Analysis and Design Principles of Microwave Antennas
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Lecture – 10
Effective Aperture of an Antenna

Welcome to this lecture, now we will see a very important property of an antenna of a receiving antenna, by which again we can find out lot of things particularly how much power an antenna is receiving, that will be help if we can find this parameter.

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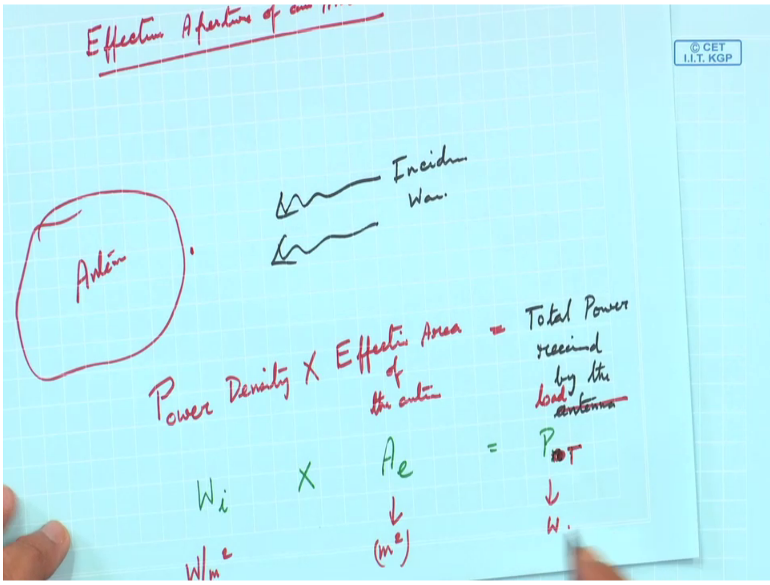

This is called Effective Aperture of an Antenna, actually every antenna has an effective aperture, but you see that when we talk of wire antenna, we say that I really that has only a length.

So, it does not have any width so, it is area is 0 but, please remember that the it is physical area maybe 0, or very small because its width is small, but to be an antenna it should have a high effective area. That effective area is actually an electrical, concept it is not a physical area thing.

So, effective area if it is not high, then it is not a (Refer Time: 01:44) antenna and, it would not be able to take the power take from there actually, any antennas job is suppose there are lot of here waves going on so, to extract power so, that it would not be able to do, if it does not have a sizeable effective area.

Now, for other aperture antennas it is not difficult to visualize this effective aperture, because you see that larger and larger, if I make a aperture, then aperture will take more power and obviously, all the powers I cannot take because, then I require uniform illumination over the whole aperture.

So, because of the non-uniformness in the illumination, the effective aperture of a aperture antenna is always less than the physical one. So, we will have to find out from the knowledge of field theory, that how much is the effective aperture, how much it is less than one, but so, now we can unify that whether it is an wire type antenna, or aperture type antenna, or a mixture of that anything.

So, that has an always has an aperture. So, and that what is the concept of that aperture we considered, that it is the area which, because any place we know in the field theory, ultimately if I know the antenna is either radiating or receiving.

So, at this point what is the radiation what we call, the power density that we can always find out, because we know the what is the electric and magnetic field from the we can always from the power density at a point.

Now, point is whatever total power the antenna is taking that is nothing, but that power density multiplied by the effective area. So, I can say that suppose I have a this is an antenna and some electromagnetic wave incident wave is coming.

So, at this point I have some power density, now this power density, it is unit is watts per metre square. Now, that multiplied by effective area of the antenna that should give me the total power received by the antenna received, or extracted by the antenna.

So, I can say that if power density we generally have this expression W i symbol, effective area is A e so, this is I can say total power. So, this is the formula for effective area. So, effective area its generally it will be unit is metre square, this is watts per metre square and this is watts.

So, total power received by the antenna, or you can say how do I measure total power received by the antenna, you can say total power received by the antenna and delivered to the load, or total power received by the load, because ultimately the antenna will be connected to a load.

So, whatever total power received, because I cannot measure what is the power received by the antenna, that we can measure across a load. So, better we can correct the a thing the total power received by the load, or delivered to the load ok. So, now let us see that for an antenna how to find it.

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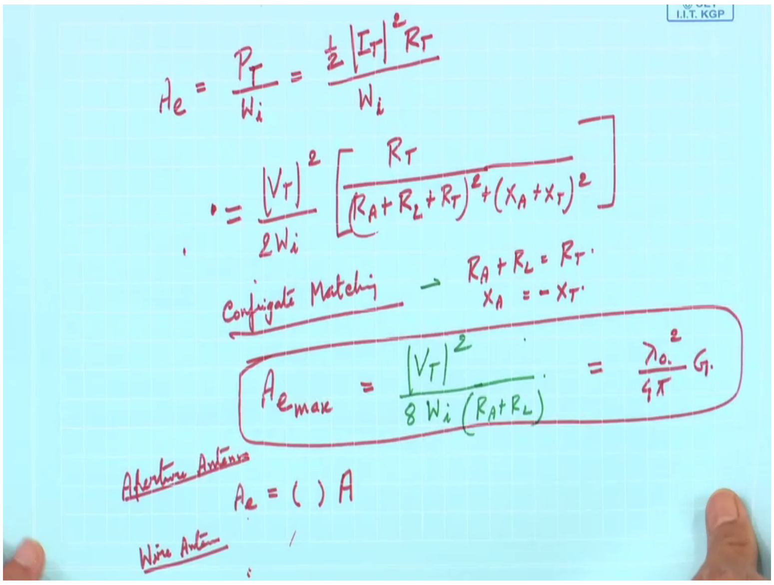

So, I can now write that effective area A e is P T, or I can there will be some confusion with the latent nomenclature so let us make it P T. So, A e is equal to P T by W i is half so, what is P T half I T I T star that is I T mod square, this I T have seen in the case of input impedance, that in the receiving mode this is the in the Thevenin equivalent circuit. This is the current that is flowing and, R T is that there I think I have that slide here.

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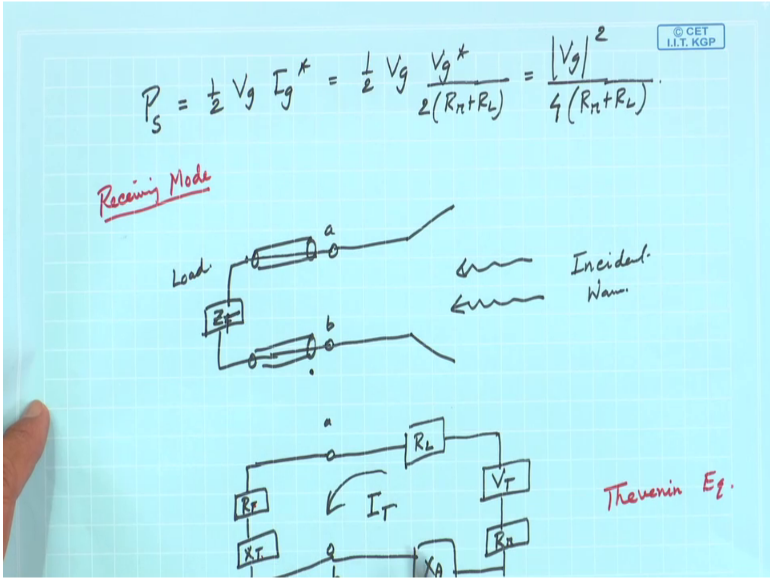

This is the Thevenin circuit so, R T X T so, how much will be the power in the load that we can calculate from here, half I T square I T mod square into R T, that is what I am writing.

So, this we have already seen so, then what is now I have all this expressions there. So, can I write directly that half I T square so, A e will be V T square by 2 R T by R A again under conjugate matching, when we say that we are getting maximum power to the load, under conjugate matching.

So, this A e effective area obviously, maximum power will be delivered there. So, we can that time call the A e as A e max so, effective aperture become maximum. So, you see that it is a electrical concept that under conjugate matching the A e value is changing, under conjugate matching its value is changed to maximum.

So, A e of a antenna is not a fix thing, but it changes if you have matching with the external, or generator impedances etcetera or load impedance etcetera.

And so, its value will be if you put it conjugate matching means what is the condition of conjugate matching, let me write that R A plus R L will be equal to R T and X A will be equal to minus X T. So, then this thing will be V T square by 8 W i, then say R A plus R L.

So, actually if the if there is no polarization mismatch, then just later will show, by a thought experiment that these actually A e max value this will be equal to lambda naught square by 4 pi into G this will prove. But you write here that this is the maximum value.

So, you see that if you know the operating frequency, you know lambda not and then actually this effective maximum effective area, of any antenna is related to another measurable quantity of the antenna that is G. What is the G? G is the maximum value of G theta that means the gain of the antenna.

Now, gain if the antenna we always define as a transmitting thing. So, you see that this is a connection between the receiving characteristic and the transmitting characteristic of the antenna. So, transmitting characteristic is G, we know how to measure G, you find the power pattern.

You find what is the power distribution at various point and from that you can find out what is the maximum G. So, you can now find if we the same antenna we use as receiving antenna, what is the relation what will be it is effective area. So, that the same antenna which you have characterized with it is special gain.

Now, you know that if I use it as receiving antenna, how much power it will be able to find out. So, I have already said that this effective area, it is a some portion of the physical area, or a that means the for a aperture antenna.

Aperture antenna the effective area A e, we can say some fraction of the physical area of the antenna, always it is less than the physical area. And this thing we have also seen that actually the electrical distance of the antenna determines it is area.

In case of wire antenna, it is as I already said that it is very much greater than, it is physical area. So, the case is opposite in case of aperture antenna, you the effective area is less than the physical area in case of wire antenna the physical area, the effective area is much greater than the physical area otherwise it cannot be used as a thing.

Now, let us go to prove that point this is a very important relationship various times, we will use this in most of the models of communication radar etcetera this formula is used but to prove this we need to find out one thing.

This effective are concept we have introduced actually when we have introduced in the start of the lectures the current element, we have found some of the parameters antenna parameters of current element, but we have not found its effective area.

So, let us find that because effective area of a current element is a very useful quantity, because that time you have seen that it is not a very good radiator that means, we said that its radiation efficiency is not very high that we calculated that time.

But otherwise as a directive characteristic of current element is very good, that we have not found, because we have neither found the gain nor found the effective area of the current element. The reason is basically that time we have not introduced this concept.

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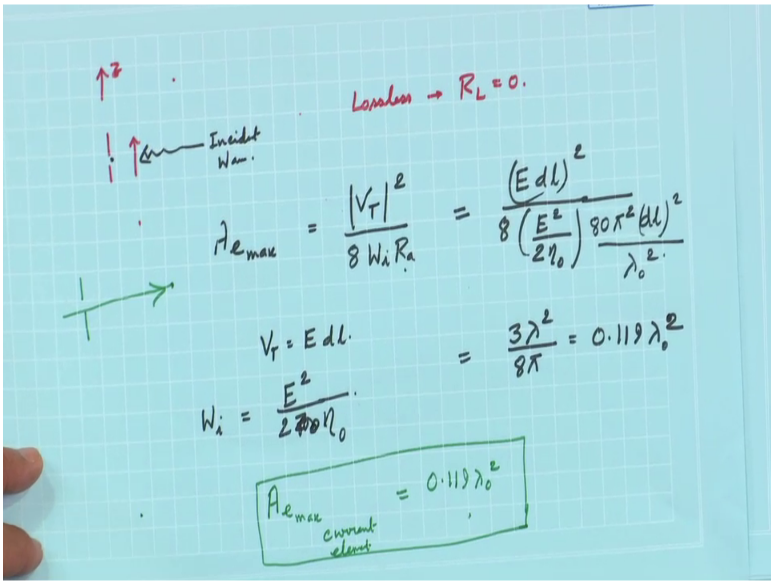

So, let us do it now so, fine suppose I have a current element and, there is a incident wave coming. And we assume that this incident wave, this current element is Z directed and incident wave is also Z directed.

So, there is no polarization mismatch between the current element and, the incident wave also we assume it is a lossless current element that means its R L is 0. So, what will be it is A e max from our relation, because we have last found this relation that A e max is V T square by 8 W i R A plus R L. So, here if I put so it will be V T square by 8, W i R a, because I am taking R L is equal to 0.

Now, what is V T for a current element, you see I have an E electric field and, I know that this distance is thing so, can I say that V T is E into d l and what is W i. Now, here we are assuming that this incident wave is coming from a source which is at the far field. So, actually it is a plane wave is coming so, what is W i for plane wave we know for plane wave, this power density at this point at this point on the antenna, where I am capturing.

What is the power density? Can I write it is E square by 2 Z naught, where Z naught is the or earlier, we use to call it sorry it is eta naught is better to say E square by 2 eta naught, where eta naught is the intrinsic impedance of free space, that is 377 ohm ok.

So, let us put it now so, it is E dl square by 8 E square by 2 eta not and this value we found, for current element. What is R a? If you see your notes, it is 80 pi square dl square by lambda naught square. So, if you simplify this will come to 3 lambda square by 8 pi is equal to 0.119 lambda naught square. So, the value is only valid that means, we have got this.

So, let me right again A e max of a current element, this is 0.119 lambda naught square. So, this is for a lossless current element and; obviously, the losses for a this type of current element is quite small, but if it loss is sizable; that means, if actually for current element the radiation resistance, this 80 pi square dl square by lambda not square is also very high.

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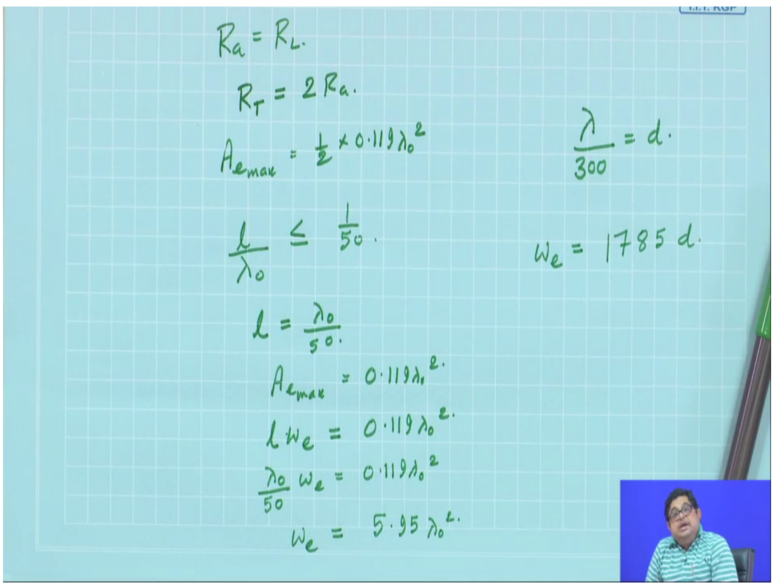

So, if we take that that means, there are losses and let us say that R A is equal to R L, in that case the total R T, that will be 2 R A and so, that time the A e max value will become half of this. So, for a lossy antenna lossy current element, the e max will be less also.

Now, let us have some typical values actually, suppose we will see later that the dipole, it can be considered a current element a dipole can be considered as a current element, if it is length by the frequency of operation is less than 1 by 50.

So, if I have a dipole of length lambda not by 50, then its A e max I am assuming lossless. So, A e max is 0.199 lambda not square, now what is A e max l into width effective width is w e that is 0.199 lambda naught square. Now, length I have assumed lambda naught by 50 into w e is 0.199 lambda not square, so, if you do that what is the value of effective width 5.95 lambda naught square.

So, physical generally dipoles, when we design we will see that dipoles, the physical dimension a physical width of a dipole is generally the then a physical diameters of dipole are lambda by 300. So, it becomes the w e this is actually 1785 d.

So, this is the thing actually effective width, or electrical width of a dipole or a current element, if that is almost 1500 times it is physical wave and gives you is the effective area, as I said that for current watt type of antennas, the effective area is much small much larger than. It is physical area basically the width get is let up otherwise it cannot be used.

Now, let us do another trick that if you believe actually we will just make prove this formula, but if you believe this so, I have already found out the A e max for current element so, what is the value of the gain of a current element.

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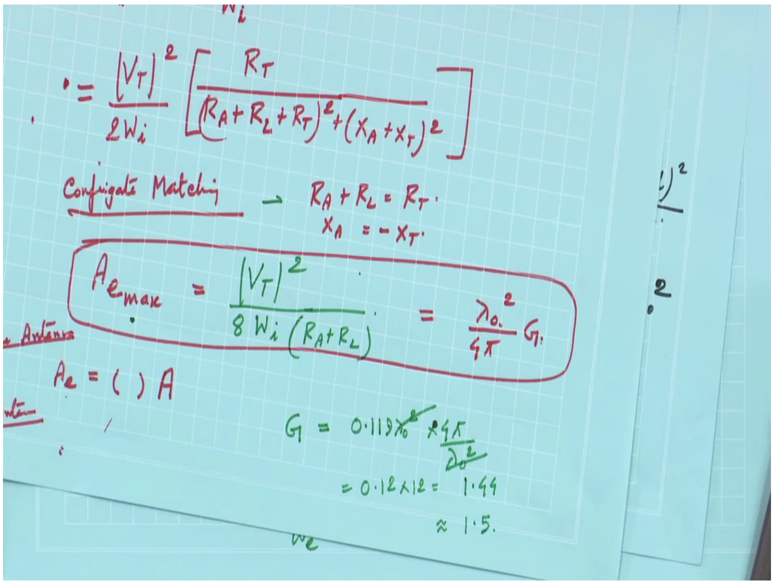

That also we have not found generally we found find it and, I think in assignment it will be given that find the gain of a current element, find the gain of a current element.

So, G find out G from here so, what will be the value of G? A e max is for current element 0.119 lambda naught square into 4 pi by lambda naught square. So, lambda not square will cancel 4 pi roughly 12 into 0.12, 0.12 into 12.

So, it is 1.44 the actual value is if you do it will be 1.5. So, roughly the gain of a thing is 1.5, that is compared to an isotropic antenna the maximum direction when current element we have seen, where is the maximum direction of a current elements radiation, the radiation pattern of current element is something that in these direction it is having maximum radiation.

So, compared to an isotropic antenna it forms, 1.5 times power in this direction and, that is not bad a practical antenna like dipole resonant dipole half wave dipole that has a gain of 1.67. So, we can say that current element earlier we have discussed, when we introduced the thing that current element is a hopeless radiator.

Because it is radiation resistance is very low, but it has a very good characteristic it has a very good directive characteristic. So, that is why current element we studied we studied for many other things that time we said that it is same gives us, very good picture about what happens for other complicated antennas, for all complicated antennas some general things we derived from current element.

But it also is a very good radiator, because of it is structure because of is geometry and that is proved here. So, this value actual again I will use in the next one. So, now I will find out, I will do a thought experiment to prove to you this formula that means, we will have to prove this formula because this I have not proved A e max that means, what is the relation between transmitting gain and effective aperture so, that I will prove now,

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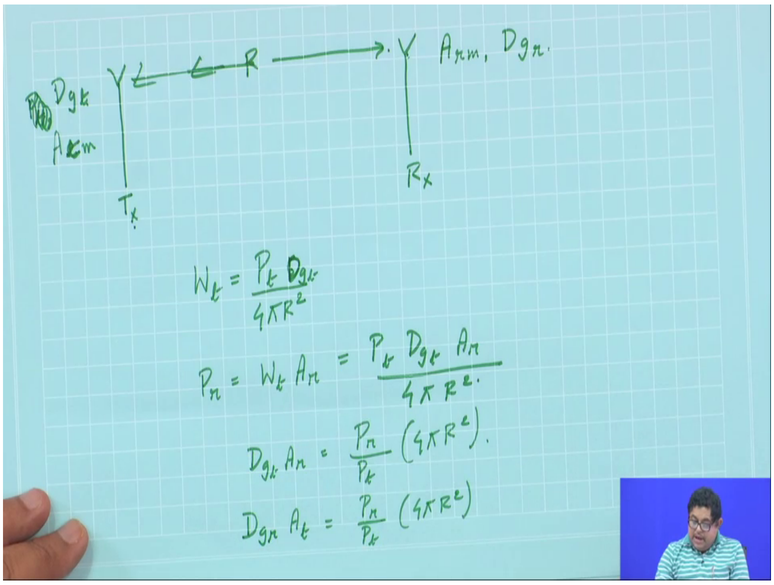

So, for that consider two antennas, separated by a distance R. So, this an antenna this is an antenna separated by a distance R. There in the far field of each other and let us say that this is a transmitting antenna this is a receiving antenna.

So, whatever this is transmitting this is receiving, how much it is transmitting let us say that it is transmitting P t amount of power, then it is directivity is D g t in this direction, or I can say maximum directivity in this direction, or directivity in this direction, if it is it is a V that is D g t.

You see T subscript I am using because it is a transmitter and, it is area effective area is A e, or I can write A t m a maximum effective area for this case. And the corresponding things for these antennas are A r m and D g r ok.

So, that means, basically this P t is later comes actually antennas are these are the characteristic A t m and, D g t for this A r m D g r for this, because these are receiving antenna these are transmitting antenna.

Now, what is a power density at this point, if this antenna is connected to a transmitter of power P t, can I say that W t here, that will be that P t by 4 pi R square, because this is a far field spherical wave into D g t, we are assuming not gain we are saying directivity that means, lossless antenna and efficiency one ok.

Now, so the how much power this antenna will collect power collected will be P r is equal to W t into A r. So, that will be I am not writing m because m is a maximum of that, I am writing in an earlier. So, that is P t D g t A r by 4 pi R square, from here can I write D g t A r is equal to P r by P t, 4 pi R square ok.

Now, I invoke a theorem called reciprocity theorem. So, I say that if antenna two is now used as a transmitter, antenna this one as a receiver and the intervening medium is linear passive anisotropy that means, whatever is called simple medium.

Then can I write that you do this thing that, now this is transmitting P t, this is transmit receiving P r the things are same. So, if we again write we can prove that D g r A t will be equal to P r by P t 4 pi R square. So, from this we say that D g t A r is equal to D g r A t.

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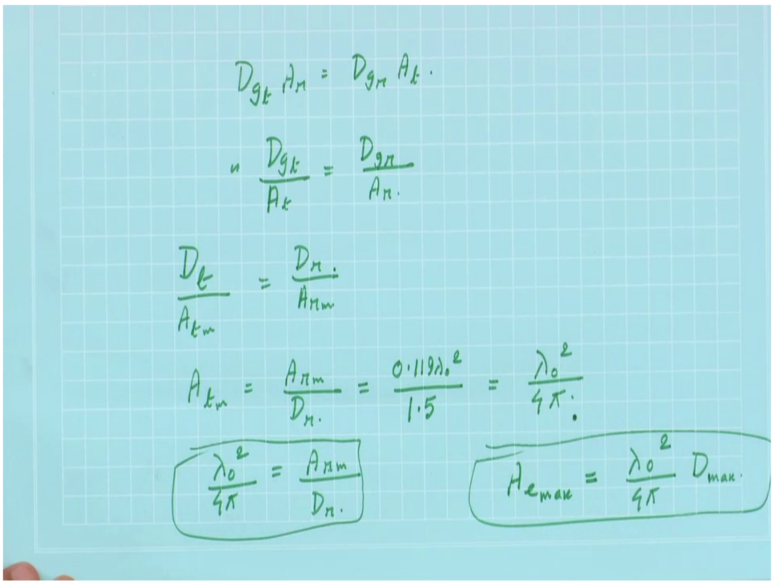

So, I will write that that D g t A r is equal to D g a r A t, or D g t by A t is equal to D g r by A r, this relation says that increasing directivity of an antenna, increases its effective aperture in the same proportion.

So, if A t m and A r m are maximum effective aperture of antenna 1 and 2 respectively, then we can write D t by A t m is equal to D r by A r m, where D t and D r are the maximum values of the directivities.

Now, you see this relation is valid for all the antennas, considered that the first one is an isotropic antenna. If it is an isotropic antenna its maximum directivity is 1 so, we can write A t m is equal to A r m by D r. So, what is the maximum effective area of an isotropic antenna, it is equal to the ratio of the maximum effective area, to maximum directivity of any other antenna. I repeat the maximum effective area of an isotropic antenna is equal to the ratio of the maximum effective area to maximum directivity of any other antenna.

Let me choose that any other antenna to be current element, because I know these two value for the current element. So, put it so what is the maximum effective area of the current element? It is 0.199 lambda not square and what is D r that is 1.5. So, find out this is nothing, but so, this value is how much lambda naught square by 4 pi.

So, we can say that lambda naught square by 4 pi is equal to A r m by D r. So, from here, we can say that A e max of any antenna, because now r subscript I am leaving.

So, this is true for any general antenna. So, A e max is nothing, but lambda naught square by 4 pi into D max or something, now many times you will see that that time I have written you the gain so, it is a gain. So, this formula is proved that A e max is lambda naught square 4 pi instead of D max we are calling it gain.

If there are those efficiencies involved that should be affected into here. So, this is a very useful thing but if obviously, if this there are there here, we have assumed that the antenna are lossless, the antennas are there are no polarization mismatch, there are no impedance mismatch etcetera.

If that is there all that should be factored here, as the efficiencies. But this is a very useful relation because, if I know transmission characteristic of any antenna, this relates me to find the receiving characteristic of the antenna. So, this route is through the effect area of the antenna that is why, it is a very important parameter for the antenna.

Thank you.