Digital Circuits

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Lecture – 10
Boolean Algebra (Contd.)

So, in case of Boolean expression so, we can represent it in 2 different forms.

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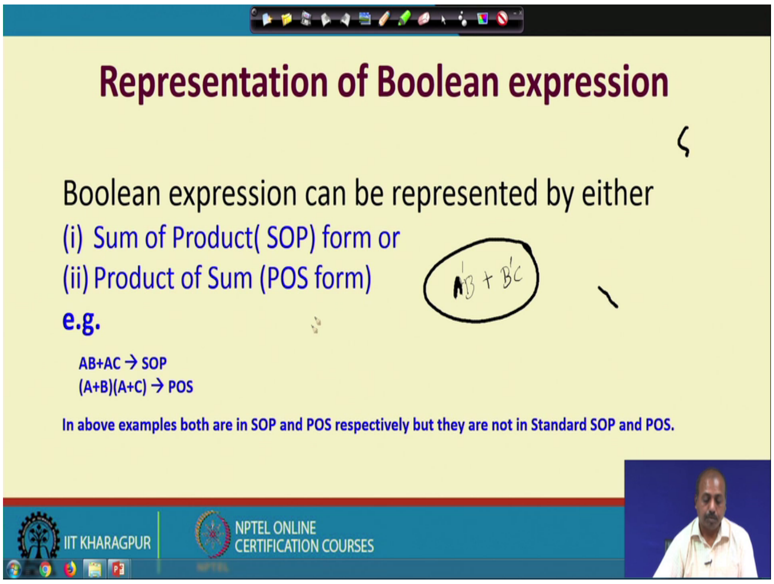

One is known as sum of product form, another is called product of sum form. So, as a name suggest sum of product means it is a summation of product terms. For example, this AB plus AC so, AB is A and B. So, as we said that it is obvious so, we do not write the dot in between to represent the and so, AB is the first product term, AC is the second product term. So, overall expression is the summation of the terms AB and AC so, that it is in sum of product form.

Other alternative for this Boolean expression representation is product of sum, where we take the sum as individual components like A plus B or A plus C. So, these are individual component and then we take the product of those components to come to the Boolean expression.

Now, instead of simply being the variables so, it can be the complemented forms of that also like, this is also a correct forms. So, A bar B plus B bar C so, that is also sum sort of expression where it is so, this is also a sum of product form, but here this A bar B. So, these individual terms they are slightly they are the bit. So, individual terms so, they are in sum complemented forms so, that can happen. So, whatever it is so, based on that we can we can have different forms for the differ the functions.

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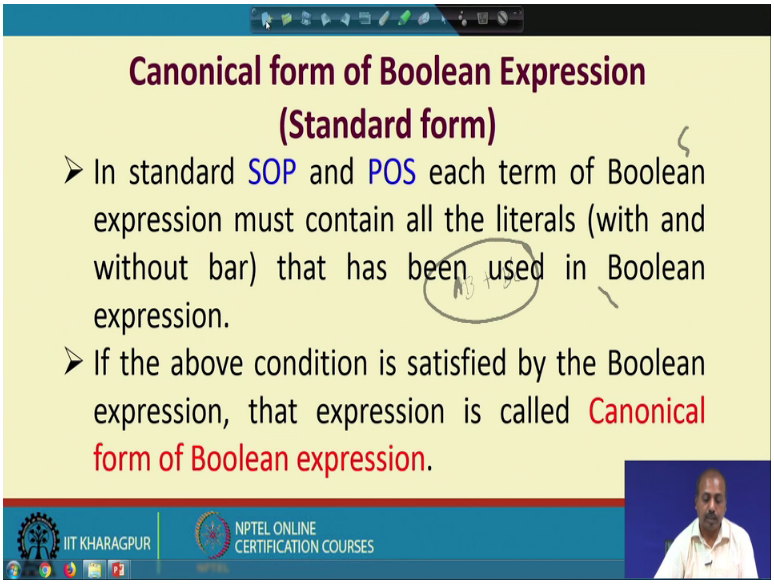

Now, but the problem with the these forms is that it is not it is not unique. For example, for the same expression so, we can have different forms like I will take an example and try to explain. Like if I have the term say AB bar plus B bar C or say BC bar so, this is one form.

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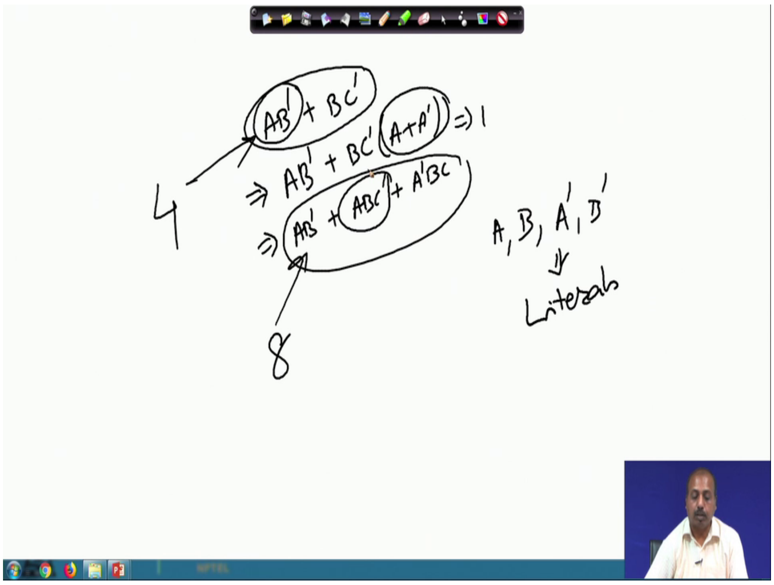

Now, if you expand this AB bar so, it can be written as say a say, we can write it as say somebody may write it as AB B bar plus B into C bar into A plus A bar. So, if you expand this you get like AB bar plus AB C bar plus A bar B C bar. Because is A plus A bar is equal to 1. So, we can always write in this form.

So, what is happening is that you see this is also sum of product form; this is also sum of product form. So, the problem with these forms is that one of them may be sum sort of minimum form and sum other one is not that minimize. So, it has got 3 terms in it the first one has got 2 terms in it. So, it is not mandatory that this minimized form they will be unique ok.

So, that is the problem with Boolean expression. So, if the minimum sum minimum term expression may not be unique, there will multiple by expressions with the similar type of cost, in terms of number of product terms, number of variables. So, this AB etcetera so, these or the compliment like A bar B bar. So, they are called literals.

So, this number of literals in this expressions may be so, in this case in this in this expression we have got 4 literals, in this expression we have got one 2 3 4 5, 6 7 8 literals. So, number of literals may vary so, literals means the variable or it is complemented form. Then we have got this term. So, this AB bar is a term, similarly AB C bar is a term, ok. So, we have got literals term, and then the expression the whole expression.

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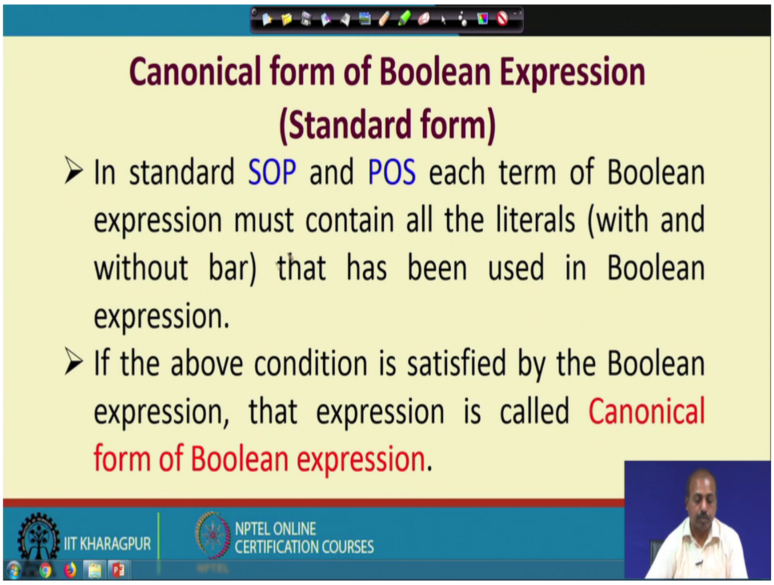

So, coming back to our discussion. So, we have got so, how to check like 2 expression whether they are same or not. So, for doing that there is a canonical representation. So, anything that is canonical means so, all expressions that are equivalent they will boil down to the same canonical form, ok. So, that is the beauty of this canonical form of expression. So, this standard sum of product or product of sum each term of Boolean expression must contain all the literals, with and without bar that has been used in the Boolean expression.

So, it says that when your taking the sum of product or product of sum. So, individual terms they should so, all the literals should appear in the individual terms. If which is not, then it is not a not in the canonical form. So, in the if the condition is satisfied, then the Boolean expression is set to be in canonical form of Boolean expression. So, it is called canonical, because now it becomes unique.

So, for a particular Boolean expression, so, you can have only one canonical form canonical sum of product form and another canonical product of sum form. So, you cannot have more than one canonical form for the same expression. So, if you are given to expression and to check whether they are equivalent or not they are same or not. So, what you need to do is to convert them into their canonical forms, and then check whether the terms are same in both the expressions.

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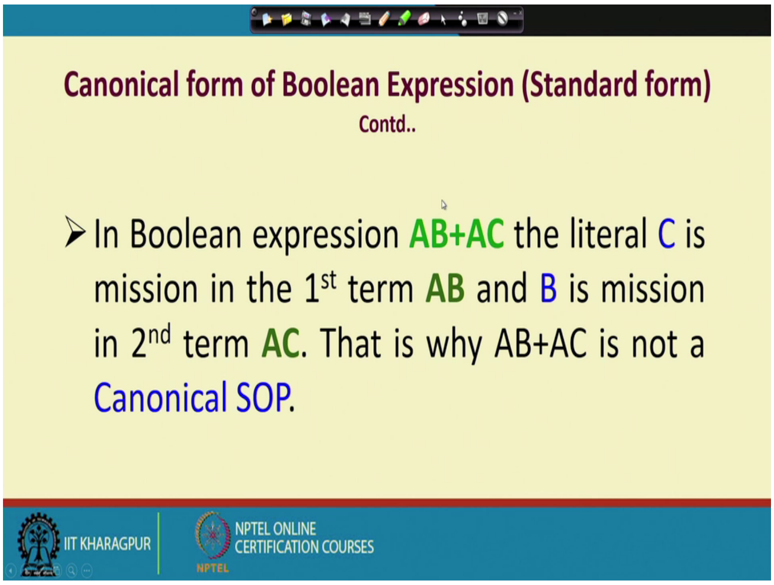

So, so, if AB plus AC. So, here then the first term the literal C is not there. Similarly, in the second term if this B is missing here. So, it is A and C will this to literals are present so, B is missing. So, this is why this AB plus AC is not a canonical sum of product expression.

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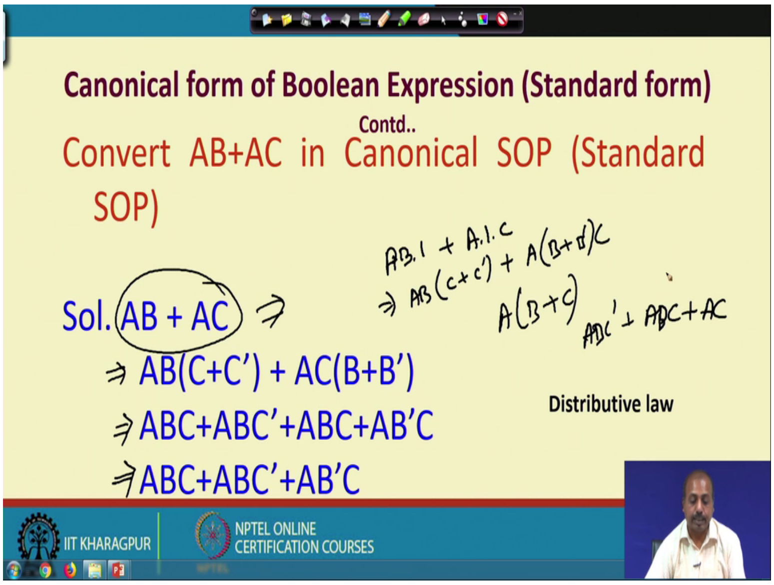

So, how to convert this AB plus AC non canonical from into a canonical sum of product form? So, this is simple, by using the Boolean identities. So, what we do? We write this AB as AB dot one we write it as AB dot 1, plus A dot 1 dot C. Then what happens is this

AB is so, AB is that this 1 can be re written as C plus C dash C bar, and this can be written as B plus B bar into C so, that is what is done here. So, we substitute this in place of one we write here C plus C bar in place of one here we write B plus B bar.

Then if we expand so, you will get this form ABC plus ABC bar plus ABC plus ABC bar. So, if these are all equivalent so, from here we are getting this, from this we are getting this. Now you see that sum of the terms are appearing more than once the term ABC C as appeared more than once. So, you can just remove the duplicate and write it as ABC plus ABC bar plus AB bar C so, this is the canonical form. So, whatever be the different forms in which you write this, like somebody may write it ASA into B plus C, somebody may write it as say AB C bar plus ABC plus AC.

So, in this way different term different people can write the same first expression in different form. But if you convert it into canonical form then all of them will become same. That is the beauty of the canonical form.

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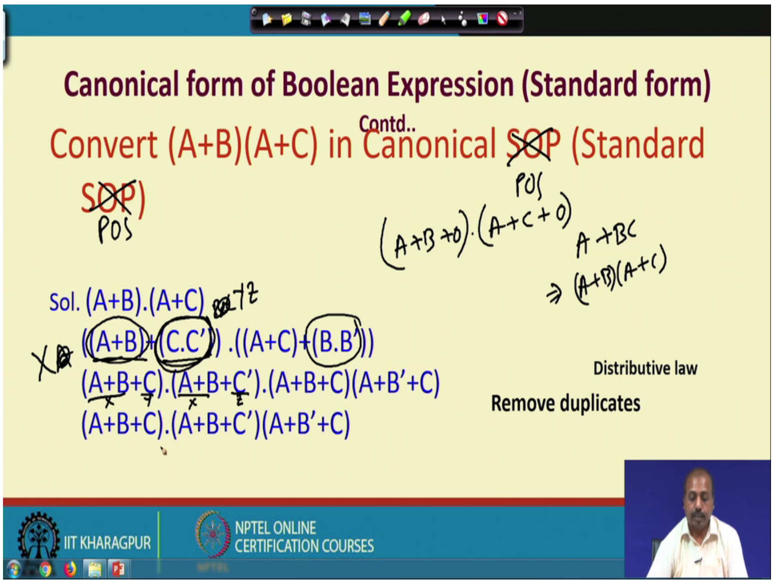

So, another expression A so, this is this AB so, AB plus AC. So, this is converting into canonical POS form. So, this is not SOP so, this is POS so, this is POS form product of sum form from, which is also known as the standard product of sum form ok. So, we have got the POS form. So, how do you do it? So, AB plus AC so, this can be written as; so, A plus A plus B plus 0 into A plus C plus 0.

Now, this 0's can be written as CC bar here, and BB bar here then we apply the distributive law, where this is the first term this is the second term. So, you remember the distributive law A plus BC is can be written as A plus B into A plus C. So, this is if this whole thing is consider as A, and this is consider as say BC , then it is A plus B, ok.

So, that is A plus B plus C in this case and you can write it as X and Y Z better. So, this is say X and this is say Y Z. So, this is this is written as so, this part is my X, this is Y. So, this part is my x so, this is z, so and distribute the thing. Similarly for the second term also. So, you can distribute this BB bar into the 2 constituents.

So, that way finally, then if you remove the duplicates. So, you get so, this A plus B plus is coming twice. So, you can remove them, and you can get it once. So, this way we can convert the product of some expression into their canonical forms.

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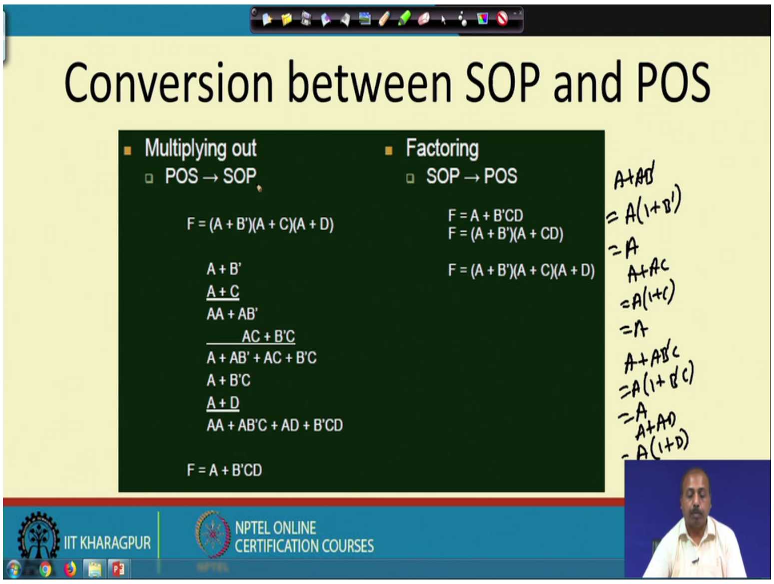

Next so, how to convert from product of sum to sum of product or sum of product to product of sum is like this. Product of sum to sum of products so, these expression is very simple so, you just go on the multiplying the thing like say yeah I have got an expression like A plus B bar into A plus C into A plus D. So, what you do you just multiply A plus B bar by A plus C. So, these gives us this term A plus AB bar plus AC plus B bar C, then that is that is equivalent to A plus B bar C, why? Like say, this one sorry like say this one so, A plus A plus AB bar. So, it can be written as A into 1 plus B bar that is equal to A.

So, the first part A plus AB bar can be converted into A. Now this A plus AC so, it can be again be converted into A into 1 plus C, that is equal to A. So, the first 3 terms of the expression here. So, it boils down to a so, ultimately you have got A plus B bar C. And then we can multiply it by A plus D as a result we get this expression AA plus AB bar C plus AD plus B bar CD. Again you have to take common and so, AA is A, then this A plus AB bar C.

So, by a similar logic A plus AB bar C is equal to A into 1 plus B bar C. So, this one will be so, that gives by to a. So, a so, this part A plus AB bar C gives me A. And with that if you do A plus AD so, if you do A plus AD, then you will be getting again a common. So, 1 plus D so, that way I will get A finally, again from here I will get an A. So, it is A plus B bar CD.

So, that is the whole multiplication that we have done. So, sum product of sum to sum of product conversion we can do it like this. Similarly sum of product to product of sum conversion. So, you have to factor out. So, this is A plus B bar CD. So, it can be written as A plus B bar into A plus CD. And again if you factor out so, A plus B bar is A plus it remains as A plus B bar, but A plus CD can be re written as A plus C into A plus D.

So, in this way we can convert from sum of product to product of sum or from product of sum to sum of product, of course, they are not canonical in nature. So, to convert to canonical so, you have to again do it you have to, you have to convert to canonical form like say A plus, I am sorry, A plus B bar CD.

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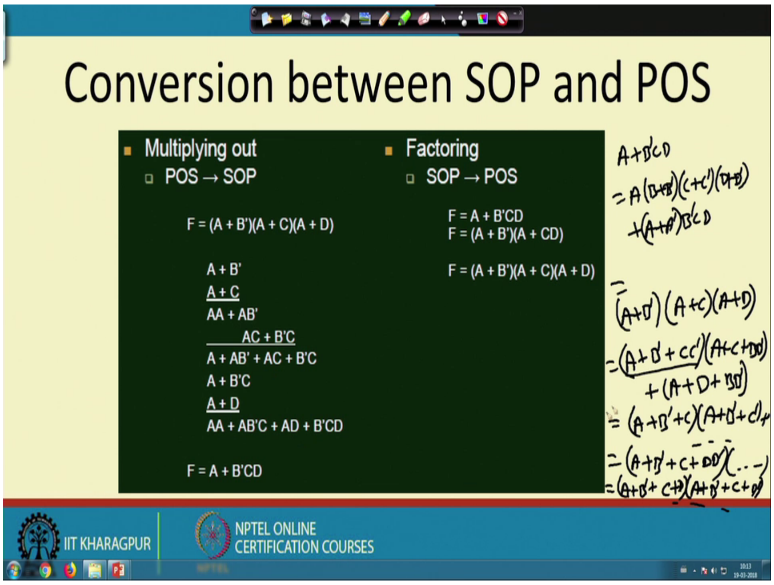

If you have to convert so, you have to write it has A into B plus B bar into C plus C bar into D plus D bar plus A plus A bar for the second term A plus A bar into B bar CD.

Now, if you just go on multiplying this so, you will get the canonical sum of product form and similarly for the second expression like say A plus B bar into A plus C into A plus D. So, you have to write it as like A plus B bar plus CC bar into A plus C plus BB bar plus A plus D plus BB bar. Then that is not the end, because after this, after you have factor out this. So, you have got likes a terms like A plus B bar plus C into A plus B bar plus C bar, plus the other term that I am not writing here.

So, again these as to be factored out , this is this C so, you have to write the first term as A plus B bar plus C plus DD bar. Again the same thing will be happen for others. And then it will be re written as A plus B bar plus C plus D into A plus B bar plus C plus D bar so, etcetera. So, this way you can convert first you convert from sum of product to product of sum or product of sum to sum up product. And then if you want to convert to canonical from so, you have to expand like that.

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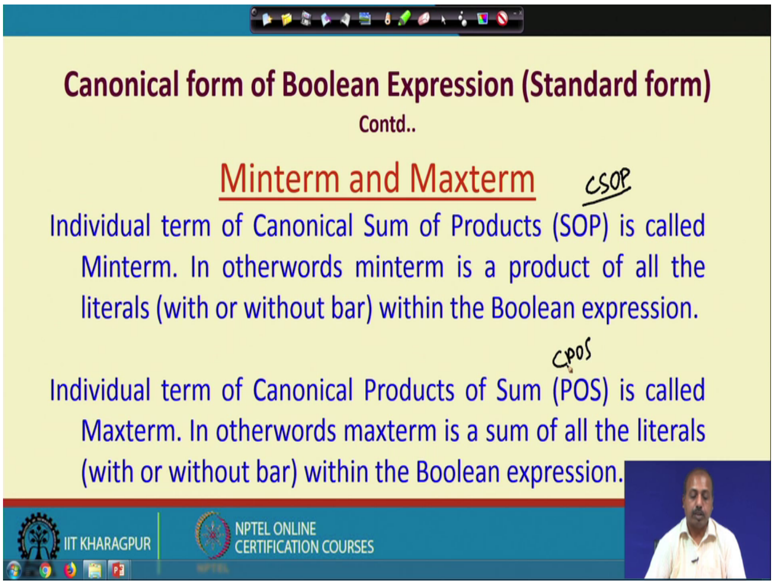

So, when your converted and expression to this canonical form product of sum or sum of product. So, we can we come across 2 new terminologies, one is called minterm another is called maxterm. So, individual term of this canonical sum of product expression. So, they are also known as C SOP this canonical sum of product we often write tell it as CSOP canonical sum of products. Similarly here we will be canonical product of sum we write as CPOS. So, this was a CSOP form. So, it is so, individual terms of this canonical SOP form so, they are called minterms.

So, minterm is a product of all literals with; in the Boolean expression, with or without the complementation. And in case of this product of sum expression the canonical product of sum expression. So, individual terms so, they will be called individual sum terms so, they will be called max terms. So, max term is a sum of all the literals with or without bar within the Boolean expression.

So, in this way we define minterm and maxterm or Boolean expressions.

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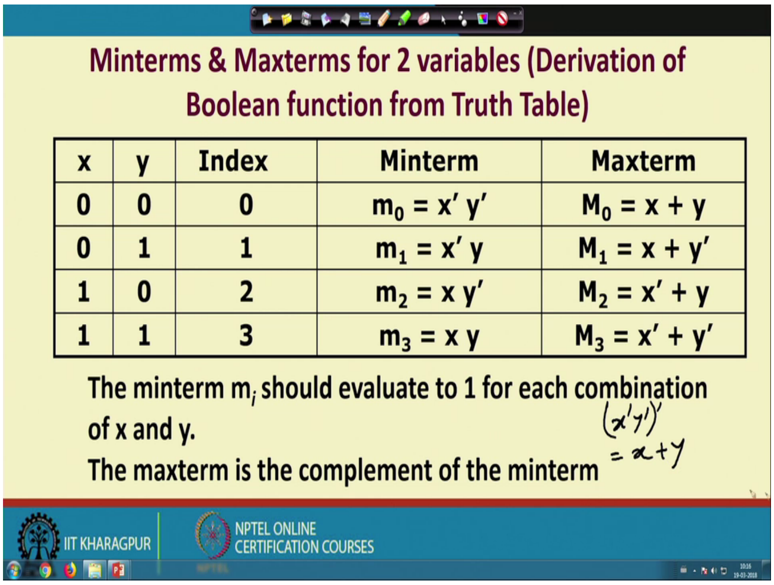

So, this is a thing like say 0 0. So, this is a these the corresponding minterm is X bar Y bar. So, 0 1 corresponding minterm is X one Y like that 1 0 and 1 1. So, XY bar and XY as far as max term is concerned. So, they are written as m 0 so, this is written as X plus y. So, you note here that for while writing the max term. So, for if the value of variable is 0, we are taking it as un complemented form. So, that is the convention that you have to follow. And whenever the value of variable is one so, you have taking it in the complemented form. Whereas, for writing minterm when the variable is in true form so, we are writing it in true form, and when it is in complemented form like say 0 0 so, you are writing it as X bar Y bar.

So, minterm was so, all the minterms mi should evaluate to one for each combination of X and Y. And the maxterm is the complement of the minterm. So, this is the complement of this. So, basically X bar Y bar if you take a compliment so, this X bar Y bar so, if you take compliment then by Demorgans law it becomes equal to X plus y. So, maxterm is the complement of minterm.

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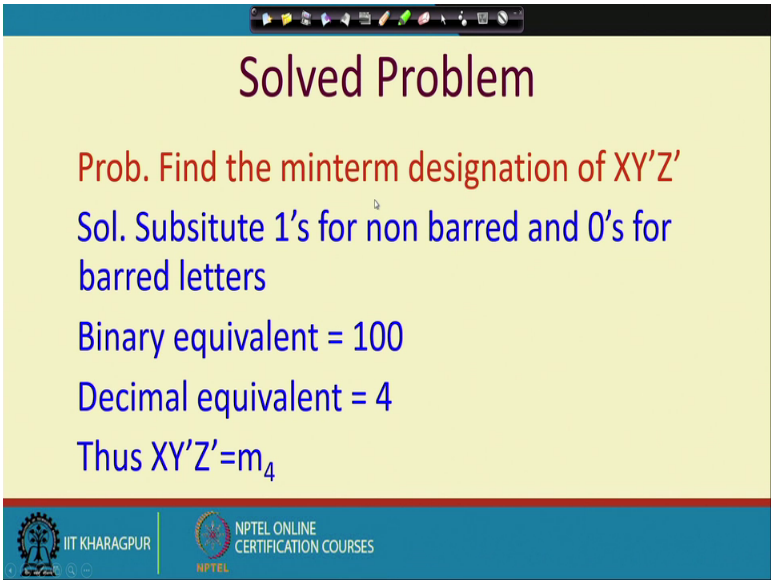

So, next we will see some problem. So, find the minterm designation of XY bar Z bar. So, we have took so, it is binary equivalent is 1 0 0. So, decimal equivalent is 4 so, it is the m 4. So, this is this is the standard notation that we are using. So, this is say if this value evaluate to 0 so, will write it as m 0. So, if this value evaluate is 2 ah. So, 1 0 so, that evaluates 2 so, we write it as m 2.

So, by giving this number or index, we can directly understand what is the corresponding minterm. So, that is why this indexing is done. Similarly, here the indexing is done, we know that whatever be the values so, it is say a say a one. So, that is 0 1. So, X should be taken into true form and Y should be taken into complemented form. For the max term for the minterm, should be taken in this X should be taken in complemented form and yb taken in true form.

So, this X Y bar Z is 1 0 0 so, decimal equivalent is 4 and so, this is the this is m 4.

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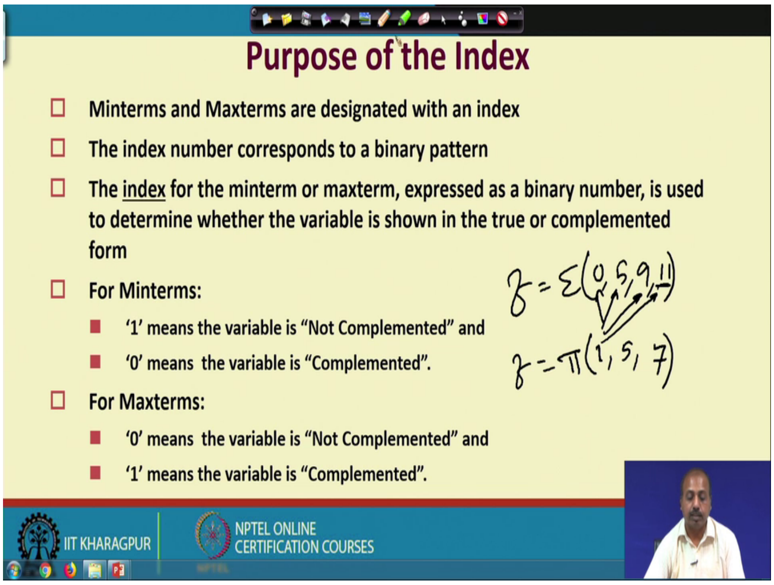

So, why do we need to do this indexing? So, minterms and max and maxterms are designated with an index, and this index number corresponds to a binary pattern. So, if you are if you are talking about a particular function, so, one possibilities that you write down the sum of product or product of sum expression. Otherwise you can just still that what are the indexes like for the minterms or maxterms. So, that way you can also tell what is the function. So, that is why this index number is associated.

So, index number for minterm or maxterm expressed as a binary number, it is used to determine whether the variable is in true form or in complemented form. So, for minterms one means the variable is not complemented and 0 means the variable is complemented. And from max term so, 0 means the variable is not complemented and one means the variable is complemented.

So, this is the convention that is followed for this minterm and maxterm. So, indexing actually helps us to tell the function like you can simply say, that my function f is the is the summation of the minterm so, 0 5 9 and 11. So, this immediately says that since I am going. So, it is 4 variable function, because I am going up to 11 so, I will need 4 bits to represented. And out of that the minterm 0 5 9 and 11 for them, the function value is one, where as others the function value is 0.

So, this way this is very standard rotation similarly, I can have the max term representation which is written as this pi or product. So, 1 5 7. So, this means that for this product terms 1 5 and 7 so, this I have to take the sum of product form sorry product of sum form to express the function.

So, talking about so, sum of product form will Boolean function represented by the truth table.

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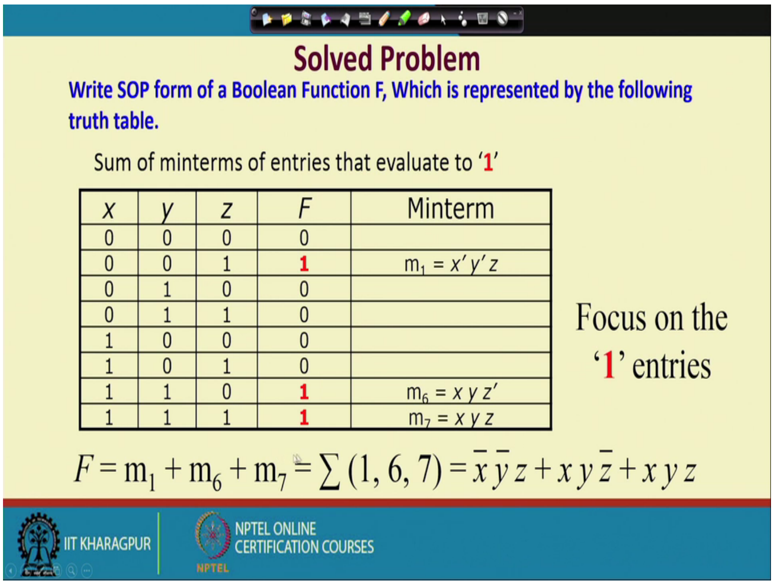

x	y	z	F	Minterm
0	0	0	0	
0	0	1	1	$m_1 = x'y'z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$m_6 = xyz'$
1	1	1	1	$m_7 = xyz$

$$F = m_1 + m_6 + m_7 = \sum(1, 6, 7) = \bar{x}\bar{y}z + xy\bar{z} + xyz$$

So, this is a so, a truth table means I have already said. So, you have to tell what are the input assignment. And corresponding to that what is the function the output column. So, what will be the value of the function for this one? Like say this is the truth table for this function. So, 0 0 0 is 0 0 0 1 is 1 so, like that.

So, if you consider only this values, where the function f evaluates to 1, ok. So, this is for this is for this 0 0 1, 1 0 1 sorry 1 1 0 and 1 1 1. So, 0 0 1 the corresponding index is one 1 1 0 index is 6 and 1 1 1 index is 7. So, by overall function is m 1 plus m 6 plus m 7 ok, in that sum of product form. So, this is see these are standard notation where put a sigma, and within bracket we give the terms or you can write it explicitly. So, it is X bar Y bar Z plus X Y Z bar X Y Z bar plus xyz so, this way also you can write it.

So, this is the from truth table to minterm to sum of product expression.

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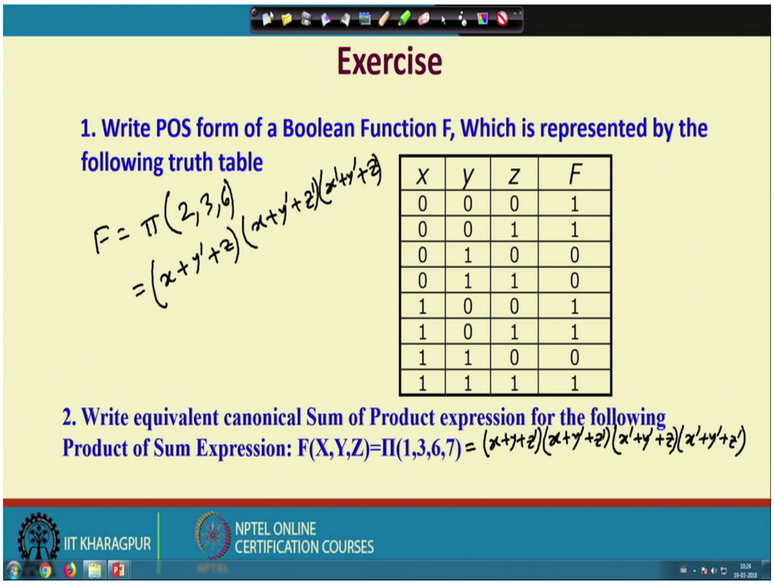

So, POS form for this one. So, the so, so if you if you want to write the POS form, then we have to concentrate on the 0’s. So, this is basically the where the function value is becoming 0, I have to take those functions. So, those terms so, this is ah. So, this is 0 1 0. So, this is 0 1 0. So, I have got so, for 0 1 0 the value is one so, I have to the function f. So, the function f in this case is function f in this case is the product is the product of sum form of this one.

So, this is 0 1 0; that is, value is 2, then we have got 0 1 1 that is 3, then we have got 1 1 0; that is 6 2 3 6. So, which essentially means that the function when I am writing it. So, for 0 while I will write in true form. So, X plus Y bar plus Z into 3. So, the 3 is X plus Y bar plus Z bar into 6. So, this is X bar Y bar or z.

So, this is the sum of product this is this is the product of sum expression. So, this is represented by max term like this. And then we can we can representative in pi from, and then we can write down this thing. So, similarly the second problem that it says that write down the sum of product of sum expression for this. So, we can understand what we can do this here. So, this is was one is 0 0 1 so, I have to write like X plus Y plus Z bar into the 3. So, 3 is 0 1 1 so, X plus Y bar plus Z bar into 6 that is 1 1 0. So, X bar plus Y bar plus Z into 7. So, that is 1 1 1 X bar plus Y bar plus Z bar,.

So, see apparently it seems that it will how will it give the function for different forms. So, this is this can be obtained like say this product of sum so, if you expand it. So, you will get the vary the if you convert it into sum of product form, then you will get the original terms, the for where the values are values of the variable is 1.

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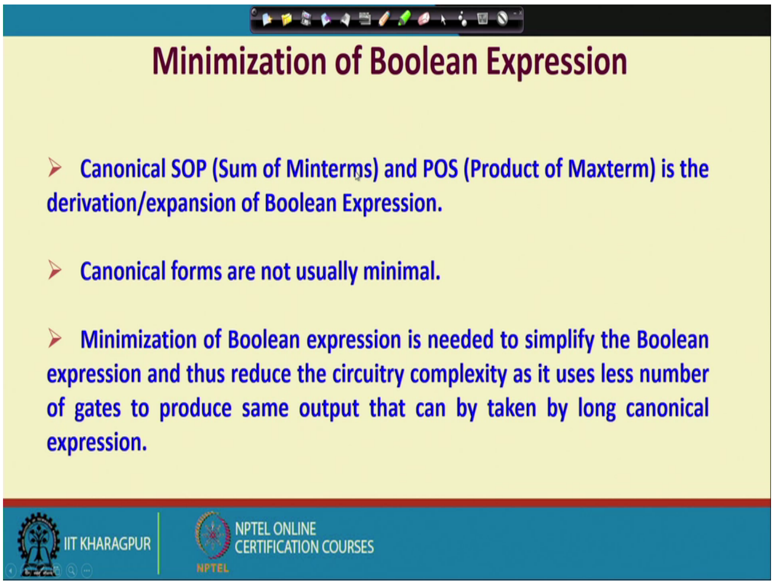

Next will see how to minimize this Boolean expression. So, canonical sum of product or canonical product of sum is the derivation or expansion of Boolean function. So, it essentially we have got less number of very literally (Refer Time: 24:52) term, and that we have expanding to have more number of literals and that converts into canonical form. But apart from the application in which you need to establish equivalence between Boolean expression. So, this is not much in not much useful because while realizing the function. So, what will look for is the minimum size realization in terms of less number of gates or less number of connections like that.

So, canonical forms are usually not minimal. So, they are may be cases where the canonical form itself is a minimal form, but it is a in general that is not the case. So, minimization of this Boolean expression so you needed to it is required to simplify the Boolean expression, and thus reduce the circuit complexity so, that I can use lees number of gates logic gates to produce the same output, that can be obtained by taking the canonical expression.

So, this minimization of Boolean function is by itself a very important topic. And we will see some technics for that, but keeping in mind that this a minimization process is not very simple. So, in our course so, will be consists considering Boolean expression with less number of variables only 4 variable 5 variable, at most 6 variable. And we will see that they are exist exact method for getting the exact minimum form corresponding to those function.

However as the those methods that will discuss. So, they are not applicable for number of variables say more than 6,. So, if you are looking for larger number of variables then exact method do exist, but the problem is that they take enormous amount of time, even if you think of writing a computer program for those minimization process, it will take enormous amount of time.

So, keeping that in mind so, it is so many a many a time what we do is we do some heuristic minimization and be satisfied with that any way so, that is not the purview of this course. So, will be restricting our self to Boolean expression of less number of variables up to 6 variable, and will see some technic by which we can do the exact minimization.