Digital Circuits
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Lecture – 11
Boolean Algebra (Contd.)

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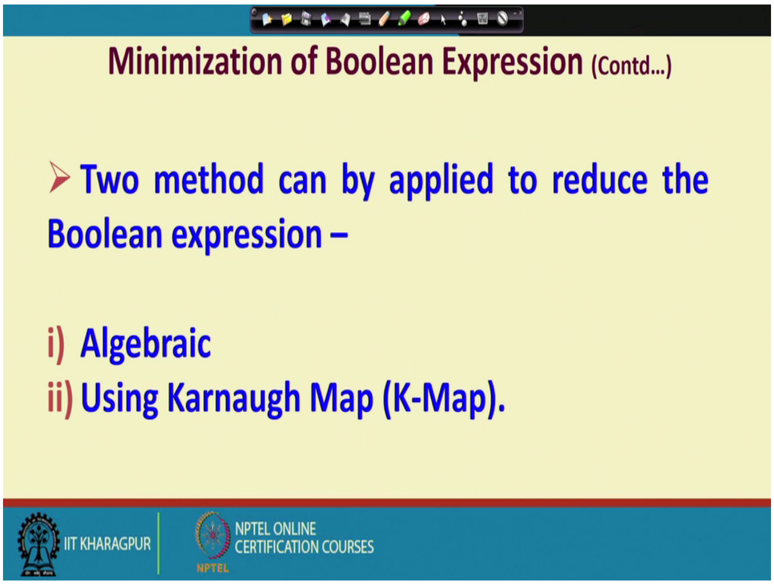

So minimization of Boolean expression; so, there can be 2 method that can be applied for doing this minimization. One method is known as algebraic minimization and other one is using a mapping map based method which is known as Karnaugh map, or K-map in short. So, this algebraic method so, we have already seen some Boolean expression conversion using algebraic method; that uses the Boolean identities for reducing the or converting one Boolean expression to another Boolean expression.

And this Karnaugh map is another a map based method which is much simpler compare to algebraic method. So, both of them can give us minimum from, but this algebraic method so, there is no as harden first rule that you have to, you have to apply the algebraic manipulations in this order to get the minimums also that makes it difficult. Whereas, in Karnaugh map so, there are some rules if you follow those rules, then you will definitely get a minimum form expression.

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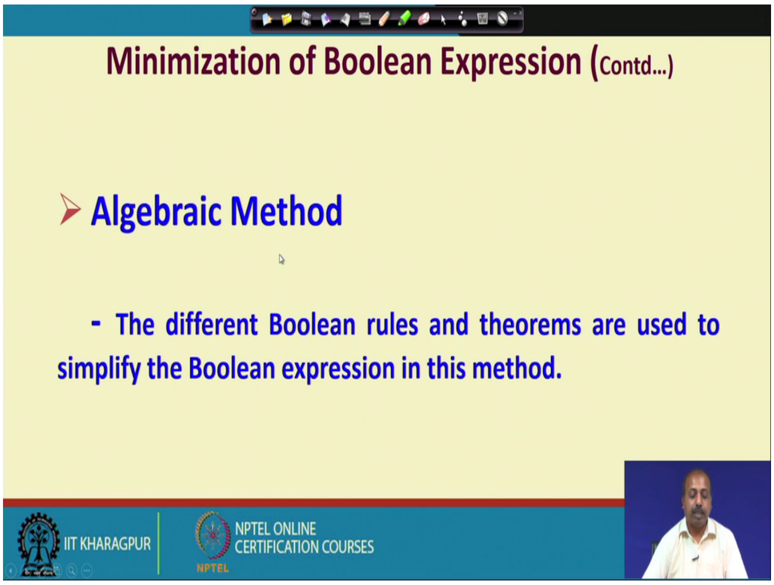

So, this algebraic method the different Boolean rules and theorems, that we have discussed in our previous classes. So, they can be applied for simplifying the Boolean expression in this method so, this is the algebraic method.

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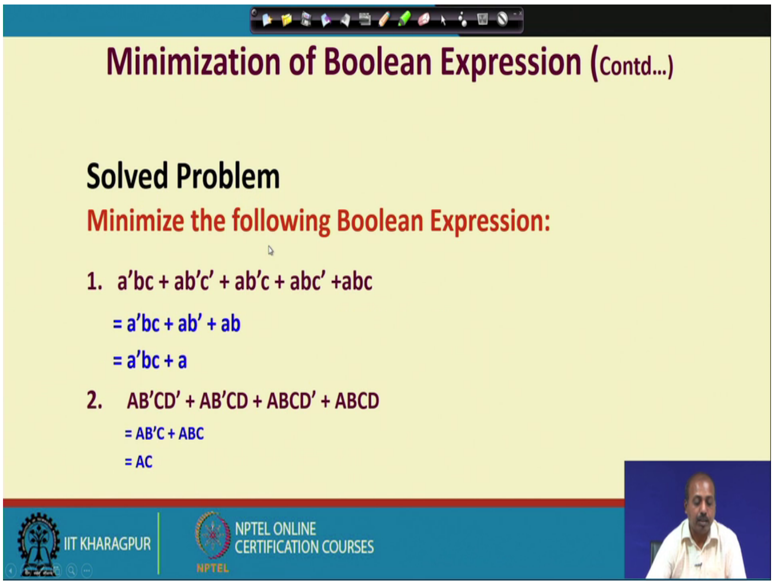

So, we will take some example; like say, this is a Boolean expression. So, this in fact, in canonical sum or product form. So, if you want to simplify this then this a b bar c bar and a b bar c. So, if you take ab bar common from these 2 terms so, you will get c bar plus c.

So, c bar plus c is equal to 1. So, as a result these 2 terms if you combine them so, you will get like ab bar.

Similarly, between these 2 terms; so, if you take ab common you will again get c bar plus c so, that c bar plus c evaluates to 1 so, you get ab bar, sorry ab. And then between these 2 terms so, if you take a common. So, you will get b bar plus b that that is equal to 1. So, this is also get cancelled so, you will get a. So, in this way we can convert this Boolean expression into a corresponding simplified form a bar bc plus a. Of course, there is no guarantee that if you if you are applying this Boolean rules you. So, there is no proof that this is the exact minimum form, ok.

In this in this particular example that maybe the case, but in general there is no such guarantee. Similarly if you take say another example. So, AB bar CD bar plus AB bar CD plus ABCD bar plus ABCD. So, here also if you take AB bar C common from the first 2 term you will get like A B bar C into D bar plus D. So, D bar plus D is equal to 1 so, you get A B bar C. Similarly between these 2 terms so, you take ABC common so, it is D bar plus D. So, this ah that is equal to 1. So, after getting this so, if you take AC common from these 2 terms so, you will get B. B bar plus B. So, b bar plus B evaluates to 1 so, you get AC.

So, in this form, you can apply these Boolean identities to convert this Boolean expressions into some minimized form. Of course, as i said that it is not proved to be absolute minimum ok. So, we can do some exercise this is say this x bar y bar z plus this thing.

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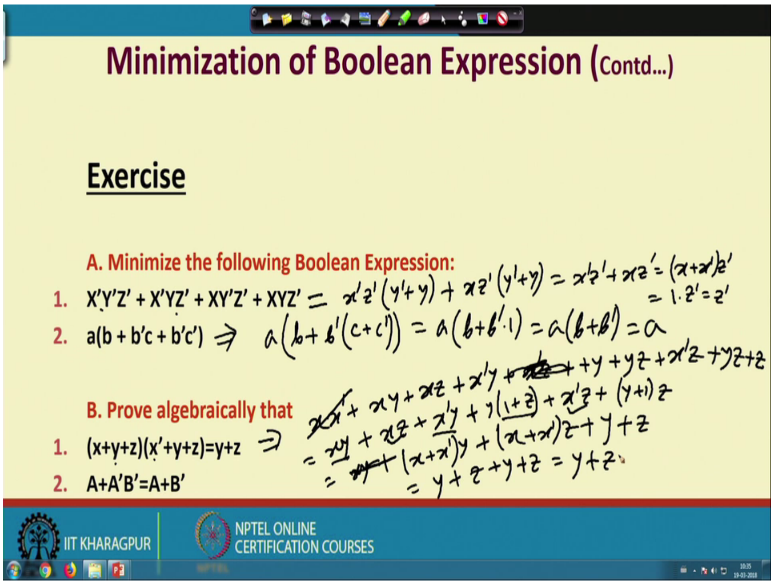

So, if you want to do this so, you can say. So, you between the first 2 terms so, you can say I can take x bar z bar common and inside is y bar plus y.

Similarly, from the second 2 terms. So, I can take xz bar common, and I get y bar plus y. So, that boils down to x bar z bar plus xz bar. And again between these 2 terms so, we can take z bar common. So, we can write it as x plus x bar into z bar. So, x plus x bar evaluates to 1. So, one into z bar that is equal to z bar so, that way you can do the simplification and this one so, you want to simplify, then a in so, you can do many things; like, you can say that I will take ah so, this b is there. So, this b bar c plus b bar c bar. So, we can take b bar common. So, it take it get inside as c bar c plus c bar.

And then this c plus c bar is equal to 1. So, a into b plus b bar into 1, that is a into b plus b bar. So, b plus b bar is again equal to 1 so, you get a. So, this way you can simplify the Boolean expression. So, you can try to verify these rules. So, this x plus y plus z into x bar plus y plus z equal to y plus z. So, how to do this? So, you can convert then you can do a straightforward multiplication; like say if you multiply, so, you will get like x x bar plus x y plus xz plus yx bar y plus x bar z plus so, x bar y plus so, I have taken xx bar plus xy plus xz plus x bar y yy bar is 0, ok.

Let us write in a proper form, otherwise this will be difficult. So, we have got so, this now I multiplying by y. So, yx bar is x bar y plus yy so, that is equal to y yy is y, plus y yz, ok, plus then this z into x bar. So, this is x bar z plus this yz plus z. So, xx bar is 0 so, this cancels out. So, you have got xy plus xz plus x bar y. So, this y plus yz so, this is basically if you take y common so, it is 1 plus z.

Similar the then this is x bar z plus, again if I take z common so, it is y plus 1 into z. So, this is xy plus so, if you take these 2 terms together, this xy and this x bar y. So, this is x plus x bar into y, plus again if you take this xz and x bar z together, to you take that x plus x bar into z plus, this now this y into 1 plus z. So, the 1 plus z is equal to 1 so, you get a y here and this 1 plus y equal to 1 so, get a z here.

So, this is ah so, this is y plus a z plus y plus z that is equal to y plus z. So, this way you can verify this so, this is proved. So, you can use Boolean identities for a getting this thing.

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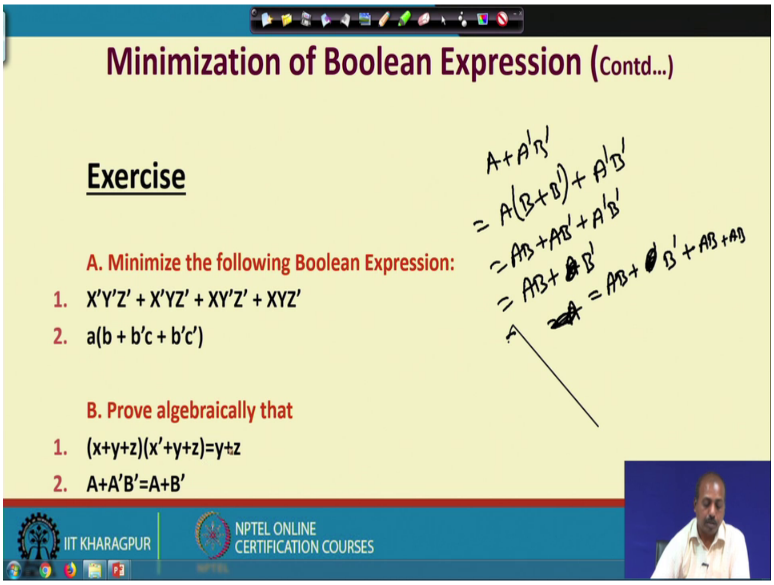

Similarly, A plus A bar B bar ok. So, what you can do? So, you can just convert it into so, B plus B bar plus A bar B bar. So, you get like AB plus AB bar plus A bar B bar so, you get like. So, this is so, this is AB plus AB plus B bar.

So, AB plus B bar so, if you so, AB plus B bar. So, then the from there you can reach, this a plus B bar actually this B bar it can be written as so, it is you can you can show it, or you can start from the other side like at this point. You can just AB plus b bar plus you can take these terms twice. So, AB plus AB bar so, AB plus AB like, that and then you can do some simplifications to come to this form. Or you can you can take a truth table, and then show that this A plus A bar B bar is equal to a plus b bar.

So, next we will see this Karnaugh map method.

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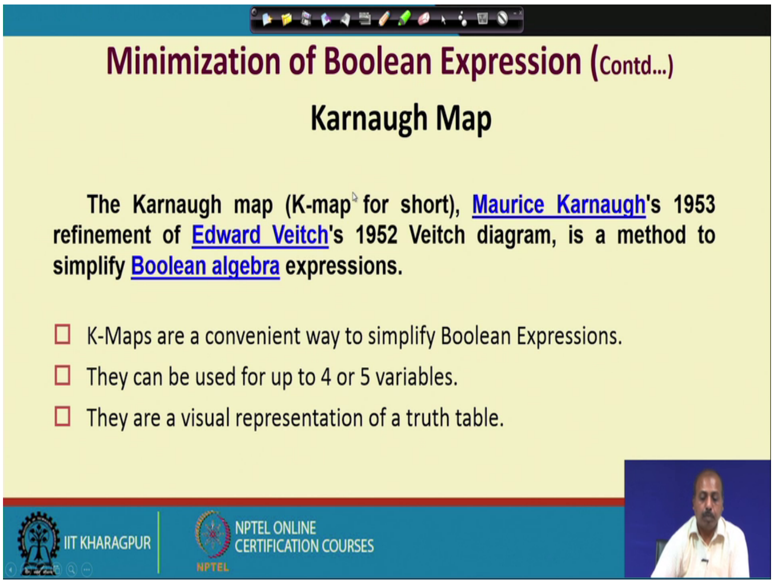

So, this Karnaugh map or K-map so, this was invented by maurice Karnaugh in 1953. So, this is the refinement of weights 1952 Veitch diagram, which is a method to simplify Boolean algebra expressions. So, K-maps so, they are a very convenient way to express Boolean expressions. They can be used for up to 4 or 5 variables. So, in some cases you can expand to 6 variables as well, and they are the visual representation of a truth table.

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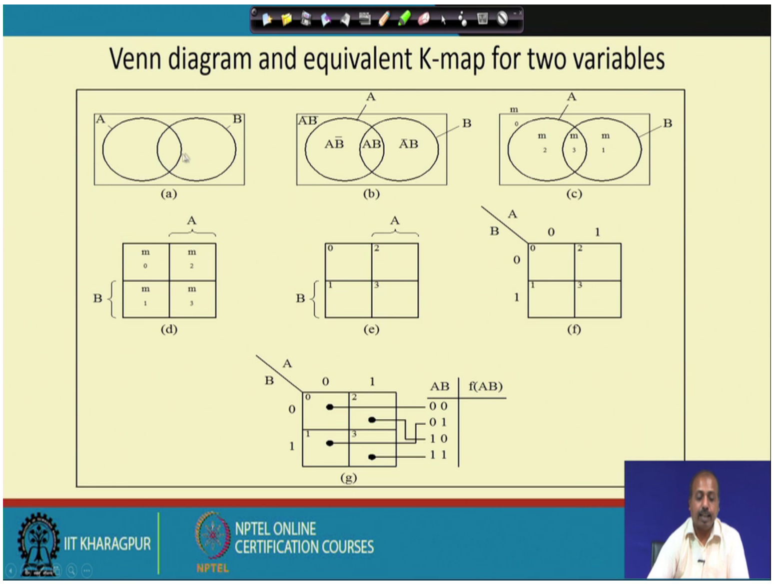

So, we will see how they look like. So, this Venn diagram, so, this we are all familiar with Venn diagram. So, that is for set you can represent it. So, this is the universe, and suppose I have got A and B as 2 variables or they are representing 2 sets so, there may be intersections and some portions maybe common. Now outside this region of these 2 sets, so, I have got the region which is represented as A bar B bar.

Similarly, this part which is common between A and B so, that we represent it as represent as AB, as if both of them are there both of them are true. This part A is there, B is not there so, it is AB bar. And this part A is not there and B is there so, it is A bar B. So, you can also tell, if you write down the corresponding index like A bar B bar the index value is 0, AB bar index value is 2, AB index value is 3, and A bar B index value is 1.

So, this way I can write down the minterm expressions; like, m 0, m 1, m 2 and m 3. So, this is if; this can be represented in the form of a rectangular region, which is divided into 2 in each side, since I am considering only 2 sets or say 2 variables. So, I can represent it as one ah this part this side I represent A and this side I represent B. So, when I am on this part so, this is A both A and B are equal to 0. Here A is equal to 0, but B is equal to 1.

So, this is A equal to 1 and B equal to 0 and here both of them are equal to 1 so, I can represent it like this. So, for the sake of simplicity I can just drop this terms m, and I can write it as 0 2 1 3. So, this this is also this AB. So, this is 1 th form in which I am writing it. So, you can also this is also a standard representation, where we put a diagonal line here, and this side we write A, this side we write B. And then we write like 0 plus the this A, equal to 0 and 1, B equal to 0 and 1.

So, if this is 0 1 0 1 so, like that. So, you can say like, say this is my full so, this is this is another way of representation like say this one. So, first term so, this is 0 0. So, whatever be the value of the function so, we can write here ok. So, this is if this value is say, if this value is say 1 so, this is 0, this is 1 and this is 0. So, we will what we will do? So, at this place we will write a 1, then this 0 1. So, we will write a 0 there. Then 1 0 so, this is this we will write as a 1 here, and 1 1 we will write a 0 there.

So, this way we can fill up this table ok. So, this is known as the Karnaugh map for a function. So, this is, this is a Boolean function, and we can make the corresponding Karnaugh map like this.

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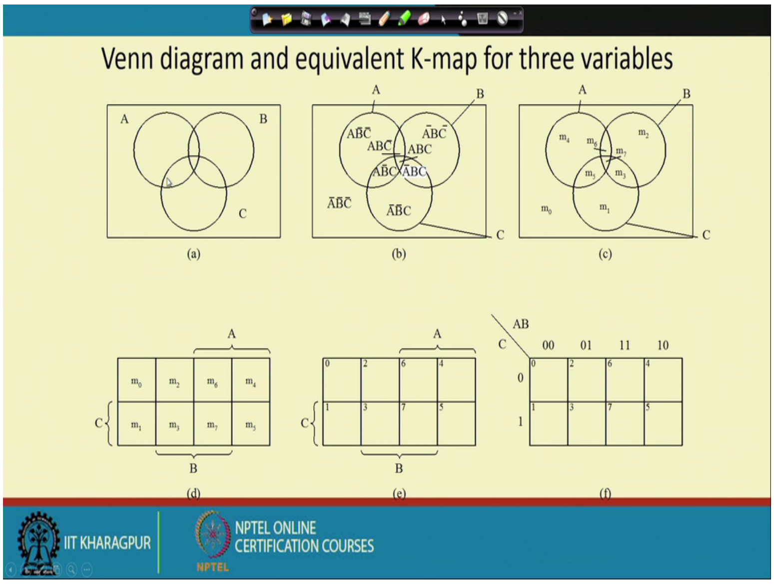

So, this helps in representing the functions like, now what happens to the 3 variables? So, if I have got 3 variables so, in the Venn diagram notation we have got AB and C as 3 surface. And there if you see in that overlapping so, this part is here a all the 3 sets are present so, it is ABC, ok.

So, outside is A bar B bar C bar. So, here I have got A bar B bar C ok. So, the because the A and B are not there only C is there. So, this way for every region, you can figure out the corresponding part of the set which is present there. And now in the same fashion so, we convert it into this minterms. So, outside is m 0 so, this is m 1 and this is m 2. So, like that we can write down the corresponding minterms. So, in A tabular notation so, a so, A and B they are put on the on the row side, and C is put on the column side.

So, 2 variables will be put on the row side and one variable will be taken as the column side. So, that way so, this ab is 0 0 0 1 1 1 and 1 0. So, you note the you take the notation that, this A is represented by these 2 row, these 2 columns and B is B equal to 1 spans over these 2 columns. So, A equal to 1 spans over these 2 columns, B equal to 1 spans over these 2 columns. C equal to 1 spans over this particular row.

So, this part, this particular column A and B both are 0. And here A is 0 B is 1, ok. So, that way, we can have different rows and columns representing different parts of the ah of this diagram. And ultimately we have got these rectangular block, rectangular box, where we can this is also a notation a standard notation; that is you write AB here and C here on this side we enumerate the combinations 0 0, 0 1, 1 1, 1 0. This side we enumerate the combination for C 0 and 1.

Now, given the function so, we will be, we will populate these entries. And accordingly we can fill it up.

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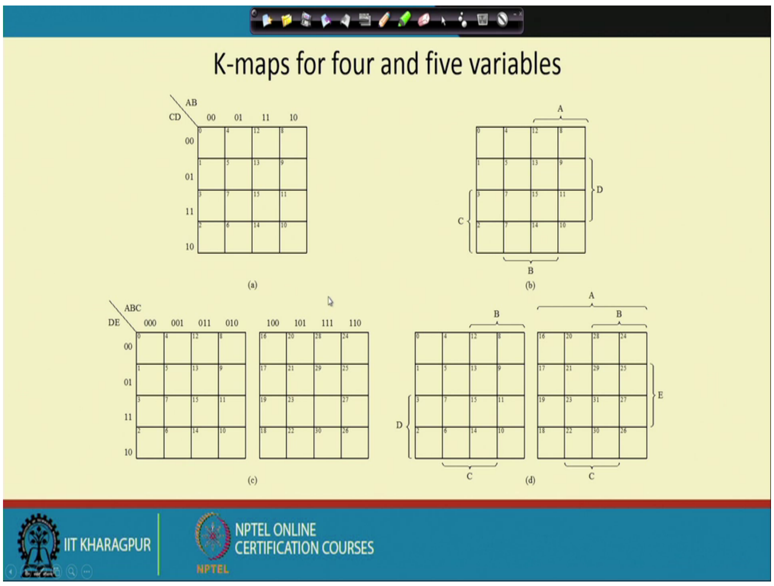

So, 4 variables so, if we are taking 4 variables then this AB so, we were using the row side for representing 2 variables. Now we will be using the column side also for representing 2 variables so, AB and CD. So, 0 0, 0 1, 1 1, 1 0 and CD is 0 0, 0 1, 1 1, 1 0. And so, in a, sometimes we write it in this format also, and for these 2 columns A is 1, for these 2 columns B is 1. So, like that is also a standard notation.

So, generally we will be using this notation for most of the cases. But sometimes we will be going to this notation as well. So, for 5 variable what we do? We take maps, ok. So, in one of the map A is equal to 0. So, you see all the combinations that we have; here A is equal to 0. So, this is 0 so, here also A is 0, here also A is 0. And on this side I have taken all the maps where a all the permutations where all the combinations where A is equal to 1. So, that way, we as if we split the map into 2, 1 for A equal to 0 other for A equal to 1. So, this way we can go for 5 variable maps, ok.

So, though they are not a single map, but we can go for in this way. Or we can also visualize it as if this is a 3D; so, one is tack over the other. So, here it was a 2D up to 4 variable the map remains 2D, but when you go to 5 variable so, map becomes 3D. So, you have to expand in another direction on the vertical direction.

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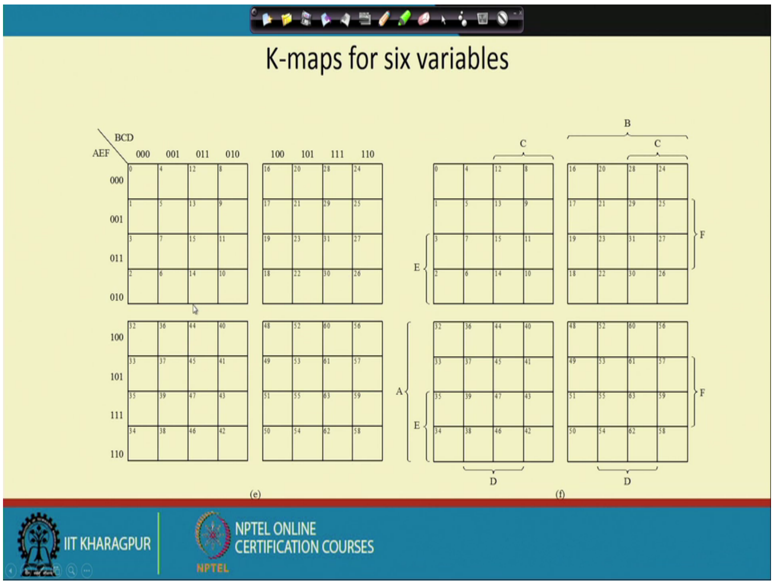

6 variable so, again so, as you can understand, that I have to have 4 maps ok. So, for this map A equal to 0 and B equal to 0. For this map A equal to 0, B equal to 1. Here A equal to 1, B equal to 0. And for this map, I have got A equal to 1 and B equal to 1.

So, this way the minterms get distributed over, the number of ah number of maps number of tables. And then we can try to do some manipulation with those numbers and get the minimized form. And similarly so, this is also another notation of the same map, 6 variable map.

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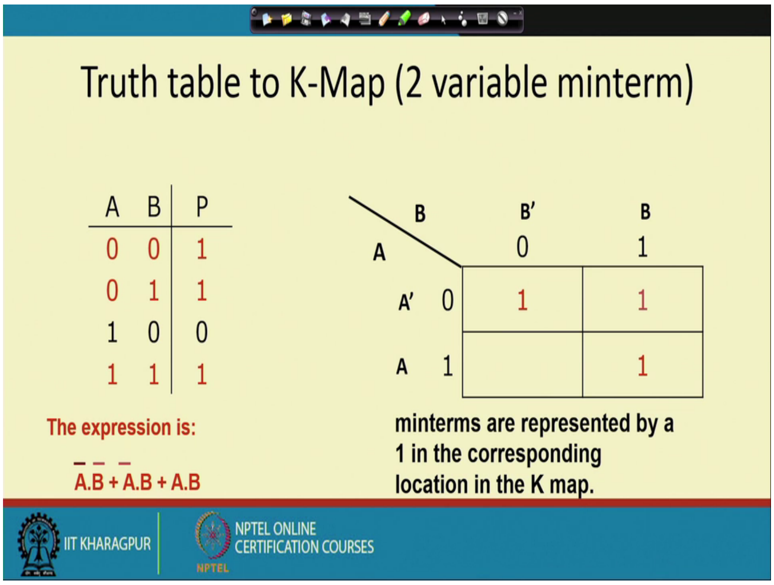

Now, how the truth table gets represented in the form of a Karnaugh map, ok. So, 2 variable suppose this is the function so, AB 0 0 0 1 1 0 and 1 1 for 0 0 0 1 and 1 1, the value of the variable the value of the function is 1, otherwise it is 0.

So, in the Karnaugh map so, we represent the minterms at by putting a one at the corresponding location of the Karnaugh map. So, at so, these 3 ones were there so, these 3 ones we are put here, ok. So, this is the this is the representation of this truth table in the form of Karnaugh map. So, minterms are represented by a one in the corresponding location in the Karnaugh map.

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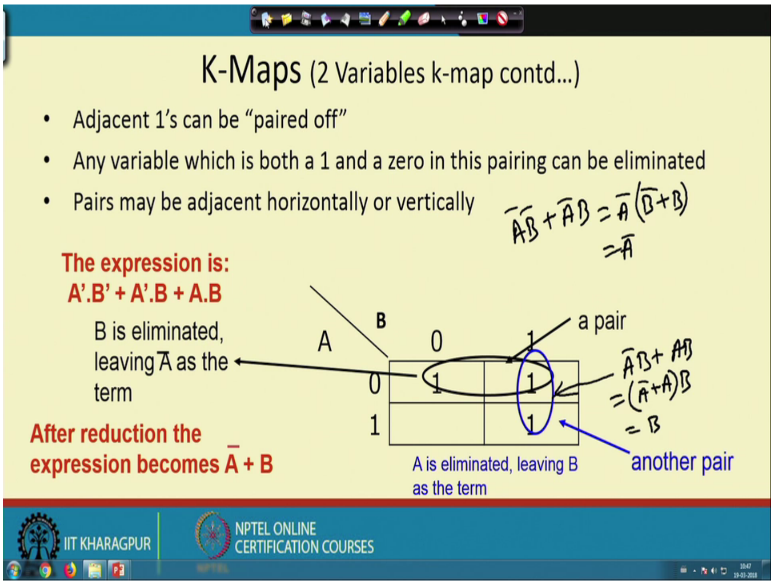

So, 2 variable Karnaugh map so, it is adjacent ones can be paired of so, we can do it like this.

So, this is one and this is one. So, these 2 can be paired together, and we can get a pairing of them. So, this is. So, in this so, the term that we have is so, if we look into this so, it is A bar B bar plus A bar B. So, here it is A bar B bar plus A bar B. So, if I do a simplification so, A bar B bar plus A bar B. So, here if I take A bar out so, I will get B bar plus B so, this B bar plus B is equal to 1. So, this simplifies to A bar, so, that means, if I have got ones or 2 successive places, and I do. So, do circle them, combine them together then, one of the variables get eliminated.

So, in this case B is changing from 0 to 1. So, whichever variable change it is polarity so, that variable gets eliminated. So, this is the variable B has got eliminated. Similarly if you look into this term, then what is happening is; so, this is nothing but A bar B plus AB. So, in a similar form so, you can say take a this B comma A bar plus A into B and A bar plus A equal to 1 so, you get a B here. So, here this the term a has got eliminated.

So, in this way, once we have represented the function in Karnaugh map so, we automatically do this type of grouping and eliminate variables for for getting the minimum form. So, this is the beauty of Karnaugh map so, once you have drawn it. So, you can just go on grouping them and getting the minimum form. And there is a proof that these are going to be the minimum one. So, there are certain rules so, if you follow that. So, there is a proof that you will be be definitely getting a minimum sum or product sum.

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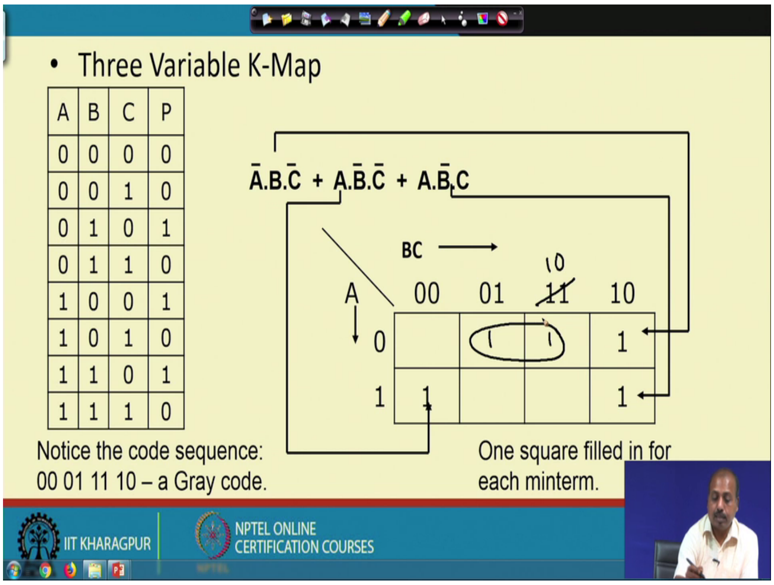

Let us take a 3 variable function. So, this ABC combination varying from 0 0 0 to 1 1 1, and this is the output column that the function the function P. Now in a so, this is one at 0 1 0 1 0 0 and 1 1 0. So, this is 0 1 0 so, 0 1 0 is this one, then we have got 1 0 0 so, 1 0 0 is this one and 1 1 0 is that one.

So, in in this if you see that I can this is the representation. And then I have to I can do a or I can do something called this grouping as you are done doing in the previous example. So, one square is filled in for each minterm. And here you see one thing that the way we are writing these indices. So, this is 0 0 0 1 1 1 and 1 0. So, it is you just keep in mind that it is not like 0 0 0 and 1 0 1 1.

So, it is not the binary ascending order, but it is in some sort of gray code, ok. So, this we will see later that this is something called this is a, this type of coding where only one bit changes between successive terms so, that is called a gray code. So, it is 0 0 to 0 1 only the lsb is changing. Similarly, 0 1 to 1 1 only the second bit is changing, and 1 1 to 1 0 again only the lsb is changing. So, when only one bit is changing so, we get a gray code.

So, this gray coded form so, this is useful because when you are grouping. So, if I have got 2 successive terms. So, if you write here so, so, instead of this row B this column B in 1 1 so, if this was say 1 0. Then if you are trying to combine these 2, then the problem is that the more than one variable is changing, but in that case it will be a wrong one. So, we cannot take more than one variable changing between the successive terms. So, that is that is the rule of this Karnaugh map; so, you have to take it in the gray coded format.

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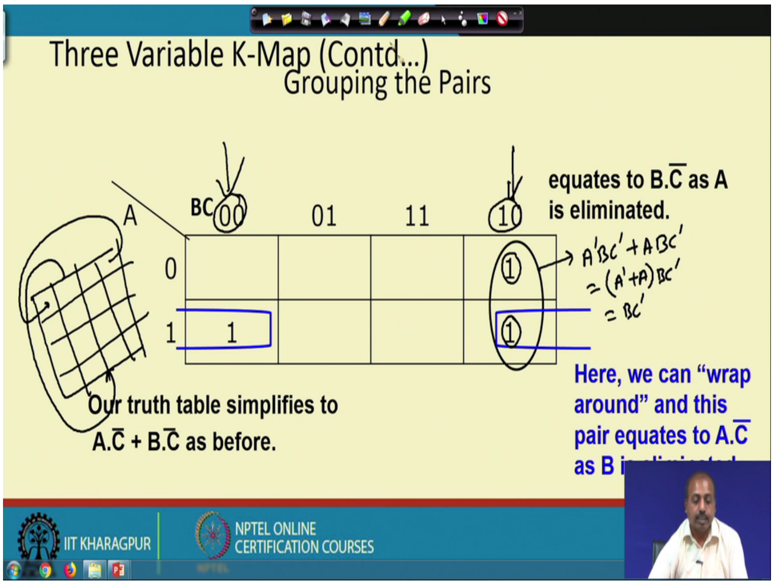

Now the grouping part; so, this first the example that we have. So, say for that so, you can do a grouping like this. So, where I have got a ones at 2 successive places in the vertical direction. And the term that gets eliminated is given by so, this is this can be written as say this is A bar. So, A bar sorry, this is basically A bar B C bar plus A B C bar ok. So, this one is ABC bar, and this one is A bar B C bar.

Now, if you combine them so, this is a this BC bar is common between them. So, we get A bar plus A into B C bar. So, this A bar plus A equal to 1 so, you get BC bar here. Also while you can think this Karnaugh map to be some sort of folded one. So, this last row or the last column you can take it as if the whole table has been folded. So, you get this last column adjacent to the first column. Similarly if you have got more the higher sized Karnaugh map say 4 by 4 Karnaugh map, then also you can take this folding.

So, you can take as if this part is at this row is adjacent to this row, and again this column is adjacent to this column. Because it is folded in the both in the row wise and the column wise direction. Because of very simple like if you look into these 2 patterns. So, this is 0 0 and so, this is 1 0. So, only one bit is changing between them so, whenever only one bit is changing the columns are considered to be adjacent. Similarly, between the rows if only one bit is changing. So, we will consider the rows to be adjacent.

So, here the so, for by following that rule so, this 0 0 and 1 0. So, these 2 columns are adjacent to each other so, we can do a grouping taking the corresponding ones. So, if you do that then you will be getting like.

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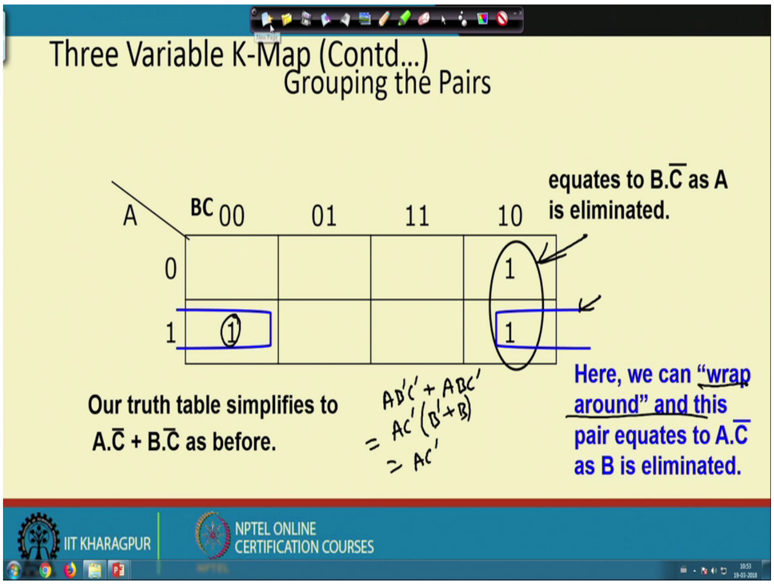

So, this is so, this term is A B bar C bar. And this term is A B C bar so, if you do that. So, if you take this AC bar common, AC bar common. So, you will get B bar plus B so, B bar plus B is equal to 1 so, you get AC bar.

So, ultimately so, we have got this ac bar term from the, from this blue square, and we have got this B bar C term from the BC bar term from the first one. So, the overall function is so, this is that is what I was telling, but it is a rap around connection. So, rap around so, we can rap around, and we can say that we can get more number of minimizations for this part, ok.

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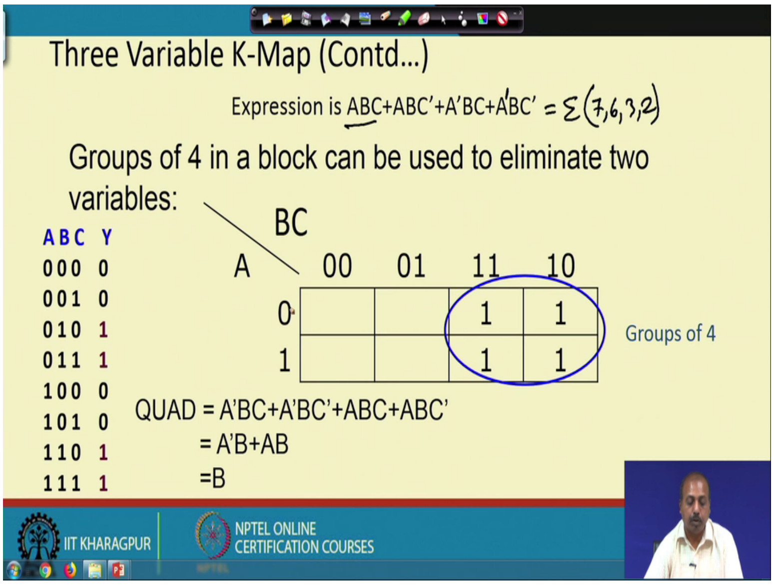

So, next we will see another example; suppose, you have got an expression like say ABC plus ABC bar plus A bar BC plus ABC bar. So, in the index notation. So, this is basically so, this is 7 then this is ABC bar. So, that is 6, then we have got A bar BC that is 3, and ABC bar is so, ABC bar is written twice, ok. So, that is something wrong here in the example. So, this is A bar BC plus A bar BC bar. So, this should be A bar B C bar, this is not A ABC bar A bar B C bar. So, A bar BC bar is 2, so, these are the 4 terms in index notation. So, you have got 4 terms like this.

Now, so, this is the function so, the you have got so, if you if you look into the corresponding Karnaugh map; so, say 3 variable map and we will have situation like this. So, this A bar B C bar. So, a bar so, it will be on this row B C bar. So, this one so, this is this is 1, then 0 1 1. So, A bar BC so, A bar BC. So, this is also equal to 1, then we have got AB C bar. So, A BC bar so, this so, so this one. So, this is one and then we have got ABC so, this is one.

So, ultimately what you get is a 4 ones which are close to each other. So, they are all one hop away from each other. So, you can see that this expression. So, you can whenever we have got normal 4 1. So, you can club them together into a group of 4 1's. And then if you do a minimization by using Boolean identities, then you see that this whole term it reduces to B. So, in terms of Karnaugh map you can what you can say in this case, you see A is changing it is polarity from 0 to 1, if you consider this 4 terms A is changing it is polarity. And your C is also changing it is polarity so, it is becoming 1 to 0.

So, what is not changing is the polarity of B. So, this whole quad. So, it can be retain as the term the variable which is not changing it is polarity, that is B. So, without doing this algebraic manipulations so, we can directly write that, the term corresponding to this quad will be B. So, we just note down the variables which are not changing it is polarity.