phi means minus ax sin phi plus ay cos phi naught the primed one. So, we have got a; so, once we got a we can we know we can find the magnetic field; magnetic field will be 1 by mu naught del cross A. So, if you do that in this case it will be minus 1 by mu naught r del; del r r since I have only a phi. So, it will be a theta directed field; so, if you solve because now you have this thing. So, you

can write it in terms of magnetic dipole moments as where M we have used that magnetic dipole moments μ r square a i . So, this is the far field of magnetic field.

So, from we know the structure of electric field and magnetic field relation in the far field.

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$$\begin{aligned} \vec{E} &= -z_0 \hat{a}_r \times \vec{H} \\ &= -z_0 \hat{a}_r \times -H_\theta \hat{a}_\theta \\ &= z_0 H_\theta \hat{a}_\phi \\ &= \frac{M z_0 k_0^2 \sin \theta}{4\pi r} e^{-jk_0 r} \hat{a}_\phi \end{aligned}$$

$$\begin{aligned} P_r &= \frac{1}{2} \operatorname{Re} \int_0^{2\pi} \int_0^\pi E_\phi H_\theta^* r^2 \sin \theta \, d\theta \, d\phi \\ &= \frac{M^2 z_0 k_0^4}{16\pi^2} \int_0^{2\pi} \int_0^\pi \sin^2 \theta \, d\theta \, d\phi \\ &= \frac{M^2 z_0 k_0^4}{12\pi} = \frac{1}{2} |I|^2 R_r \end{aligned}$$

So, electric field in the far field will be minus z naught a r cross H . So, this will be minus z naught a r cross minus H theta a theta. So, that will be z naught H phi; a phi and that value will be M z naught k naught square \sin theta by 4 pi r e to the power minus j k naught r a phi.

So, if you just compare that now the for a loop; the E field has a phi component and H field it has a theta component just dual 2; that means, electric and magnetic field got just interchange with respect to the current element or Hertzian dipole, but since these are things. So, can we say you see the variation with respect to theta is only theta in both the cases.

So, same as the current element; so, from this we can say that radiation pattern will be same and directivity also will be same. So, now let us calculate what is the other thing; that means, directional characteristic is same but what is the radiation resistance? So, for that we need to calculate the total radiated power P_r . So, total radiated power will be half real E phi H theta star r square \sin theta d theta d phi.

So, this will be $M^2 \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$; $16 \pi^2$; now this P_r ; we need to equate to half $I^2 R_a$ here I is constant. So, we are taking the adamant's value; that means this. So, if we compare we get the radiation resistance.

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$$R_a = 320\pi^6 \left(\frac{r_0}{\lambda_0}\right)^4$$

$r_0 = 10 \text{ cm}$ at 1 MHz .

$$R_a = 3.8 \times 10^{-9} \Omega$$

N turns $\rightarrow R_a = N^2 R_{a \text{ single turn}}$

Ferrite core μ $R_a = \mu^2 N^2 R_{a \text{ single turn}}$

The radiation resistance is $320 \pi^6 r^6 \lambda^{-4}$; so, this is the radiation resistance. So, π^6 means roughly 10; so, this gives you 1000. So, you will be thinking that this is very high value, but this r^6 is already we assumed it is a small current element. So this to the power 4 that makes it; so, let us do a take suppose the radius of the loop is 10 centimeter and at 1 megahertz.

So, you can calculate the radiation resistance turns out to be 3.8×10^{-9} ohm; again very poor radiator and ohmic resistance will be much larger than this. So, very inefficient radiation efficiency is very inefficient. So, gain is very poor for this loop then why it is used? But this can be increased a bit if we instead of a single turn of a loop; if we use N turns, the radiation resistance will increase by n square.

So, we can make that N turns; so, R_a will be n square into R_a of single turn then also to put more people use some ferrite core if you do. So, the radiation resistance will again increase by so, if you have a ferrite core whose permeability is μ ; then radiation resistance will be $\mu^2 N^2 R_a$ single turn etcetera by that you can put.

But this generally is heavily used actually in early days; nowadays the planar antennas are available, but before that advent of planar antennas, all radio receivers, radio receivers they were having this loop in the receiver; that means, a users box the radio receiver that used to have a coil. Now, question is why? It is a very interesting question is as that why where this loop antenna which is such a bad radiator is so, heavily used in this.

Actually in generally in transmitting mode no loop antenna is used anywhere, but in pocket radios they are used. Because the reason is that low frequencies atmospheric noise is dominates; that means, the whole radio deception that is limited by this atmospheric noise.

Now, we have already seen when we have seen the antenna noise temperature or antenna temperature that whatever noise you are getting that gets waited by the gain of the antenna. So, if you use a good antenna then actually you are inviting more noise. So, you should have a very poor antenna to receive the signal because then your noise will be less. Because all these normal radio communication they are mostly bothered with what is the SNR that is received. And that is why the SNR for this antenna will be very good because its gain is very poor. And in actually in radio communication of voice this signal power is quite higher.

Actually the; whatever since they have a huge transmitter stations and huge power they use to pump. So, there are plenty of voice power signal power available. So, efficiency is not a concern there, but SNR is a concern. So, to defeat noise they have chosen these and that has successfully been used for yeah. Now this is an very good example that you see inefficiency sometimes are used. We used to say that sometimes in the whole this human community also; you will see that sometimes the non intelligent persons they serve some very useful purpose this is one of the way of looking at it. So, do not always say that intelligence is good sometimes non intelligent persons becomes very beautiful human beings.

So, that is the beauty of nature and actually the size of a small loop was quite attractive. And also its, since it is you see that in a dipole or other things you have the polarization problem because the reception needs to be in that polarization, but for a loop only the

axis. So, whatever way you turn your radio receiver the axis is same; so, its reception is quite good.

So, because of their small size etcetera ferrite loop antennas of few turns wound around a small ferrite rod are used widely in pocket transmitter radio. The antenna is usually connected in parallel with RF amplifier tuning capacitance for tuning; in addition to acting as antenna it furnishes the inductance to give tune circuit. Because inductance is obtained by ferrite the loss resistance of few turns is quite small thus the Q of such antennas is quite high and results in high selectivity and greater induced voltage at the radio receiver.

So, with this we see these antennas and many times you will see that a loop sometimes gives a much needed thing. Because many times you cannot have a wire antenna of proper size or the space is not there or the mechanical strength is not there; loop can replace that in many cases, in many high and things also a loop antenna sometimes are used not always very rarely, but in some applications they are good applications as I said.