Analysis and Design Principles of Microwave Antennas
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Lecture – 11
Friss Transmission Equation and Antenna Temperature

Welcome to this lecture. Actually if you recall that I started the antenna analysis this course with first introducing some potentials, vector potentials etcetera and then we say that actually if I know the current distribution of an antenna at its surface then I can find the electrical magnetic field. But this is easily said, but we said that instead of solving this problem we want to intermediately introduce the potential so that if we will first find the potential and after that we will solve the things.

But what was the advantage? There are some advantage for going to potentials etcetera, but only for very simple antennas we can find the current distribution across the antenna. It is very difficult to find for complicated antennas, modern antennas it is not easy to find the, what is the current distribution along the antennas because there will be various modes, there will be various types of currents, you know the currents sometimes where the specimen current, sometimes there conduction current etcetera. So, it is not so easy, though today you can say that with the help of modern computers there are numerical methods. So, you can find out you can now visualize that is a good thing and that is being used.

But if you cannot find this current distribution on a thing, but then cannot you use the antenna. The answer is yes, you can approximately use the antenna and the results soon very much provided you take this approach Friis transmission equation where you need to know that instead of the exact field of radiated by antenna you should perform, or find out what is the gain of the antenna in various directions where you are interested particularly in the from the transmitter to the receiver the direction in where the receiver is receiving a thing power.

So, there if you can find the gain then you can find out how much power it will receive and actually links, communication links are established based on these long back even when the antennas were not much understood etcetera based on this equations.

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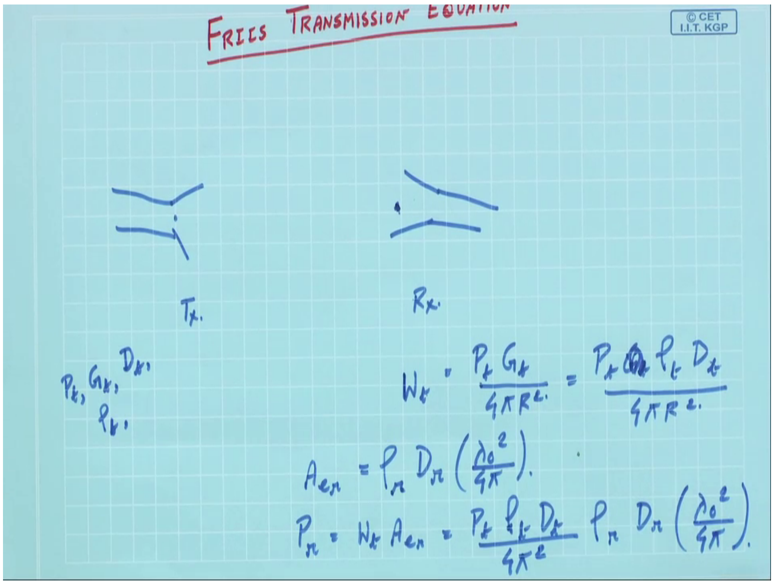

FRIIS TRANSMISSION EQUATION

$P_t, G_t, D_t, \rho_t,$

$$W_t = \frac{P_t G_t}{4\pi R^2} = \frac{P_t \rho_t D_t}{4\pi R^2}$$

$$A_{er} = \rho_r D_r \left(\frac{\lambda_0^2}{4\pi}\right).$$

$$P_r = W_t A_{er} = \frac{P_t \rho_t D_t}{4\pi r^2} \rho_r D_r \left(\frac{\lambda_0^2}{4\pi}\right).$$

So, we will see that the, was Friis transmission equation. So, let us see the two antennas. So, this is a transmitting antenna, this is a receiving antenna, and. So if this one is connected or radiating power P t then what is the power density at the receiving antenna? We can easily write that W t is P t G t, actually it should be retain as an angle, but I am writing that as if I know the value here. So, P t G t by 4 pi r square if you want this G t means actually P t G t then if there is efficiency total then sorry P t D t by 4 pi r square.

Now, this antenna has an effective area, so the receiving antenna has an effective area that is rho r D r lambda naught just in the last lecture we have derived this lambda naught square 2 pi. So, how much power it will collect? Power collect will be P r is equal to W t into this. So, I can say W t into A e r. So, that will be P t rho t D t by 4 pi r square into rho r D r lambda naught square 4 pi, ok.

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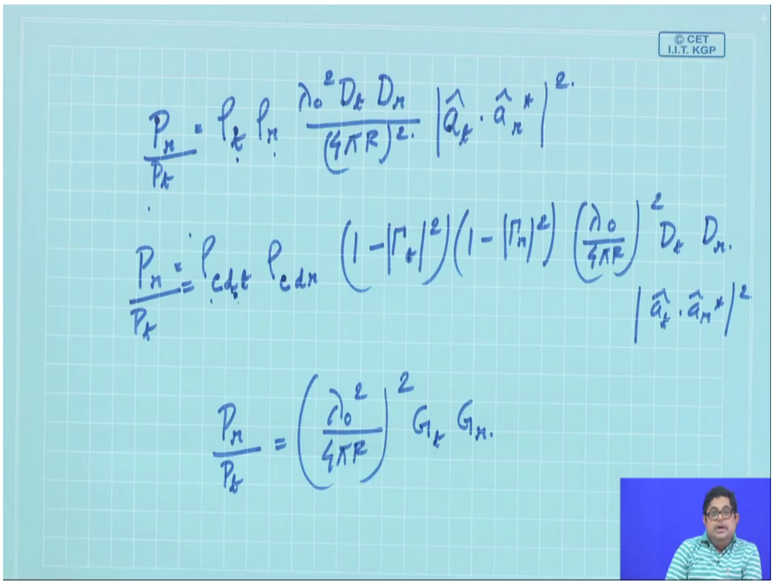

$$\frac{P_r}{P_t} = \rho_t \rho_r \frac{\lambda_0^2 D_t D_r}{(4\pi R)^2} \left|\hat{a}_t \cdot \hat{a}_r^*\right|^2$$

$$\frac{P_r}{P_t} = \rho_{cdt} \rho_{cdr} \left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda_0}{4\pi R}\right)^2 D_t D_r \left|\hat{a}_t \cdot \hat{a}_r^*\right|^2$$

$$\frac{P_r}{P_t} = \left(\frac{\lambda_0^2}{4\pi R}\right)^2 G_t G_r.$$

So, we can say that P r is equal to rho t rho r lambda naught square D t D r by 4 pi r whole square. Now, if we introduce the polarization loss factor then we can write here that a t char a t r this thing we have earlier derived, or if we break it into this efficiencies can be broken into mismatch the impedance mismatch, into the conductor loss, into the dielectric loss etcetera. So, P r can be written as the rho cdt rho cdr 1 minus gamma t square 1 minus gamma square lambda naught by 4 pi r square D t D r, ok.

So, this is this formula is called Friis transmission equation. So, you see that if you know various parameters. Actually here only you see the conductor dielectric loss together has been taken, it can be also broken into conductor loss and transmission loss, but actually it is generally taken together because it is very difficult to from measurement to find out what is conductor loss and what is dielectric loss. So, the two losses can be put together that is why it is conductor dielectric together.

So, these parameters are known we can find what is the, oh ho there is a P t should be coming here. So, actually this equation is P r by P t I am somewhere from here P t has been gone, P t D t you see all these are actually should be P r is equal to W t A e r, there was a P t that P t here was a P t, but after that that P t has been lost. So, this is also P r by P t this is also P r by P t.

So, this equation is called Friis transmission equation. And you see if there are impedance match then this two terms will go. Similarly if there is polarization match this term will go, if it is a loss less antenna these two term will go, that is why generally we have under ideal condition lambda naught square by 4 pi r whole square G t G r.

This is we are familiar with, but in actual this is the formula, this formula is called Friis transmission formula many time we use this the assumptions here are the transmitter and receiver antennas they are separated by the far field distance. We the we know at the direction of the receiver from the transmitter, the gains similarly for the receiving antenna we know the gain to the trans transmitter that direction and we know all other these the losses then we can find this thing.

A similar equation can be derived for radar range equation they are also this type of formulas can be used, that only thing they are is instead of a receiver there is a target which takes the energy and scatters back. So, that in radar classes we will come across that if we take they also there is a new term called radar cross section gets into news there and by that again what is the back scattered power that comes from these type of simple models you can find. So, knowing the gain you see instead of analyzing the antenna also you can find out what is the power required. So, you can establish communication links etcetera with that for that, you not need not do all analysis of the antenna.

So, sometimes actually if this theory analysis is overkill that it gives all the information, but all that information is not always necessary, so if a communication engineering these information is enough. So, he need not bother what is a various feels etcetera, at various points, what is that polarization that if he has all these loss factors, which are simply stabilize the link. So, this is one thing that we wanted to say. And another thing we wanted to say is about the noise that, you see antenna whenever it is wa it is always looking towards the free space and so it is a source of noise.

Now, how to do we characterize that? Many times we ignore that, but if you want to do suppose radio astronomy or if you want to have a very, if you want to you know that suppose you want to see the whether or ocean etcetera if you want to monitor you need to find out ocean temperature by a receiver. So, or you have some scatterometer, some sensors are there who senses various whatever passive radiation coming. So, for that whatever antenna is receiving in the noise that is actually information. So, they need to have another parameter of antenna, very important parameter that is called antenna temperature. So, we will discuss this next, antenna temperature.

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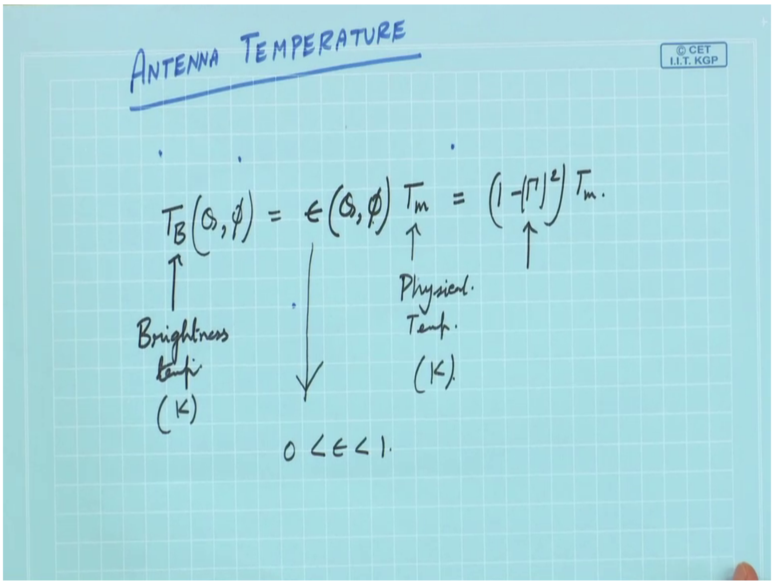

ANTENNA TEMPERATURE

$$T_B(\theta,\phi) = \epsilon(\theta,\phi)\, T_m = (1-|\Gamma|^2)\, T_m .$$

Brightness temp (K)

Physical Temp. (K)

$0 < \epsilon < 1.$

All of you have idea of receiver noise or receiver noise temperature or any electronic circuit that has a noise temperature. Now, antenna it is a passive device it does not create its own noise, but it is a receiver of noise because it receives noise, because it is an open it is opening its door. So, noise also comes apart from signal noise also comes, that is why it also should have that how much it can take this antenna temperature.

Now, we start that every object which is not at absolute 0; that means, not at minus 273 db radius energy. We know this fact. The amount of energy radiated is usually represented by an equivalent temperature T B. So, we can write that T B and also whenever radiation means that is a radiation it is a function that theta phi in various directions it will have various things.

So, that is equal to a function like this. So, what is this epsilon? This is an emissivity function it is a dimensionless quantity. So, actually it is a model that a any object it has an emissivity, so what is the emissivity of various things that we know. And what is T B? T B is called brightness temperature. And what is T m? This is the physical temperature, so both these temperature their unit is Kelvin emissivity is dimensionless.

And this, what is this reflection coefficient? It is a reflection coefficient of the surface for the polarization of the wave because you know that when the wave come from any surface there will be a reflection, so it is there. Now the emissivity values obviously, they are varying from 0 to 1. So, the maximum value the brightness temperature can achieve is equal to the physical temperature now in our case; that means, in microwave region natural emitters of microwave frequency apart from I think the stars etcetera they are our radio stars they are but apart from that there are two nearby sources of radiation, one is the ground.

Ground is the source of this radiation and its grounds equivalent temperature is around 300 degree Kelvin. And also the sky, sky is have a temperature and this brightness temperature of around 5 degree, 5 Kelvin when looking towards any and about 100 to 150 degree in the horizon. Obviously in the horizon it is more noisy and in the zenith it does not have any noise actually that is fortunate for radars etcetera, that in the zenith.

Now, you see the brightness this temperature or this noise emitted by the different sources is intercepted by our antenna, and so it appears at the terminals as a ambient temperature. Now, only thing is antenna also has a gain function. So, it will wait the noises coming from major directions by its own gain function so that is why antenna noise temperature is not exactly this T B it will be a waiting of that.

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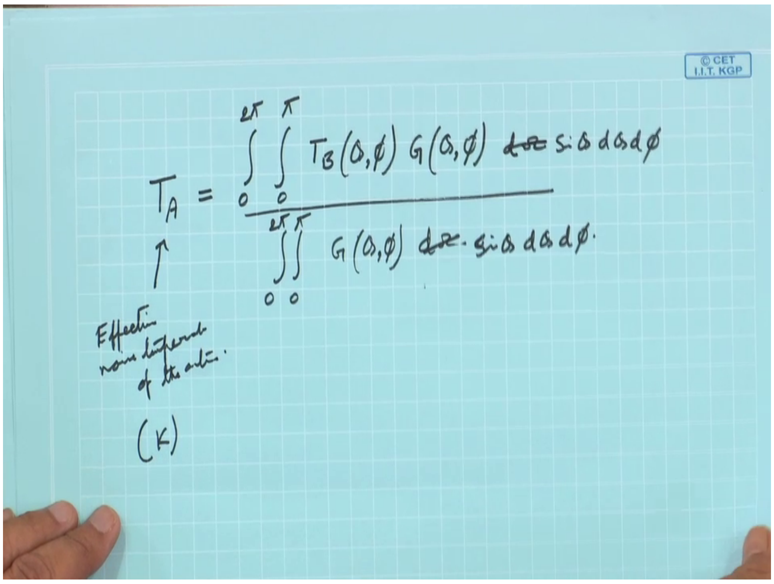

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\, G(\theta,\phi)\, \cancel{d\Omega}\, \sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\, \cancel{d\Omega} \cdot \sin\theta\, d\theta\, d\phi}$$

Effective noise temperature of the antenna.

(K)

So, that is why the antenna noise temperature T A is nothing but the I can say T B theta phi that will be waited by again theta phi and then d the solid angle divided by I will have to find out what is the total of these that is the average because these T A is an average. So, you see G theta phi d gamma; so the source as a T B brightness temperature that I will have to pass through this antenna which is having an n.

So, now, I can complete this, this will be 0 to 2 pi, this will be 0 to pi, this will be also to 2 pi, this will be 0 to pi and instead of that I will write sin theta d theta d phi. So, T A is the effective noise temperature of the antenna and G theta phi is the power gain pattern. So, now, if we assume now let us see the, this is clear and this unit will be again the will be Kelvin.

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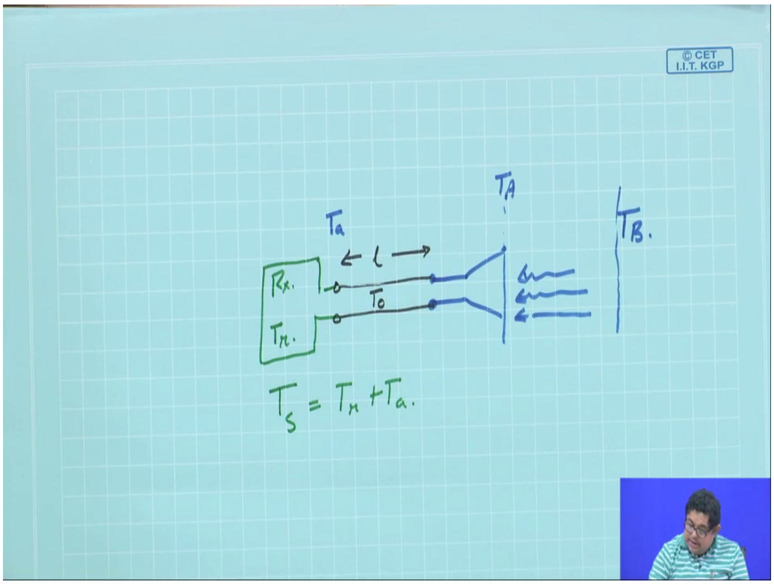

Now, let us see the this is the antenna. So, the source is here T B, now that when it comes here this T B gets converted to T A by that formula. Now, after that what happens? Suppose this is the antennas, I think.

Here I have a transmission line there is this there will be a transmission line, that transmission line let us say length of l and that as a temperature let us say T naught it is lossy transmission line. So, that will make this change because this T A here then there will be one thing is due to antenna loss this will change, now here there will be loss again in the transmission line. So, this temperature T A will be change here to something called T small a from T capital A to T small a some other temperature. And then we it will be connected to whom? It will be connected to a receiver, that receiver has its own noise temperature T r. So, I can say that the whole thing ultimately we will have a T s, which is T r plus T a.

Because we will see temperatures are additive because actually these are equivalent power. So, noise power total system noise power will be whatever antenna has brought up to here, plus the noise added by the receiver. So, let us with this let us modulate that. So, antenna is giving how much power can I say K T A delta f.

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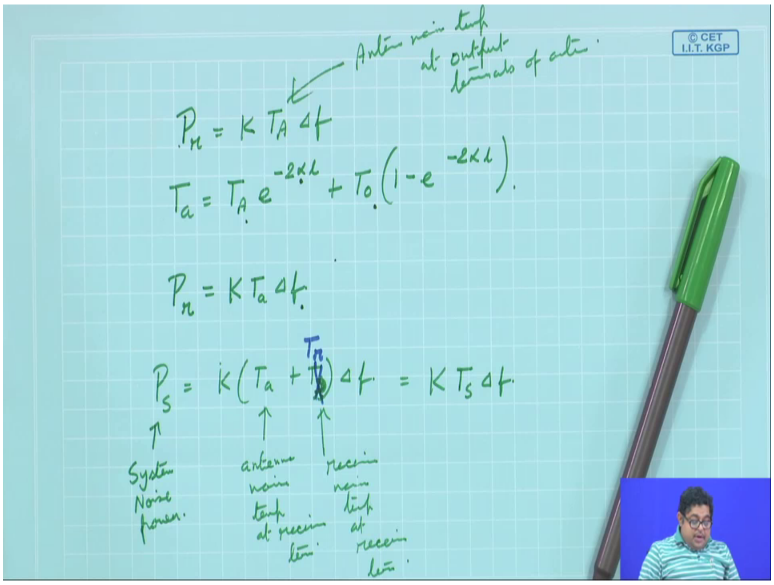

$P_n = K T_A \Delta f$

$T_a = T_A e^{-2\alpha l} + T_0 \left(1 - e^{-2\alpha l}\right)$.

$P_n = K T_a \Delta f$.

$P_s = K(T_a + T_R)\Delta f = K T_s \Delta f$

So, from here antenna is giving this is this much power K T A delta f, you understand K is Boltzmann constant, T A is the antenna temperature here and delta f is the antenna delta f I can say is the noise equivalent bandwidth.

Now, what is the relation between small t and capital T A? Can I write T A is T A e to the power minus 2 alpha l plus T 0 1 minus e to the power minus 2 alpha l. Why? Because this is one thing that this much length with this attenuation constant of the line that has brought here and this is for this is due to the, this transmission line thing, the thing has a T A temperature the physical temperature of this is T 0.

So, T A is nothing but this is the minus given by the, this physical temperature and this is from the antenna attenuates, antennas now attenuation T A. And what else I need to say? So, now, I need to then modify that. So, now, what is the power given here? To the receiver a T A small T a delta f, so this has become with the addition of this a T a and then what is P s, K T A plus T s delta f is equal to you can say K not T s this will be T r, because this temperature is T r, T r. So, I can say K T s delta f. So, P s is system noise power at receiver terminal T A is antenna noise temperature, noise temperature, at receiver terminal T r receiver noise temperature at receiver terminal. And what is T A? T A is antenna noise temperature at I can say output terminals of antenna or at the (Refer Time: 27:09) end of the antenna, whatever it is. So, this is an useful thing.

Particularly you know that you need to calibrate it. Actually suppose you will receive black body radiation. So, those antennas suppose an antenna will see how much is coming from the ocean. So, how it will see unless and until it is calibrated that how much it receives originally. So, for that you need to point it to the thing, actually generally it is pointed to the zenith because that is 5 degree Kelvin almost 0 degree Kelvin you can say. So, you point it and then find out these values. So, that is the calibration. Then, you point to an actual body may be a ground, may be a some the sky which is at horizon which is of invisible noise and then you find out, so a calibrated one can easily find out (Refer Time: 28:14). So, that is called antenna noise temperature, ok.

So, with that we now close the basic antenna parameter. So, we have seen all. Next we will see some of the very noble wire antennas where you can easily design wire antennas and for many cases it suffices. So, we will see wire antennas in few lectures. After that we will go to aperture antennas with some of the very simple aperture antennas, and then we will see the general method of analysis of antenna, ok.

Thank you.