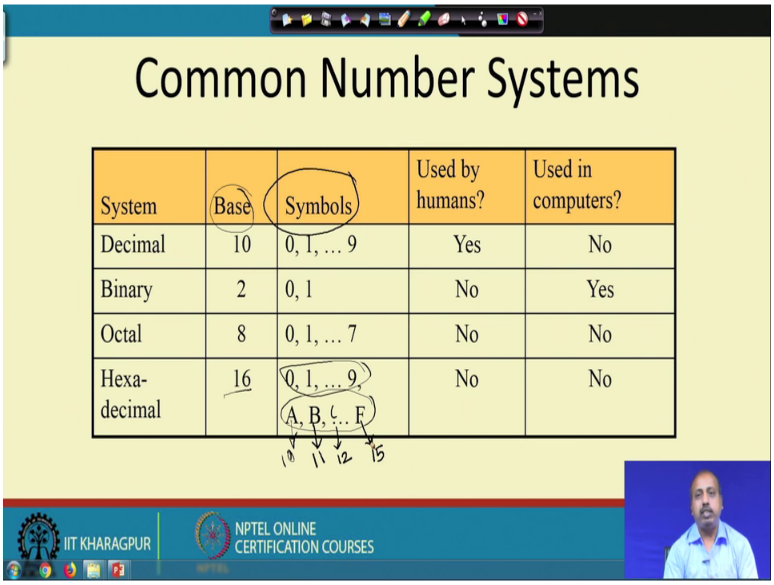
Digital Circuits
Prof. Santanu Chattopadhyay
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 03
Number System

So, we will be looking into the number system. So, Number System that tells us how can we store or represent numbers in a digital system. So, this is a very important part because this will tell us how can we do processing over those numbers and all, and what are the a limitations and what are the advantages that we can get of different number systems, that we can think about. Or what are what are in general the number systems that are available in the in a computer, or say for the sake of simple mathematical interest also, you can look into this topic which will be telling about different ways in which you can represent numbers.

(Refer Slide Time: 01:01)



So, any number system that we look into so, it will have 2 important components in it. One is called the base of the number system, so, where we talk about some quantity that we represent like how many digits, can be the how many symbols, can be there different types of symbols and the other part is the symbols.

So, any number system that we talk about. So, these are the 2 things that we have to tell. So, to tell me a base so, as soon as you tell me a base we can assume that there will be so many different types of symbols that you can have. Now a number system maybe a consisting of different number of digits. Like I can have say a decimal number system where base is 10. So, as soon as we know that the base is 10, we know that there are 10 symbols, and the those symbols are conveniently represented as the say 0, 1, 2 up to 9 ok.

So, if you are talking about a binary number system, the base is 2, and in that case the symbols only 2 symbols are needed so, we represent them as 0 and 1. So, so, the we that there is of course, some value associated with this. So, the convention that we follow is that in a decimal number system. So, 0 we represent the integer value 0, 1 represents the integer value 1 and similarly the 9 represent the integer value 9. In a binary number system the base is 2, and the symbols that we have are 0 and 1. So, here also we have got the assumption that 0 represents the integer 0 integer value 0, and 1 represents the integer value 1.

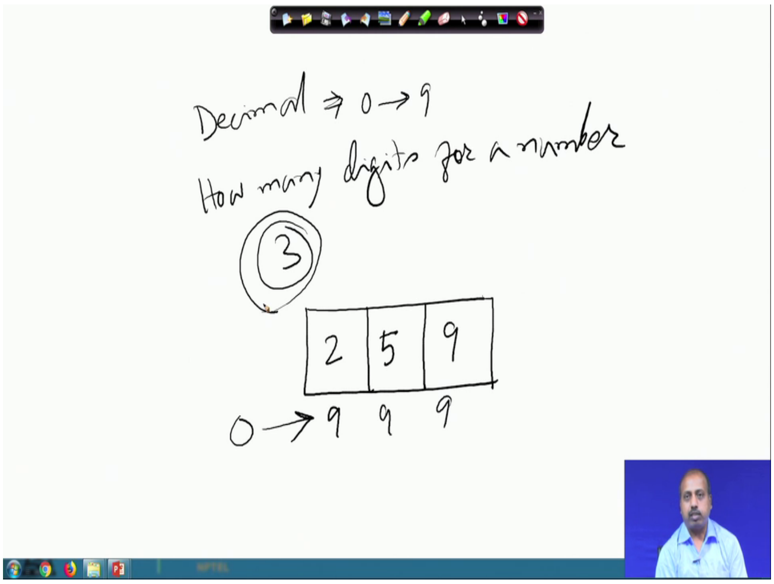
In an octal number system base is 8, and naturally there will be 8 symbols and we have got the symbols 0 to 7. Hexadecimal number system the base is 16, now here is the problem. Because so far whatever we had so, we have sufficient number of symbols to represent individual values of the symbols, now we do not have that. So now, you see that up to 0 to 9 we take. So, these are these are the 10 numbers that we have after that so, we do not know any symbols. So, what is done is that this ABCDEF so, they have been taken as symbols.

And here actually comes the thing that I was talking about the value associated with the symbols. So, this A it is associated with the value 10, the integer value 10. Similarly this B so, this is associated with the integer value 11. So, C associated with integer value 12, that way f associated with integer value 15. So, you can you can say that instead of this ABCD, I will be using some other notation alpha beta gamma delta like that.

So, there is nothing wrong in using some different notations for these symbols, but what is required is that you have to tell what is the corresponding integer value, for the symbols otherwise we will not be able to represent the some quantity in that number system ok.

The next thing that we need to know or need to understand is like, whenever you are representing some numbers so, you have to tell how many digits are you allowing for the numbers. Like if I say that I have a, if I say that I have got decimal number system ok.

(Refer Slide Time: 04:29)



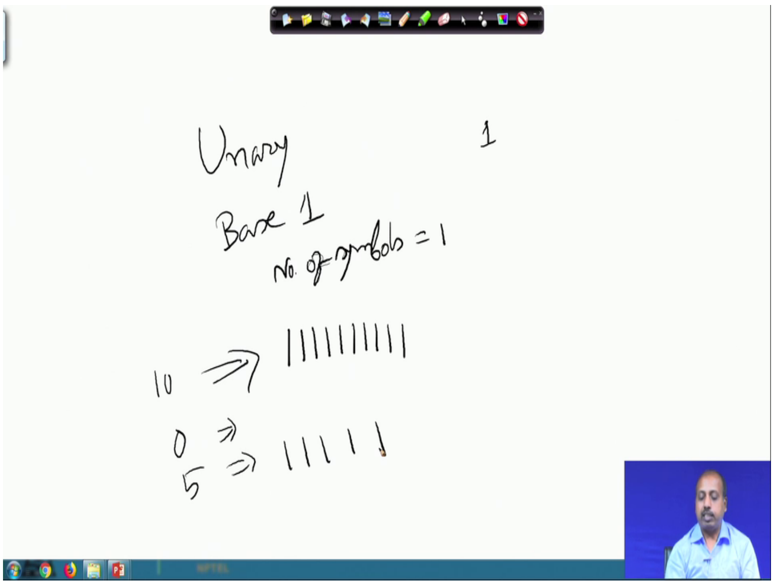
So, as soon as I say decimal. So, I know that the digits that I can have are 0 to 9, the next question is how many how many digits I can use, how many digits for a number. So, if you are trying to say that so, if I say that the I will be using 3 digits. So, you can say as if I have box like this and in this box so, there are 3 places ok. So, in the first place you can put a digit say 2.

So, here you can you can put a symbol 2, here you can put a symbol 5, here you can put a symbol 9. So, as soon as I limit this value to 3 so, you can understand the maximum value that I can have in this system is 999, not more than that. So, I can the this system can represent numbers from 0 to 999. So, this number of um digits that we are using for representation that is vital. So, whenever we are talking about. So, these computers or the processors we see that it is a 16 bit processor or 32 bit processor. So, essentially what it means is the information that it can handle the size of the data that it can handle as the basic processing limit. So, that is the that is going to limit the number of digits here.

So, going back to the point that we are discussing, we have got this number systems the their base values and their symbols are like this. Then we can say answer like whether they are use by human or not. So, of course, this is a bit tricky like somebody may be very comfortable with hexadecimal number system, and may be always writing in terms in terms in terms of the hexadecimal digits. So, that is very much possible, but normally it is not so. So, from our school days we are familiar with decimal number systems so, we will be we are we say that it is used by humans whereas, the others binary octal and hexadecimal are not, ok.

Then we convey so, so they are used in computers. So, decimal number system is not used in computers, and then we can have this binary number systems used octal and hexadecimal number system. Now you see that while talking about this one thing I have I have purposefully avoided; where can we have a number system with base 1, so, that we have not talked about. So, answer is definitely yes we can have a base one number system as well.

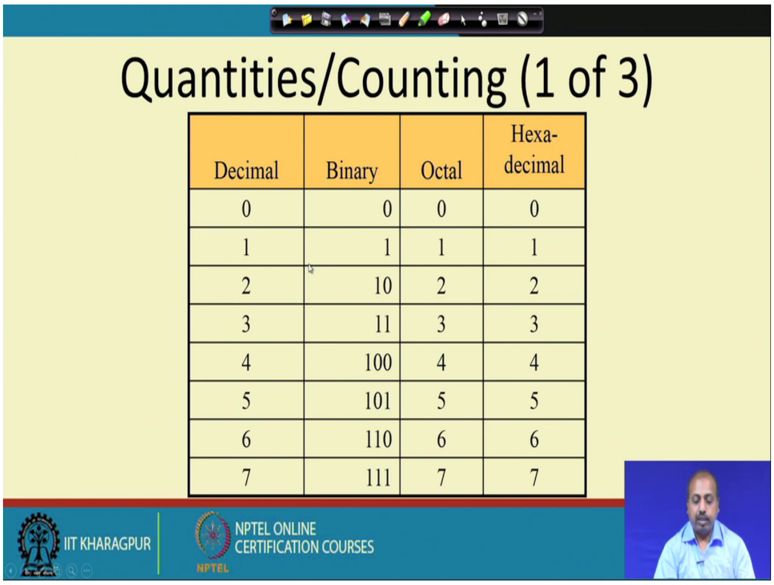
(Refer Slide Time: 07:14)



So, so, here if I say it is a base one number system so, that is then that is a unary number system. So, since base is 1, base is 1. So, I can have only a single symbol so, number of symbols, number of symbols that you can have is equal to 1. So, if I decide that, but the symbol that we will be using is also 1. Then if I ask you to represent 10 the quantity 10 in this number system, then what we will do? You will write 10 such ones, fine? Because I do not have any other symbols. So, as many value I want so, I needed a quantity 10 to be represented. So, I put 10 such 1's, I need to if I need to represent the quantity 0. So, I do not put any of these things so, the none of the ones are there.

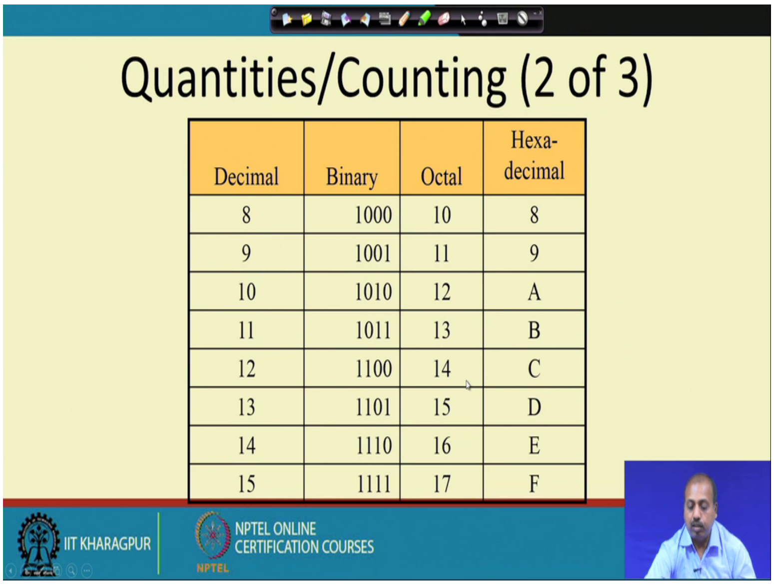
So, if I want to represent 5, then 5 such ones are there. So, this is unary number system so, it can be used, but this is not used, because of it does not does not make it amenable to the processing that we have possible with the other number system the other basis basically. So, that is the reason why this is not done, but it is very much possible the unary number system ok. So, in our discussion we will be concentrating mostly on these number systems and so, this decimal number system.

(Refer Slide Time: 08:44)



So, if we say the quantity 0, 1, 2, 3, 7. So, the 0 in binary will be represented as 0 in octal also 0 hexadecimal also 0. 1 will also be represented same, but 2 onwards there will be problem, because 2 cannot be represented by the symbols of binary number system. So, I need to use 2 symbols, 1 0 to represent 2, similarly once 2 is done. So, 2 3, they can be represented by binary number systems. And then octal will be there octal and hexadecimal they do not find problem till 7, so, we have got enough symbols to represent the numbers.

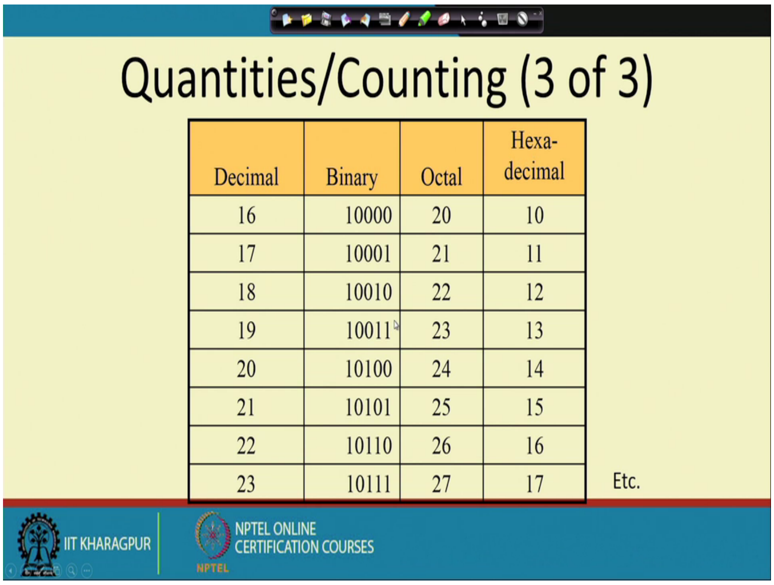
(Refer Slide Time: 09:25)



Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

But after that of course, problems starts coming like octal faces problem for 8 onwards. So, 8 has to be represented by 2 symbols 2 digit like 1 0 9 by 1, 1 like that. So, that way till 15 hexadecimal does not face any problem.

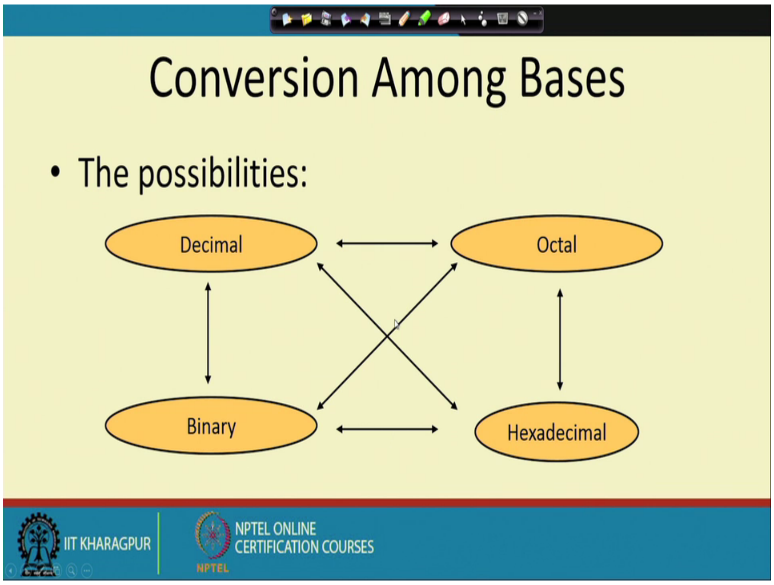
(Refer Slide Time: 09:47)



Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

But if you are going to 16 then of course, hexadecimal will have to have 2 digits now ok. So, this will be 16 so, that way it will grow. So, this way this numbers are or the quantities they will be represented in different number systems.

(Refer Slide Time: 09:58)

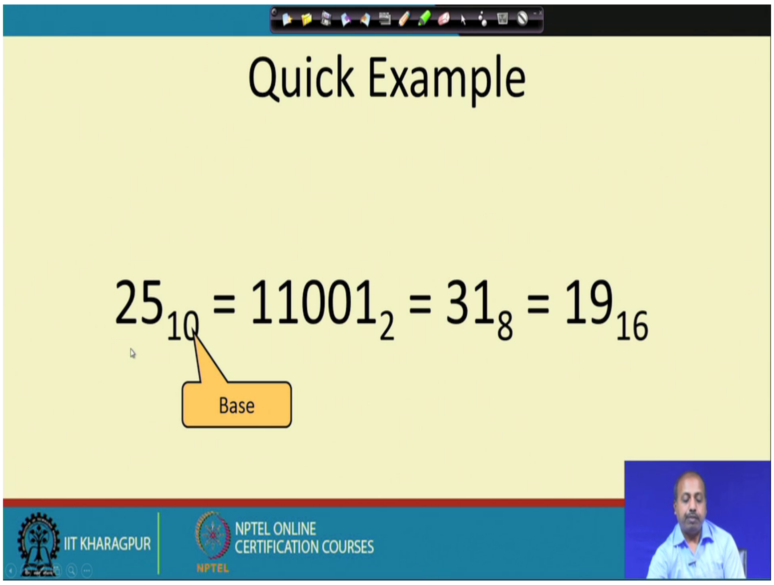


So, this talks about the conversion between the bases ok. So, these are the possible conversion like all these number systems we can convert from one form to another form. So, decimal to binary, binary to decimal, decimal to octal, octal to decimal. So, like that we can convert between this number systems very easily, and the so, ultimately in a computer system so, it understands the binary number systems.

So, they are of course, it does not have any bearing other number system it does not have any bearing so, truly speaking, if we are learning about this digital circuit class. So, digital circuit course we do not need to learn about the other number systems ok. So, that is just for our understanding of the binary number systems.

So, truly speaking so, if we know the binary number system well, we are done we do not need to know others, but because we are familiar with the other number systems. So, we will be looking into the conversion to make us feel comfortable with the number system.

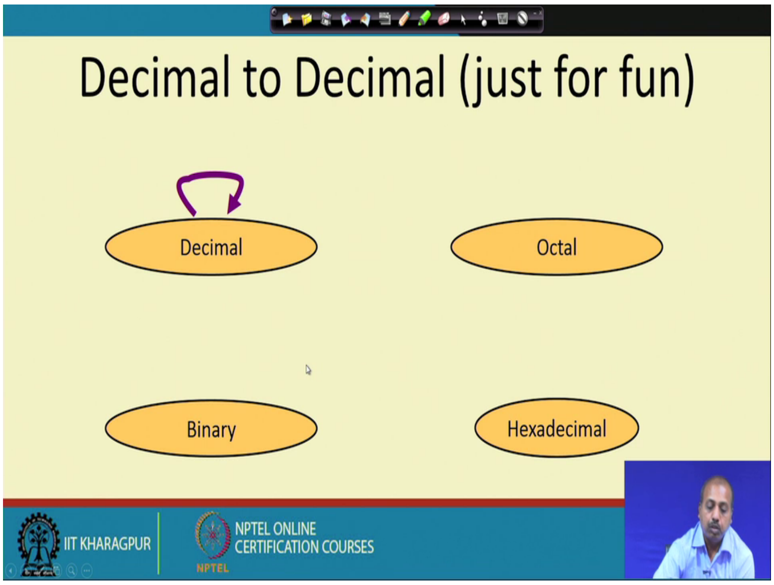
(Refer Slide Time: 10:59)



So, 25 so, normally while writing the numbers so, we do not write this base part, when the because in our day to day life or from our school days with the numbers that we know they are all base 10 number system.

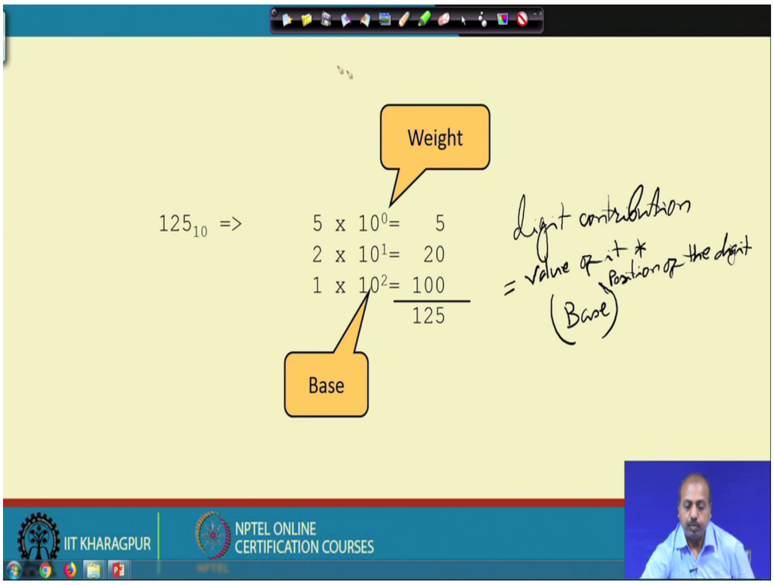
So, we will be ah, but if you if you want to make it explicit, then we do it like this after the number at the suffix we write the base. So, this 25 to the base 10 is equivalent to 1 1 0 0 1 to the base 2. So, in a binary number system so, it will be 1 1 0 0 1 it is equivalent to 31 to the base 8 octal number system and it is again equivalent to 19 to the base 6 16 that is hexadecimal 19 so, all these numbers are equivalent.

(Refer Slide Time: 11:46)



So, we will be looking into some conversions so, between these number systems. So, just to understand this conversion process so, we will be looking into the conversion of decimal to decimal, the decimal numbers how is it converted to decimal.

(Refer Slide Time: 12:03)



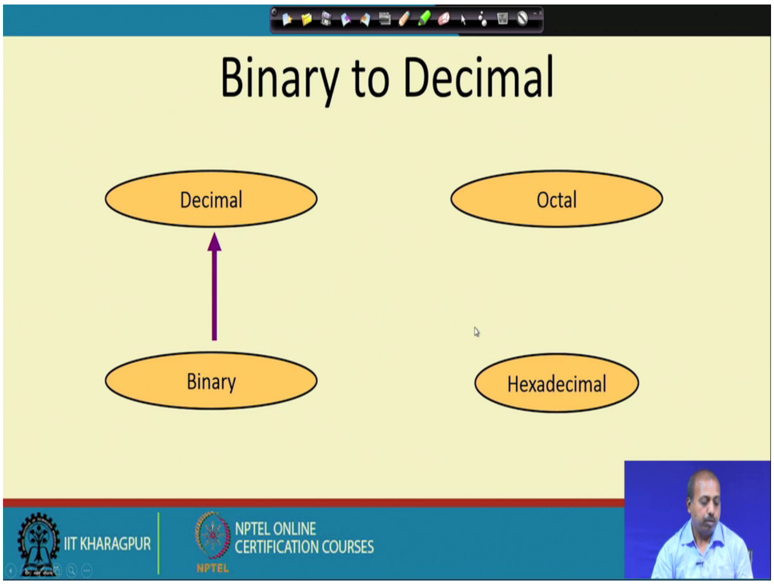
So, this will give us the understanding so, you see that we are not going to so, decimal to decimal conversion as such is meaningless. But this will this is just give us a recapitulation of our school days knowledge, about how this numbers are actually representing some quantity.

So, in this in a number like 125, this the that there are 3 digits 1 2 and 5. But all of them are not representing values of equal strength ok. So, the number 5 so, when you are looking into the number 5, say actually it is representing the number 5 into 10 to the power of 0 that is equal to 5. So, this 2 here so, this is actually representing 2 into 10 to the power of 1, that is 20, and this 1 here is representing one into 10 to the power of 2 that is 100.

So, the symbol that we have so, the value of it multiplied by the some weighted value of the base. So, that is going to be the contribution of a particular digit. So, a digit is; so, whenever we are talking about a digit so, digit contribution digit contribution is equal to the value of the digit, value of it multiplied by base taken to the power of the position of the digit. So, this is the overall formula that we are getting.

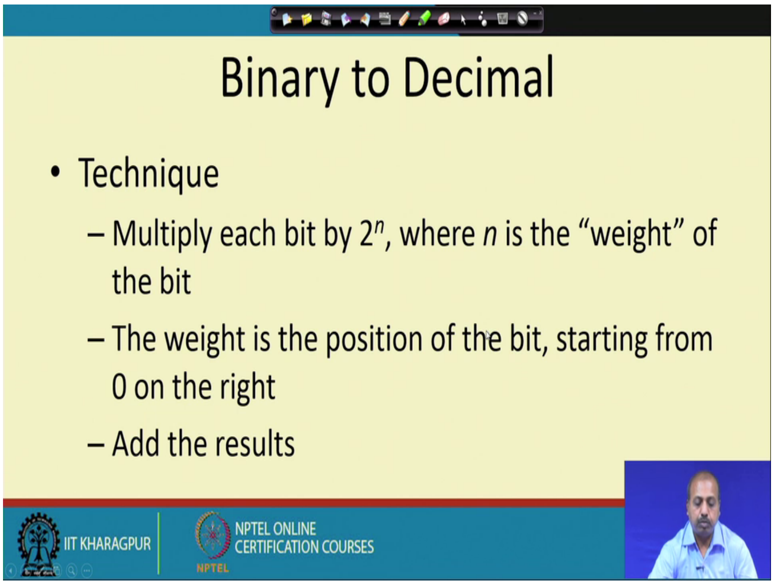
So, this is digit contribution is value of it multiplied by base to the power of position of the digit. So, this position is 0 so, it is contribution base to the power 0 this 2 is a position 1. So, this is 10 to the power 1, and this one is a position 2. So, it is 10 to the power 2. So, this is a weight varies with the position, ok. So, so, next we will look into so, so, this will tell us like how do how do we do the conversion.

(Refer Slide Time: 14:25)



Now, when we are converting say binary to decimal.

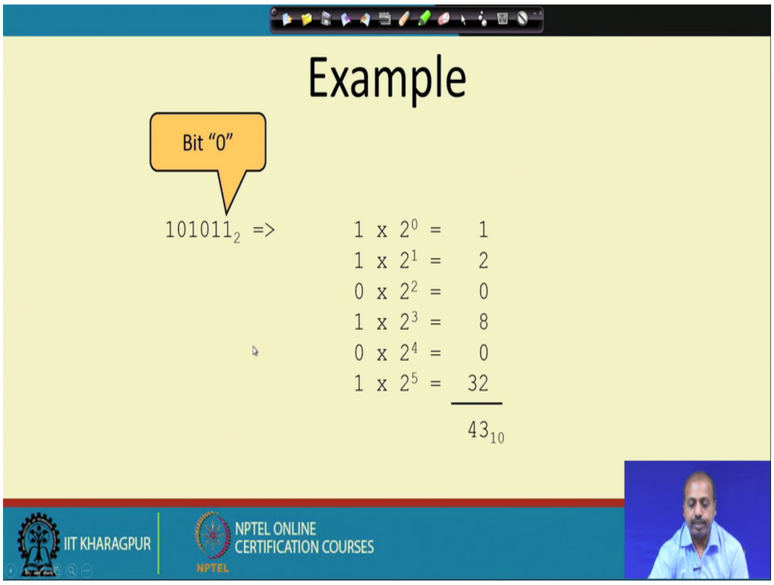
(Refer Slide Time: 14:33)



So, the technique that we have to follow is multiply each bit by 2 to the power n, where n is the weight of the bit. As I said that any number, when we are trying to get the value of a contribution of a bit or a digit it is the value of the digit multiplied by weight of the digit. So, the weight of the digit is coming from the base to the power n; where n is the position of the digit

Now, in case of binary number system so, we have got only 2 digits possible 2 symbols possible 0 and 1, and anything multiplied by 0 gives 0 only. So, we can say that why it is a so, we can just write it multiply each bit by 2 power n. So, these weights is the position of the bit starting from 0 on the right and then add the results.

(Refer Slide Time: 15:21)

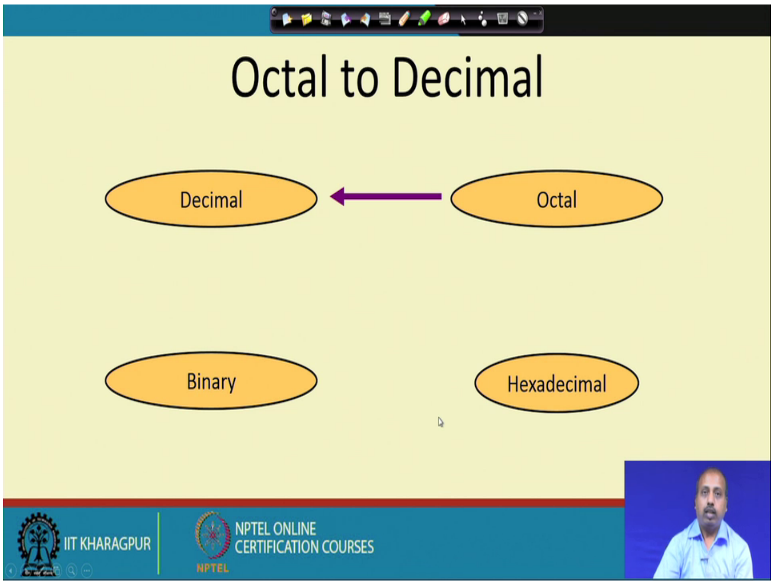


So, here is an example so, this is the, this is your binary number, say a 6 bit binary number so, this is the bit 0, ok.

So, this 1 into 2 to the power 0 so that the position 0, then 1 into 2 to the power 1 then 0 into 2 to the power 2. So, that way it is growing so, if you just saw I will get the corresponding contribution. So, this is one this is 2 this is multiplied by 0 so, the contribution of the bit 2 is 0. Then contribution of bit 4 is also 0, others are contributed to some value. So, that way it gives 43 in the decimal number system, because this values that we are taking so, they are the decimal values ok.

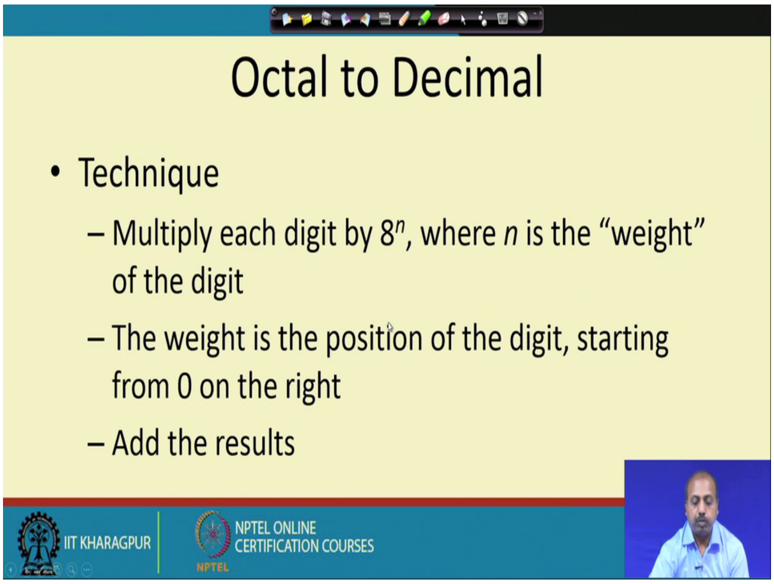
So, since they are the decimal values so, that is ultimately giving me a decimal number 43.

(Refer Slide Time: 16:08)



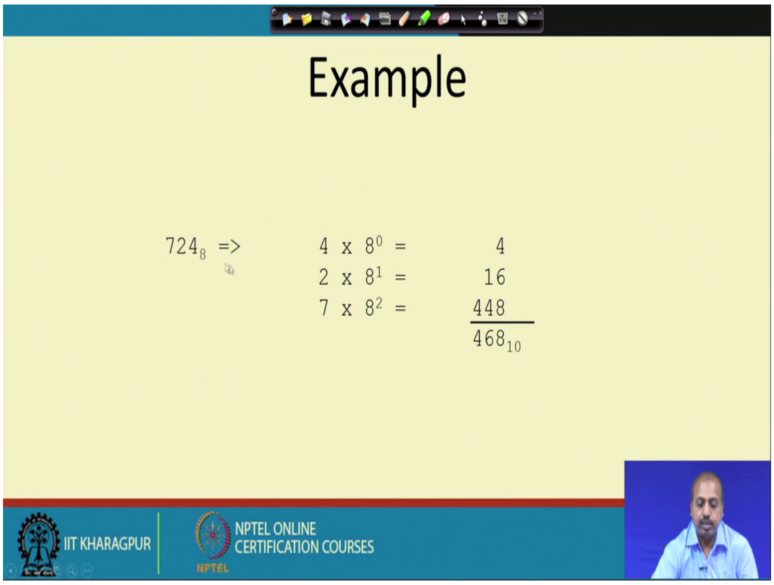
Now, in a similar line so, we can convert octal number to decimal number. So, multiply each digit by 8 to the power n. So, in case of octal number system so, base is 8.

(Refer Slide Time: 16:14)



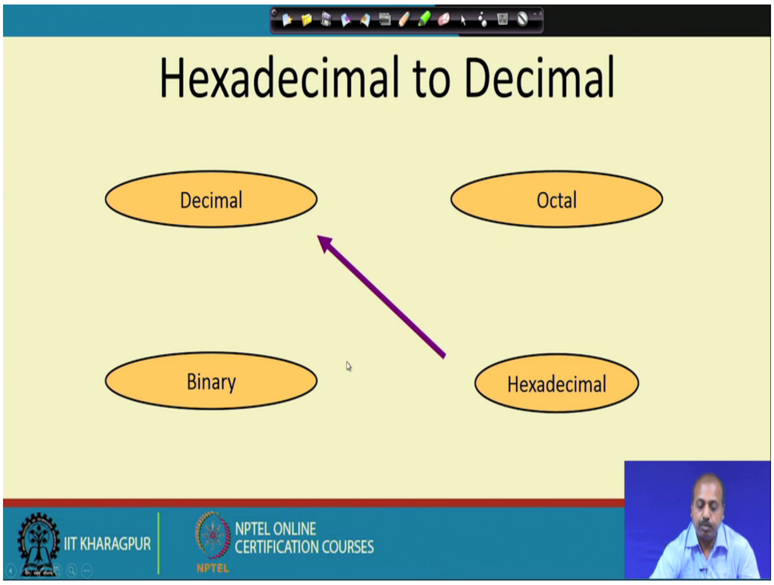
So, we multiply each digit by 8 to the power n; where n is the weight of the digit, and the again the same thing that weight is the position of the digit starting from 0 on the right, and then we add all the results.

(Refer Slide Time: 16:33)



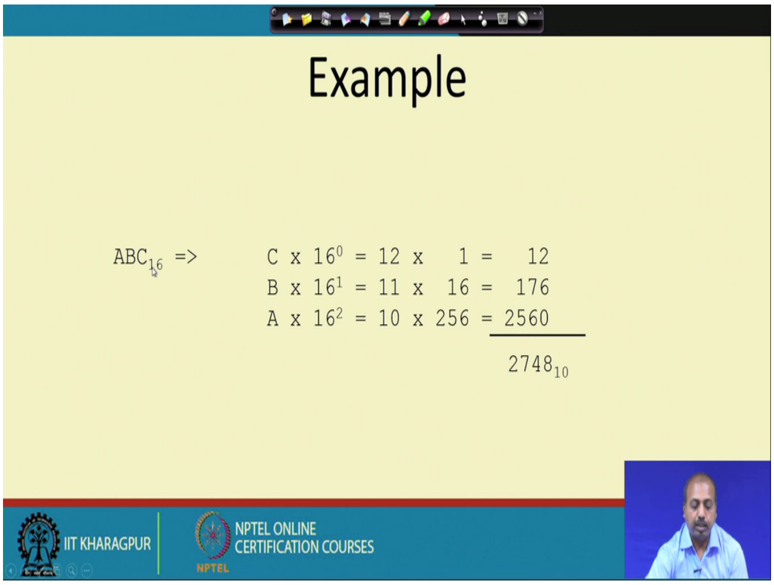
So, this is 724 to the base 8 so, 4 into 8 to the power 0, that is a contribution is 4, 2 into 8 to the power one contribution is 16, 7 into 8 to the power 2 the contribution is 4 14. So, you sum them up so, it becomes 468.

(Refer Slide Time: 16:58)



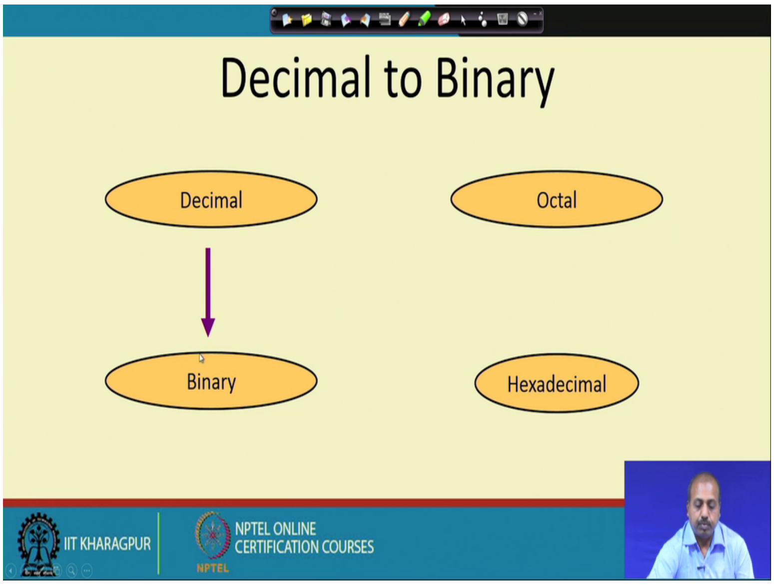
So, that way we can convert this um, we can convert an octal number into a decimal number. Hexadecimal to decimal again the similar way we can do multiply each digit by 16 to the power n. So, base is 16 so, we take it to the power n; where n is the weight of the digit, and then we the weight. So, that the then we just add the results.

(Refer Slide Time: 17:17)



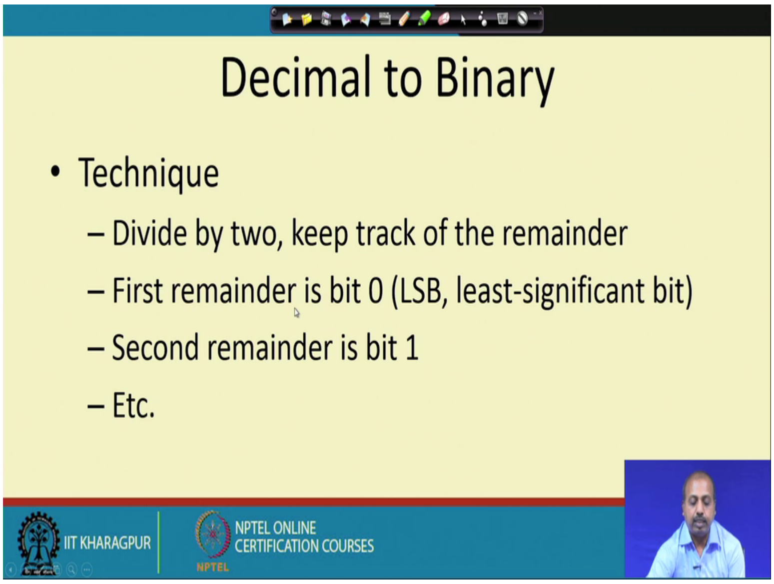
So, in a similar way say ABC to the base 16 so, a 6 C into 16 to the power 0, and the value of a C is 12 ok. So, the as I said that each symbol it has got some corresponding integer value, in the number system; so, and for hexadecimal number system so, a is 10 B is the integer value is 11 and CS integer value is 12. So, that way so, this values are multiplied by the powers are 16 so, that way we get the value 2 7 4 8. So, this is the corresponding decimal value for this ABC to the base 16 in hexadecimal system, ok.

(Refer Slide Time: 17:55)



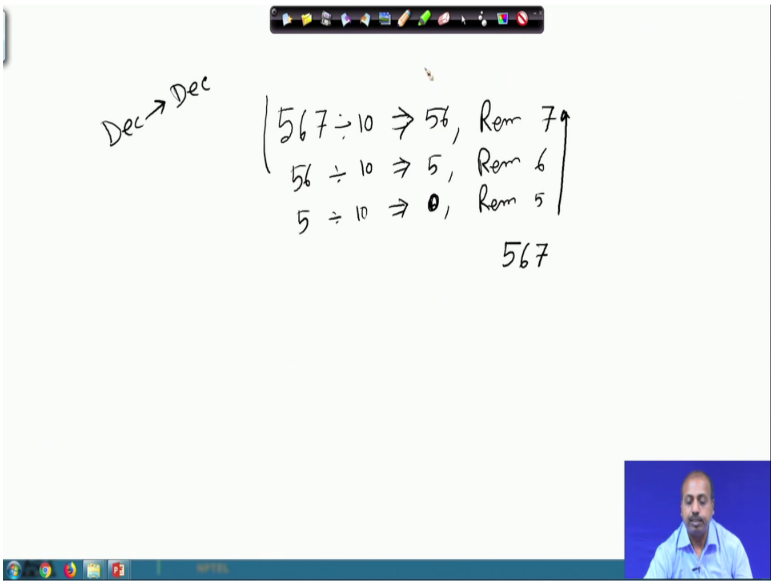
So, that way if you want to go to the decimal so, you can do it in this fashion. So, you can we can multiply by the, we can multiply by the weight of the base, and then we can go to the decimal system.

(Refer Slide Time: 18:18)



But the other side the decimal to binary octal and hexadecimal conversion so, that the for that we need to divide for decimal to binary. If you want to convert so, you can you can convert into we can divide by 2, and we can keep track of the remainder. So, the first reminder is bit 0 second remainder is bit one etcetera. So, for the sake of a understanding, if we think about a decimal to decimal conversion, how do we do this thing? Ok so, we have seen by multiplication technique, but you can we can if you thing about this decimal to decimal conversion, say the number say 567, ok.

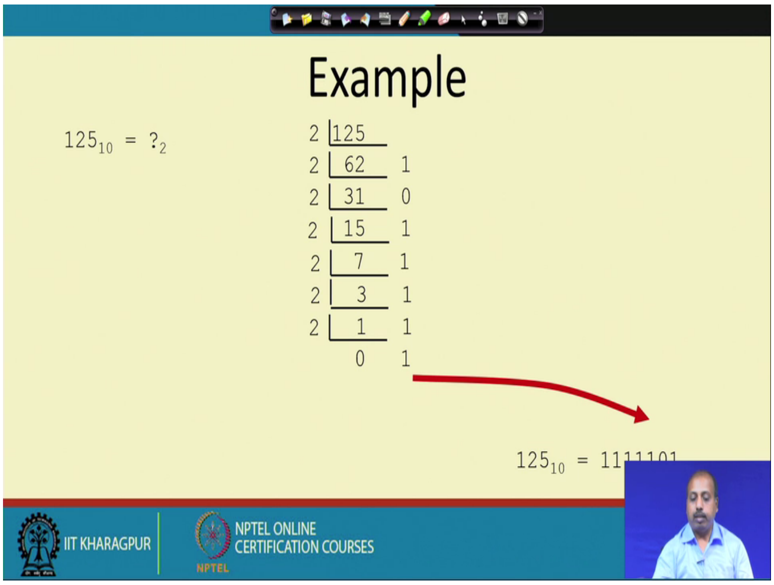
(Refer Slide Time: 18:57)



So, 567 so, I am looking for decimal to decimal conversion. So, what it says is that you divide by the base of the number system. So, if you divide this 567 by 10 so, what you will get the value 56, as the quotient and the remainder is 7. Then you divide again 56 by 10 so, what you get is 5 with remainder as 6, and what this 5 divided by 10? So, you get you get this 0, and the remainder is 5.

Now, what you do for getting the converted form. So, you just write down this remainders from this side first remainder of the latest remainder is 5, previous to that 6 and previous to that 7. So, this is the way we are converting the numbers, so, same thing applies to any other number system. So, this is with respect to decimal so, you can do it with respect to any other number system. So, we will see that while we are converting from decimal to binary. So, in case in the since in binary system, the base is 2 so, we will go on dividing by 2, and we will track the reminder value. Say, what is 125 which decimal to binary, ok.

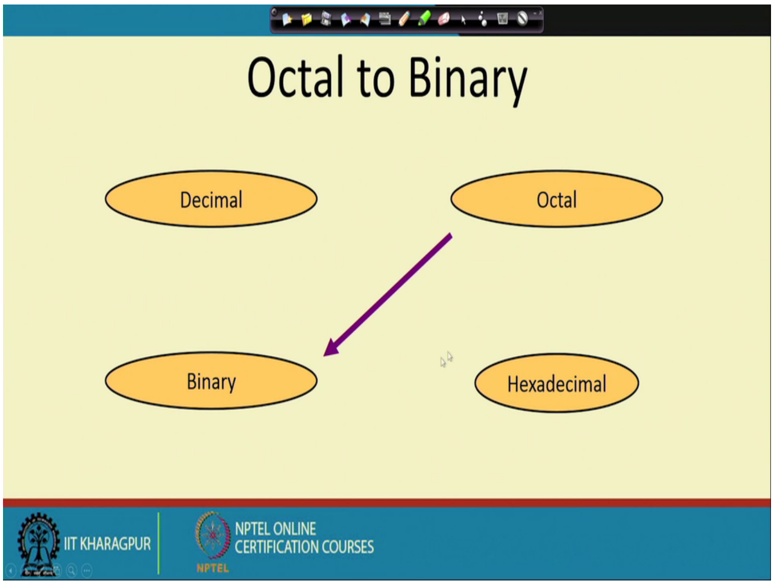
(Refer Slide Time: 20:28)



So, you divide 125 by 2 so, 62 is the quotient and 1 is the remainder. So, we divide again 62 by 2 so, you get 31 as the quotient and 0 as remainder, divide 31 by 2 you so, you get 15 as quotient and 1 as remainder. Divide 15 by 7, 15 by 2 you get 7 as quotient 1 as remainder divided by 2, 3 as quotient, 1 as remainder, then again divide by 2, you get one as quotient and one as remainder. Again divided by 2 the quotient becomes 0 and remainder is 1.

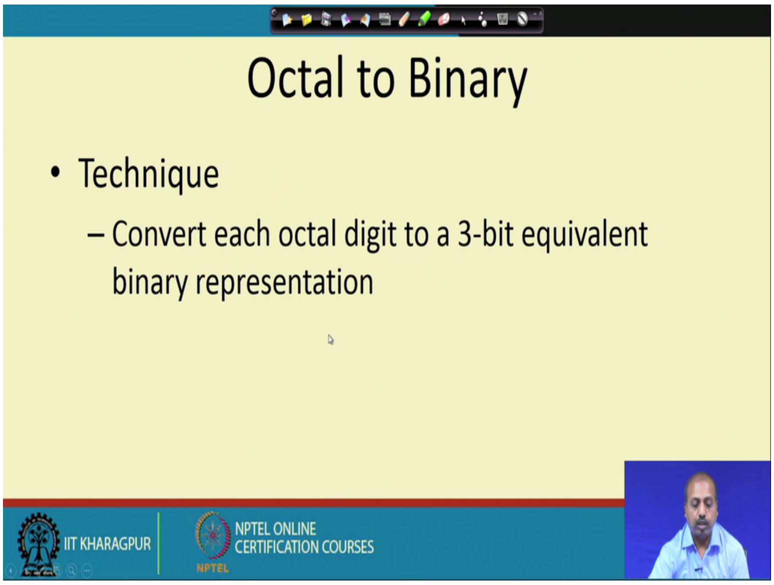
So, at this point we stop, the similarly what we did in the in the decimal number system. So, we do the same thing here and so, we get this so now, we start writing from this side. So, the 1 1 1 1 1 0 1 so, this is the thing so, this is the conversion. So, this way we can convert a decimal number into binary number.

(Refer Slide Time: 21:35)



So, if we looking for other way the octal to binary number system so, conversion so, one thing that you can do is octal. So, you can you can come this way so, you can convert this octal to decimal, and then from decimal we can come to binary.

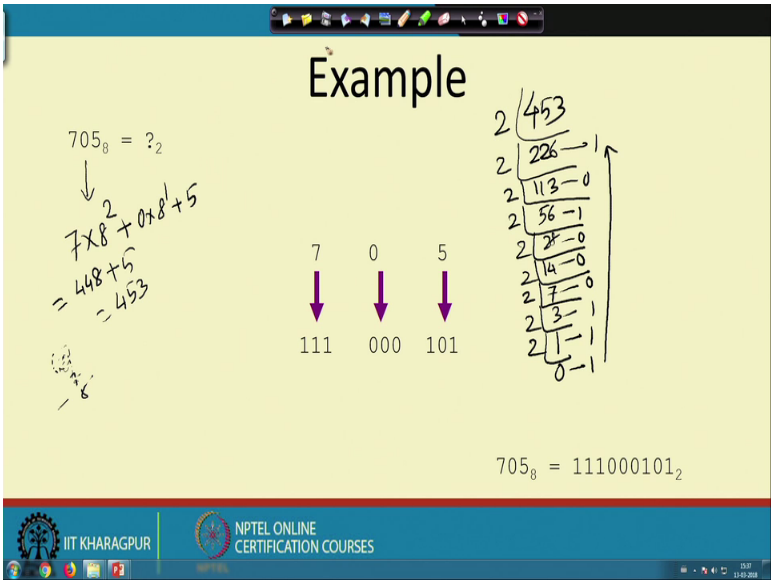
(Refer Slide Time: 21:39)



So, this is one avenue other straightforward avenue for octal to binary conversion is you can do it some sort of grouping of bits, because in octal system the base is 8, and in binary system the base is 2. So, you can group 3 bits of binary numbers, sorry, you can convert each octal digit by a 3 bit binary number. And because of this the divisibility of this 2, basis of this base 2 of binary and base 8 of octal.

So, this octal digit boundaries are similar to this binary number boundary. So, that that is why so, this each octal digit will form a 3 bit boundary for then binary number system so, you can convert octal to binary. So, you can directly convert individual digits of this octal number system into binary number and get the result.

(Refer Slide Time: 22:57)

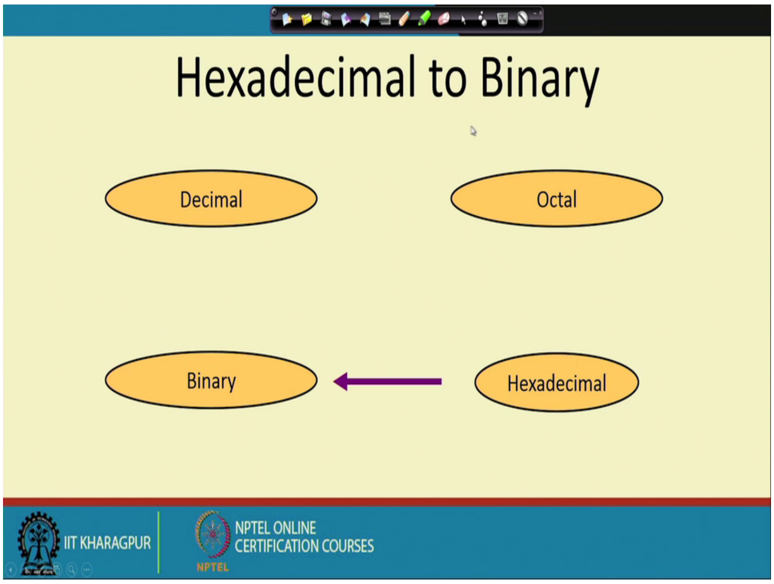


So, what is 7 0 5 to the base 2 so, 7 is 1 1 1, 0 is 0 0 0 and 5 is 0 0 1 so, you can get it like this. So, or other way you can say that ok, I can convert it in the so, 7 0 5. So, if you convert this 7 0 5 to decimal so, you will get 7 into 8 to the power 2 plus 0 into 8 to the power 1 plus 5 so, it is 64 into 7, ok.

So, so, it is 64 into 7 yeah, ok. So, this is 448 plus we have got this plus 5. So, that is 4 453. Now this 453 so, so this 453, as I was telling. So, you can convert it into binary. So, this is 22, 13 that is 6. So, 1 then 2 1 1 3 0 then 2 so, this is 56 1, then 2 28, 0 then 2 14 0, then 2 divided by 2 7 0, remainder is 0. So, this is 3 1, this is 2 1 1, this is 2, ok.

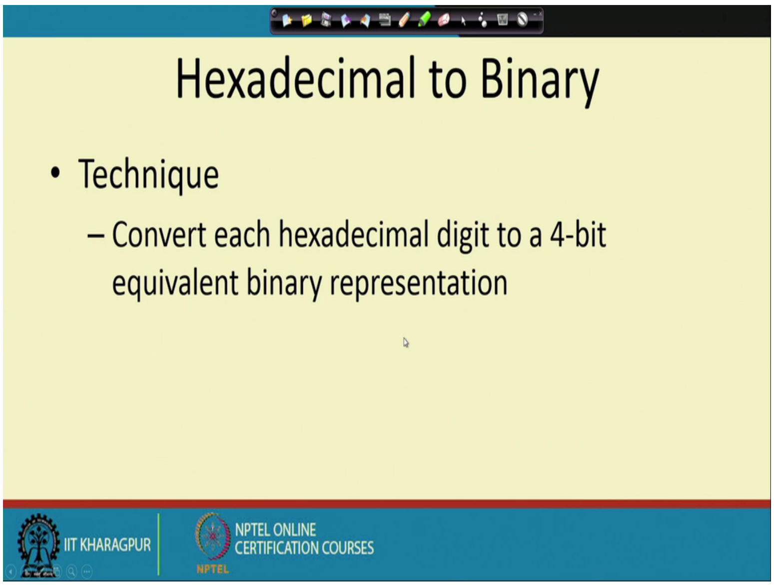
So, then we have to write in this order for the conversion into a binary so, this is a 1 1 1 0 0 0 1 0 1. So, either you can do it like this so, we can take this individual digits, and then convert them directly into 3 bit binaries. So, we can get it directly from octal to binary, or you can just you can convert to decimal number system, and then from the decimal number system again divided by 2. So, it is a slightly longer way of doing it ah, but that we are we are doing it for the sake of our confidence that this is the method is correct, ok.

(Refer Slide Time: 25:43)



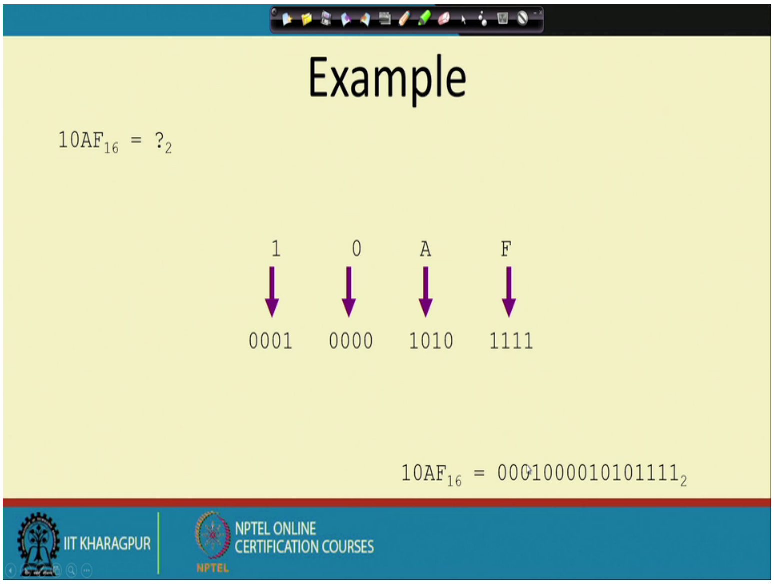
So, next we will next we will look into this hexadecimal to binary conversion. So, here just like the octal systems so, we took 3 bits each octal digit was taken separately and they converted into 3 bit pattern.

(Refer Slide Time: 25:46)



So, here also we will be taking the individual hexadecimal digits and then convert it into 4 bit equivalent because that hexadecimal 16. So, that can we can have it can represent a values from 0 to 15, and we will need 4 bit for representing this 0 to 15 in the binary number system.

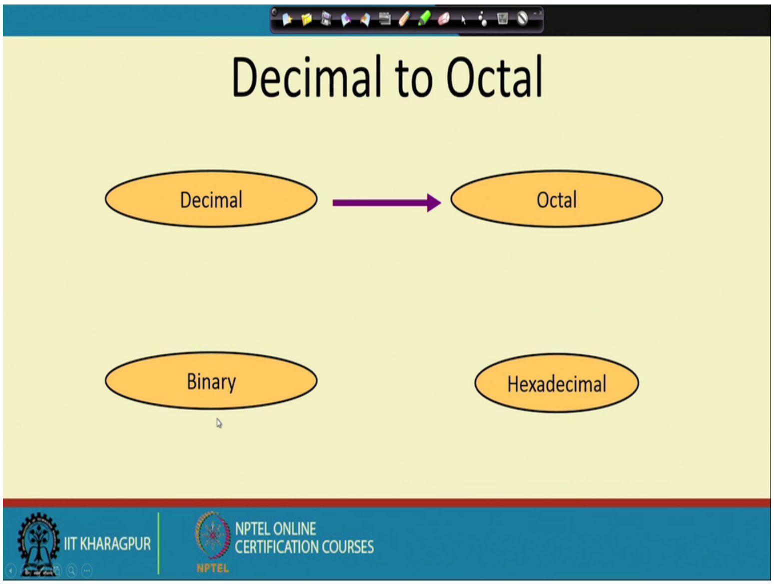
(Refer Slide Time: 26:15)



So, the same thing that is suppose 1 0 AF to the base 16. So, what is the value in the binary number system? So, you can just take the 4 digits and then convert them into binary numbers 4 bit binary numbers and that is the result. So, you can just verify this by doing the similar exercise that is converting this into decimal and from decimal converting it into binary, but that is a lengthy process so, we are not doing it. Of course, we can say that this 0 0 0. So, this 3 0es are not necessary, because they are the leading 0es and they do not have any contribution to the overall value of the numbers so, we can ignore them.

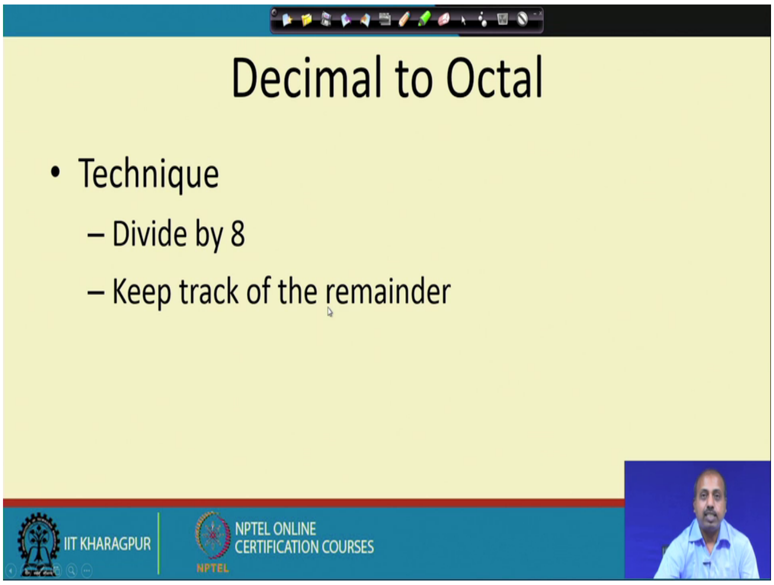
So,, but this is just for the sake of completeness so, it has been done like this.

(Refer Slide Time: 27:02)



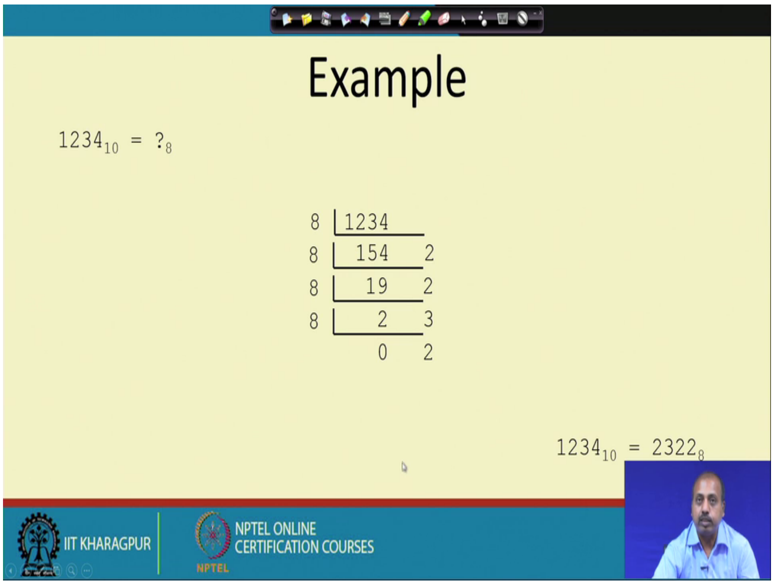
Now, how to convert decimal to octal?

(Refer Slide Time: 27:07)



Ok, so, just like decimal to binary we divided by 2 and kept track for the remainder. So, here also we do the same thing we divide by 8 and keep a track for the remainder.

(Refer Slide Time: 27:21)



So, this is a 1 2 3 4 to the base 10 converting it to hexadecimal sorry, octal number system divided by 8. So, remainder is 2, proceed another step, remainder is 2, then another step, remainder is 3 and then another step, so, remainder is 2.

So, the number that we get is 1 2 3 4 to the base 10, the decimal number is basically 2 3 2 2 to the base 8 in the hexadecimal number system in the octal number system. So, in this way we can convert between the number systems, and by dividing by the base of the number system so, and it keeping track of the reminders.