

Fundamentals of Electrical Engineering
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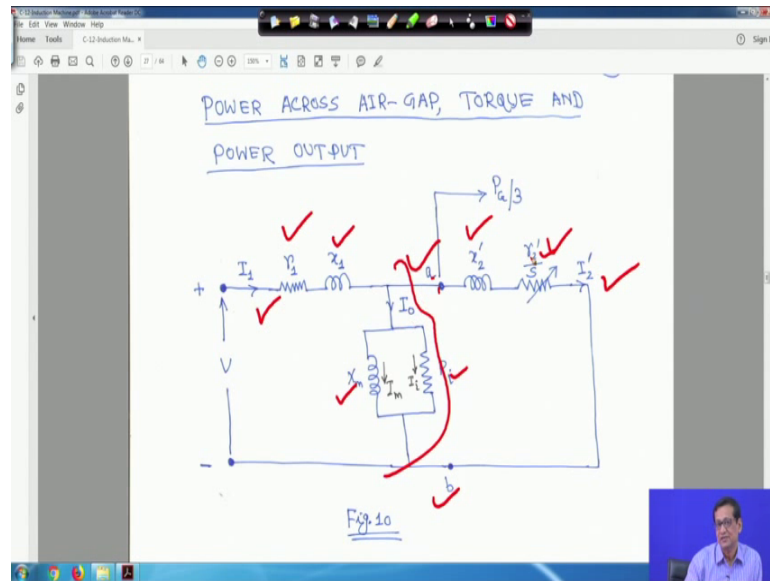
Lecture – 60
Three Phase Induction Motors (Contd.)

So, again next is earlier we have seen that your, what you call that the development of circuit model, now you have to see the power across air gap torque and power output. So, before that that regarding principle of operation of induction machine we have discussed that, when the rotor is short circuited it is a this; thing it is a we assume that rotor is short circuited itself, then it tends to move right in the direction of the stator magnetic field right.

But it will never achieve n is equal to n_s , then torque will be actually 0 right. And that interaction between what you call that field flux for pole p and your what you call that route they are stationary with respect to each other and the torque will be developed in the direction of your f_r if you see the diagram right. And that finally, your rotor rotates in the direction of the stator magnetic field with that is with speed n , but n less than n_s , it cannot be at the, what you call and it reaches a what you call a steady speed.

So, now, after that you have seen the development of circuit model, actually things are not difficult just a little bit of you know our understanding is required little bit and that is all.

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So, now power across air gap torque and power output actually why we call power across air gap, that you have a stator, then you have a rotor in between stator and rotor there is a gap right and power actually input power if you think to the rotor, that is the that is the input power to the stator right minus the stator loss, that power will be actually transferred through the air gap that is where you have that magnetic field right.

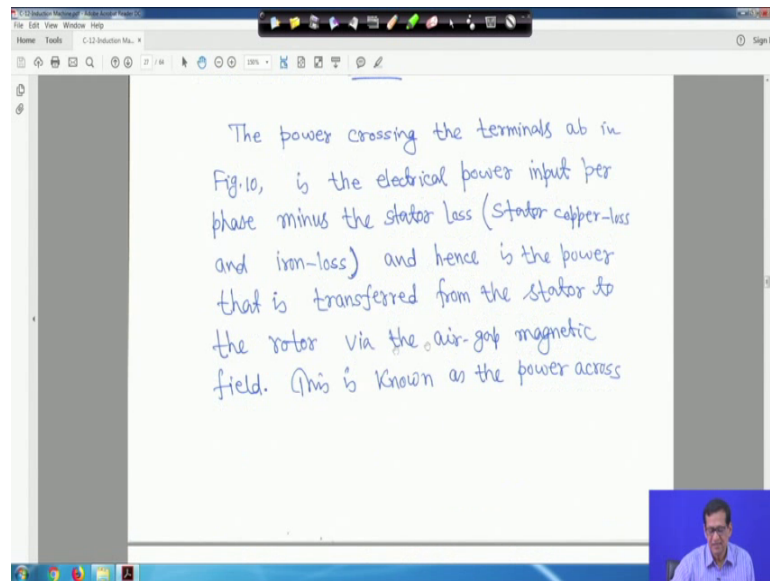
So, and that will be the input to the your what you call to the rotor. So, that is why we call power across air gap; that means, power actually is what you call transferred right through the air gap the where you have the magnetic field right. So, that is why you call power across air gap. So, there is no such confusion of this terminology that power across air gap right. So, if you look into this diagram right, we will come to the nomenclature and other thing.

If you look into the diagram, this one point is here a and b right and this side this part your what you call, and this is your stator resistance and reactance, this is your core loss component and this is the magnetizing component and this is a and this is b; when you make P G by 3. So, it is your power phase power we will come what is P a, what is P G right. And this side is the rotor side, this is reactance and this is your r_2 dash by S that your that because that your this part, this part is a variable part it depends on the slip right. So, r_2 dash by S; now this is your actually whatever power crosses that a b

whatever we are (Refer Time: 03:11) this is the power actually input to the your rotor side.

So, and this is your and this part that this is the current I_1 this is current I_2 dash and this is I_0 ; quite similar to the transformer circuit right only here it is r_2 dash by S because it is a rotating device right.

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So, in this case you are now this is the circuit; now the power crossing the terminals a b this is the terminal I told you look at the mark right in figure 10 is the electrical power input per phase minus the stator loss right that is stator copper loss and iron loss right. So, this is here it is $I_i^2 R_i$ is that your iron loss and here $I_1^2 r_1$ is the stator copper loss and hence is the power that is transferred from the stator to the rotor via the air gap magnetic field this is known as the power across the air gap.

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the air-gap and abs- three-phase is symbolized as P_g . (28)

From Fig.10,

$$P_g = 3(I_2')^2 \left(\frac{r_2'}{s} \right)$$

$$\therefore P_g = \frac{3(I_2')^2 r_2'}{s} \dots (18)$$

And it is its 3 phase is symbolized as P G right. So, therefore, from figure 10 P G will be 3 I 2 square I 2 dash square into r 2 dash by S. If you look into the diagram this is the P G 3 phase power; P G by 3 means power phase. So, 3 phase power will be 3 into your I 2 dash square into r 2 dash by S, I mean this side this side right.

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$$\therefore P_g = \frac{3(I_2')^2 r_2'}{s} \dots (18)$$

\therefore Power across air-gap = $\frac{\text{rotor copper-loss}}{\text{slip}}$
 $(P_g) \dots (19)$

OR Rotor copper-loss = $(\text{slip}) (P_g) = s P_g$

$$\therefore P_{cp} = s P_g = 3(I_2')^2 r_2' \dots (20)$$

Mechanical power output (gross),

$$P_m = P_g - 3(I_2')^2 r_2' = 3(I_2')^2 r_2' \left(\frac{1}{s} - 1 \right)$$

So; that means, this means that is a power that P G is actually P G is the power across the air gap, it is written here P G is the power across the air gap is equal to this 3 I 2 dash square into r 2 dash this is basically rotor copper loss right so; that means, this is rotor

copper loss divided by slip right there for or rotor copper loss is equal to slip into P G right.

Rotor copper loss is equal to slip into P G the slip is S. So, is equal to S P G right the or the rotor copper actually rotor copper loss we are representing by P c r right is equal to S into P G and that S into P G is thing, but if your cross multiply S into P G is equal to 3 I 2 dash square into I 2 dash. Therefore, here you can write p rotor copper loss S P G is equal to 3 I 2 dash square into r 2 dash this is equation 20.

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OR Rotor copper-loss = (slip)(P_g) = s P_g

∴ P_{cr} = s P_g = 3(I₂')² r₂' --- (20)

Mechanical power output (P_m),

$$P_m = P_g - 3(I_2')^2 r_2' = \frac{3(I_2')^2 r_2'}{s} - 3(I_2')^2 r_2'$$

∴ P_m = $\frac{3(I_2')^2 r_2'}{s} (1-s) = (1-s) P_g$ --- (21)

Now, mechanical power output that is the gross power right that P m is equal to P G that is your power across the air gap minus this copper loss rotor copper loss that will be mechanical power output. So, it will be P m is equal to P G minus 3 I 2 dash square into r 2 dash right, but P G is equal to from these here, P G is equal to 3 I 2 dash square r 2 dash by S, here it is this P G you substitute here this P G you substitute here. If you do so, and take your 3 I 2 dash square r 2 dash upon is common, it will be 1 minus S and this part is P G, because P G is equal to P G is equal to 3 I 2 dash square r 2 dash upon S. So, it is this part is P G. So, is equal to 1 minus S into P G, this is equation 21 right.

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(23)

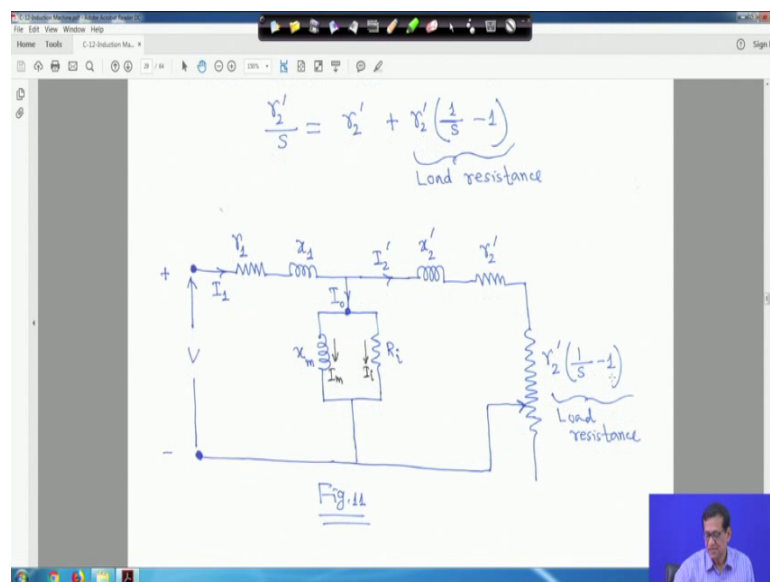
This means that the gross mechanical power output is three times (3-phase) the electrical power absorbed in resistance $r_2' \left(\frac{1}{s} - 1\right)$.

Fig.10 can therefore be drawn as in Fig.11 where $\frac{r_2'}{s}$ is represented as

$$\frac{r_2'}{s} = r_2' + \underbrace{r_2' \left(\frac{1}{s} - 1\right)}_{\text{Load resistance}}$$

Now, this means that the gross mechanical power output in these 3 times that is 3 phase actually the electrical power absorbed in the resistance, r_2' into $\frac{1}{s} - 1$. So, this is actually what you call that is that your gross mechanical power output.

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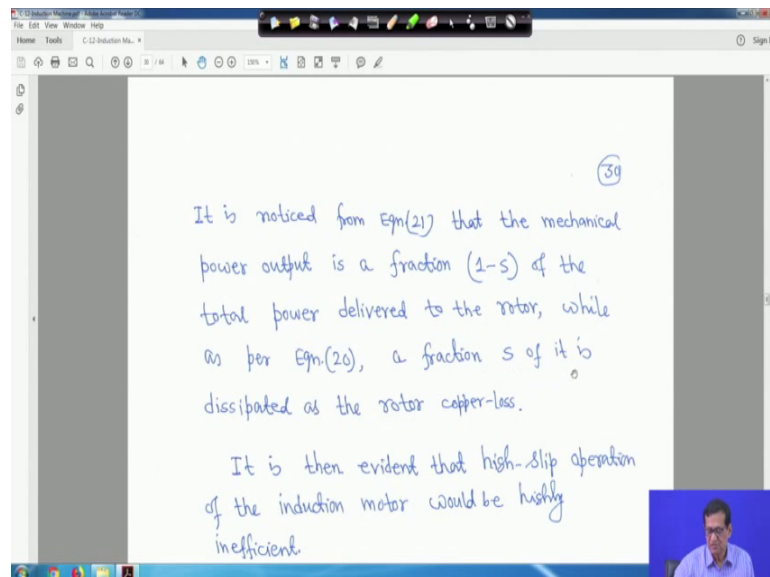


We will see that from figure 10 can therefore, be drawn as in figure 11 where r_2' is represented as, this r_2' upon s you can write r_2' plus r_2' into one upon s minus 1; that means, you add r_2' subtract r_2' . So, this part will be the load

resistance right. Therefore, if we put that this is my r_2 dash this is my r_2 dash rotor resistance.

And this other thing rest of the thing r_2 dash into 1 upon S minus 1 actually it is load resistance here right. So, I mean nothing this from this r_2 dash by S only from this circuit, that your r_2 dash by S only written as r_2 dash plus your r_2 dash into 1 upon S minus 1 . So, this way you can mathematically you can represent like this. So, this is my this is our load resistance therefore, this is it is marked also load resistance right this in figure 11.

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Now, it is noticed from equation 21, that the mechanical power output is a factor of 1 minus S of the total power delivered to the rotor, we have seen P_m is equal to 1 minus S P_G .

Here P_m is equal to equation 21 1 minus S P_G , P_G is the air gap power and p_v is the mechanical power output that is 1 minus S P_G . So, that that thing I am just writing in language right of that of while as per equation 20 a fraction of S of it is dissipated in the rotor copper loss right. Even if you come to equation 20, your equation 20 this is equation 20. So, copper loss is equal to S into P_G where fraction of air and gap power will be the copper loss and this is P_m will be 1 minus rest will be 1 minus S into your what we call your P_G right. So, basically your, what you call this your it will be P_G minus S P_G . So, basically it will be P_G minus the rotor copper loss right.

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from Eqn(5)

$$s = \left(1 - \frac{n}{n_s}\right) = 1 - \frac{\omega}{\omega_s}$$

$$\therefore \omega = (1-s)\omega_s \text{ rad(mech)/sec. --(22)}$$

Electromagnetic torque developed is then given by

$$\omega T = (1-s)\omega_s T = P_m = (1-s)P_g$$

$$\therefore T = \frac{P_g}{\omega_s} = \frac{3V^2 \cdot (1/s)}{\omega_s} \text{ Nm} \quad \text{--- (1-s)}$$

$(1-s)\omega_s T = (P_g/s) \text{ Nm}$

So, it is there, it is then evident that high slip operation of induction motor would be highly inefficient and if you operate at a very high slip. So, it will be your what you call that inefficient in the sense first thing is your S into P G, if S is high that is copper loss S is high then copper loss will be more right. So, from equation 5, we know S is equal to 1 minus n upon n s. So, same thing you can write 1 minus omega by omega S also you multiply numerator and denominator by 2 pi. So, it will be 2 pi n is omega and 2 pi n S is omega S the synchronous speed. Therefore, omega is equal to you can write 1 minus S omega that is radian mechanical radian per second right this is equation 22.

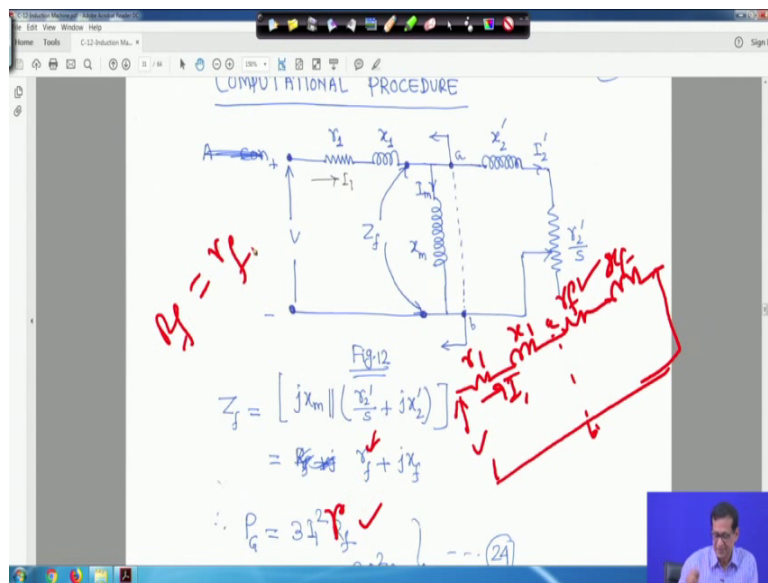
Now electromagnetic torque you know power is equal to torque into angular speed, this relationship you know. So, electromagnetic torque is developed and given by if it if it is omega is equal to 1 minus S omega S both side you multiply by t right; that means, omega T will be 1 minus S omega S T if you go back to that the that schematic diagram on the stator rotor where f 1 f 2 f r and torque is torque direction of torque everything is pointed out I am not just going back again right just look into that here it is given. So, omega T is equal to 1 minus S omega S divided by both side by T and you and you know that this is actually is equal to P m because P m is equal to omega into T. So, and also P m is equal to you know omega 1 minus S P G right.

Therefore, therefore, your this torque is equal to that this one is equal to say 1 minus S is equal to this one right; that means, 1 minus S your omega S T is equal to 1 minus S your

P G. So, $1 - S$ will be cancelled. So, $\omega_s T$ is equal to P G therefore, T is equal to P G upon ω_s right. So, let me clear it so; that means, in my P G is equal to you know $3 I_2^2$ dash square into r_2 dash upon S by ω_s Newton metre this is equation 23 right.

Now next is a computational procedure. So, these are the certain relationship you have to keep it in your mind. If you have understood this thing, then there is no need to keep it in mind easily you can make it right now computational procedure.

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So, in this case what we have done is that, that core loss component is removed right and this is your what you call this is the stator side, this is the magnetizing part and this is x_2 dash this is r_2 dash by S right. So, if you if you just look into the circuit. So, despite this a b part is given this power across air gap P G whatever I have explained right a b is given. So, if you try to find out what is my; this thing Z f.

So, this is actually it is given Z f right because this is actually series parallel circuit. So, this x_2 dash into r_2 dash upon S is, r_2 dash upon S plus the x_2 dash right it is parallel with $j x_m$.

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Fig. 12

$$Z_f = \left[jX_m \parallel \left(\frac{R_2}{s} + jX_2 \right) \right]$$

$$= R_f + jX_f$$

$$\therefore P_G = 3I_1^2 R_f$$

and $T = \frac{P_G}{\omega_s} = \frac{3I_1^2 R_f}{\omega_s}$ (24)

So, it is $j \times m$ parallel to R_2 dash upon S plus $j \times 2$ dash. If you make it equivalent say if you make R_f plus $j \times f$ in this form you can separate real after making this computation you can separate the real and imaginary part. I am not doing it I request you please do this right. So, this you can make it already you have studied that single phase ac circuit same thing. So, P_G is equal to now this is R_f plus $j \times f$.

So, P_G is equal to your $3 I_1^2$ square into R_f because when you make this your, what you call when you make this equivalent of this one right. So, power across the air gap will be $3 I_1^2$ square R_f because at that time I mean if I make it here if I make it here. So, at that what will happen? You make an equivalent circuit, this is my r_1 , this is my x_1 and this is my r_f and this is my x_f right. So, and this is your circuit is closed say circuit is closed and this is the current is voltage this is the voltage v and this is the current flowing I_1 right. So, this is my r_1 this is my x_1 this is my r_f and this is my your x_f right. So, j I am not putting. so, $j \times 1$, $j \times x$, $j \times f$ it is understandable.

So; that means, your what you call that P_G will be this point this point actually this point is a and this point will be b right. So, P_G actually the same I_1 current now flowing after making this your what to call you equivalent one; so, same I_1 . So, P_G will be $3 I_1^2$ square R_f right because this is my r_f here your here it is here it is small r_f I have written. So, better you make it small r_f right. So, I mean there should not be any confusion r_f is equal to smaller r_f right. So, this is the small r_f , $3 I_1^2$ square r_f . So, let

me clear it right. So, ; that means, and torque is equal to P G upon omega S. So, you can write 3 I 1 square R f by omega S look here it is you know here it is here it is small r f here it is small r f. So, that should not be any confusion right. So, this is equation 24.

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TORQUE-SLIP CHARACTERISTIC

The expression for torque-slip characteristic is easily obtained by finding the Thevenin equivalent of the circuit to the left of ab in Fig.12, o

$$Z_{TH} = (r_1 + jx_1) \parallel (jx_m) = r_{th} + jx_{th}$$

$$\therefore V_{TH} = V \left[\frac{jx_m}{r_1 + j(x_1 + x_m)} \right] \dots (25)$$

So; that means, the next is the torque slip characteristics. So, in this case what we do, the expression for the torque slip characteristic is easily you can obtain by finding Thevenin equivalent of the circuit to the left of the a b in figure 12. So, this is your figure 12, this is our figure 12. So, if you try to find out the your what you call Thevenin voltage and your what you call Thevenin equivalent impedance, as we the this side as if we have studied in your say the maximum power transfer theorem or your Thevenin's theorem we have studied know, maximum power will come later.

So, Thevenin's theorem we have studied. So, that here what you call the branch which are interested in to find the current that is taken out after that we computed Thevenin voltage and Thevenin impedance right or Thevenin for d c circuit Thevenin resistance right; same way we will do it. So, left side of this; so, left side of this circuit we will see this that is why arrow is marked like this left side of this one right. So, if you if you do so, look at the circuit again and again I am not going to the circuit; when you listen to these you please draw the simple circuit and the way it has been drawn here and accordingly what you do that your this thing that find out j Thevenin literally r one plus j x 1 parallel to j x m.

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$$Z_{TH} = (r_1 + jx_2) \parallel (jx_m) = r_{th} + jx_{th}$$

$$\therefore V_{TH} = V \left[\frac{jx_m}{r_1 + j(x_1 + x_m)} \right] \dots (25)$$

The circuit then reduces to Fig.13 in which it is convenient to take V_{TH} as the reference voltage.

The diagram shows a circuit with a voltage source V_{TH} on the left. The circuit consists of a series combination of r_{th} and x_{th} in the top wire. This is followed by a node 'a'. From node 'a', the circuit splits into two parallel branches: one with x_2' and another with $x_2' / (s-1)$. Both branches recombine at the bottom wire.

So, its equivalent is r Thevenin plus j x Thevenin right. Once you get r Thevenin plus j x Thevenin right then what you do that you compute your what you call, you compute the current. So, V Thevenin will be v into j x m upon r 1 plus j x 1e plus x m.

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COMPUTATIONAL PROCEDURE

The diagram shows a circuit with a voltage source V on the left. The circuit consists of a series combination of r_1 and x_1 in the top wire. This is followed by a node 'a'. From node 'a', the circuit splits into two parallel branches: one with x_2' and another with $x_2' / (s-1)$. Both branches recombine at the bottom wire.

$$Z_f = [jx_m \parallel (\frac{x_2'}{s-1} + jx_2)]$$

$$= r_f + jx_f$$

$$\therefore P_g = 3I_f^2 R_f \dots (24)$$

Handwritten equations in red:

$$i = \frac{V}{r_1 + j(x_1 + x_m)}$$

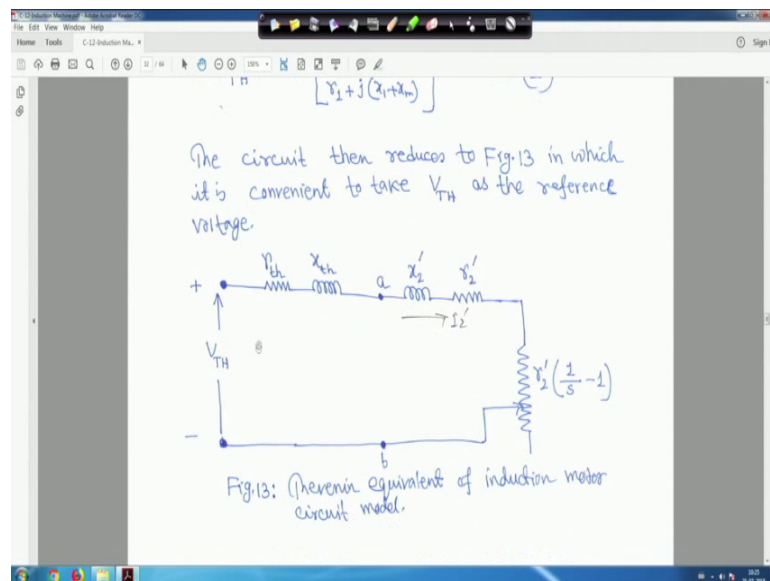
$$V_{TH} = \frac{jx_m \times V}{r_1 + j(x_1 + x_m)}$$

So, in this case what happened that as soon as soon as you I mean it is something like this, this side we are interested. So, forget about this one as this side is not there because we have to calculate the V Thevenin means, the voltage across your what to call your v a b right what will be the what will be the voltage here right V a b is equal to b Thevenin.

So, in that case if this is not there. So, it is not there say current is flowing say this is my I right. Therefore, my I is equal to forget about other thing this is part not there, this is not there one nothing is there. So, I is equal to actually v upon your r 1 plus j x 1 plus x m this is my I; therefore, b Thevenin.

Where v a b is equal to is equal to this is my j x m forget about this one nothing is there this part is not there only I am considering these series part right. So, my V Thevenin will be is equal to this is x m. So, this is the reactance x m into this I so; that means, into v divided by r 1 plus j x 1 plus x m right that is why V Thevenin right this is why V Thevenin. So, same thing same thing I am writing here this V Thevenin is equal to v j x m upon r one plus j x 1 plus x m now I showed you this is equation 25. The circuit then reduces to figure 13 in which it is convenient to taken V Thevenin as the reference voltage.

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Now, it is that Thevenin equivalent. This is my V Thevenin this is my r Thevenin and x Thevenin and this r 2 dash this is r 2 dash by S actually this is your r 2 dash by S this is r 2 dash by S this r 2 dash by S we are writing like this same circuit r 2 dash plus r 2 dash 1 upon S minus 1 this is my load resistance part right and this is shown a variable as this S can change because slip can change. So, this is r Thevenin x Thevenin and this is the point a b and then this is connected basically it is x 2 dash r 2 dash by S, but r 2 dash by

So we have written like this right. So, so this is my I_2' the current and this is my voltage V_{TH} understandable right.

So, now, from figure 13 if you write your I_2' is equal to; then you can write your r_{TH} plus r_2' by S plus $j x_{TH}$ plus x_2' right and if you do it and take the magnitude and of the current and then the square then you will get $I_2'^2$ square is equal to V_{TH}^2 .

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From Fig. 13

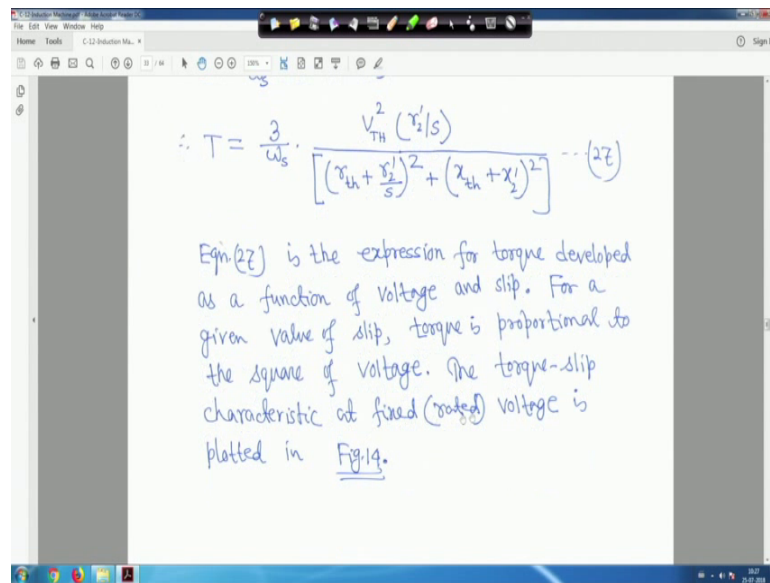
$$(I_2')^2 = \frac{V_{TH}^2}{\left(r_{th} + \frac{r_2'}{s}\right)^2 + \left(x_{th} + x_2'\right)^2} \quad \text{---(26)}$$

$$T = \frac{3}{\omega_s} \cdot (I_2')^2 \left(\frac{r_2'}{s}\right)$$

$$\therefore T = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 \left(\frac{r_2'}{s}\right)}{\left[\left(r_{th} + \frac{r_2'}{s}\right)^2 + \left(x_{th} + x_2'\right)^2\right]} \quad \text{---(27)}$$

Already we have studied single phase circuit directly we are writing that magnitude of this current I_2' square is equal to V_{TH}^2 , r_{TH} plus r_2' upon S whole square plus x_{TH} plus x_2' upon whole square this equation 26.

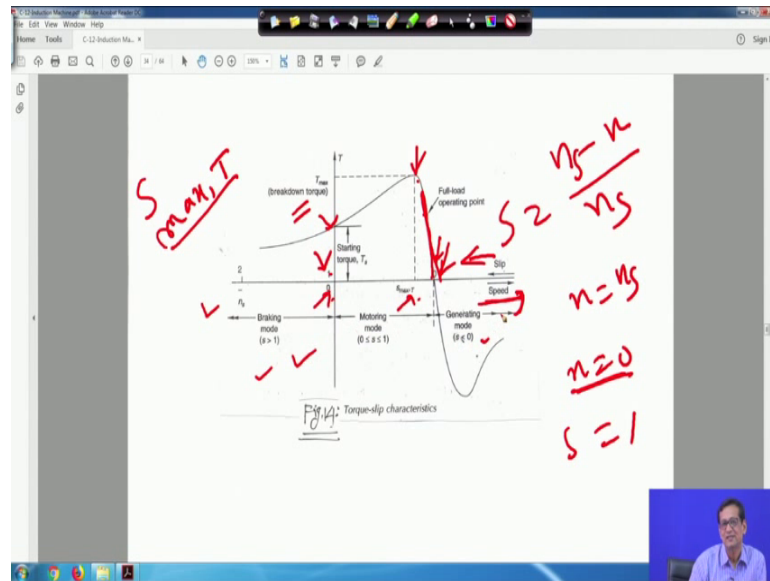
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$$\therefore T = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 (r_2'/s)}{[(r_{th} + \frac{r_2'}{s})^2 + (x_{th} + x_2')^2]} \quad \dots (27)$$

Eqn.(27) is the expression for torque developed as a function of voltage and slip. For a given value of slip, torque is proportional to the square of voltage. The torque-slip characteristic at fixed (rated) voltage is plotted in Fig.14.

Now, we know torque is equal to $3 \cdot \frac{P_G}{\omega_s}$ right sorry torque is equal to $\frac{P_G}{\omega_s}$. So, P_G is equal to $3 I^2 r_2' s \omega_s$. So, this is my $3 \omega_s$ then $I^2 r_2' s$ from this expression you substitute here you substitute here into your $r_2' s$. So, this is my torque expression this is equation 27 right. So, if you if you look into equation 27, is the expression for the torque developed as a function of voltage and slip right. For a given value of slip if slip is known, the torque is then proportional to the square of the, your voltage that is Thevenin voltage right. If slip is constant if you suppose slip is given, because all are the parameters will remain constant. So, the torque slip characteristic at fixed voltage is plotted in figure 14. So, actually we learn a little bit only about motoring part breaking part or generating part, but it is given, but we will not we are not interested into that.

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So, this is I have taken from a book this diagram, this part actually is a motoring part it is written this side breaking part and this is breaking mode and this is generating more and this is motoring mode.

So, we will not study generating mode, we will not study breaking mode also only this part. So, we know we know one thing we know one thing that slip is equal to your $n_s - n$ divided by n_s right. Look at this look at this diagram the speed is in this the speed is like this and slip is direction is this right. Now when n is equal to say your n is equal to n_s suppose when n is equal to n_s , then slip is equal to your what to call that slip is equal to 0. So, here it is here it is for 0 slip is equal to 0 and this side it is and when your n is equal to 0 that is the starting standstill condition at that time slip is equal to 1.

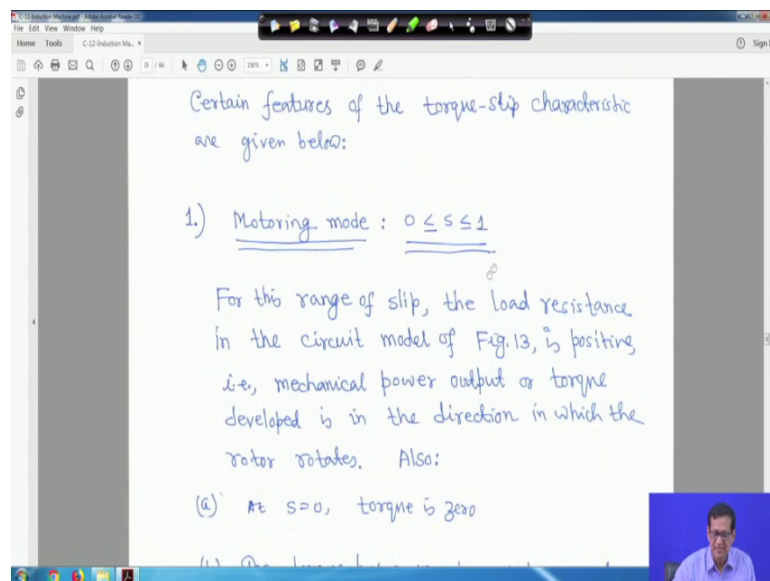
So, when n this is my n is equal to 0, this is my n is equal to 0 and this is the place that S is equal to one right. That is why speed is this direction and slip is your this direction if slip is moving from right to left and speed is your what you call left to right because speed is increasing speed is increasing means that your what you call this suppose if n is increasing. So, if you increase the speed. So, when n is equal to n_s . So, this is the part that slip is equal to 0. So, slip is this one and when n is equal to your, what you call your 0 that is stand still condition slip is one. So, it is one and here speed is 0 this side need not bother and this side breaking mode and this side is generating mode a little bit explanation only will be given and that is all right.

And at this point when slip is equal to 1 so, we will find whatever torque it is this is the starting torque T_s right and another thing is that that this is the this slip it is it is actually given it is right like we write like this is $S_{max} T$; I mean this is the value of slip for which that this torque is maximum this is called breakdown torque right. So, this is the value of slip for which the torque is maximum. So, we write the $S_{max} T$ the T_{max} means this is the value of T for which the it has maximum torque and this is called your breakdown torque and this is actually some point here full load operating point.

So, at the low value of slip you will see this region you will find something like a straight line design right. And you will find; that means, you will operate in between your what you call this point to this point the slip value right somewhere it is full load operating point. This is the this is your what you call the your what you call that torque slip characteristic if you draw right. So, and this side we will only give a because I have taken from a book. So, when I am showing it I think I have to show it for the sake of completeness I have to show this one, that is why little bit of right up is there, but I am not explaining that right because at this first year level there is no I mean no need.

When you will go to the second or third year you will learn much more than this one and everything in detail right. So, that is that is what I said sometimes this T_{max} we call break down sometimes we call it as TBD that maximum torque sometimes we call TBD right that breakdown torque this one the maximum torque right.

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Certain features of the torque-slip characteristic are given below:

- 1.) Motoring mode : $0 \leq s \leq 1$

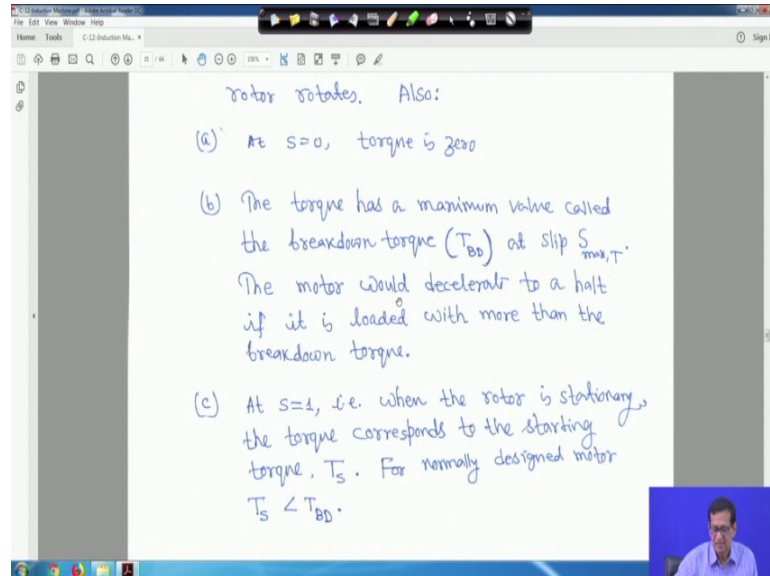
For this range of slip, the load resistance in the circuit model of Fig.13, is positive i.e., mechanical power output or torque developed is in the direction in which the rotor rotates. Also:

(a) At $s=0$, torque is zero

(b) At $s=1$, torque is zero

So, that is why some write up is here, the motoring mode and slip will lie in between 0 and one from this diagram you see slip will lie in between 0 and here it is 1 right.

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So, for this range of slip, the load resistance in the circuit model; figure 13 is positive; obviously, there is a mechanical power output that torque developed in the direction in which the rotor rotates right also at S is equal to 0 torque is your what you call 0 right. If you come here this is my S is equal to 0. So, at this point torque is 0 right because this y axis this side is torque and this the speed of the slip. So, when this is your torque is 0 at this point. So, that torque has a maximum torque I told you or the breakdown torque T_{BD} I told you, that slip $S_{max,T}$ the rotor motor would decelerate to a halt or if it is loaded with more than the breakdown torque right.

So, another thing is at S is equal to one that is your standstill condition that is when the rotor is stationary, that is standstill condition torque corresponds to the starting torque T_s I showed you in the diagram for normally design motor the starting torque will be less than your breakdown on maximum torque right.

(Refer Slide Time: 23:16)

(d) The normal operating point is located well below T_{BD} . The full-load slip is usually 2% - 8%.

(e) The torque-slip characteristic from no-load to somewhat beyond full-load is almost linear.

2) Generating Mode: $s < 0$

Negative slip implies rotor running at super-synchronous speed ($n > n_s$). The load

So, the normal operating point is located well below TBD that is the maximum torque the full load slip is usually 2 to 8 percent right. So, that that TBD is the maximum torque right and the torque slip characteristic for no load somewhat given full load is linear. So, I showed you the linear portion.

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2) Generating Mode: $s < 0$

Negative slip implies rotor running at super-synchronous speed ($n > n_s$). The load resistance is negative in the circuit model of Fig.13 which means that mechanical power must be put in while electrical power is put out at the machine terminals.

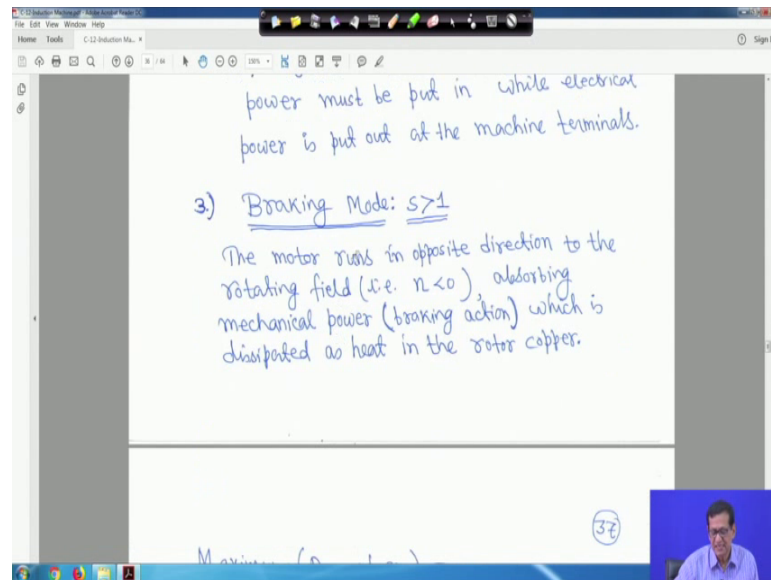
3) Braking Mode: $s > 1$

The motor runs in opposite direction to the rotating field (i.e. $n < 0$) absorbing

See another thing is the, this is actually the, this is just for the sake of compliment generating mode and slip is negative. So, negative slip implies rotor running at super synchronous speed n greater than n_s ; that means, motor is running as a generator right.

The load resistance is negative in the circuit model of figure 3, which means that the mechanical power must be put in while electrical power is put out at the machine terminals only this much more details. We will learn in your machine course for second or third year and braking mode when slip greater than one.

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The motor runs in opposite direction to the rotating field that is less than 0, negative means is running in opposite direction to the rotating field that is $n < 0$ negative means it is running in the opposite direction right absorbing mechanical power that is braking action which is dissipated as heat in the rotor copper right only.

This much for first year level nothing else.

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(27)

Maximum (Breakdown) Torque

From Eqn (27),

$$T = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 \cdot \left(\frac{r_2'}{s}\right)}{\left[r_{th} + \frac{r_2'}{s}\right]^2 + (x_{th} + x_2')^2}$$

For maximum torque, $\frac{dT}{ds} = 0$

Next is maximum or breakdown torque how to find out.

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(27)

For maximum torque, $\frac{dT}{ds} = 0$

$$s = s_{max,T}$$

$$\therefore s = s_{max,T} = \frac{r_2'}{\sqrt{r_{th}^2 + (x_{th} + x_2')^2}} \quad \text{--- (28)}$$

From Eqns (27) & (28)

$$T = T_{max} = \frac{3}{\omega_s} \cdot \frac{0.5 V_{TH}^2}{\left[r_{th} + \sqrt{r_{th}^2 + (x_{th} + x_2')^2}\right]} \quad \text{--- (29)}$$

From Eqn (29), it can be observed that the maximum torque is independent of the rotor resistance (r_2')

So, from equation 27 this is my equation for maximum torque d T upon d S is equal to 0 is equal for which we will get S is equal to S max T. If you set d T upon d S or d T upon d S is equal to 0 and simplify you will get S is equal to actually S max T is equal to r 2 dash and divided by under root r Thevenin square plus x Thevenin plus x 2 dash square right this is equation 20 eight right. Now if you substitute this is S is equal to your what

to call your S max T is equal to your this much in this expression if you substitute if you substitute here.

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From Eqns (27) & (28)

$$T = T_{max} = \frac{3}{\omega_s} \cdot \frac{0.5 V_{TH}^2}{\left[r_{TH} + \sqrt{r_{TH}^2 + (x_{TH} + x'_2)^2} \right]} \quad (29)$$

From Eqn (29), it can be observed that the maximum torque is independent of the rotor resistance (r'_2) while the slip (Eqn 28) at which it occurs is directly proportional to it.

And simplify you will get T is equal to T max is equal to 3 upon omega S into all this things right.

Ah I mean 0.5 V Thevenin square divided by r Thevenin plus root of r Thevenin square plus x Thevenin plus x 2 dash square whole square this is equation 29. So, equation 20 nine from equation 29, it can be observed that the maximum torque is independent of the rotor resistance right r 2 dash while the slip at which it occurs is directly proportional to it right. So, if you look it here; that means, the slip is directly proportional to r 2 dash right we and this is this and this part is independent of the rotor resistance r 2 dash right. So, this is equation 29 next is the starting torque at.

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STARTING TORQUE

At start $s=1$,

From Eqn. (27), we get

$$T = T_{\text{start}} = \frac{3}{\omega_b} \cdot \frac{V_{th}^2 r_2'}{[(r_{th} + r_2')^2 + (x_{th} + x_2')^2]} \quad \dots (30)$$

Starting torque increases by adding resistance in the rotor circuit.

From Eqn. (28), the maximum starting torque

Start slip is equal to one right therefore, from the equation 27 you put S is equal to 1, then you will get this is expression for the starting torque. Starting torque increases your what you call by adding resistance in the rotor circuit. If you add resistance rotor circuit then this is actually r_2 dash is here. So, gradually it can be increased.

(Refer Slide Time: 26:05)

Starting torque increases by adding resistance in the rotor circuit.

From Eqn. (28), the maximum starting torque is achieved for $(S_{\text{max},T} = 1)$, i.e.,

$$r_2' = \sqrt{r_{th}^2 + (x_{th} + x_2')^2} \quad \dots (31)$$

From Eqn. (26)

$$I_2' = \frac{V_{th}}{\sqrt{(r_{th} + \frac{r_2'}{s})^2 + (x_{th} + x_2')^2}} \quad \dots (32)$$

At start $s=1$,

From equation 28 the maximum starting torque is achieved for $S_{\text{max},T}$ is equal to 1 that is r_2 dash will be is equal to root over r Thevenin square plus x Thevenin plus x_2 dash square. The equation for the maximum starting torque is achieved, that is from equation

28 if you come to that if you come to that the maximum starting torque can be achieved right if you set S is equal to 1 right.

Because this is the slip expression for maximum we got. So, that starts V Thevenin is equal to 1; if you put S is equal to 1, then your what you call, then your starting torque is achieved for r_2 dash is equal to root over r Thevenin square plus x Thevenin plus x 2 dash square right that is your from equation 28 this is my equation 20, this is my equation 28 right.

(Refer Slide Time: 26:58)

is achieved for $(S_{max, T} = 1)$, i.e.,

$$r_2' = \sqrt{r_{th}^2 + (x_{th} + x_2')^2} \quad \dots (31)$$

From Eqn. (26)

$$I_2' = \frac{V_{TH}}{\sqrt{(r_{th} + \frac{r_2'}{s})^2 + (x_{th} + x_2')^2}} \quad \dots (32)$$

At start $s=1$,

$$\therefore I_{2start}' = \frac{V_{TH}}{\sqrt{(r_{th} + r_2')^2 + (x_{th} + x_2')^2}} \quad \dots (33)$$

And from equation 26, then I_2 dash will be V Thevenin upon root over this expression this we know right already we know. So, at start S is equal to 1 when slip is equal to 1. So, I_2 dash starting the starting current will be V Thevenin up here you put S is equal to one here we put S is equal to 1. So, this is the expression I mean everything is straight forward just you have to keep it one or 2 things in mind and rest is solving like a solving like your 3 phase circuit right a sorry single phase circuit right.

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(39)

From Eqn (33), it is clear that starting current will reduce.

This indeed is the advantage of the slip-ring induction motor in which a high starting torque is obtained at low starting current.

An Approximation

Sometimes for getting a feel (rough answer) of the operational characteristic, it is

Now, for equation 33 it is clear that starting current will reduce right. So, next is this indeed is the advantage of the slip ring induction motor, in which high starting torque is obtained at low starting current. Now an approximation now we will go for approximation.

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Sometimes for getting a feel (rough answer) of the operational characteristic, it is convenient to assume the stator impedance to be negligible, i.e.,

$$r_{th} = 0, \quad x_{th} = 0 \quad [\text{Fig. 13}]$$

Therefore, $V_{Th} = V$ [See Fig. 12, Fig. 13 and Eqn. (25)]

Substituting $r_{th} = 0, \quad x_{th} = 0$ in Eqn. (32)

$$I_2' = \frac{V}{\sqrt{\left(\frac{r_2}{s}\right)^2 + (x_2')^2}} \quad \dots \quad (34)$$

Now, sometimes for getting a feel that is a rough answer rough answer at the, of the operational characteristic it is convenient to assume that the stator impedance to be negligible that is r_{Th} is equal to 0, x_{Th} is equal to 0 if you neglect that.

That is figure 13 you go back figure 13, I am not going. In that case what will happen you will find $V_{Thevenin}$ is equal to V . Here I have written see figure 12, figure 13 and equation 25. Please go back to that just with this with this assumption you put it there right and you substituting $r_{Thevenin}$ and $x_{Thevenin}$ and in equation 32, you will get I_2 is equal to V upon root over r_2 dash upon S square plus x_2 dash square this is equation 34 right.

(Refer Slide Time: 28:28)

Substituting $r_{Th}=0$, $x_{Th}=0$ and $V_{Th}=V$ in Eqn.(27), we get

$$T = \frac{3}{\omega_s} \cdot \frac{V^2 \left(\frac{r_2'}{s}\right)}{\left[\left(\frac{r_2'}{s}\right)^2 + (x_2')^2\right]} \quad \dots (35)$$

Also,

$$s = s_{maxT} = \frac{r_2'}{x_2'} = \frac{\text{rotor resistance}}{\text{standstill rotor reactance}} \quad \dots (36)$$

$$\therefore T_{max} = \frac{3}{\omega_s} \cdot \left[\frac{0.5V^2}{\dots} \right] \quad \dots (37)$$

So, substituting $r_{Thevenin}$ and $x_{Thevenin}$ and $V_{Thevenin}$ is equal to V in equation 27 you put, it you will get torque is equal to 3 upon ω_s into V square into r_2 dash upon S divided by these expression right. So, this is equation 35.

(Refer Slide Time: 28:44)

$$s = s_{max} = \frac{r_2'}{x_2'} = \frac{\text{rotor resistance}}{\text{standstill rotor reactance}} \quad \dots (36)$$

$$\therefore T_{max} = \frac{3}{\omega_s} \cdot \left[\frac{0.5 V^2}{x_2'} \right] \quad \dots (37)$$

$$T_{start} = \frac{3}{\omega_s} \cdot \frac{V^2 r_2'}{[r_2']^2 + (x_2')^2} \quad \dots (38)$$

Maximum starting torque ($s_{max} = 1$) is achieved under the condition,

$$r_2' = x_2'$$

Now, also s is equal to s_{max} in that case if you do so, then s is equal to it will become r_2' over x_2' . If you mean based in this assumption you substitute those your s_{max} in your T_{max} expression, you put r_2' Thevenin voltage and s_{max} Thevenin voltage you will get s is equal to s_{max} T_{max} is equal to r_2' over x_2' . So, this will be required for sometimes for solving numerical. Therefore, rotor resistance by stand still rotor reactor and T_{max} if you put in that expression, you will get 3 upon ω_s into $0.5 V^2$ upon x_2' this is equation 37.

And similarly in that expression of the starting torque if you put the same thing, only r_2' Thevenin voltage and s_{max} Thevenin voltage is equal to 1 you will get starting torque is equal to 3 upon ω_s onto $V^2 r_2'$ upon this one this is equation 38. This is first we developed and then we make some simplified assumptions right.

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$$\max T_{start} = \frac{3}{s} \left[\frac{V^2 r_2'}{r_2'^2 + (x_2')^2} \right] \dots (37)$$

$$T_{start} = \frac{3}{s} \left[\frac{V^2 r_2'}{r_2'^2 + (x_2')^2} \right] \dots (38)$$

Maximum starting torque ($s_{start} = 1$) is achieved under the condition,

$$r_2' = x_2'$$

and

$$T_{start(min)} = T_{max} = \frac{3}{s} \left[\frac{0.5 V^2}{x_2'} \right] \dots (39)$$

So, maximum starting torque $S_{max} T$ is equal to 1 is achieved under the condition r_2 in this case r_2 dash is equal to x_2 dash and I mean if you assume S is equal to 1 at starting then it will be r_2 dash is equal to S_2 dash right. So, and T starting maximum if you put in that expression T_{max} will be 3 upon ω_s $0.5 V$ square upon x_2 dash. So, these are all simple things just put one upon another and you will get this expression right.

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Some Approximate Relationships at Low slip

Around the rated (full-load) speed, slip of the induction motor is very small such that

$$\frac{r_2'}{s} \gg x_2'$$

So that x_2' can be altogether neglected in a simplified analysis Eqn(34) then simplify to

$$I_2' = \frac{sV}{r_2'} \dots (40)$$

So, some approximate relationship at low slip; so, around the rated full load slip of the induction motor is very small right such that are 2 dash upon S much greater than x_2

dash. So, in this so, that x_2 dash can be altogether neglected. So, in a simplified analysis; so, therefore, equation 34, then simplified to I_2 dash will be $S v$ upon r_2 dash I mean we are see neglecting x_2 dash. Because we are assuming r_2 dash by S is much greater than your x_2 dash, further is simplification and then therefore, I am not going to this equation again and again, just you write and put it you will get it right. And similarly equation 35, then simplifies to based on this neglecting I will put x_2 dash is equal to 0 and that is all right.

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$\frac{1}{s} \gg x_2'$

So that x_2' can be altogether neglected in a simplified analysis. Eqn.(34) then simplify to

$$I_2' = \frac{SV}{x_2'} \quad \dots \dots (40)$$

and Eqn.(35) then simplify to,

$$T = \frac{3}{\omega_s} \cdot \frac{SV^2}{x_2'} \quad \dots \dots (41)$$

From Eqn.(41), it can be observed that the torque-slip relationship is nearly linear in the region of low slip.

So, then T is equal to 3 upon ω_s , you will get $S v$ square upon r_2 dash this is equation 41. So, equation 41 it can be observed that the torque slip relationship is nearly linear in the region of low slip and when slip is very small then torque slip relationship is quite your linear right.

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Maximum Power Output

Since the speed of the induction motor reduces with load, the maximum mechanical power output does not correspond to the speed (slip) at which maximum torque is developed.

For maximum mechanical power output, from Fig. 13, condition is

$$r_2' \left(\frac{1}{s} - 1 \right) = \sqrt{(r_{th} + r_2')^2 + (x_{th} + x_2')^2}$$

So, will see this that maximum power output; now all though maximum power will see what is not varying for induction machine right. We will see this see the speed of the induction motor reduces with load the maximum mechanical power output does not correspond to the speed that is the slip at which maximum torque is developed. For maximum mechanical power output from figure, the conditional will be we have seen know that your what you call in the maximum power transfer theorem say in the d c circuit we have seen, in a c circuit we have seen; in that case the load resistance; this is load resistance the r_2' is equal to $1/s - 1$ it has to be equal to your r_{th} plus r_2' square plus x_{th} plus x_2' square.

We go to figure 13 right and figure 13 r_2' and these things you right and you just look at the circuit that for maximum your what you call mechanical power output this load resistance must be is equal to this one, but it is physically not possible.

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For maximum mechanical power output,
from Fig.13, condition is

$$r_2' \left(\frac{1}{s} - 1 \right) = \sqrt{(r_{th} + r_2')^2 + (x_{th} + x_2')^2}$$

--- (42)

The maximum power output can then be found corresponding to the slip defined by Eqn. (42). However, this condition corresponds to very low efficiency and very large current and is well beyond the normal operating region of the motor.

So, maximum power output can then be found corresponding to the slip defined by equation 42; however, this condition corresponds to very low efficiency and very large current and is well beyond the normal operating region of the motor.

So, it is physically not possible. You use maximum power transfer theorem go to figure 3 and from your condition you can make it right. We have also we have also seen know that your for maximum power test (Refer Time: 32:26) theorem in DC circuits that r_{load} is equal to $r_{Thevenin}$ is not it. So, here also because r_2' is added x_2' is added here it is an AC circuit. So, that is why impedance part is there right; so now, ok.

Thank you very much we will be back again.