Digital Circuits
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Lecture - 13
Boolean Algebra (Contd.)

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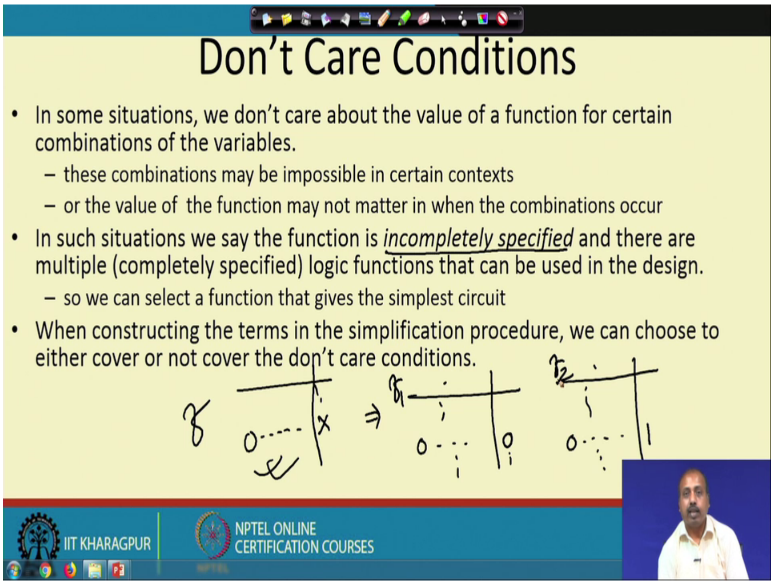

So, in case of this do not care conditions so, what is happening is that; the original function that we are given is incompletely specified function. So, this is an incompletely specified function, because for some of the combinations of input so, we did not mention what is the output. So, if the output can be anything so, the designer is not bothered about the output for those cases.

So, in that sense I can get multiple functions. So, truly speaking so, if this is the truth table, then suppose for say this particular combination the output value is do not care. Then it essentially says that I can have 2 different truth tables. So, in one case the rest of the row column so, they get the values from here, but for this particular column. So, if the value was x. So, I make it 0, rest of the thing remain same as this truth table. And in another case so, I take this one and for this particular combination so, I take the value as one.

So, essentially this is one function this is another function. So, this is the original incomplete function f, from their I can derive 2 completely specified function f1 and f2.

And we can minimize them, and see whichever gives as the minimum result. So, while we are taking this x is in our grouping so, it will automatically take the representation that we will have the minimum simplest represent, simplest representation in terms of number of literals and number of term.

So, we can have we can select a function that gives the way the simplest form. And when constructing the terms in the simplification process, we can choose to either cover or not cover the do not care conditions. So, that way we can choose between the alternate functions.

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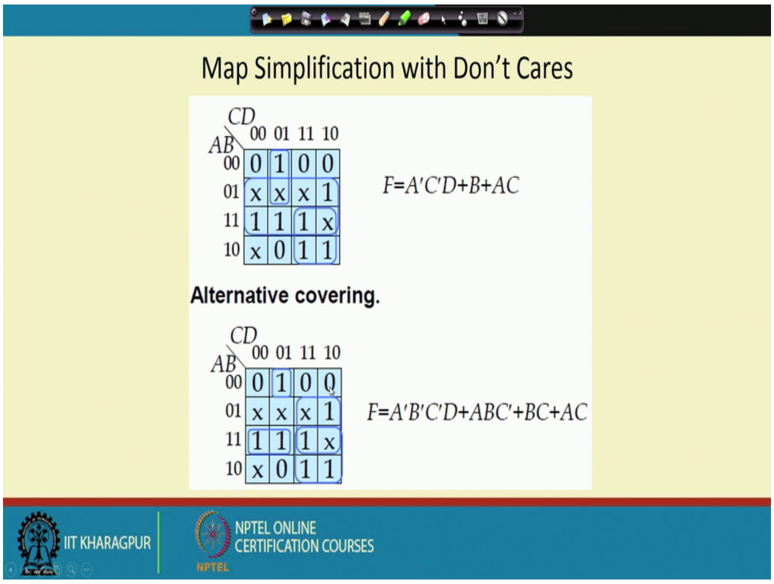

So, this is map with some do not care. So, you see that in this case so, so, this do not cares we have fruitfully exploited to make a quad here ok. So if so, that way this quad gives me like a only B is not changing it is polarity all the other variable they are changing their polarity so, that is B.

Similarly, this quad so, this sorry, sorry this so, this is an octet so, containing 8 1. So, this is an octet so, that gives me B and this is A quad. So, that here A is change not changing it is polarity, and C is not changing it is polarity so, this is AC. So, B plus AC and this pair so, A is not changing it is polarity. And your with A is not changing it is polarity, and this C and D, they are not changing their polarity. So, AC bar D is for this pair so, accordingly we can do this thing.

Another possible or covering may be like this; so, somewhat it has a covering in this fashion ok. So, this is this is one singleton, singleton one so, that is A bar B bar C bar D like this, then we have got this AB C bar. So, we take this pair so, it is AB C bar then we have say BC like say this if I take this term. So, A is changing it is polarity B is not changing so, B here and then between these 2 columns C is not changing it is polarity. So, you get a quad BC, and for this one you get a quad AC. So, this way we can write down a set of quads and singletons and pairs so, that way we get it like this.

So, definitely we can understand that this grouping is not good, because it does not fruitfully exploit the do not cares here, ok. So, that way the resulting solution is not very good.

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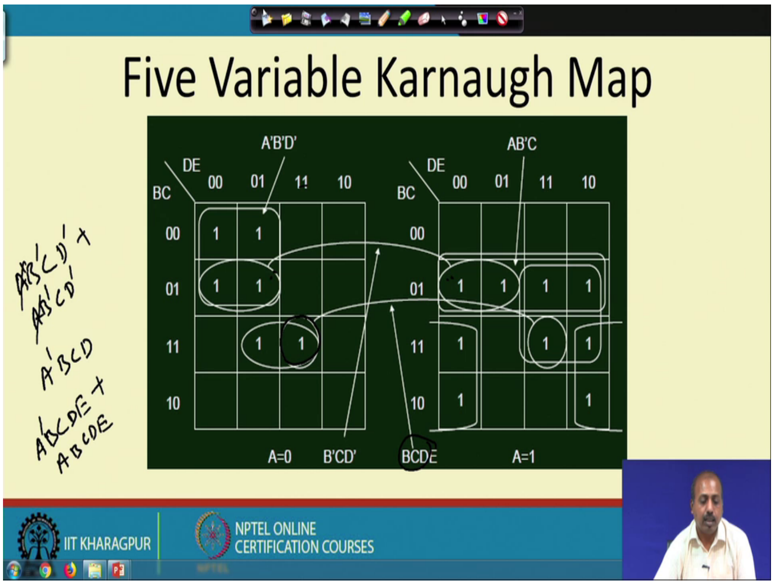

Next we will look into 5 variable Karnaugh map. So, for we have look seen up to 4 variables. So, for 5 variable Karnaugh map what is done? We take groups of we take groups of 2 Karnaugh maps, one is for a equal to 0 another is called a equal to 1.

So, for a equal to 0 so, we have got a this particular map or a equal to 1 we have got a map like this. And then we have this one so, this is map is A bar B bar D, because for all these terms here A is equal to 0. So, they will come as A bar A bar terms A bar B bar D bar, and this will come as A bar B A bar BC. So, this will come as so, A is the A is remaining unchain so, A is becoming a remaining an A bar. So, between these 2 columns E is changing E is not changing it is polarity. So, this term E remains and between a also between these 2 between these 2 rows our B B is changing it is polarity, but C is not changing.

So, if we consider only this map. So, you get this term this pair leading to the term A bar, then it is the between these from here you will we are getting say B BC A bar BC, and from here your will be getting D. So, A bar is definitely their then this particular it is BC and between these 2 columns. So, we have got so, D is not changing it is polarity so, D is coming as D. So, you were getting BCD, A bar BCD.

Now, now this A bar BCD so, then you can find that from this map. So, this BCD is also their so, if we if you take this term. So, if you take. So, if you for example, if you take say this singleton. So, this is your BC DE, now this BCDE, and this BCDE so, here also in for a equal to 1 map we have got BCDE. So, this from the first map so, I am getting it as A bar BCDE. For the second map I am getting it as ABCDE. So, when they are combined so this, a plus A bar will make it one so, ultimately we will get this BCDE. So, in this way by combining these ones from 2 different map. So, if the ones are at same position so, you can combine them to make pairs, ok.

So, similarly say this so, this term is this pair this quad is way of itself. So, it is A bar BD A bar B bar D bar, similarly so, this these pair these pair and these pair. So, they are located at the same place so, they are from the first map so, we were getting the term as B bar C and here the D is not changing it is sign. So, B bar C D bar so, A bar in front of that because it is from the first map, and then from the second from this one. So, for this term you are getting the thing like B bar C B bar C and D bar and a in front of that.

So, if you combine these 2, if you combine these 2 then this A bars will cancel out. So, you will get the term B bar CD. So, by combining this 2 pairs, we can form a quad and you can get the term B bar CD bar. So, this is likely cumbersome, but it is a still we can used it we can use it for combination across the tables.

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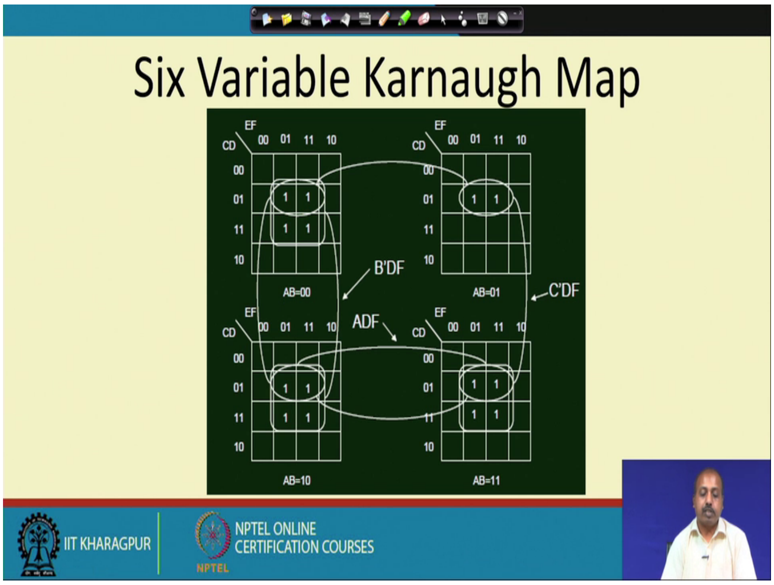

But it is the not that elegant you can say, but it will be it is useful. Going to 6 variable map, so, you can; so, we have got 4 such maps, where AB equal to 0 0 0 1 1 0, and 1 1 they are 4 maps. And again after making the individual grouping of pairs quads etcetera. So, you have to see whether there exist a similar pair or quad or singleton, or octet in the other maps also. So, if it there then you can combine them together to get a minimized form ok. So, that way we can use this 6 variable map for doing the minimization.

So, here you see that these 2 these 2 pairs. So, 1 1 here and 1 1 here so, they are at the same place. So, we can combine them, similarly say this octet and this quad this quad. So, this quad is present in a here as well as their so, we can combine these 2 quads together. Similarly, this quad is present at in this map and this map, and between these 2 maps only one variable is changing it is polarity. So, you can combine this, but you cannot combine say this quad with this quad, because here AB is 0 0, and here AB is 1 1. So, the both the variables are changing their polarity. So, you cannot do this combination.

But you can do that as long as their within the so, within similar within the quads are with the same locations of 2 maps which are varying only in one bit, ok. So, that way we can do that.

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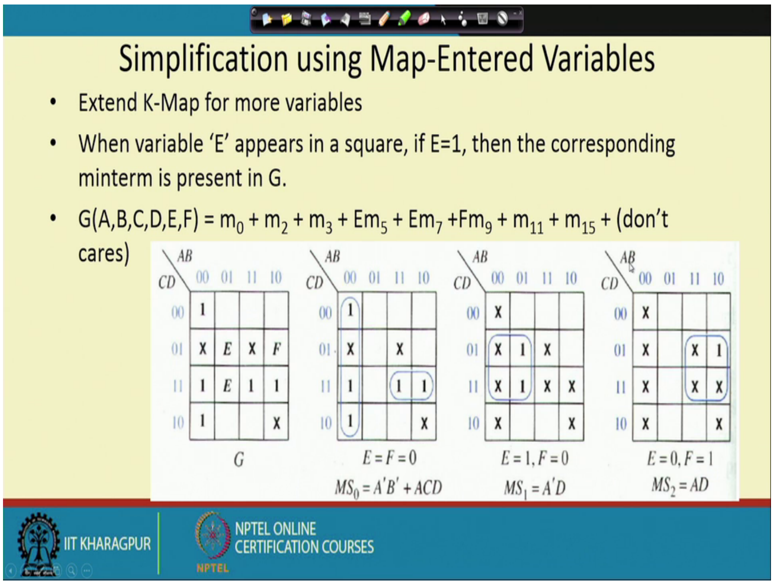

Sometimes, we can do this simplification using some map entered variable. So, if we have got more number of variables, you can do that using some map entered variable concept. So, so this is a situation like I have got a 6 variable map, where the terms are expressed like this. So, m 0 plus m 2 plus m 3 plus Em 5, that is E, E equal to 1, then m 5 will be coming if E equal to 7, then if E equal to 1 then m 7 will come. If F equal to 1 then m 9 will come, then m 11 m 15.

So, this is represented as 4 variable terms like m 0 to m 15, but here I have got the other variables introduce the E F etcetera, so, this is actually a 6 variable function, but while writing the function. So, we are writing it in we can we can write it in next less number of variable so, E m 5 Em 7 so, in the map we write it like this. So, this do not cares are not written here explicit the do not cares are there. So, these 2 these 3 are the do not cares that we have. So, they are retained as it is, but this E the in place of m 5 we entered E in place of Fm 9 so, entered F and in place of m 7 also we entered E.

Now, we consider the case where E and F both are equal to 0. If E and F both are equal to 0 so, you get so, you get a map and we do the grouping like this. So, you get A bar B bar plus ACD. Now if E equal to 1, if E equal to 1 and F equal to 0 so, we get a map now here, what we are doing is that we are not considering these 1's. So, we are taking it as do not care, because this one is already covered by these situations. So, this m 0 term so it does not depend on the value of E and F; so that has already been covered here, so, they are taken as do not cares.

So, as soon as some ones are covered so, we can take them as do not cares in the successive formulation.

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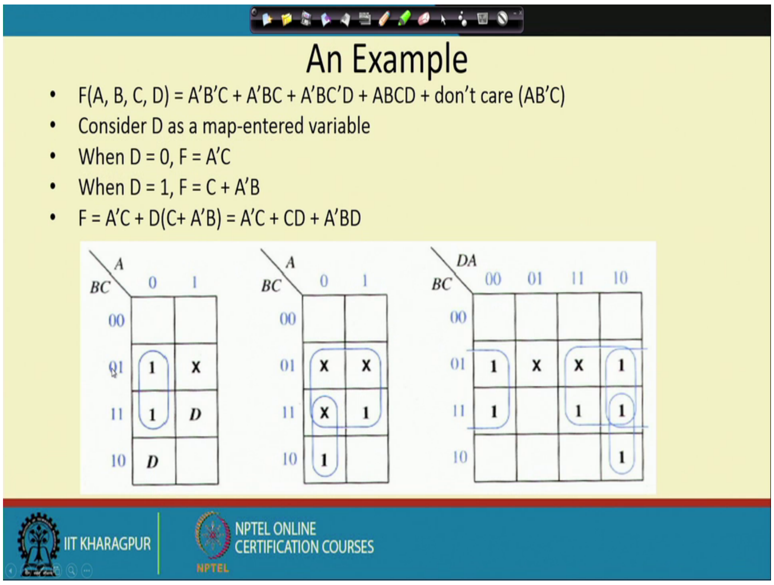

So, this is another example. So, ABCD is say this one, A bar B bar C plus A bar BC plus A BC bar A bar BC bar D plus ABCD, plus some do not care term AB bar C so, this is if we consider D as the map entered variable. So, then when D equal to 0 so, if put D equal to 0 here so, you will get the function A bar B is a so, you get the function as A bar C like if we just take this variable if we take this function and put D equal to 0.

So, these 2 terms will cancel out so, you are getting A bar B bar C plus A bar BC. So, this B plus B bar will cancel out so, get A bar C, and when D equal to 1. So, we are getting F equal to C plus A bar B so, this is also obtained from this one, say when we take in a map entered variable. So, when D equal to 1 so, these 2 terms are coming into picture the first 2 term A bar B bar C. So, A bar B bar C is coming here so, this is A bar, and this is B bar C so, this is one.

Similarly, A bar BC so, A bar and BC so, these 2 are ones. So, we have got AB bar C as do not care that is there, but for the other 2 terms so, we call them map entered terms. So, A bar B C bar D so, when D equal to 1, then this is coming. So, this A bar so, this D actually so, A bar B C bar so, that is coming here and for D equal to 1 it is ABC. So, we get the map like this and then we do a minimization so, first in this map so, we take this 2 as ones because they are always present. So, we take them out so, it is independent of so, that will give as the term A bar C. So, A bar C will on definitely come, then for the second term so, we get this one it is a map entered variable and D. So, these ones are to converted to do not cares, because they are already covered by the map where D is not D is not considered at all.

So, we do not need to do it again so, D so, for these ones are taken as do not cares here. So, you get now D equal to 1 so, you get these 2 D is as ones ok. So now, if do a grouping so, you get one term as this one, which is the nothing but C ok. So, that it is nothing but C, and from this pair you will get A bar B. So, C plus A bar B, and then that is. So, this map is coming when D equal to 1 so, it is multiplied by D.

So, then if we expand it you will get A bar C plus CD plus A bar BD. So, this way we can. So now, if we consider the 4 variable map for the original function, fine? So, this one so, if you take the 4 variable map so, you see that A bar B bar C. So, it is irrespective of D when it is 0 0 a AB is A is 0 and B is 0, then it is a independent of this as D value the corresponding bits will be turned on. So, this is the corresponding Karnaugh map. And here, if you do the grouping you will see that we will be getting back with this original term so, original these 3 terms always.

So, this 4 variable k maps so, we can solve it using a series of 3 variable, k maps is it taking help of this map entered variable concept. Though of course, with the increasing number of variables so this will also become cumbersome. But up to certain lengths so, up to certain number of variables so, you can try it out.

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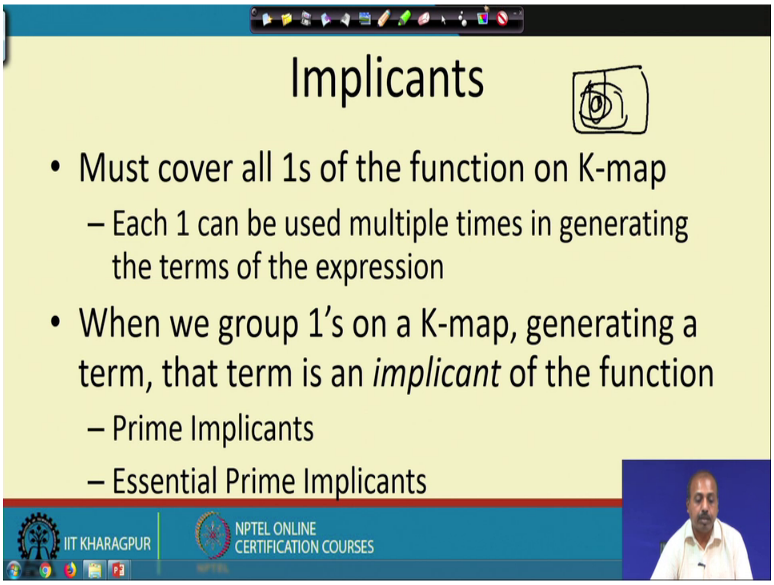

So, another view of k map so, we can we can view this k map as collection of implicants. So, an implicant so, whenever we are doing this Karnaugh map covering so, we must cover all ones of the function on the Karnaugh map. So, you while doing the minimization or writing down the corresponding Boolean expression for a Karnaugh map. So, you cannot say that some of the ones will remain uncovered I cannot take the term such that some of the ones are uncovered.

So, each one can be used multiple times in generating the terms of the expression. So, that can happen because were one. So, if you look into a Karnaugh map, then if this is a Karnaugh map and if this particular empty is one so, it may be covered by several pairs. So, it may be covered like this also it may be covered like this it may be covered in a bigger quad or like that. So, that may be that way. So, this particular one may be generated by all the terms. So, here it is covered by 3 terms so, whenever is so these all the 3 terms can generate this particular one, that is why it said that each one can be used multiple times in generating the terms of the expression.

Now, when we group ones on a Karnaugh map generating a term that term is called an implicant of the function. So, what is an implicant? So, implicant means that so, it is that term can generate some one in the Karnaugh map. And this implicants may be we can classified them into prime implicants and essential prime implicants so, we will see what do they mean.

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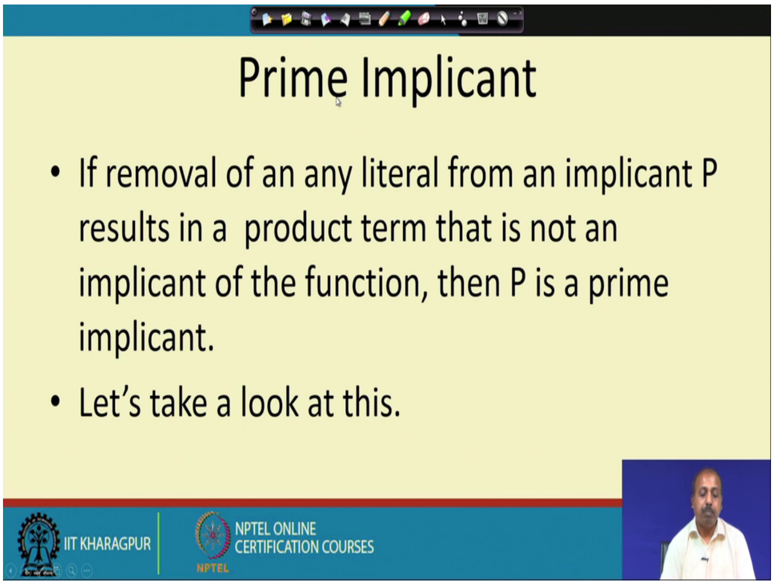

So, a prime implicant is one if we remove any literal from the implicant then it will result in a product term that is not an implicant of the function so, in that case p is called a prime implicant. So, implicant means something that generate some one now if you remove some terms.

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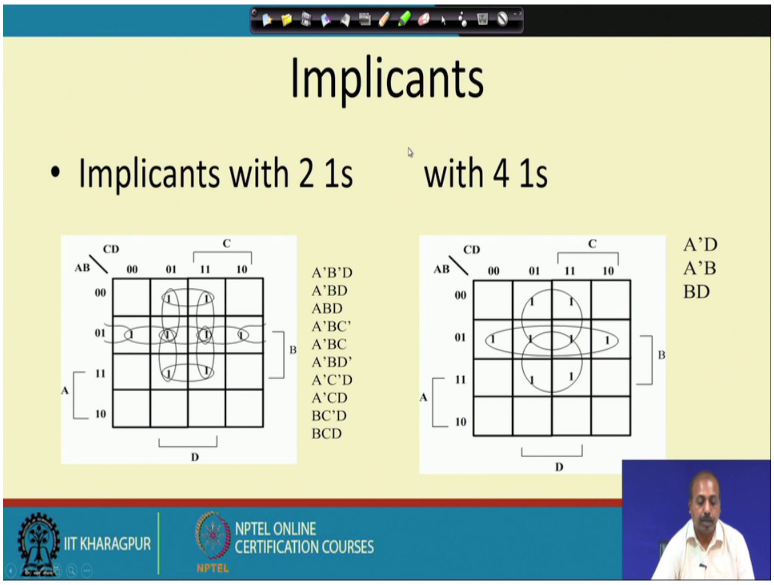

So, if it leave some of the so, ones uncovered then it will be called a prime implicants. Like see these are the implicant like say suppose this is a Karnaugh map. So, with implicants with 2 ones these are all the implicants like that, ok. And if I so, here I have purposefully group them into pairs. So, that I can all the implicants are covering exactly 2 ones here, ok. So, these are the implicants so, these are all implicants.

Now, if you take groups of 4 then we will be getting like this. So, these are also implicant because they are covering some of the ones ok, but whether they are essential or not that we will see. So, consider this term A bar B bar D the first one. So, A bar B bar D so, this pair we are talking about. Now, so, if we remove A bar we get B bar D. So, if I take this B bar D, if I take this B bar D, then what will happen is that so, B bar D so, B bar so, B bar is this row and while so B bar is this one and this one and D is this one. So, B bar D means these ones will also come into picture so, they are actually not there in the function, fine.

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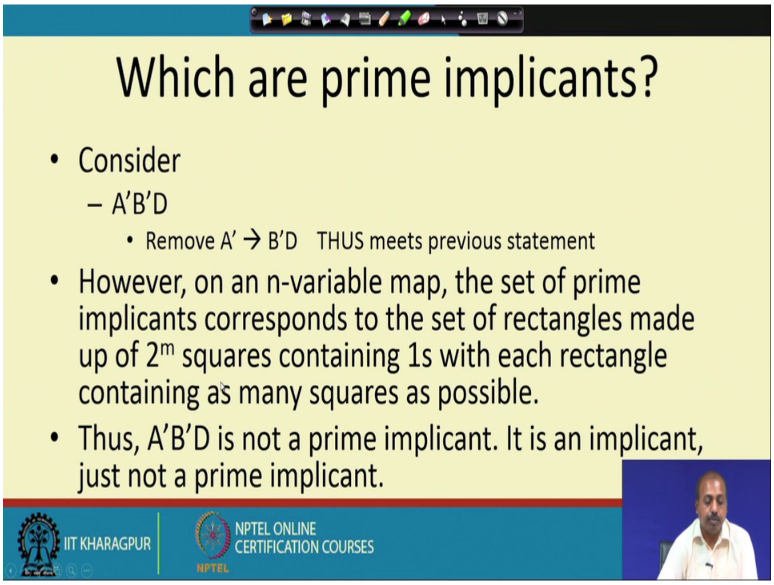

So, it says that if we are removing this A bar getting B bar D. So, B bar D will cover the original mean terms like B bar D also covers these 2 ones. So, that way it covers ones so, it can be an implicant. However, on a n variable map the set of prime implicants corresponds to the set of rectangles made up to 2 power n squares containing ones with each rectangle containing as many squares as possible. So, A bar B bar D is not a prime implicant. So, because if we remove this, then the it will be it will be generating a bigger sized bigger sized quad in fact, we can say like here it was a pair A bar B bar D so, if I remove A bar so, I am getting B bar D. So, this is this corresponds to a quad like say this means that these 2 ones and so, I am talking about these 2 1's and these 2 1's.

So, that way so, that will be giving rise to a quad so, the size increases. So, as a number of literals reduces the size of the covering increases, ok. Size of the rectangles increases so, it says that this is not a prime implicant, because if you take it as if you remove, if you remove this then some of the implicants are some of the ones are getting introduce so, which is not there in the original function. So, this is an implicant, but not a prime implicant. So, A bar B bar D is an implicant, but not a prime implicant so, if you do this if you take it is for a prime implicant.

So, we should have the set of rectangles made up to 2 power m squares, containing ones with each rectangle containing as many squares as possible. So, you cannot maximize it further that will be called a prime implicant.

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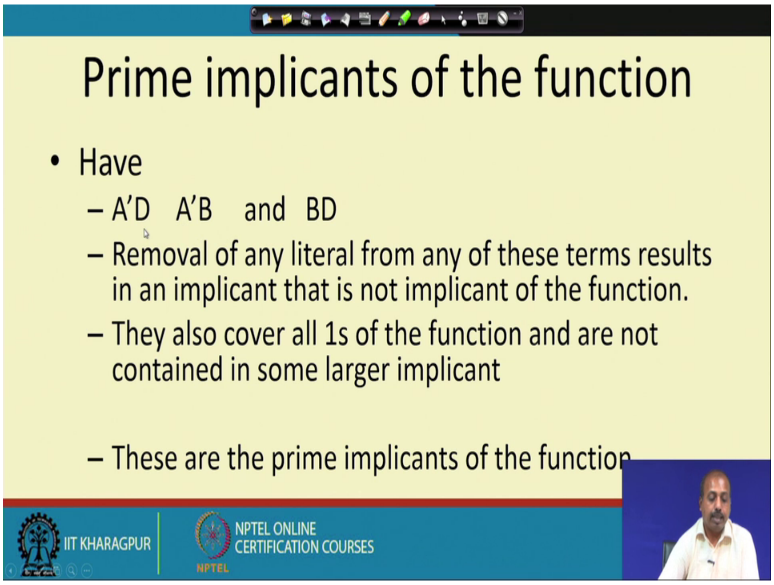

So, this A bar D A bar B and BD; so, if you remove any literal from this terms it will results into an implicant that is not an implicant of the function, ok. So, they cover all ones of the function and are not contained in some larger implicant so, these are the prime implicant so, we will go back to the example and see. Like you see that these are implicants so, these are also implicants, but this particular implicant. So, these A bar B bar D is contained in the larger implicant A bar D. So, this is that A bar D implicant so, this is covering this lower sized implicant. So, for an implicant to be a prime implicant, it should not be covered by; it is it should not be possible that we cover it by a larger sized implicant.

So, that is why this A bar B bar D is not a prime implicant, but A bar D is a prime implicant. So, if you remove any of the literal from A bar D. So, it will no more remain an implicant of the function, because some 0s will bit now taken as one so, that is not correct. How and also we cannot reduce it you cannot reduce it further like this if the size of this A bar D cannot be reduce further without un defining the function, but this A bar B bar D so, it is contained in this A bar D in this rectangle, and we have got this we so, this is this is a this is an implicant A bar B bar D is an implicant, but is not a prime implicant, but A bar D is also a prime implicant.

So, this A bar D A bar B and BD so, they are prime implicant because removal of any literal from any of these terms results in an implicant, that is not implicant of the function, and they cover all ones of the function and are not contained in some larger implicant, ok. So, they these A bar D A bar B and BD so, they are covering all ones, but none of the ones are remaining uncovered and they are not contained in some large sized implicant so, these are called prime implicant of the function.

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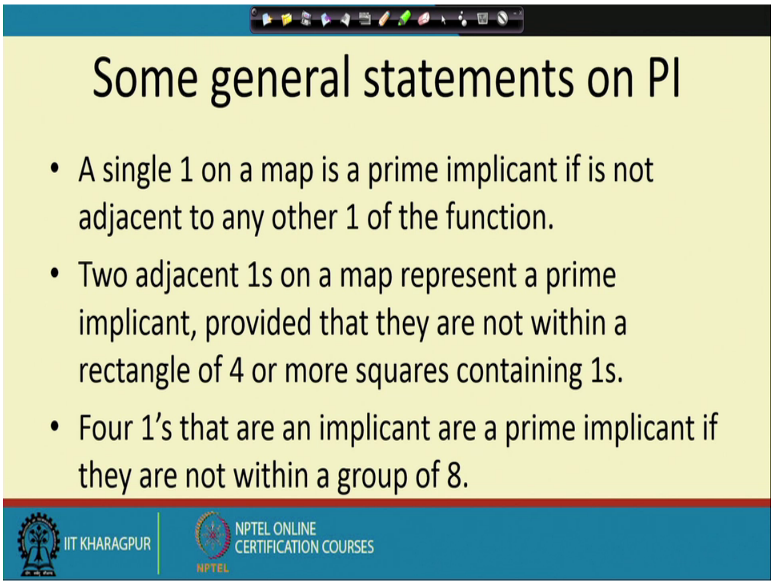

So, this way I can say that Karnaugh map is nothing but a collection of implicants, out of them are prime implicants and some of them are essential prime implicant. So, what you have to do is that while after drawing the Karnaugh map; so the ones that we are putting in the Karnaugh map. So, they are actually the implicants, now you have to try to do grouping. So, such that you can get some large sized implicants, ok. And then you will learned up into a set of prime implicants, and this prime implicant means from they are if you cannot remove anything. So, ultimately we are looking for getting the prime implicants.

So, 2 adjacent ones on a map represent a prime implicant provided, they are not contained within a rectangle of 4 or more squares of squares containing ones. Similarly, 4 ones that are an implicant are a prime implicant, if they are not covered within a group of 8. So, you can just go back to this example so, these are all implicants. So,, but it is not a prime implicant, because it is covered in some larger sized implicant.

So, similarly this 4 1's so, this is forming a quad so, this is a prime implicant, because it is not covered into some some sort of octet. So, if it is for example, if the in the is original function if these 2 bits were also one. So, these 2 combinations are also one then these whole thing would have formed a formed an octet.

So, in that case A bar D and so this one and this one so they will not remain your prime implicant anymore, but they will be simply implicants. So, that is said here that 2 adjacent ones on a map they will represent a prime implicant provided they are not within a rectangle of 4 or more squares containing ones. And similarly 4 ones that are with that are prime implicant, that are that are implicant, so that can that can be a prime implicant only if it is not containing within a group of 8. So, this way it goes on this prime implicant will go on.

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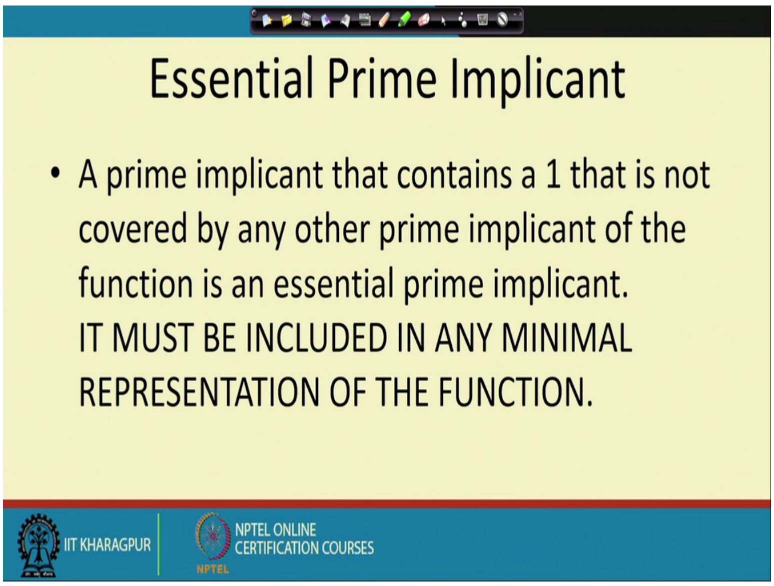

Essential prime implicant a prime implicant that contains a 1, that is not covered by any other prime implicant of the function is an essential prime implicant. And it must be included in any minimal representation of the function. So, this prime implicants they must be included in any minimal, so, you cannot you cannot replace them. So, you cannot avoid them in the minimization process so, they are called essential prime implicants.

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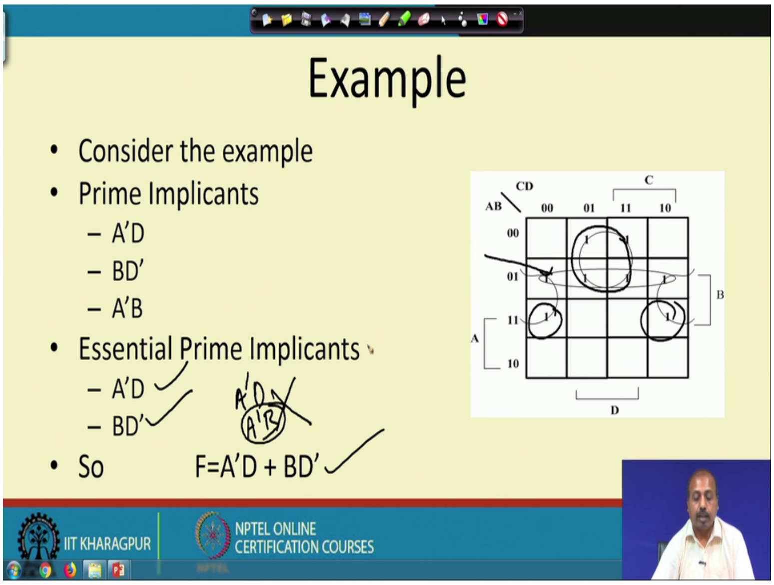

So, like in this case we have got prime implicant A bar D BD bar and A bar B. So, these are all prime implicants out of that say this A bar D and BD bar so, these are they are essential prime implicant. So, you see that while doing this grouping where, this is somebody has done a grouping like this a grouping like this as well this grouping. So, from that map itself you can understand that there is some redundancy here like, so for example, the ones have been covered by more than so these 2 ones. So, they have been covered by this as well as by this.

So, similarly these ones have been covered by, sorry. So, these ones have been this one has been covered by this as well as by this. So, it may be possible that we can get rid of some of these groupings and we can still get the minimized form. So, that is what is told that these are all prime implicants. So, A bar D BD bar and A bar B, because they are containing ones in them, but they are not essential, because you can get rid of some of them like. So, it is like if you say this A bar D. So, A bar A bar A bar D is this one. So, A bar D A bar A bar D is so, this grouping is A bar D in the first one. So, this grouping is A bar D so, if this A bar D and this BD bar if you take then; so this is B and if you take this 2 so, they actually this particular grouping. So, this is BD bar so, if you take this 2 it is only. So, this all the ones are getting covered, but in that case we do not need this particular we do not need this particular grouping of A bar B. So, A bar B is not essential so, you can take A bar D and BD bar.

But if you take say A bar D and A bar D, and this A bar B you take A bar D and A bar B then also it is fine that may be another possibility so, that way. So, this is also not unique so, I can have I can also take another one like say A bar D and A bar B. Because A bar D will give me this 1, ok, and A bar B will give me A bar B now A bar D A bar B. So, this is not essential this will not give me give me full covering. So, this is my A bar D so, these ones will remain uncovered. So, these are not these cannot be taken as essential prime implicant. So, A bar B is not essential so, we have got A bar D and BD bar are essential prime implicant so, the minimum form is A bar D plus BD bar.

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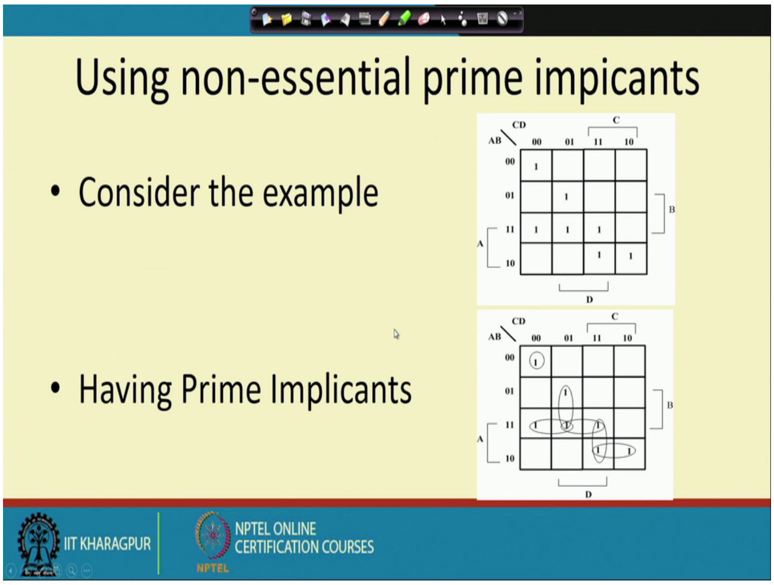

So, essential prime implicants we cannot remove. So, non-essential prime implicants sometimes if you take this example you see; that if you take only prime implicants so, you will get a grouping like this ok. But you see that these ones it is; so, this particular this one is covered by more than ones like that. So, that way it is we are we are taking some non-essential prime implicants.

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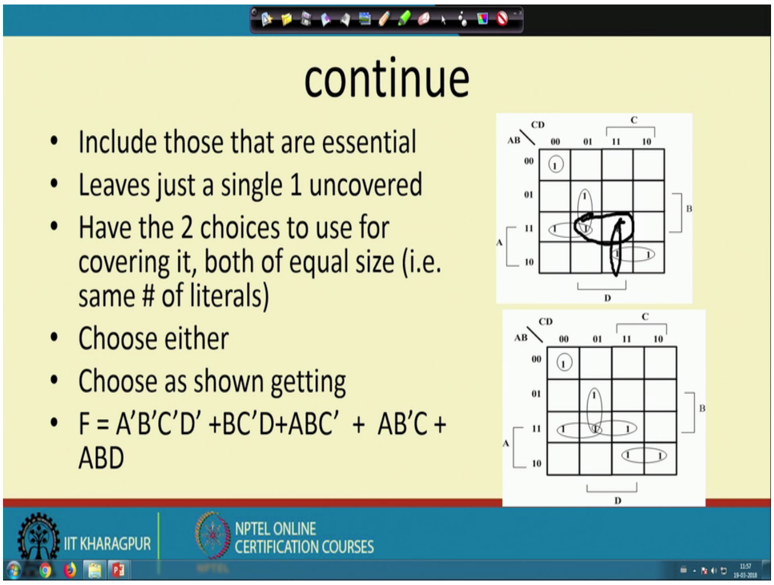

So, include those that are essential so, if I just so, in the previous case. So, this is covering this is showing all the implicants that I can make, ok. So, these are all the implicant that I can make from here I have to choose the essential set.

Now, how to we choose the essential set? For we take though that the take ones that are essential. So, these are essential so, I take this thing. So, this was not essential, because this is already covered by that. But the problem is that after taking the essential one. So, this one is left this one is left uncovered. So, in that case we have a choice. So, we can we can take say either this one or that one. So, we have got 2 choices of covering it both of equal size so, we can check either of them and ultimately come to this representation.

So, by taking after choosing the essential one; so we have to check we have to check some others which if some ones are still left, and then we can we can have a choice. So, I can group this one with this one or this one. So, this one can be group like this or so, I can group it in this fashion or I can group it in this fashion. So, both are possible, but in whatever you do ultimately you get the same type of cost so, that way it is same.

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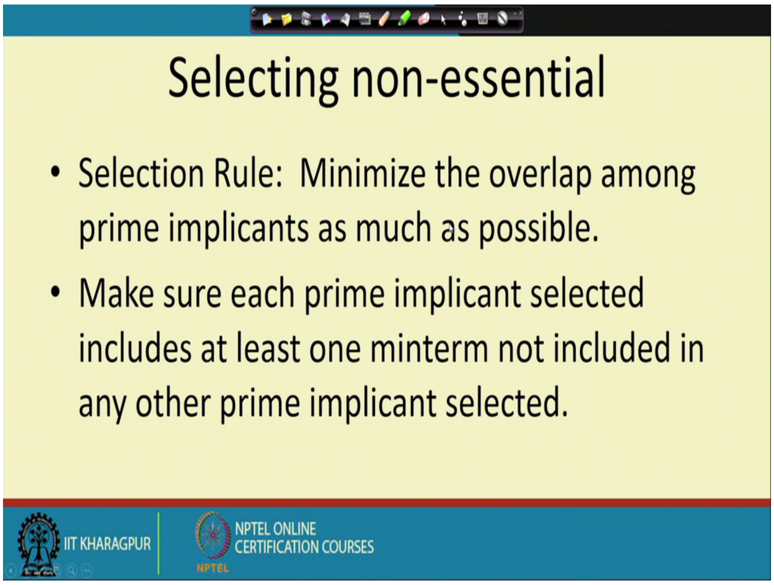

So, we want to minimize the over lapping so, that way we can we can try to do as much minimization as possible by doing this selecting in such a fashion.

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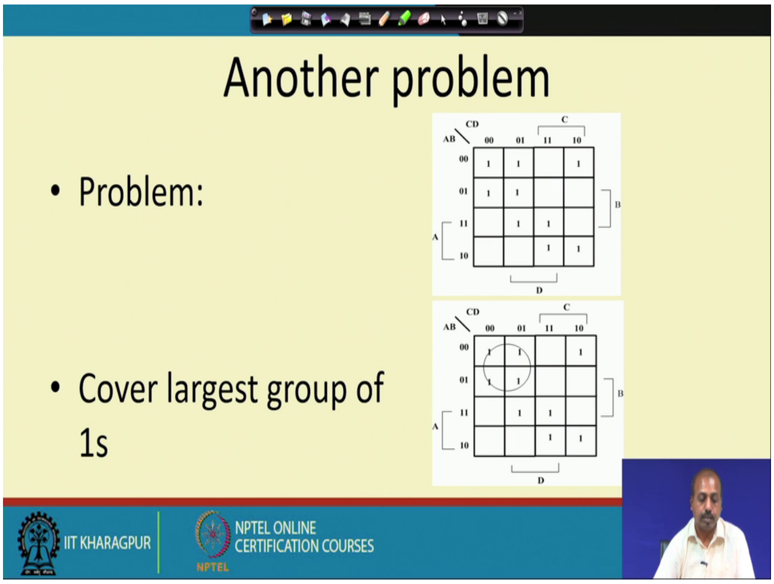

So, this is another example we have got this 1's, and then we have we cover the largest group of one's so, this is the there.

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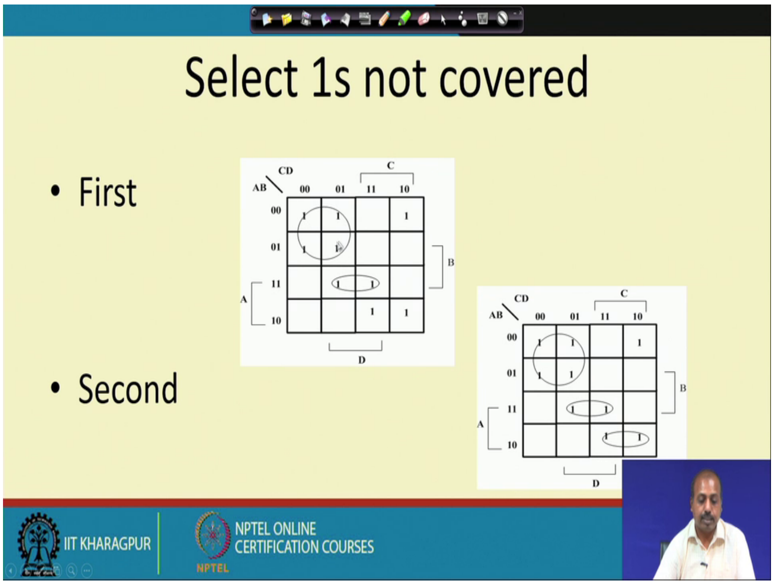

And after that, we can after selecting this select the ones that are not covered yet. So, this is the one that is not yet covered you select that one. Then we select another group that is not covered yet. So, we have so, essential we have essential part we have done.

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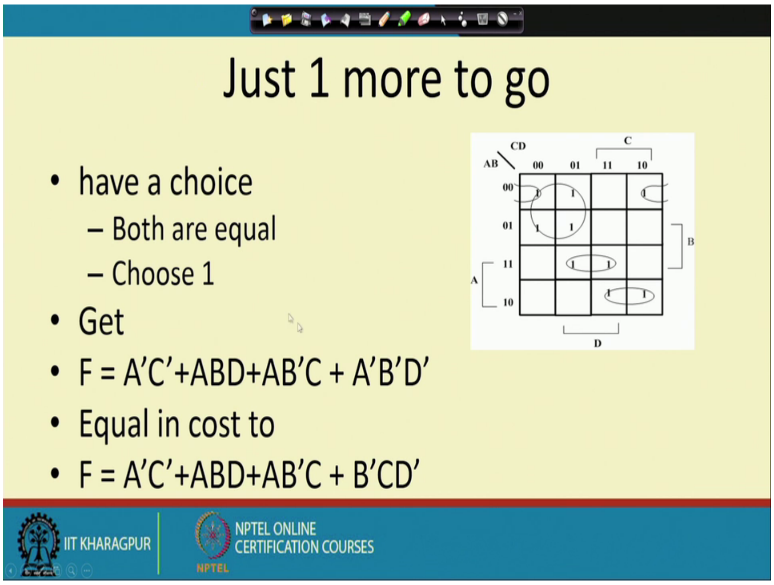

Now, we have got a choice so, these one may be covered with this or it may be covered with this both are having equal cost. So, if we take the example that is shown here so, it is giving me this particular expression whereas, if I do a grouping with say this one and this one, ok.

So, these two ones a group so, we will get the second configuration, but both of them are having same number of literals so, they are just same getting the same; so that way in Boolean expression minimization so, we can use Karnaugh map, and we can have different views it may be viewed from a set theory approach it may be viewed from prime the implicants approach. So, whatever you do ultimately it is a matter of covering. So, if you in the covering process, if you are not taking the redundant 1's, then you will definitely get the minimum form with minimum number of literals and minimum number of terms.