Digital Circuits
Prof. Santanu Chattopadhyay
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 09
Boolean Algebra (Contd.)

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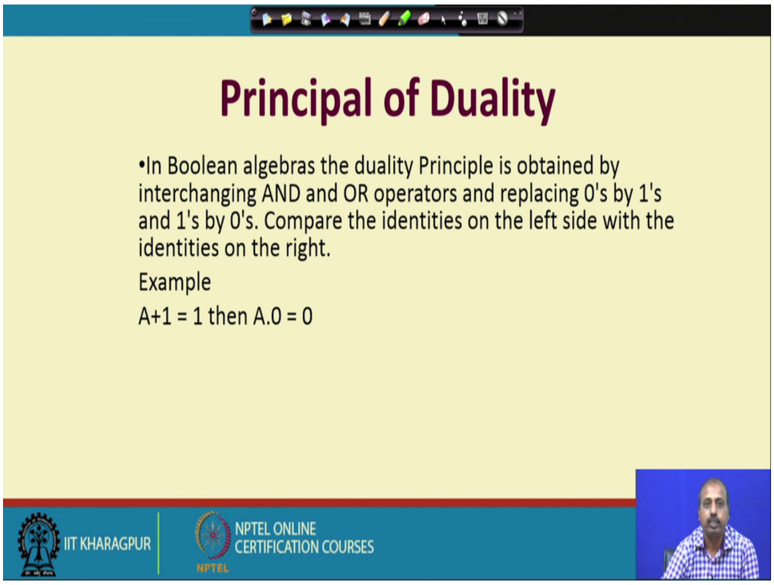

So, next will be looking into something called principle of duality so, in case of in this, what it says is that in case of a Boolean algebra the duality principle. So, it says that you can interchange this AND and OR operators, and at the same time you have to replace 0's by 1's and 1's by 0's, then you get the relations.

For example, this is a Boolean expression A plus 1 equal to 1. So, we will see its proof later. So, then if this is true, then you can replace this 1 by 0 and these or by and so, this A dot 0 will be equal to 0. So, if A or 1 equal to 1, then a and 0 will be equal to 0. So, this is called principle of duality. So, you can interchange the OR and AND operator and accordingly you have to interchange the that 1's and 0's

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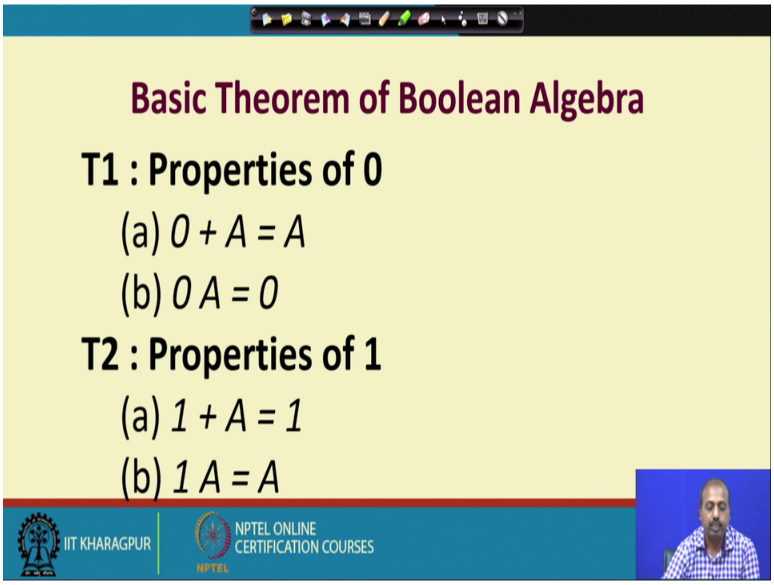

So, we will them again and again. So, some of the properties of Boolean algebra so, if we look into the basic theorems are like this, the first theorem is the properties of 0. Say is the 0 plus A equal to A. So, so if we if we have a variable A, then you can understand that if you audit with 0 the result will be equal to A. So, these are known as Boolean identity, these are Boolean theorems like that.

Similarly, 0 and A equal to 0 then 1 plus A equal to 1, then 1 dot 1 and A equal to 1 ok. So, these are some of the properties of Boolean algebra.

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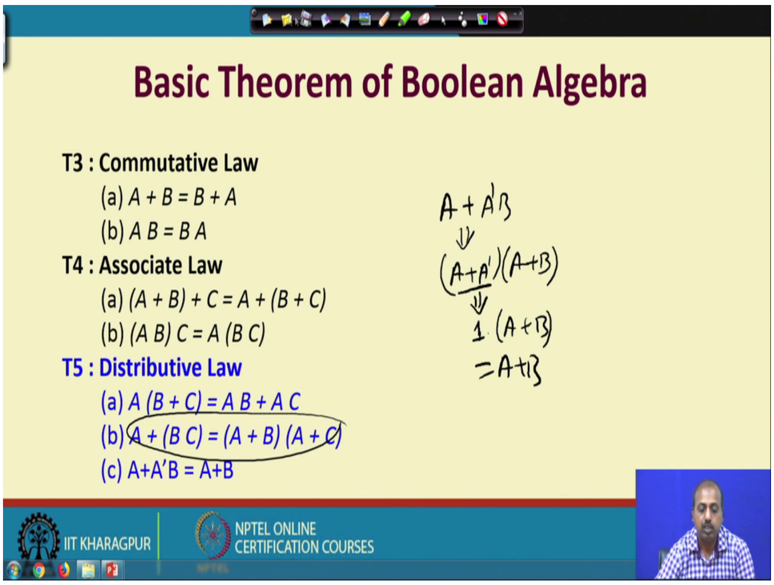

Then there is a commutative law it says that A plus B A or B equal to B or A; similarly so, by means of duality so, we can replace this plus by dot. So, A dot B will be equal to B dot A, then there is a associativity associativity law it says that A plus B plus C. So, that is I am doing or of three variables out of that I in the first case I do or of A and B first and then with the result I or C, the second case I or B plus C B and C first and then we were do the or of A with that. So, the both the cases the result will be same. So, A plus B plus C is equal to A plus B plus C.

Similarly, from the AND operation also A and B and C is equal to A and B and C. so, that is we can we can the that is on a associativity law. Then there is a distributive law which says that A and B or C equal to A and B or A and C and by means of duality. So, if you replace this dots by or pluses and pluses by dots that is or by and and and by or.

So, you get from this relation A, the relation B says that A plus BC will be equal to A plus B into A plus C. There is another distributive law which say that A plus A bar B equal to A plus B. So, you can get a proof of this very easily say this one. So, this is actually A plus A bar B. So, if you allow if you if you look into say this rule, it says that I can write it as A plus A bar into A plus B by using the distributive law.

Now, this A plus A bar so, this is this is equal to 1, because this is always true so, A plus A bar into A plus B. So, this is a tautology. So, previously we have seen that p plus p bar is a always equal to 1. So, that is tautology. So, this is this is equal to A plus B. So, A or A bar B is equal to A plus B. So, this way we can have a number of properties of this Boolean algebra.

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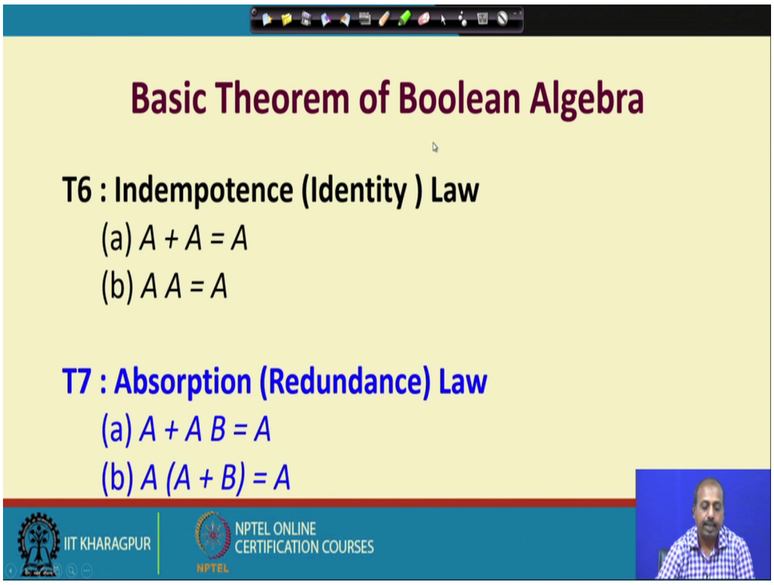

Then there is indempotence or identity law which says that A plus A equal to A, and AA equal to A. So, that is A variable plus itself. So, x plus x in our normal arithmetic we have equal to 2 x, but here it will be equal to x only. So, here a plus A equal to A similarly A and A is equal to A.

Then there is absorption law or redundance law it says that A plus AB equal to A and by duality A dot A plus B equal to A. So, here also the same thing like you can say. So, the if you try to analyze it logically. So, it is A plus AB. So, I A is true or AB is true. So, for this one also A has to be equal to true so, naturally that is equal to A.

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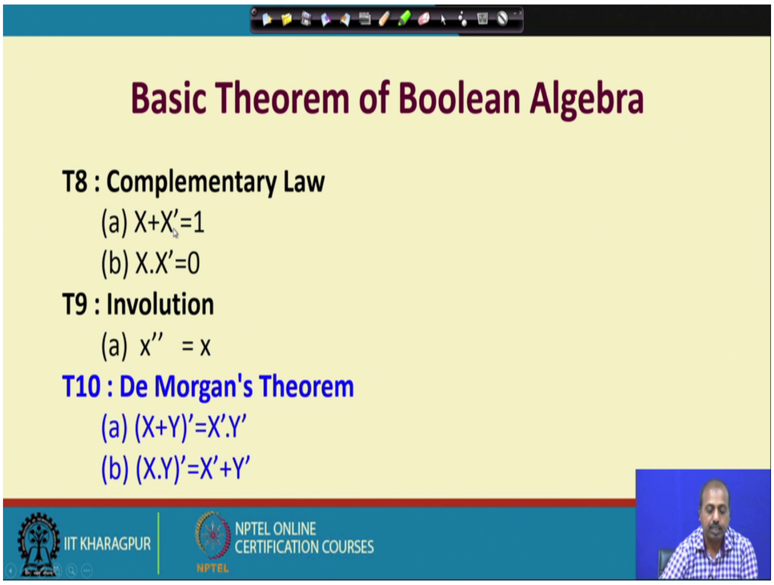

So, this way we can have a number of rules for this Boolean algebra, then this X plus X bar equal to 1. So, that is complementary law. So, that is the that is the tautology then X X bar equal to 0 that is the fallacy that we have seen previously then involution. Involution says if you do complementation twice X X bar is the X dash is the complement of X and then if you take complement again, then you will get back the original variable X. So, this is involution.

And there is another pair of rules which are known as De Morgan’s theorems; it says that X or Y bar equal to X bar dot Y bar. So, similarly by duality so, X Y bar equal to X bar plus Y bar. So, these are De Morgan’s theorems. So, all these identities that we have written here they are very much useful for the sake of Boolean simplification.

Like if you want to get some if you have a complex expression and you want to simplify, then you may use some of these rules to do the simplification process ok.

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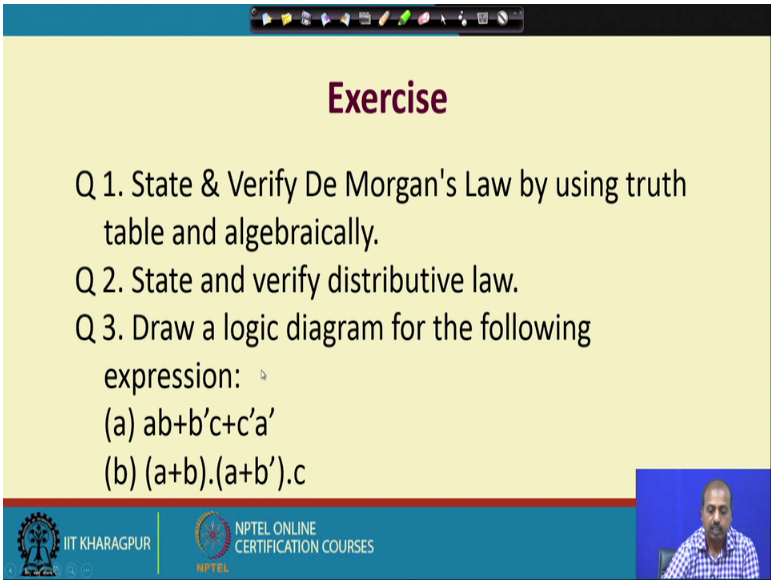

So, we will be doing a few exercises before going further ok.

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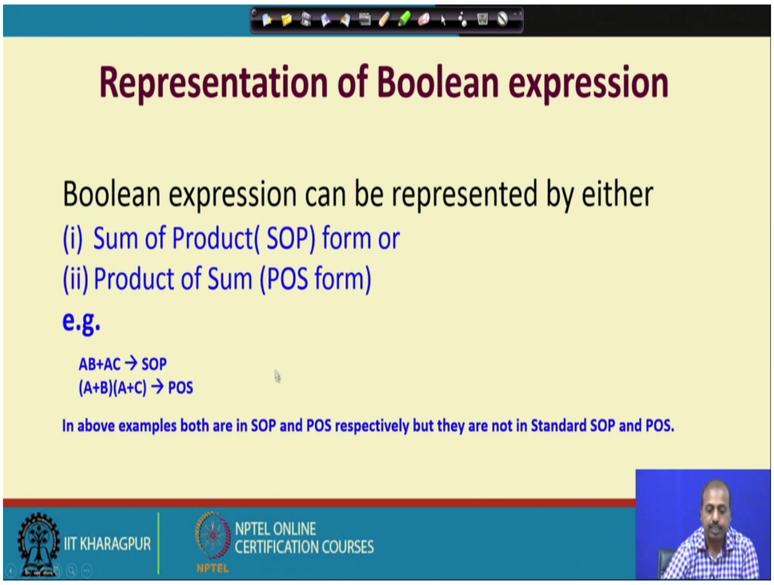

So, let us let us look into some of the properties that the first property that we will look into it says that the complement of x is unique.

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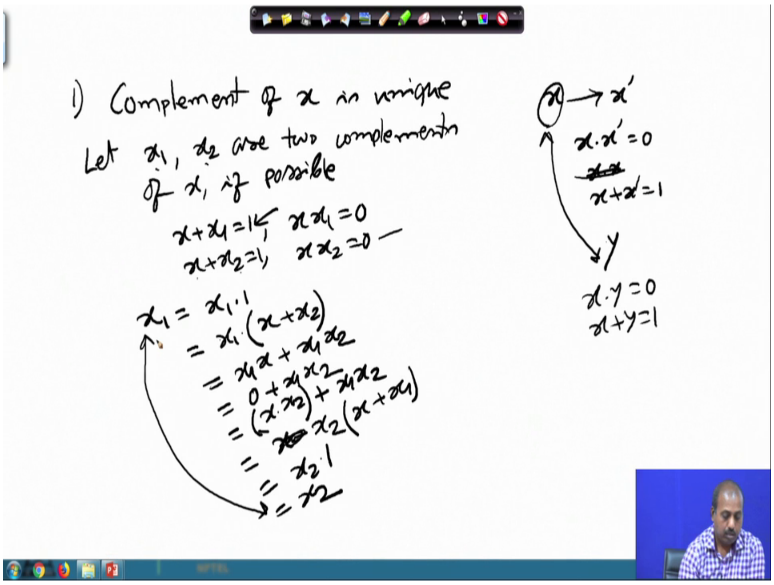

The complement of x is unique; that means, we cannot have two different complements. So, complement of x means so, if x is a variable, its complement is x we represent it as x x dash, then it says that x. So, we have seen that x dot x dash equal to 0, and x dot sorry x plus x bar x plus x dash equal to 1.

So, this is the complement then if I can for a for x, if I can find another variable x bar such that, x and x bar is 0 and x plus x bar is equal to 1 or in general instead of writing x bar to make it simple, suppose let us write it as y. So, y will be said to be complement of x. So, if I have got x dot y equal to 0 and x plus y equal to 1.

Now, the question is, can I have more than 1 y more than 1 y for x. So, can I have more than 1 complement for x. So, if possible let we have two complements one is x 1 and another 1 is called x 1 and the other is called x 2 are two complements of x if possible complements of x if possible then what we can write. So, since x 1 is a complement of x. So, we have got this property x plus x 1 equal to 1, x x 1 equal to 0. Similarly since x 2 is a complement of x. So, we have got x plus x 2 equal to 1 and x x 2 equal to 0 ok.

Now, x 1 can be written as x 1 into 1 x 1 and 1. So, because that is there in the multiplication by the identity rule. So, that says that the we can multiply we can end with the 1 now this is. So, this can be written as x 1 and now we have got this one. So, I can write it I can replace it by x plus x 2 I can replace it by x plus x 2 now if I. So, this is x 1 x plus x 1 x 2 so, x 1 so, x x 1. So, since x 1 is a complement of x. So, this is 0. So, this is

0 plus x 1, x 2. So, that is equal to. So, so it is 0 plus x 1 x 2, now this 0 can again be written as x x 2; x x 2 plus x 1 x 2 as we have.

So, 0 since this x 2 is a complement of x, x 2. So, x x 2 is equal to 0. So, this 0 I am replacing by x x 2. So, we have got x x 2 plus x 1 x 2. So, this is. So, if you distribute. So, you will get. So, this is x x 2 plus x 1 x 2. So, if I take a x 2 common if I take x 2 common from this 2. So, I will get x plus x 1 and x plus x 1 is equal to 1.

So, I can say this is x 2 into 1, and that is equal to x 2. So, it says that if I take two different if possible x 1 and x 2 are two differ two complements of x, then we finally, say that x 1 and x 2 there same. So, that proves that the complement of x must be unique ok.

So, next we will prove look into another property.

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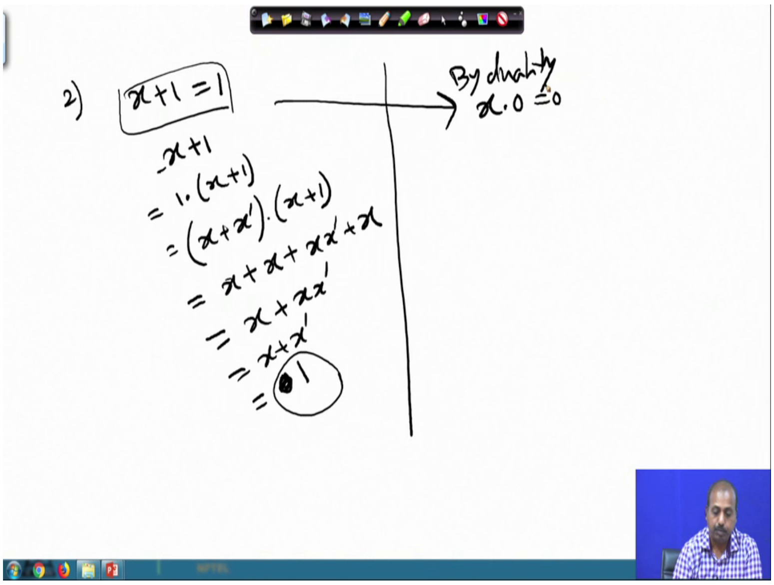

So, second property we will says that x plus 1 should be equal to 1. So, how to do this? So, to prove this so, x plus 1 it can be written as 1 into 1 and x plus 1. So, this 1 can again be written as x plus x bar into x plus 1. So, if you distribute. So, this is x. So, if you if you just multiply. So, this is this x into x will give me x, then again this x into 1 will give me x, then this is x x bar plus x ok.

So, this is what you are getting. So, this is x. So, the these 3 x's. So, that gives me x plus x x bar. So, if you x plus xx bar so, by that absorption law you know that this is nothing, but x plus x bar and that is equal to that is equal to 1. So, we started with x plus 1 and finally, we will ended into 1. So, this x plus 1 is equal to 1.

So, by duality you can say that if you replace this plus by dot that is or by and and 1 by 0. So, you can get the other property by duality so, x dot 0 equal to 0. So, this is by duality. So, you can also take as you can get a similar proof, but since we have prove the left one the so, right one is necessary. So, we can get this one directly ok.

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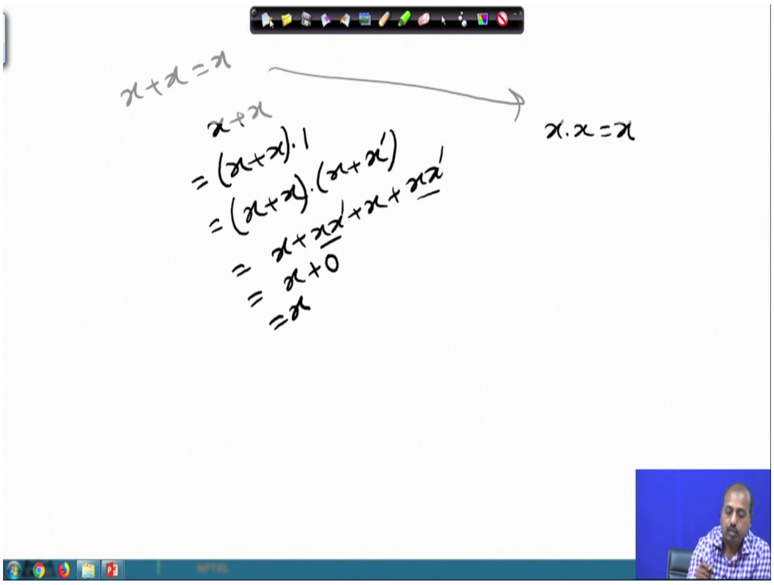

So, then we will prove that indempotent law it says that x plus x equal to x. So, for proving this we start with x plus x. So, this is equal to x plus x and 1. So, this 1 can be written as x plus x bar. So, this is and x plus x bar now if you. So, this is. So, x plus x is x into x plus.

So, we if you if you do a multiplication, then you will get x plus x x bar plus x plus x x bar; so, this x x bars are equal to 0. So, you finally, get x plus 0. So, that is equal to x. So, you get the indempotent property proved that is x plus x equal to x and again by duality. So, we can say that x dot x will also be equal to x by duality. So, this proves the indempotent property.

Next we will try to prove another result which says that the involution property.

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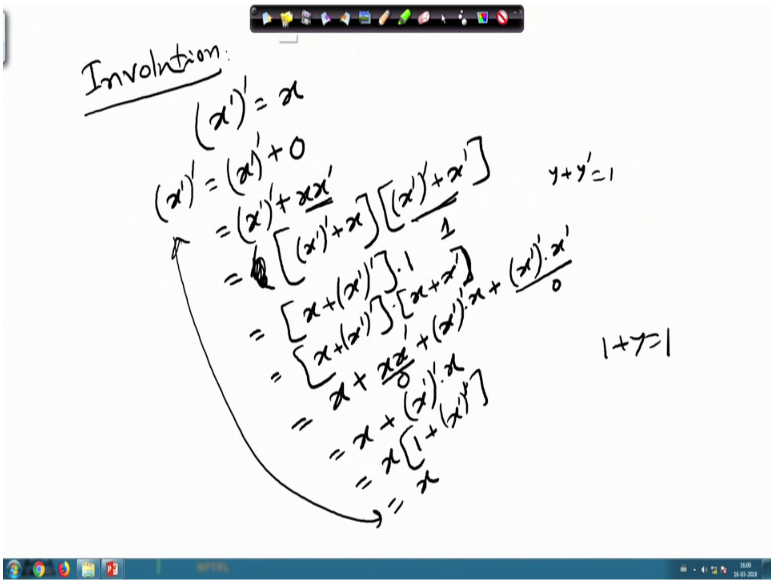

So, next we will prove the involution property, which says that x x of x double complementation of x will give you back x; x complement and com not of x not of not of x so, that gives me x. So, we do it like this, x not of not so, this can be written as x double not plus 0.

So, this 0 can be written as x x bar or x x x x dash. So, if you do distribute. So, you will get it like. So, you will get it like something. So, the if you distribute this over this. So, you will get this x bar bar plus x as 1 term and this x bar bar plus x bar as the other term

So, we distributing it so, this is so, this part so, this part is equal to 1 because this is x bar plus x bar bar. So, this is x if I write x bar equal to y. So, this is nothing, but y plus y bar and y plus y bar equal to one. So, this part becomes equal to 1. So, what we get is something like x plus x bar bar and 1 ok. So, again we do a distribution.

So, this part remains unaltered x plus x bar bar. So, this part remains unaltered and this 1 we write it as x plus x bar this 1 we write as x plus x bar then again we distribute ok. So, if you distribute this or ok so, this is since this or distribute. So, we can write from there that this is equal to so, or if you just do a flat multiplication, then you will get it like x plus x x bar plus x bar bar x plus x bar bar into x dash.

Now, out of this so, this is 0 this is 0. So, you finally, you have finally, getting like x plus x bar bar and x ok. So, this is so, you will get it like this. So, this is this is x if you take common from here. So, you will get 1 plus x bar bar now 1 plus anything is equal to 1. So, 1 plus y is equal to 1. So, this is equal to x. So, x bar bar is equal to x. So, this proves the involution property ok.

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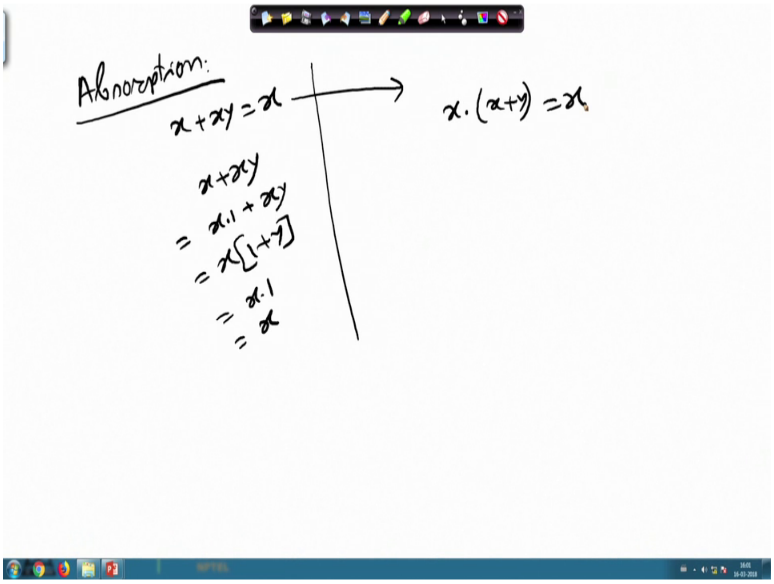

So, you can you can now prove the absorption property you can prove the absorption property, which says that x plus x y equal to x. Now x plus xy equal to x can be proved easily because x plus x y is equal to x dot 1 plus xy. So, that is if you take x common. So, we have already done this it is 1 plus y. So, 1 plus y is equal to 1.

So, that is equal to x dot 1 so, that is equal to x so, x plus xy equal to x. So, if you follow the duality, then from here I can write that x dot x plus y. So, this will also be equal to x. So, this is the duality of the absorption law v then prove another result says that x plus x bar y equal to x plus y.

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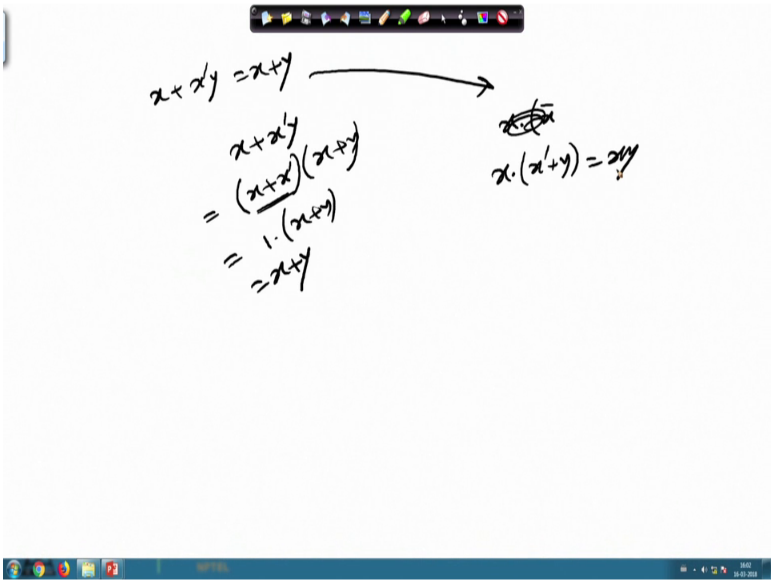

So, previously we have seen that x plus x y equal to x, now x plus x bar y equal to x plus y so, x plus x bar y so, this is. So, if you do it distribute distribution. So, this is x plus x dash into x plus y by the distribution property. So, this is x plus x dash is 1 so, this 1 into x plus y. So, that is equal to x plus y. So, this way we can prove it and again by duality you can say that x dot x bar x dot, x dash plus y equal to xy ok. So, again the dot so, that will that is by duality ok.

Then associativity if you want to show associativity then we can do it like this associativity means.

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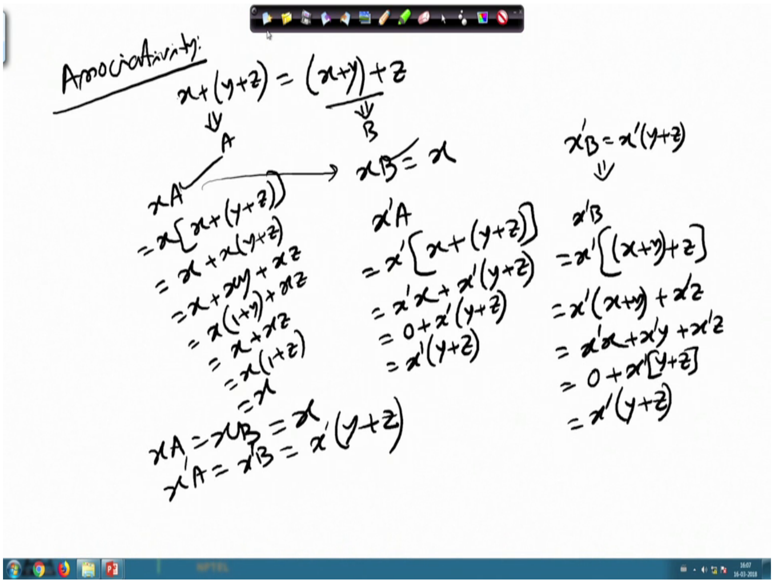

So, associativity of or say or operation associativity suppose we have. So, we need to show that x plus y plus z equal to x plus y plus z. So, in Boolean algebra how will you show that what we will do. So, this side will call A, this side will call B and then we will bring this A and B to a similar form and then we will show that they are same how will you do that.

So, see this is equal to x this is equal to A. So, we will take x and A. So, this is equal to x into x plus x and x plus y plus z. So, this is equal to by absorption law this x into x so, x into x plus y plus z. So, this is equal to x ok. So, you can if you if you want. So, you can just do 1 more step and show that this is true like you can say like if I if I do a multiplication. So, this is x plus x into y plus z.

So, this is x plus xy plus xz so, x plus xy. So, it is 1 plus y plus xz. So, that is again. So, that is x plus xz. So, that is x into 1 plus z that is equal to x. So, x A is equal to x ok.

So, similarly this x B so, if this is x A then accordingly you can show that x B is also equal to x. So, this can be similarly it can be shown now we take x bar A so, x x x dash A. So, x dash A is x dash into x plus y plus z ok. So, it is x dash x plus x dash into y plus x dash and y plus z. So, this is x x dash is 0. So, this is 0 plus x dash and y plus z. So, this will give us. So, this is. So, this is x dash y plus z. So, x dash A gives me this thing.

Similarly, you can see you can say that x dash B. So, x dash B is it can be shown similarly that x dash B is equal to x dash into y plus z. So, you can just. So, you can you can show similarly that if you take x dash B. So, then also it will lead to x dash into y plus z.

So, if you have if you can just do it ones like so, x dash B equal to x dash into x plus y plus z equal to x dash into x plus y plus x dash into z. So, this is x x dash so, x dash x plus x dash y plus x dash z. So, this x x dash is this is 0 plus x dash into y plus z. So, that is x dash into y plus z.

So, we have got this x A x B x dash A and x dash B. So, we can say. So, what is happening is that x A equal to x B equal to x. So, this is one observation. So, from here x A x B and another observation is that x dash A equal to x dash B equal to x dash into y plus z. So, with these observations so, we will be we will be proceeding like this.

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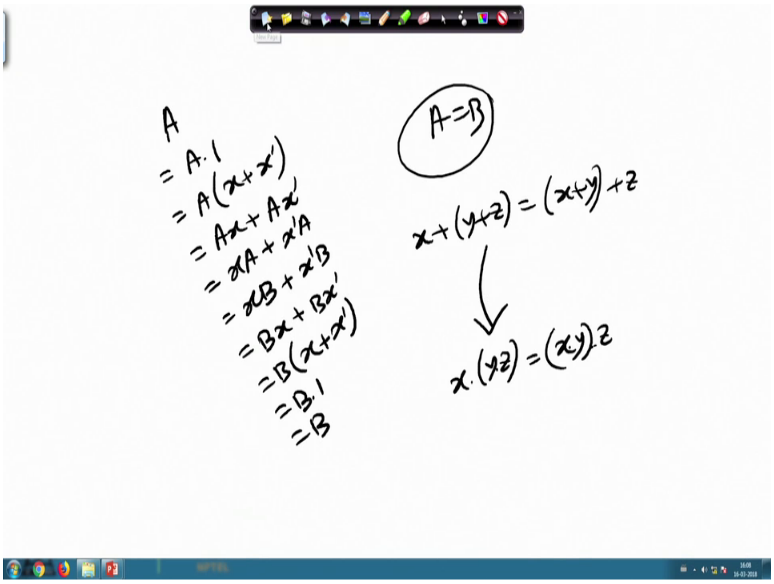

Now, we have now we will take this the variable A. So, A can be written as A and 1. So, it is A into x plus x dash. So, this is equal to A x plus A x dash now that A x is same as x A plus x dash A. Now, x A equal to x B and x dash A equal to x dash B ok.

So, I can take it can also be written as B x plus B x dash if you take B common. So, this is x plus x dash. So, that is B into 1 that is equal to B.

So, you have got A equal to B. So, that proves our result that this or is associative associativity of the or is proved. So, you can you can use the duality. So, we have got x plus y plus z is equal to x plus y plus z. So, if you use duality then it says that x dot y z will be equal to x dot y it will be x dot y dot z. So, that way you can have the duality.

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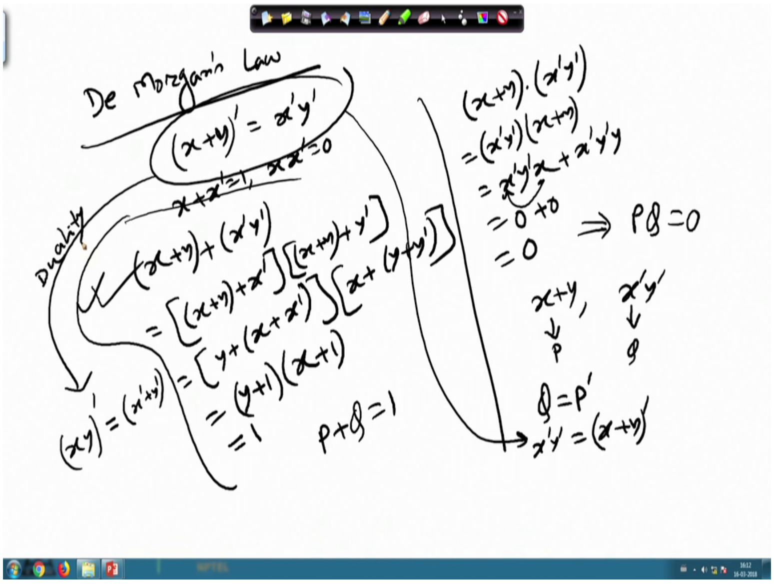

Next we will prove the De Morgan's theorem the first law. So, the De Morgan's law so, it says that x plus y bar dash equal to x dash y dash ok. So, we have got this properties known to us like x plus x bar equal to 1, then x x bar equal to 0. So, this properties are known to us similarly we can. So, what we do is that we take this sum so, x plus y plus x bar y bar ok.

So, if we do this now we use the distributive property. So, this is this can be written as x plus y plus x dash into x plus y plus y dash. So, this is nothing, but y plus. So, this can be written as y plus x plus x bar into x plus y plus y bar now this x plus x bar is equal to 1. So, I have got y plus 1 into x plus 1. So, that is equal to 1 fine. So, we have got this has equal to 1.

Also, if I take this product ok so, x plus y into x bar y bar. So, if I take this product then what will happen? So, this can be rewritten as x dash y dash and x plus y. So, this is x dash y dash x plus x dash y dash y. So, x dash y dash x so, x and x dash will make it 0. So, it is 0 plus 0 equal to 0.

Now, you compare this terms so, this term x plus y another term x dash y dash. So, what was happened? If I call it say P and let us call it Q. So, from the first property from this first proof I can say that P plus Q is equal to 1 and from this proof I can say that P Q is equal to 0 ; that means, Q is actually the invert of P, Q is P dash.

So, now, if I substitute so, Q is x dash y dash. So, x dash y dash is equal to P invert. So, P is x plus P is x plus y P is x plus y so, x plus y dash. So, we have got this De Morgan's theorem here ok.

So, x plus y dash equal to x dash y dash so, this way we can prove the De Morgan's theorem first law and by means of duality, you can claim the second law that is x y. So, if you replace this plus by dots. So, you get the dual law. So, x plus y dash equal to x dash plus y dash so, this is from here by means of duality this is by duality ok.

So, in this way so, we can have proof of this Boolean identities and of course, the other way is to draw the truth table and the show that these are all two expressions are equivalence. So, that can be done, but if it if we want to show by Boolean identities, then also we can prove it like this fashion.