

Analysis and Design Principles of Microwave Antennas
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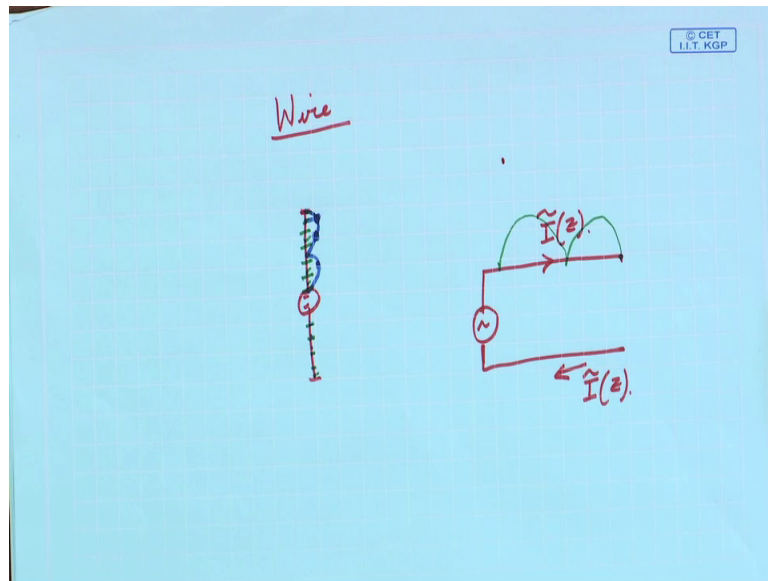
Lecture – 12
Dipole and Monopole Antenna

Welcome to this lecture. Now, we have seen various basic parameters of antennas. Today we will see a really useful antenna. Already, we have seen the elemental antenna that is hertzian dipole or current element, but we have seen the time that current element is not a practical antenna; because of several reasons, one was that its length is infinite decimal. So, it is very small and for that we have seen that its radiation resistance is very very small.

So, if we want to have a sizable amount of power radiated, we need to have a huge current in the antenna, which is not practical. Also, we have seen that in the current element the current is constant throughout. Now, that is not a physical approximation because, if we look at any length of an antenna and if throughout the current is constant; that means, at the end points also, the current is non-zero, but immediately after that there is free space. So, conduction current cannot be present there. So, where that current goes?

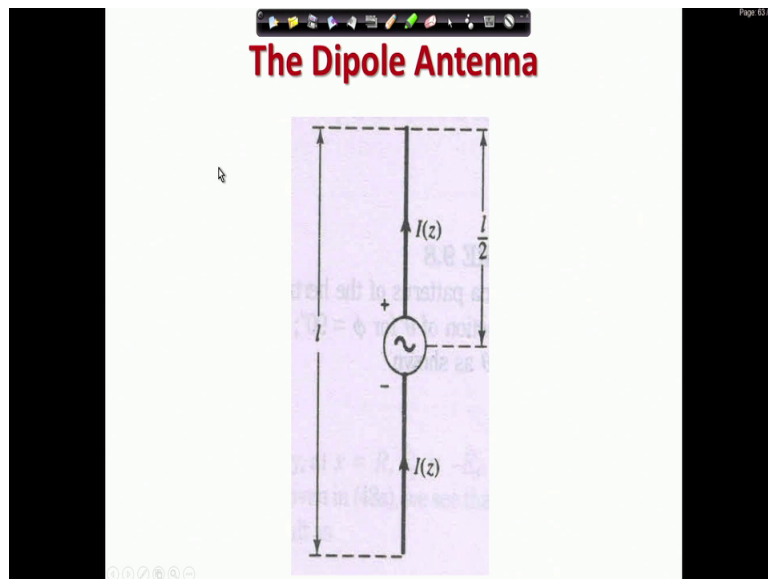
Now, we know that it goes to conduction current gets transferred to displacement current, but if throughout we assume there is same current, then there is a severe discontinuity there. So, it is not feasible to have a practical thing, but we have already noted that current element is the assigned force of antenna, it shows all the properties that any other complicated antenna have particularly the far field structure various other properties etcetera. So, that is why we have seen that, today we will see a really useful wire antenna first we are starting with wire antennas, antennas which are wires for type of wire structures; that means, it has length it does not have any width ideally.

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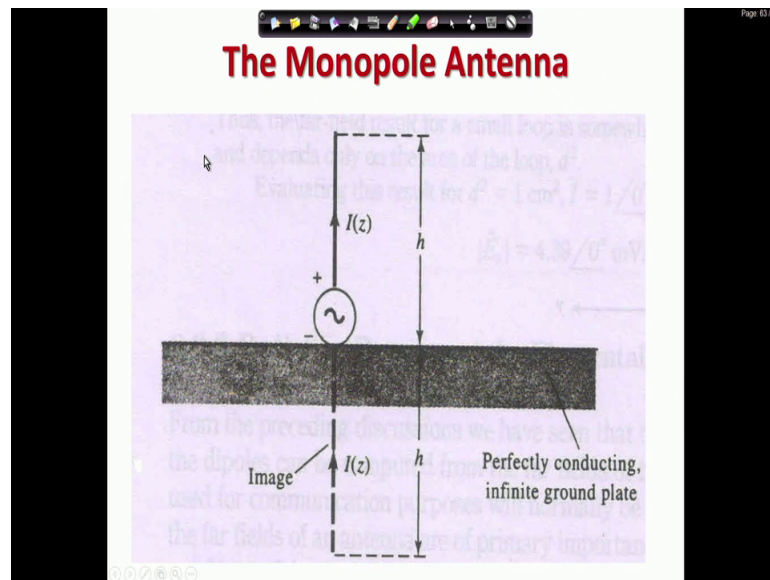
So, the 2 such examples today we will see one by one.

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The first one you can see in the view graph.

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That, sorry the first one we will see the dipole antenna, which everywhere you will be seeing that there are these dipole antenna, it is very popular it is very useful. So, it is a length l . So, it is symmetrical it is have two such parts each part is having l by 2 these are called poles of the antenna.

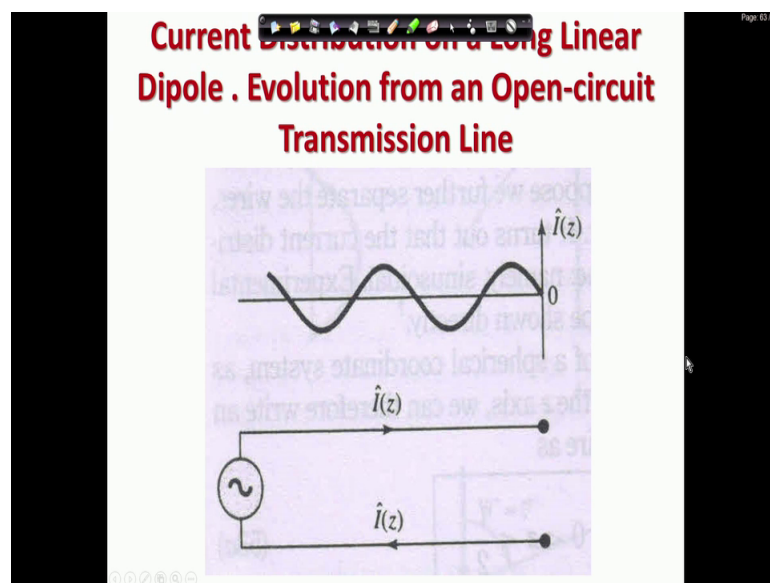
So, this is the upper pole this is the lower pole and between them there is a source ac source, ac voltage source, which is there between the 2 antennas this is the structure of dipole. Now, sometimes in particularly at lower frequencies if this length is quite high if the such a high length is not permissible, another variety of this dipole antenna which is called monopole antenna is also there.

So, in this case the source is there and from the source, there is only a single pole the upper pole is there the lower pole is not there. So, how they make it? So, there should be a infinite ground plane infinite perfect ground plane. So, that by image theory it is the dipole. So, if we place a ground plane here, then we need not have the lower pole. So, the this is called a monopole antenna of height h .

So, these two structures today we will try to analyze. First, we will start with the dipole. Now, in case of dipole how to analyze first; obviously, we will we are noted that any antenna, if we want to find it is fields radiated, we want to find the current distribution; Because, from the current distribution by solving Maxwell's equations, we can find the fields.

Now, this for so; that means, the first job will have to have what is the conduction current that is flowing on the antenna. Now, here we will have to have a gauge you see this is a long antenna. So, antenna is like this. So, we know that physically the current actually we are feeding here; that means, the current may be maximum here, but it should go to 0 at the ends. So, what can be the structure of that to understand that we take the help of a transmission line because; we have already seen the transmission line.

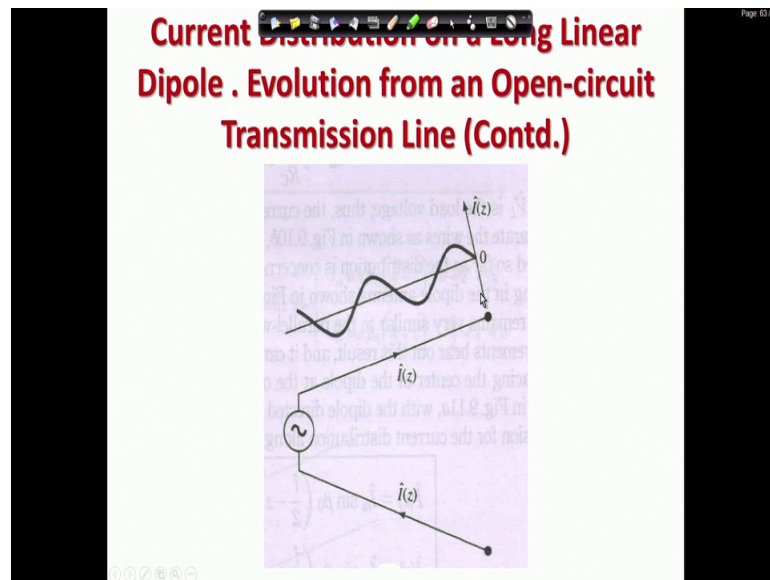
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So, let us see a transmission line which is fed and open and open circuit transmission line. So, in a transmission line we have already seen in the em theory classes, that if I have a current $I(z)$; obviously, here the radiation is taking place. So, this current conduction current is getting converted to displacement current and the return current is like this and we know how this current is distributed?

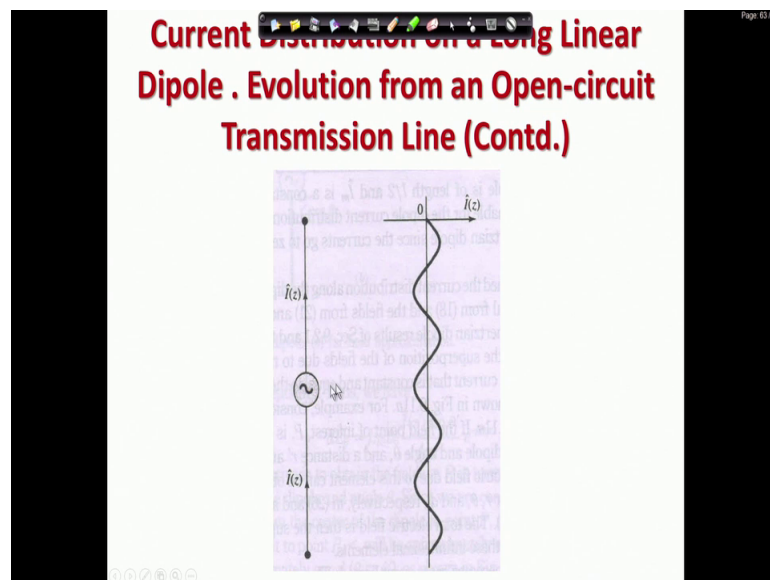
So, since this is an open circuit. So, here the current should be 0 the voltage is maximum. So, current has a minimum here; that means, we have a distribution like this that we have seen in case of the when we have seen you think. So, that you can see in the view graph, that the current distribution if I plot; that means, $I(z)$ if I plot as a function of z it is something like a sinusoidal. Now, suppose I change these 2 transmission line there may internal angle or their flare is increased, then also it is reasonable that nothing physically is getting changed.

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So, the current distribution will be something like this, because the logic from which we are say that it should be 0 here that is still remaining.

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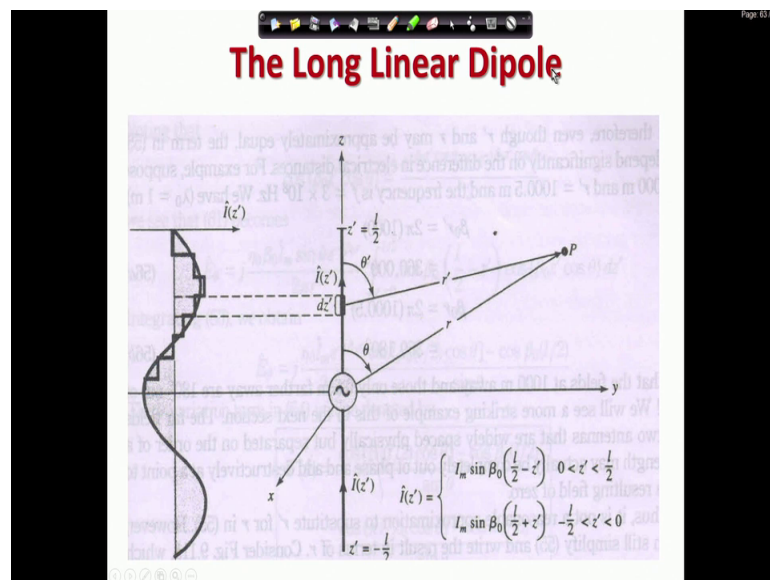


And, then if we just make it like this straight to then also this should be the current distribution. And, people have also measured and found that the current on a long dipole antenna that is indeed something like this that at the end it goes to 0, at the 2 ends of the 2 poles it goes to 0 and in between there is a sinusoidal distribution.

So, we can now say that, the we know that what is the current distribution here and from that we can solve, but if; that means, if I know the current distribution the from the Maxwell's equation we have seen in the first classes, that how to write the vector potential, magnetic vector potential, and from that how we can find the electric field and magnetic field?

But, if you can want to do this which we have done for current element, but in this case if you want to do that it will be a tough job, there will be some integrals which we only able to do. So, we will take a simplifying root, we have already say that we are mainly interested in the far field of the antenna.

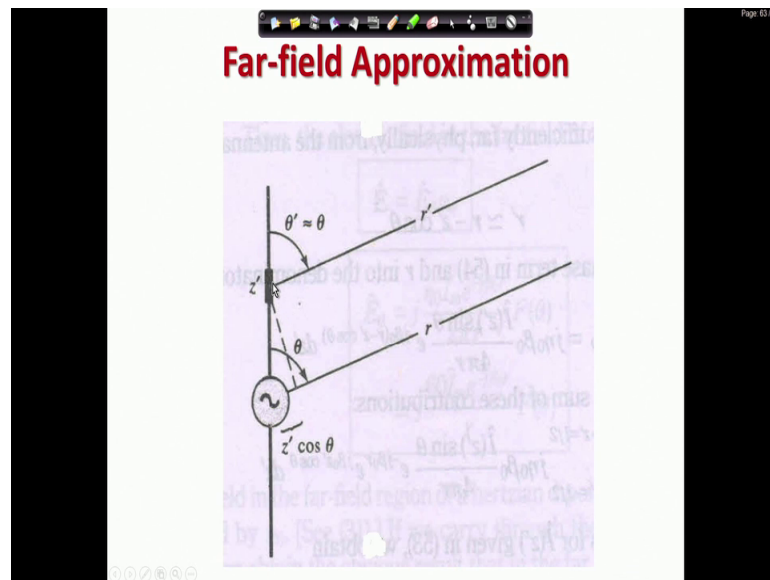
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So, we can say that the far field of the antenna if we are interested, then what we will do we will actually break this suppose this is the current distribution. Now, at we will break this whole dipole into small infinite decimal current element, current element of length dz as. So, small small current element will do; that means, here if we say we will break these into suppose this is a current element, this is a current element, this is a current element. So, this is one this is one this is one like that and here also we will make that.

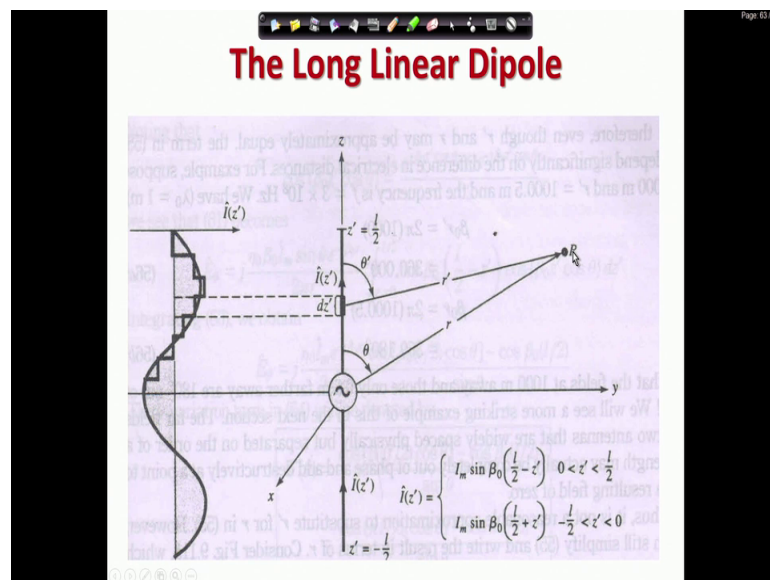
Now, I know the current distribution. So, I know the value here. So, between these 2 suppose, if this was the current distribution here. For this one I am we will assume that this is the constant value. Similarly, for the next one I will assume this is the constant current value. And, then I will go as if these are all current elements.

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So, this individual current element, now I know the result that at a distance point P to again we can see at the view graph that oh sorry.

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That at this point what is the field at least for P is in the far field: what is the far field here I know. And, then I can then I go to the next one and so, the total field here will be super position of all these fields. So; that means, basically these whole long dipole that I can break as a break into current elements distributed throughout it is length. And, each current element will have a current value that I know from the assumed current

distribution, then I will be tell you finding the what is the field at P, then I will have a superposition of those fields and I will find out.

So, to do that we need to just modify our some terminology; one thing is now you see we will have to have in the vector spherical coordinate. Actually, this source position the center of the source that will be our center of the coordinate spherical coordinate system, P is a point it will be characterized by r θ ϕ . So, P will have a radius r radius vector r then θ it is angle and then it is azimuth will ϕ . Similarly a current element at a distance z , but with a coordinate $d z$ as it will have a radius vector r' ; that means, it is a dash or flange coordinates are r' angle is θ' and the z coordinate is z' .

So, with that I can write the what is the current distribution $I(z')$, it will be $I_0 \sin \beta \sqrt{1 - z'^2/a^2}$. Because, current you see for any antenna the 2 poles these 2 currents they will be having this differential modes. So, that is why we need to break it into the 2 poles. So, here it is $1 - z'^2/a^2$ here it is $1 + z'^2/a^2$ if you do that then you see, what is the thing at the center at the center z' is equal to 0. So, this thing will be maximum here some depending on the, what is the length of the a . So, it is have some nonzero value here, but at the end at the end z' is equal to a . So, $\sin 0$ that will be 0. So, that is why in both the cases the current goes through 0 at the ends and it has some other distribution here sinusoidally distributed.

So, once we write this. So, then we can have the far field approximation and we will see that this distant point P. So, that is sufficiently away; that means, r is much greater than z' , in that case the r and r' both are parallel and r and r' are almost same. So, they are parallel and if I very minutely I want to express r' I can write as $r - z' \cos \theta$. So, r' will be $r - z' \cos \theta$. Now, as we have seen that when we will do the superposition, we need this things and what we will do that we can now write down what will be the far field for each of the current elements. So, for the one shown that these elemental one I can write what will be the far field we know, that in the far field we have only θ component of the electric field.

(Refer Slide Time: 14:53)

$$d\hat{E}_\theta = j n_0 \beta_0 \frac{\tilde{I}(z') \sin\theta}{4\pi r'^2} e^{-j\beta_0 r'} dz'$$

$\cdot P d\hat{E}_\theta$

$$e^{-j\beta_0 r'} = e^{-j \frac{2\pi r'}{\lambda_0}}$$

$r' = 1000 \text{ m}$
 $r' = 1000.5 \text{ m}$
 $f = 300 \text{ MHz}$
 $\lambda_0 = 1 \text{ m}$
 $\beta_0 r' = 2\pi \times 1000 = 3,6$
 $2\pi \times 1000.5 = 3,6$

So, I can write that due to the dz' one the $d\theta$ component that will be given by this just you can refer to your current element derivations, there we have derived that this was the case $I(z')$ is the current there is a $\sin\theta$ radiation there is a $1/r'$ variation $e^{-j\beta_0 r'}$ to the power minus $j\beta_0 r'$ dz' . So, this is the current this is the E field at the point. So, this is the point. So, at P this is the $d\theta$ and it is oriented in the θ direction and this is the value of this.

Now, here you see different points will have different r' . Now, in the as we know that in the denominator, we can since P is far away P is in the far field r and r' they are all equal, but we cannot do this here in the phase part, because phase is very much sensitive and actually it is not only r' it is $\beta_0 r'$. So, we know we can take an example, that basically let us write this phase term $e^{-j\beta_0 r'}$, this is actually the angle is $\beta_0 r'$ means 2π by λ_0 into r' . So, let us take that suppose this r' this is thousand meter.

Let us take r' is 1000.5 meter; that means, only 50 centimeter away another point and let us say the frequency of operation is 300 megahertz. So, we can find out what is the $\beta_0 r'$ value in the first. So, what is $\beta_0 r'$, if you do $\beta_0 r'$ because the λ_0 for 300 megahertz will be λ_0 is one meter and $\beta_0 r'$ that will be 2π by 1 meter into r' .

So, 2π into 1000. So, that will be 3, 60,000 degree whereas, βr that is 2π into 1000.5. So, that will be see the these angles they though the points are very near only over a distance of one kilometer, they are only 50 centimeter away, but the angle wise; that means, when we are calculating phase they are completely out of phase. So, if we superpose these 2; that means, if these 2 phases are added they will be a null phase or 0 phase.

So, that is why we cannot substitute that all r are equal here, but we can do this in the amplitude part because this will come in the amplitude part in the denominator we can do.

(Refer Slide Time: 19:02)

$$r' = r - z' \cos \theta.$$

$$d\hat{E}_\theta = j \eta_0 \beta_0 \frac{\hat{I}(z') \sin \theta}{4 \pi r} e^{-j \beta_0 (r - z' \cos \theta)} dz'$$

$$\hat{E}_\theta = \int_{z' = -l/2}^{z' = l/2} d\hat{E}_\theta$$

$$= \int_{z' = -l/2}^{z' = l/2} j \eta_0 \beta_0 \frac{\hat{I}(z') \sin \theta}{4 \pi r} e^{-j \beta_0 r} e^{j \beta_0 z' \cos \theta} dz'$$

So, here in the phase part we will do r as $r - z' \cos \theta$. So, that if we substitute in this dE the element the field due to the elemental current element, then it will be $j \eta_0 \beta_0$. So, total electric field is nothing, but superposition of electric field from all the current elements having different sizes, and these will be minus you can say z' is equal to minus $l/2$, with z' is equal to plus $l/2$, then that dE .

So, this if I do this integral, it will be z' . Now, here only thing is it is an integration of dz' . So, integration variable is z' . So, this term and this term will be present

all others will go out. And, will have to put the value of this I z dash the current distribution.

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The image shows a handwritten derivation on a blue grid background. The derivation starts with the expression for the electric field vector \tilde{E}_θ as a function of z . It consists of a prefactor $\frac{j\eta_0\beta_0 \tilde{I}_m \sin\theta e^{-j\beta_0 z}}{4\pi r}$ multiplied by two integrals. The first integral is from $z' = -\frac{l}{2}$ to 0 of $\sin\beta_0(\frac{l}{2} + z') e^{j\beta_0 z' \cos\theta} dz'$. The second integral is from $z' = 0$ to $\frac{l}{2}$ of $\sin\beta_0(\frac{l}{2} - z') e^{j\beta_0 z' \cos\theta} dz'$. The second integral is then simplified by changing the limits to $z = 0$ to $\frac{l}{2}$ and combining the exponentials into $(e^{j\beta_0 z' \cos\theta} + e^{-j\beta_0 z' \cos\theta})$.

$$\tilde{E}_\theta = \frac{j\eta_0\beta_0 \tilde{I}_m \sin\theta e^{-j\beta_0 z}}{4\pi r} \left[\int_{z'=-\frac{l}{2}}^0 \sin\beta_0 \left(\frac{l}{2} + z'\right) e^{j\beta_0 z' \cos\theta} dz' + \int_{z'=0}^{\frac{l}{2}} \sin\beta_0 \left(\frac{l}{2} - z'\right) e^{j\beta_0 z' \cos\theta} dz' \right]$$

$$= \frac{j\eta_0\beta_0 \tilde{I}_m \sin\theta e^{-j\beta_0 z}}{4\pi r} \int_{z=0}^{\frac{l}{2}} \sin\beta_0 \left(\frac{l}{2} - z'\right) \left(e^{j\beta_0 z' \cos\theta} + e^{-j\beta_0 z' \cos\theta} \right) dz'$$

So, that we have already earlier written current distribution now we will put that. And we will get the value of E theta. Now, in the first integral this z dash is negative, this is also negative we can change this limits and so, the whole thing can be written as this minus sign will come when we will change the limit here it will change. So, now, this you can note that this is nothing, but we can replace this e to the power j theta plus e to the power minus z theta means a cos term will come extra.

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$$\begin{aligned} \vec{E}_\theta &= \frac{j \eta_0 \rho_0 \tilde{I}_m \sin \theta e^{-j\beta_0 r}}{2\pi r} \int_{-a}^a \sin \beta_0 \left(\frac{r}{2} - z' \right) \cos(\beta_0 z' \cos \theta) dz' \\ &= \frac{j \eta_0 \tilde{I}_m e^{-j\beta_0 r}}{2\pi r} \left[\frac{\cos \left[\beta_0 \left(\frac{r}{2} \right) \cos \theta \right] - \cos \left[\beta_0 \left(\frac{3r}{2} \right) \right]}{\sin \theta} \right] \\ F(\theta) &= \frac{\cos \left[\beta_0 \left(\frac{r}{2} \right) \cos \theta \right] - \cos \left[\beta_0 \left(\frac{3r}{2} \right) \right]}{\sin \theta} \end{aligned}$$

So, we can write the total far electric field as j . So, these if we integrate we will get so, here you see this term is not having any theta variation all theta variation is here. So, we will be denoting this all this term this whole thing this we will be calling a function of theta. So, let me write that theta is $\cos \beta_0 r / 2 \cos \theta$ minus $\cos \beta_0 r / 2 \cos \theta$ by $\sin \theta$.

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$$\begin{aligned} \vec{E} &= \hat{a}_\theta \tilde{E}_\theta \\ \tilde{E}_\theta &= j \frac{\eta_0 \tilde{I}_m e^{-j\beta_0 r}}{2\pi r} F(\theta) \\ &\approx j \frac{60 \tilde{I}_m e^{-j\beta_0 r}}{r} F(\theta) \\ \vec{H}_\phi &= \hat{a}_\phi \tilde{H}_\phi \\ &= \frac{\tilde{E}_\theta}{\eta_0} \hat{a}_\phi \end{aligned}$$

So, thus we can write now what is the far field of a long dipole it is E_θ ; that means, it is a theta related field where what is E_θ , E_θ is time field. So, if we put this value

eta naught is the intrinsic impedance of free space which is 120π divided by 2π . So, this we can write as $j 60 I_m e^{-j \beta r} F(\theta)$. So, this is we have got the electric field.

Now in the far field, we know what will be the magnetic field? The magnetic field will be ϕ directed and it is simply in the far field, it will be we can write it in the similar fashion as this is the vector term. So, this H_ϕ and what is this is nothing, but E_θ by eta naught. So, all the θ variation is here; that means, if we want to find what is the pattern, because this is a constant term. So, pattern will be dependent on $F(\theta)$. Now, this for dipole actually the length you can choose.

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$l = \frac{\lambda_0}{2} \rightarrow \text{half wave dipole.}$
 $F(\theta) = \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta}$
 $\theta = 90^\circ$
 $F(\theta) = 1$
 E

So, if we choose a popular choice of length is $l = \lambda/2$, $\lambda/2$ is the wave length of the operating frequency. So, if we choose that then it is called a half wave dipole. So, the upper pole is $\lambda/4$, the lower pole is $\lambda/4$, the total length of the antenna dipole that is half wave. So, it is called half wave dipole, for this half wave dipole where is our $F(\theta)$. So, this $F(\theta)$ becomes $\cos \pi/2 \cos \theta / \sin \theta$ for this half wave dipole.

Now, we will see that what is the maximum of this $F(\theta)$? In which direction it is maximum you can find out that what is the θ in which this is having a maximum you know how to find maximum, it will be turn out, that this θ is 90° ; that means, what is θ you looked here in this view graph? So, this angle is θ . So, at 90°

means brought side direction this is called in antenna people call it, brought side; that means, this is the dipole length and brought side to that 90 degree to the this is a maximum and what is the value of that put that this term at 90 degree become 0 so, $\cos 0$ is 1 and this is $\sin 91$.

So, F_{θ} is 1; that means, the value is F_{91} and so, what is the maximum value for half wave dipole? We have already found this e_{θ} of E sorry where is that this E theta is here so, if F_{θ} becomes a so, what will be our E max it is $60 I_m$ by r .

(Refer Slide Time: 31:14)

Handwritten notes on a blue grid background showing the derivation of the maximum electric field for a half-wave dipole antenna. The notes include the relationship between magnetic field and electric field, the radiation pattern function $F(\theta)$, and the final expression for the maximum electric field.

$$\vec{H}_{\phi} = H_{\phi} \hat{a}_{\phi}$$

$$= \frac{E_{\theta}}{\eta_0} \hat{a}_{\phi}$$

$\sin \theta$

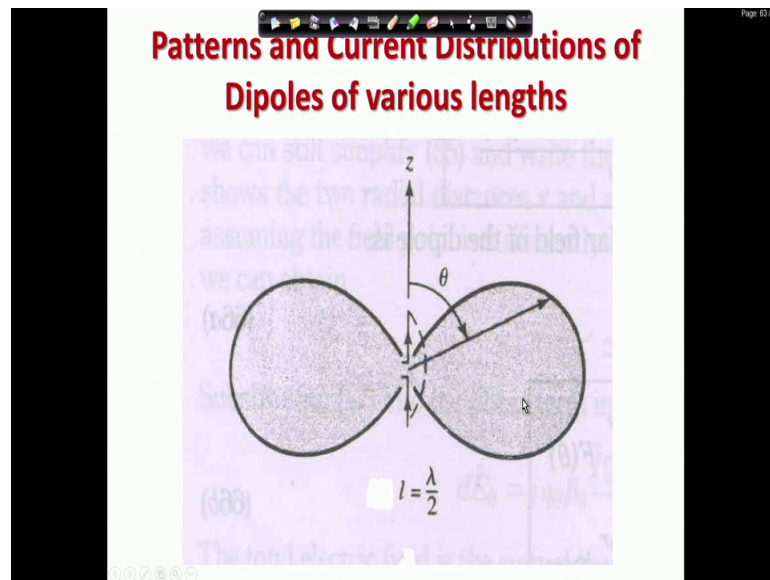
$\theta = 90^\circ$

$F(\theta) = 1$

$$\vec{E}_{\max} = \frac{60 \vec{I}_m}{r}$$

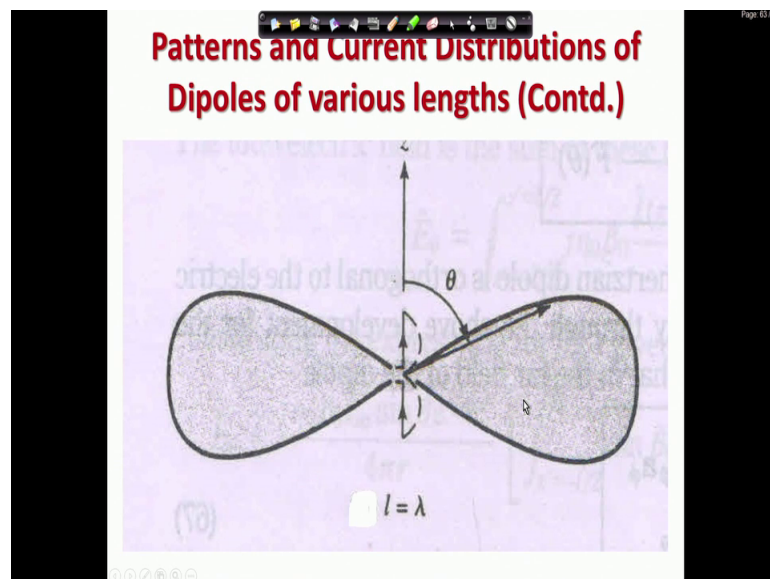
So, what is I_m ? It is a maximum current so, so, now, let us look at this F_{θ} if we plot what is this theta, we get the pattern. So, if we have a $\lambda/2$, then for this $\lambda/2$ things you see that we have a pattern something similar. And please note that the whole E field expression does not have any variation with phi; that means, in the azimuthal direction, it will be a circular pattern, in the in the elevation pattern, we have the this is called $\lambda/2$.

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If we take l is equal to λ the thing is like this.

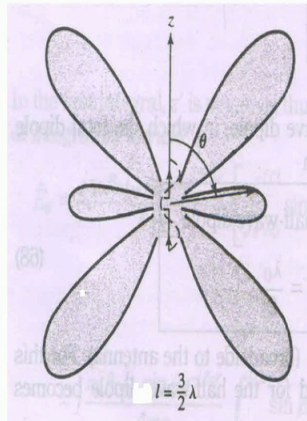
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If we take l is equal to $\frac{3}{2}\lambda$, you will see the pattern will be split in various lobes there are total 6 lobes in case of this $\frac{\lambda}{2}$ etcetera.

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Patterns and Current Distributions of Dipoles of various lengths (Contd.)



Now, if we go to the we will have to calculate that what is the radiation resistance? Because, we said that a larger antenna will give us larger radiation resistance. And, to find that we will have to first find how much power is getting radiated. So, what is the time average?

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$$\begin{aligned}
 \vec{S}_{av} &= \frac{1}{2} \text{Re} [\vec{E}_\theta \vec{H}_\phi^*] \hat{a}_n \\
 &= \frac{1}{2} \frac{|E_\theta|^2}{\eta_0} \hat{a}_n \\
 &= \frac{4.77 |\vec{I}_m|^2}{\pi r^2} F^2(\theta) \hat{a}_n \\
 P_n &= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \vec{S}_{av} \cdot \hat{a}_n r^2 \sin\theta \, d\theta \, d\phi \\
 &= \frac{\eta_0 |\vec{I}_m|^2}{4\pi} \int_{\theta=0}^{\pi} F^2(\theta) \sin\theta \, d\theta \quad \text{Watts}
 \end{aligned}$$

Power pointing vector that will be S average and we know that is half real $E_\theta H_\phi$ star in and it goes in \hat{a}_r , that we have already proved for any antenna this is true. So, here we will be finding, that what is this we have already found E_θ in each ϕ . So, it will turn out to be half E_θ^2 where η_0 a r .

So, now if we put the value of this $e^{-j\beta r}$, this will turn out to be $4.77 I_m^2$ square, by r^2 square, $F^2 \sin^2 \theta$ a r . Now, if I want to find out what is the total radiated power? This pointing vector is for density. So, we will have to integrate it over a hypothetical closed spherical surface. So, P_r is we have already seen how to do it, over an elemental area spherical thing $r^2 \sin \theta d\theta d\phi$. So, if I do this we will get $4.77 I_m^2$ square by 4π theta equal to 0 to π $F^2 \sin^2 \theta$ $d\theta d\phi$ $\sin \theta d\theta d\phi$ already 2π that is learnt and this is $30 I_m^2$ square watts.

Now, this integral further is not integrable, but so, you here you will have to put as we have already seen the value of the your length. So, if we specialize in some places it gives a value.

(Refer Slide Time: 36:05)

$L = \frac{\lambda}{2}$
 $\int_0^\pi F^2(\theta) \sin^2 \theta d\theta \approx 1.2186$
 $a=0$
 $P_r = 36.5 |I_m|^2$
 $I_{in} = I_m \sin \beta \frac{l}{2}$
 $= I_m$
 $I_{in} = I_m / \sqrt{e}$
 $P_r = 73 |I_{in}|^2$
 $R_{rad} = 73 \Omega$

So, for half way of dipole that means, if we have l is equal to λ by 2 this integration θ is equal to 0 to π $F^2 \sin^2 \theta$ $d\theta$ can be integrated and it becomes value is this. So, if we put that then P_r becomes $36.5 I_m^2$ square. Now, what are the definition of radiation resistance, the total power radiated is equal to the radiation resistance into the I_m^2 square. Now, we will have to find out what is the I_m^2 square?

We know for the half way of dipole, what is the at the input terminal let me at the input terminal what was the current so; that means, we will have to put z dash is equal to 0 in either of the expressions of I_z dash. And, if we do that you will see that it is basically becoming $I_m \sin \beta$ by 2. So, that is nothing, but I_m because this will

become a l is $\lambda/2$. So, it will this is 2π by λ . So, we will see $\sin \pi$ by 2 so, I_m .

That means I_m is our maximum value and so, what is the rms value of this? So, I_{rms} will be I_m by $\sqrt{2}$ so; that means, we can write P_r is $73 I_{rms}^2$. So, this shows that what is the radiation resistance? Radiation resistance of a half way of dipole is 73 ohm. Whereas, in previous cases we have seen for current elements it was in we have calculated various thing for various frequencies it was some milliohm etcetera, 73 ohm is quite sizable; that means, quite a good amount of radiation is taking place.

If we take the size to be $\lambda/2$ that is why half way of dipole is quite a good antenna. As far as it is radiation it can radiate good amount of power, we will see what is it is other properties in the next class.

Thank you.