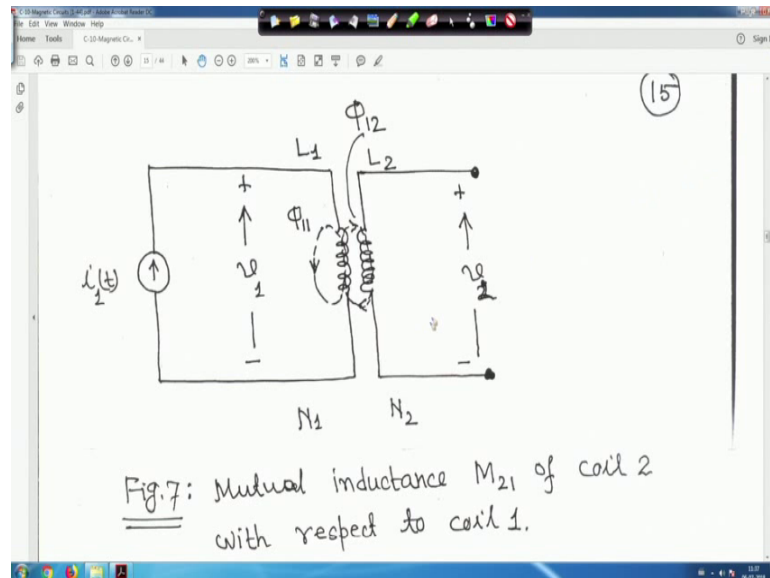


**Fundamentals of Electrical Engineering**  
**Prof. Debapriya Das**  
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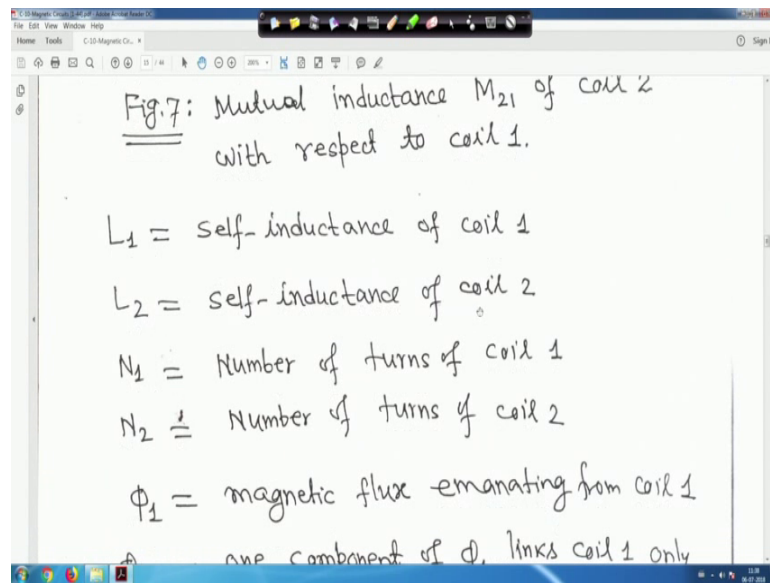
**Lecture - 52**  
**Magnetic Circuits (Contd.)**

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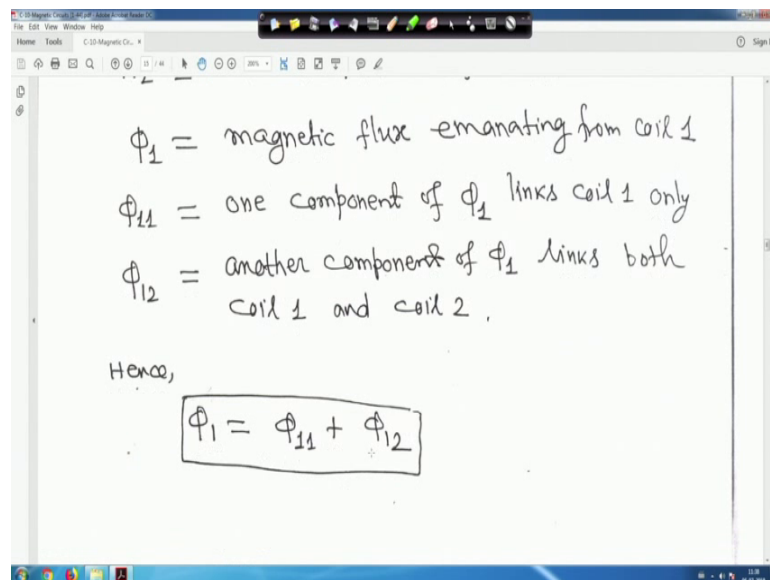
So, we are back again. So, this is now all this now all this in just one minute that all these and nomenclature your are given below right so given below. So, this is your mutual inductance of take to you want to coil to such see the meaning of this now your suffix right. So, this way you can your what you call you can find out what will be the mutual inductance right.

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So,  $L_1$  is the self inductance of coil 1 everything is given all nomenclature is given,  $L_2$  self inductance of coil 2,  $N_1$  number of turns of coil 1 given,  $N_2$  number of turns coil 2 is given.

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$\Phi_1$  is magnetic flux emanating from coil 1 and  $\Phi_{11}$  is one component of  $\Phi_1$  links coil 1 I told you and  $\Phi_{12}$  is another component of  $\Phi_1$  links both coil 1 and coil 2 that I have told you. So, hence  $\Phi_1$  is equal to  $\Phi_{11}$  plus  $\Phi_{12}$  right the total flux.

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(16)

Although the two coils are physically separated, they are said to be magnetically coupled.

Since the entire flux  $\phi_1$  links coil 1, the voltage induced in coil 1 is,

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

Now, although the 2 coils are physically separated they are said to be magnetically coupled right because, flux part of the flux is linking also the other coil. Since the entire flux  $\phi_1$  links coil 1 the voltage induce in coil 1 will be  $v_1$  is equal to  $N_1$  into  $d\phi_1$  upon  $dt$  because  $\phi_1$  is equal to  $\phi_{11}$  plus  $\phi_{12}$ . So, whole  $\phi_1$  linking the coil 1 so voltage  $v_1$  inducing coil 1 will be  $N_1$  into  $d\phi_1$  upon  $dt$  or only flux  $\phi_{12}$  links coil 2.

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only flux  $\phi_{12}$  links coil 2, so the voltage induced in coil 2 is,

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

Again, as the fluxes are caused by the current  $i_1$  flowing in coil 1, we can say

So, the voltage induced in coil 2 will be  $v_2$  will be  $N_2$  into  $d\phi_1$  upon  $d t$ , this diagram; this diagram  $v_2$  will be your this number of turns is  $N_2$  into  $d\phi_1$  upon  $d t$  because  $\phi$  want to link the coil 2 right. So, that is  $v_2$  is equal to  $N_2 d\phi_1$  upon  $d t$ .

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Again, as the fluxes are caused by the current  $i_1$  flowing in coil 1, we can write

$$v_2 = N_2 \cdot \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} = L_1 \cdot \frac{di_1}{dt}$$

where

$$L_1 = N_2 \frac{d\phi_1}{di_1} = \text{self-inductance of coil 1}$$

Now, again as the fluxes are cause by the current  $i_1$  flowing in coil 1, we can write for  $v_1$  we can write because flux  $\phi_1$  is cause by your current  $i_1$  because, your coil 2 no current source is connected. So,  $v_1$  is equal to we can write they will chain rule  $N_1$  into  $d\phi_1$  upon  $d i_1$  into  $d i_1$  upon  $d t$  this you can write  $L_1$  into  $d i_1$  upon  $d t$ . So that means,  $L_1$  is equal to actually  $L_1 d\phi_1$  upon  $d i_1$  it is actually self inductance of coil 1 right this is self inductance of coil 1.

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Similarly, (17)

$$v_2 = N_2 \cdot \frac{d\phi_{12}}{dt} \cdot \frac{di_1}{dt} = M_{21} \cdot \frac{di_1}{dt}$$

where,

$$M_{21} = N_2 \cdot \frac{d\phi_{12}}{di_1}$$

$M_{21}$  = mutual inductance of coil 2

The screenshot shows a whiteboard with a blue border. At the top right, there is a circled number '17'. The main content consists of a boxed equation, followed by the word 'where,' and another equation. At the bottom, there is a definition for  $M_{21}$ . A small video inset of a man is visible in the bottom right corner.

Now, similarly for  $v_2$  case ;  $v_2$  case we can write again chain rule  $v_2$  is equal to  $N_2 d\phi_{12}$  upon  $dt$  into  $di_1$  upon  $dt$  this actually it is actually mutual inductance. So,  $N_2 d\phi_{12}$  upon  $dt$  we are writing  $N_2 d\phi_{12}$  upon  $dt$  where,  $M_{21}$  is equal to  $N_2$  your what you call  $N_2$  into  $d\phi_{12}$  upon  $di_1$  this is actually mutual inductance right.

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$M_{21}$  = mutual inductance of coil 2 with respect to coil 1.

Subscript 21 indicates that inductance  $M_{21}$  relates the voltage induced in coil 2 to the current in coil 1

Thus, the open-circuit mutual volt

The screenshot shows a whiteboard with a blue border. The text is written in two paragraphs. The first paragraph defines  $M_{21}$  as the mutual inductance of coil 2 with respect to coil 1. The second paragraph explains that the subscript 21 indicates that the inductance relates the voltage induced in coil 2 to the current in coil 1. A small video inset of a man is visible in the bottom right corner.

So,  $M_{21}$  mutual inductance of coil 2 with respect to coil 1 this is the meaning of the suffix right. So, that here that is why I have written for you subscript 2 1 indicates the

inductance  $M_{21}$  relates the voltage induced in coil 2 to the current in coil 1 this is the meaning right, so that should not be any confusion right.

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to the current in coil 1

Thus, the open-circuit mutual voltage (or induced voltage) across coil 2 is

$$v_2 = M_{21} \frac{di_1}{dt}$$

So, thus the open circuit mutual voltage or induced voltage right across coil 2 will be  $v_2$  is equal to  $M_{21} di_1/dt$  because, flux  $i_1$  actually creating your current  $i_1$  actually creating the flux  $i_1$  and part of the flux linking the your coil 2, therefore  $v_2$  will be  $M_{21} di_1/dt$  that there it is right. So,  $v_2$  is equal to  $N_{21} di_1/dt$  it is a in your induced voltage right due to the flux main thing the other coil.

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Fig.8: Mutual inductance  $M_{12}$  of coil 1 with respect to coil 2.

Summary

So, now similarly if you this side if you suppose excited by current  $i_2$  and this side it is your  $v_1$   $L_1$   $L_2$  d down here also  $\phi_2$  is equal to,  $\phi_2$  is the total flux same as before  $\phi_2$  is the total flux is equal to  $\phi_{22}$  plus  $\phi_{21}$ . So, part of the your what you call part of the flux of your  $\phi_2$  actually linking the this coil 1 right and this is your coiling  $\phi_{21}$ , but this  $\phi_2$  is equal to  $\phi_{22}$  and  $\phi_{21}$  then both but this is a total flux. So, this  $\phi_{22}$  and  $\phi_{21}$  both linking coil 2 and this is the voltage  $v_2$  this is your same as before. This is mutual inductance  $M_1$  of coil 1 with respect to coil 2 right.

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Summary

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = \left( N_2 \frac{d\phi_2}{di_2} \right) \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = \left( N_1 \frac{d\phi_{21}}{di_2} \right) \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$\therefore v_1 = M_{12} \frac{di_2}{dt}$$

So, let it be  $\phi_2$  is equal to  $\phi_{21}$  plus  $\phi_{22}$  same as before,  $v_2$  will be coil 2 number of turns  $N_2$  into  $d\phi_2$  upon  $dt$  that is  $N_2 d\phi_2$  upon  $di_2$  into  $di_2$  upon  $dt$  chain rule, so this is  $L_2$  into  $di_2$  upon  $dt$  right. So,  $L_2$  is the self inductance of the coil 2 mainly said right and  $v_1$  that is the induce voltage due to that flux linking you are what you call in coil 1 due to the current in your coil 2. So,  $v_1$  is equal to  $N_1 d\phi_{21}$  upon  $dt$  is equal to  $N_1 d\phi_{21}$  upon  $di_2$  into  $di_2$  by  $dt$  this again chain rule. So this part; this part is call your  $M_{12}$  that is  $di_2$  upon  $dt$ , therefore  $v_1$  is equal to  $M_{12} di_2$  upon  $dt$  right.

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$$v_1 = N_1 \frac{d\phi_{21}}{dt} = \left( N_1 \frac{d\phi_{21}}{di_2} \right) \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$
$$\therefore v_1 = M_{12} \frac{di_2}{dt}$$

$M_{12}$  and  $M_{21}$  are equal, that is

$$M_{12} = M_{21} = M$$

But,  $M_{12}$  and  $M_{21}$  are equal that is  $M_{12}$  is equal to  $M_{21}$  is equal to  $M$  it cannot be different right, this way have that way mutual inductance has to be same right. So,  $M$  is actually  $M_{12}$  is equal to  $M_{21}$  right. So, this is your equation number I did not put just make step by step understandable to you right.

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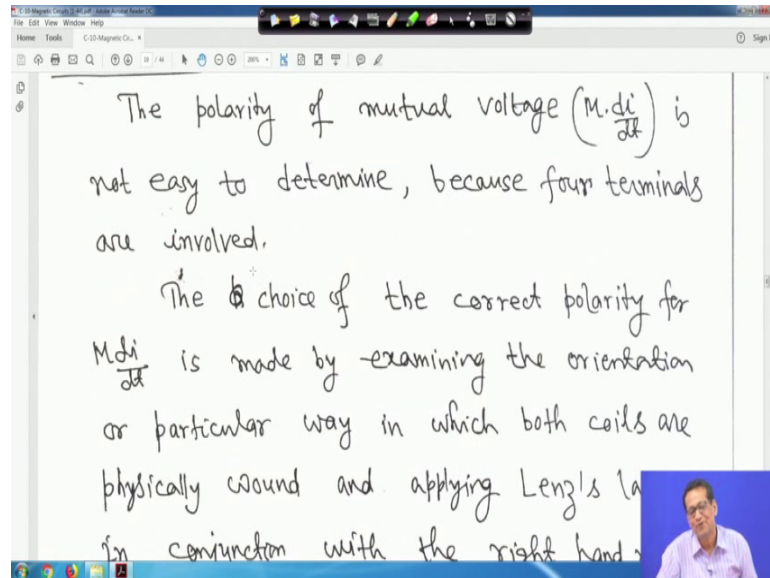
Keep in mind that mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources. Recall that inductors act like short circuit to dc.

Now, something I have written keep in mind that mutual coupling only exists when the inductors or coils are in close proximity and the circuits are driven by time varying sources. So, this is I have written for you recall that inductors act like short circuit short



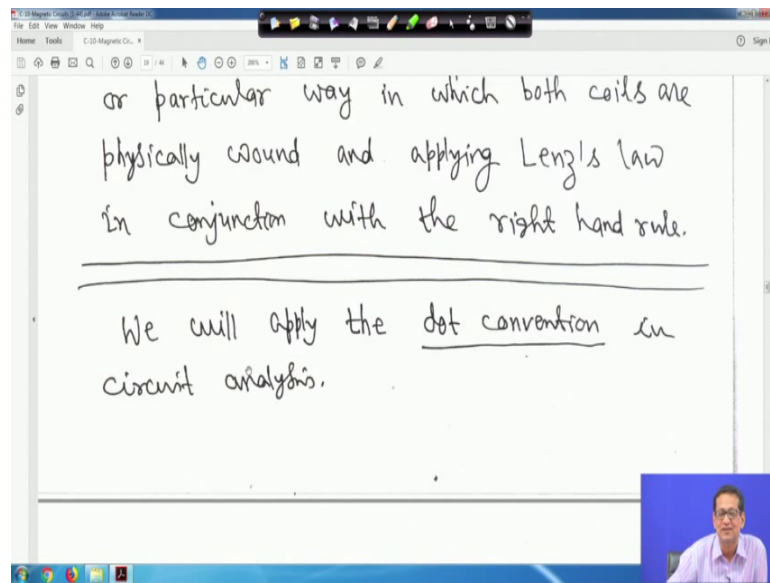
circuit to dc right. So, that is why this is written for you that keep in mind that right. Mutual coupling only exist when the inductors are coils are in close proximity and the circuits are driven by time varying sources.

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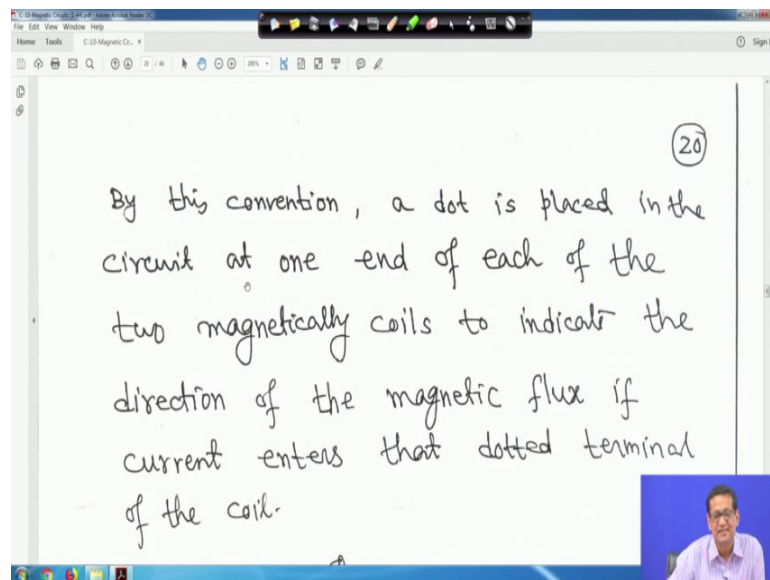
Now, the polarity of mutual voltage this is very this is the only thing we have to see the polarity of mutual voltage whether it is plus or minus  $M \frac{di}{dt}$  is not easy to determine, because four terminals are involved right. Because, your what you call that in the coil 1 side plus minus voltage terminal and in what you coil in the coil 2 side also plus minus, so four terminals are involved right. The choice of the correct polarity for  $M \frac{di}{dt}$  is made by examining the orientation or particular way in which both coils are physically wound right and applying Lenz's law in conjunction with the right hand rule this is a difficult one right.

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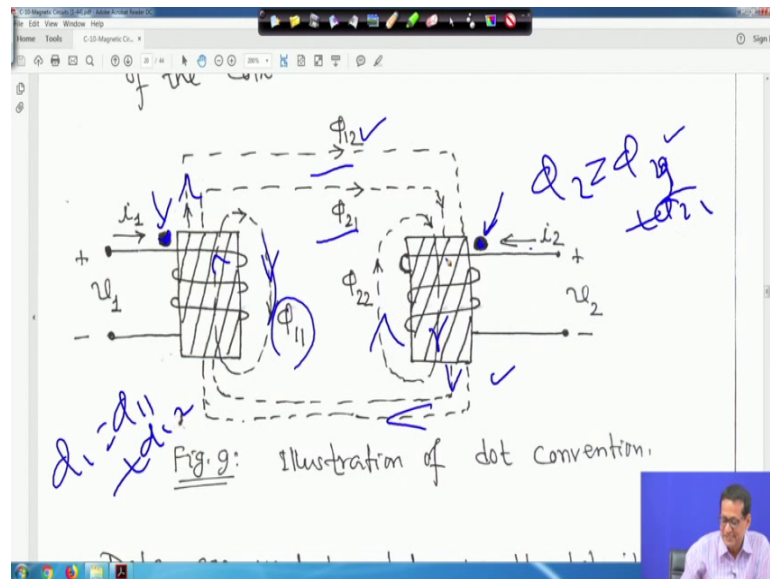


But for our circuit analysis what we will do we will apply the dot convention in circuit analysis, in our magnetic circuit analysis or this thing will apply that dot convention right.

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So, by this convention a dot is placed in the circuit and one end of each of the two magnetically coils to indicate the direction of the magnetic flux, if current enters that dotted terminal of the coil right.



Now, question is suppose this is your what you call will use dot convention, suppose this is dot is here right let us try to understand this. Suppose if dot is here you have make the dot and here also dot is there here also dot is there here also dot is there right and your illustration of dot and coils are wound here right or the coils are wounds here.

Now, question is current  $i_1$  is direction and  $i_2$  is direction (Refer Time: 08:23) direction in this direction. Now, question is that how to take plus or minus? Now, question is that this is that current entering now you grabs the your what you call you grabs the your coil right. So, in the direction of the current you grabs it right just you call it like this and this way this is that you are what you call this is the flux actually, this direction of the flux this is the direction of the flux.

So, for this coil 1 actually  $\phi_1$  is equal to  $\phi_{11}$  plus  $\phi_{12}$  because, this is a direction ultimately this the direction it is taking right you would grabs in the direction of the current that coil that your what you call in the direction of the current. That is why this coil is this coil this direction of the current this coil is wound.

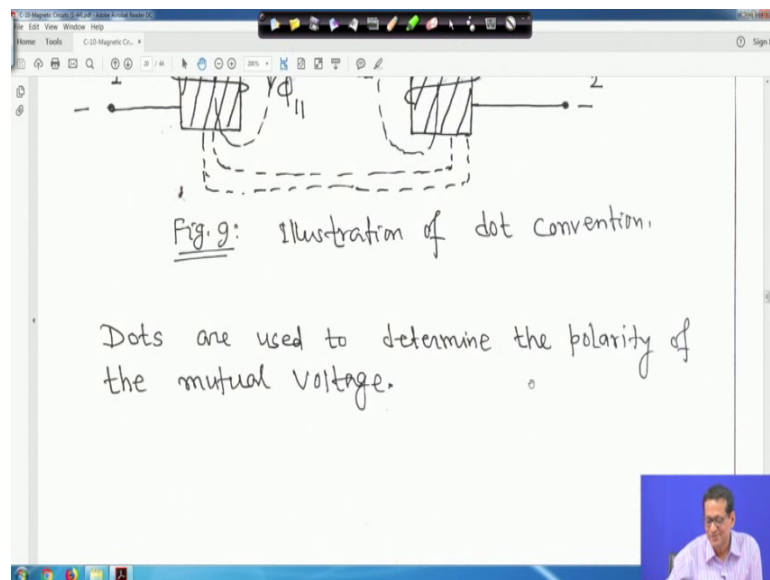
So, you grabs it and this is in the direction of the current you grabs the coil and this in the direction of the current and thumb you will indicate your direction of the flux. So, this is coming out for this is  $\phi_{12}$  and this is also coming out from this so moving like this because, this linking the coil 1  $\phi_{11}$  and part of this is coming here and linking your the coil 2. Similarly, this side this side also current is entering say  $i_2$  at dot I have place like this right so,  $\phi_2$  is equal to  $\phi_{21}$  plus  $\phi_{22}$ . Here also if you make like this

that current is moving in this direction and you grab your coil in the direction of the current and this finger is showing like this it is coming to the bottom of this your this may you are what you call the bottom of this side right in this figure.

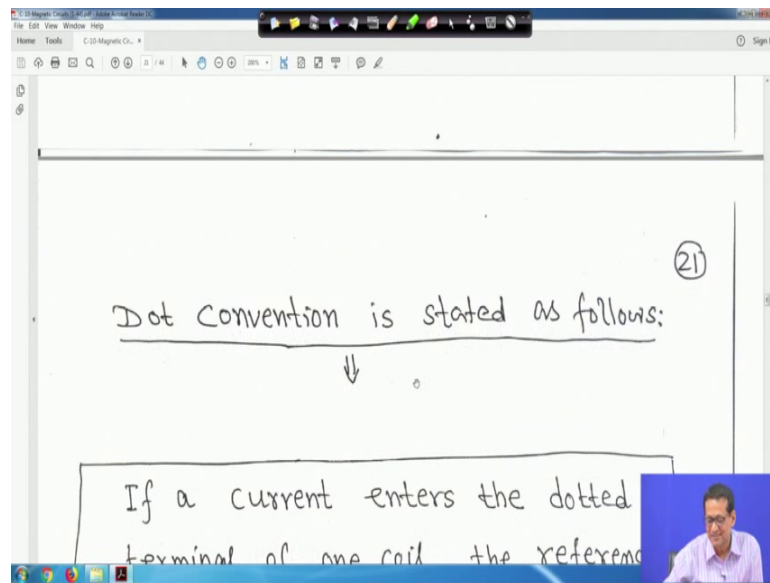
That means flux is your what you call coming out from this is coming out for this that is why this is the thing and that is why this is the direction of the flux. Now, this is actually  $\phi_{21}$  and  $\phi_{12}$  both are in the same direction the additive is not it. So, but this  $\phi_{21}$  actually linking this coil and all the  $\phi_{21}$  and your what you call  $\phi_2$  is equal to your  $\phi_{22}$  and plus  $\phi_{21}$  all this linking coil 2, so this way you have to understand the dot convention right.

So, if it is so that is illustration of dot convention that how we will do it right so let me clear it. So, hope this direction of the flux that how things are come you have understood what is there just look at the coil see that direction of the current, you grab the coil in the direction of the current and direction of the your thumb will be the direction of the flux right. And this side voltage is given in this side voltage is  $v_2$  hope you have understood this right.

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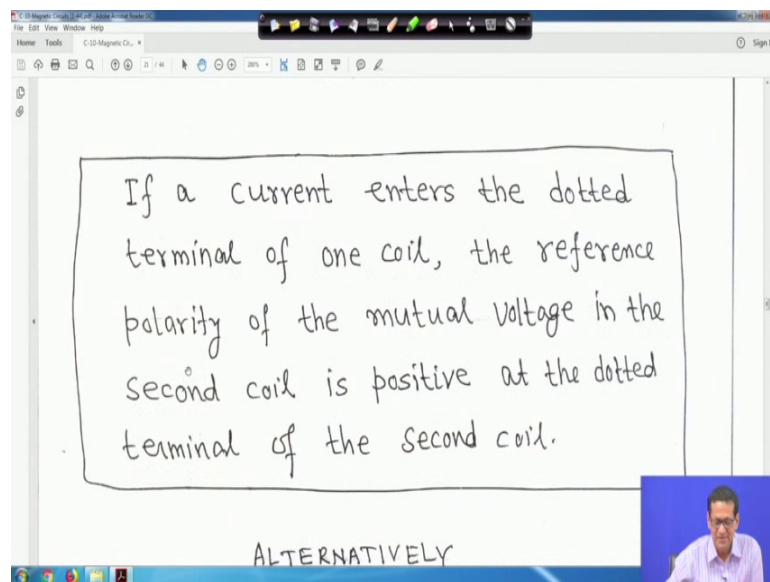


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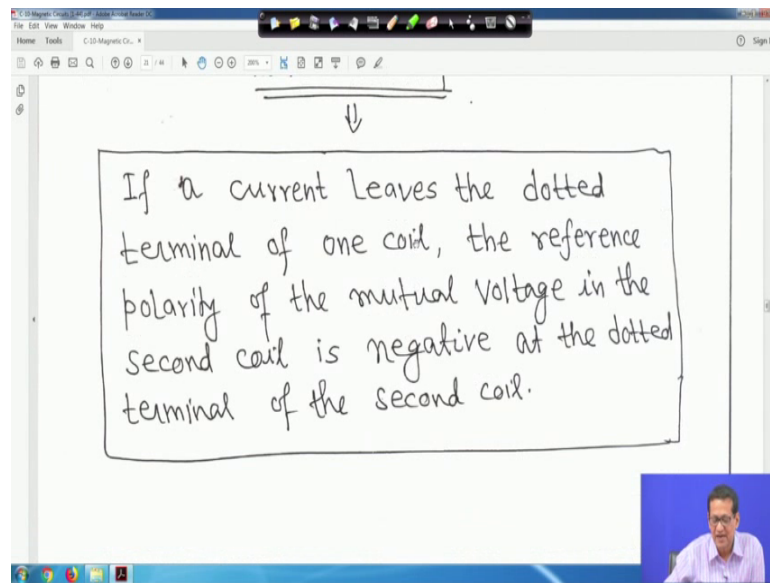
So, this way dots are used to determine the polarity of the mutual voltage using this dot. Now, question is that suppose dot convention is stated as follows.

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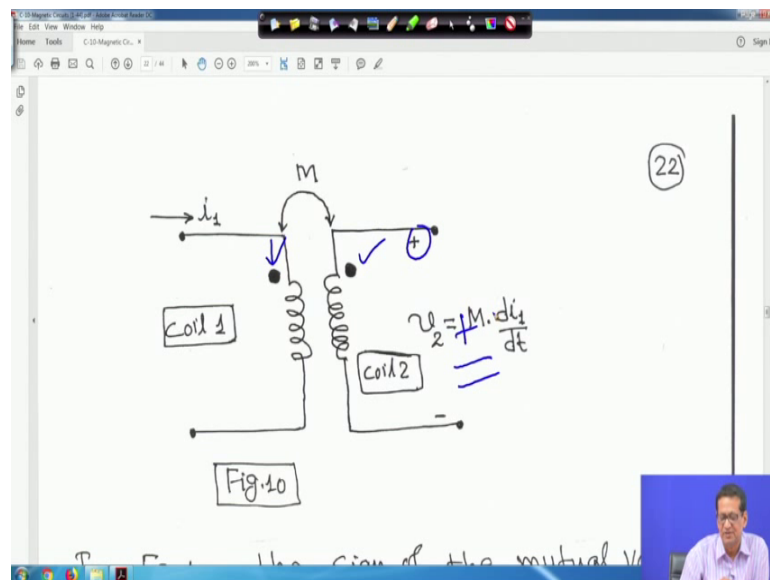
If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil, this is the language I have written that you will come to that figure and other thing. If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage right in the second coil is positive at the dotted terminal of the second coil.

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So, alternatively if a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil right; for a now if you come to this right.

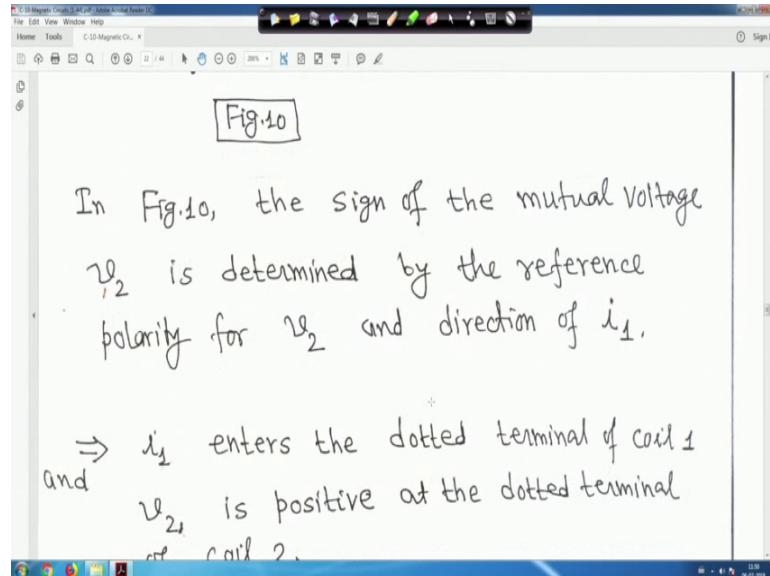
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Suppose this your what you call this current your what you call in figure 10, the 2 coils are there dot are shown this dots are shown. Here current  $i_1$  is entering and mutual inductance between this coil is shown by  $M$  here it is  $M$  a mutual inductance and this is coiled to should be written  $M$  into  $d i_1$  by  $d t$  because, current is  $i_1$  and mutual voltage

induce in this  $M$  into  $d i_1$  by  $d t$  this is a mutual inductance. We are not writing  $M_{12}$  or  $M_{21}$   $M_{12}$  is equal  $M_{21}$  is equal to  $M$ .

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Now, in figure 10 the sign of the mutual voltage  $v_2$  is determined by the reference polarity for  $v_2$  and direction of  $i_1$  right. So, in this case current is entering in their what you call this is a reference polarity is given that is your plus right and current entering into the your dot right and this is the reference polarity. So, and dot is marked here in the reference polarity plus so, this is actually  $v_2$  is equal to  $M$  into  $d i_1$  upon  $d t$  right. So, it is actually plus sign  $M d i_1$  upon  $d t$  next is so, here it is that is why written determined by the reference polarity for  $v_2$  and direction of  $i_1$ .

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$\Rightarrow i_1$  enters the dotted terminal of coil 1  
and  $v_2$  is positive at the dotted terminal of coil 2.  
Mutual Voltage is  $+ M \frac{di_1}{dt}$

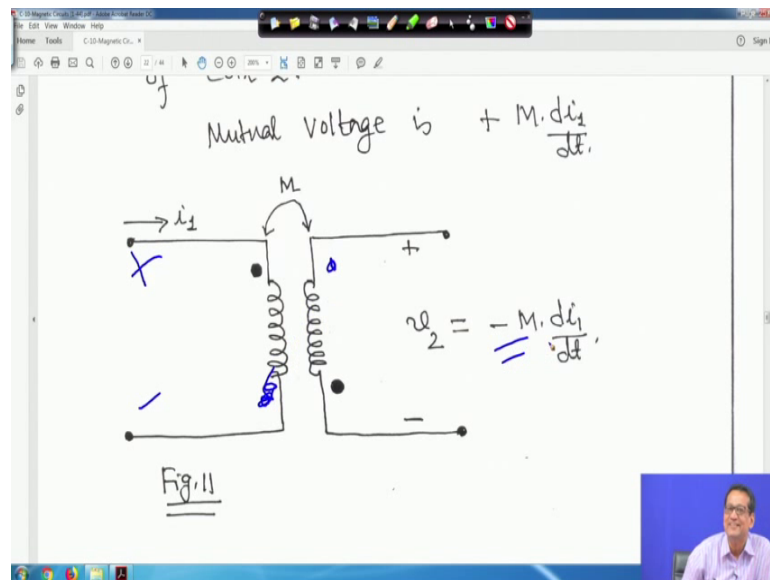
$v_2 = -M \frac{di_1}{dt}$

Now, similarly now that is I that is  $i_1$  enters the dotted terminal of coil 1. So,  $i_1$  actually entering into the dot so, this  $i_1$  actually entering into the dotted terminal right much more we will see. So, and  $v_2$  is the positive at the dotted terminal coil 2 therefore, mutual voltage is plus  $M \frac{di_1}{dt}$  upon  $dt$ . This is our reference polarity you have taken this is a reference polarity you have taken plus.

So, dot actually it is that reference polarity plus you have taken and dot is also the reference will plus. So, that is why this one  $v_2$  is equal to  $M \frac{di_1}{dt}$  that is why it is written that your what you call that  $v_2$  is positive, at the dotted terminal of coil 2 right. So, mutual voltage is plus  $M \frac{di_1}{dt}$  upon  $dt$  are so,  $i_1$  entering the dotted terminal of coil 1 and  $v_2$  and  $v_2$  is positive at the dotted terminal of coil 2 right.

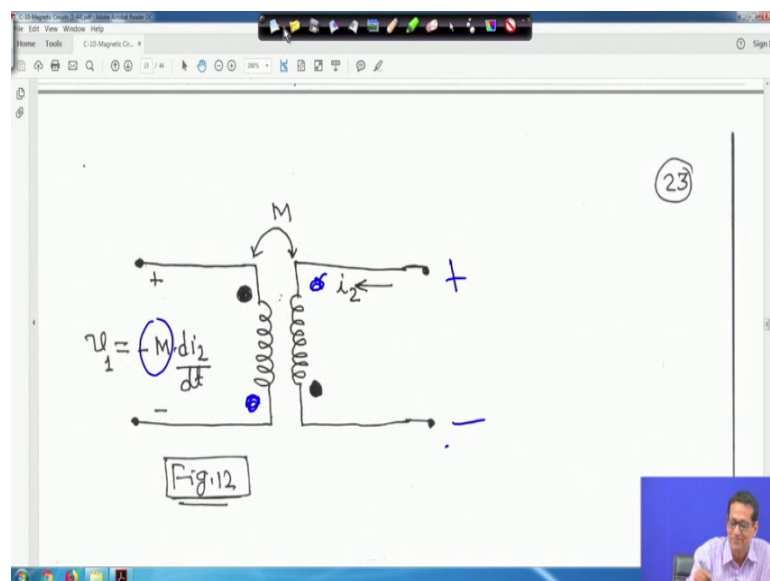


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Now, mutual voltage is plus than  $M \frac{di_1}{dt}$ . Now, next one is that second diagram so, dot is here and dot is here suppose it is marked. So, current is entering into this dot right, but in this case in a reference polarity here it is minus dot is this dot is placed to a reference polarity minus. So,  $v_2$  will be minus  $M$  into  $\frac{di_1}{dt}$ ; this is that the current all though current is entering into the dot, but dot is here a reference polarity for  $v_2$  that is minus. So, it will be minus  $M \frac{di_1}{dt}$ , when both are your both sized current are your injected we will see later right.

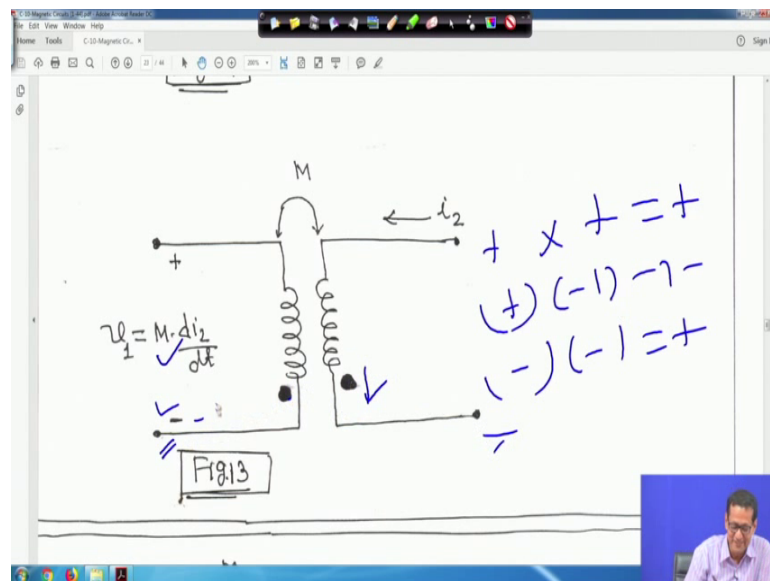
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Now, similarly other way also now this side that current  $i_2$  is your what you call your what to call current  $i_2$  is your entering here, but leaving the dot, here you will look into that this is current  $i_2$ ; current  $i_2$  actually leaving the dot right. But, question is that the reference polarity here, this is your what you call this is your reference polarity here it is plus and here it is your plus, but current leaving the dot that is why here it is minus because, you have to see here current actually leaving the dot right.

So, that means, it is your what you call, it is your minus right. So, another thing is that your that is why it is minus  $M \frac{di_2}{dt}$  because current here is  $i_2$ , but here current is if you put that current is dot is here by chance if you put. Suppose if you put dot is here right then it will be plus  $M \frac{di_2}{dt}$ , but question is that current here your what you call leaving the dot; it is leaving the dot you have to see entering the dot or leaving the dot, but reference polarity although this side is plus 1, but current is leaving the dot. So, it is minus  $M \frac{di_2}{dt}$  upon  $dt$  right.

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Similarly, if you see here, in this case if you see here current actually this  $i_2$  leaving the dot and dot is here in this minus also the reference polarity is also minus and current leaving the dot; that means, this will be plus right. So, idea is the you know they how you remember I will tell you one different rule. For example, suppose this is plus minus this you mark plus minus right.

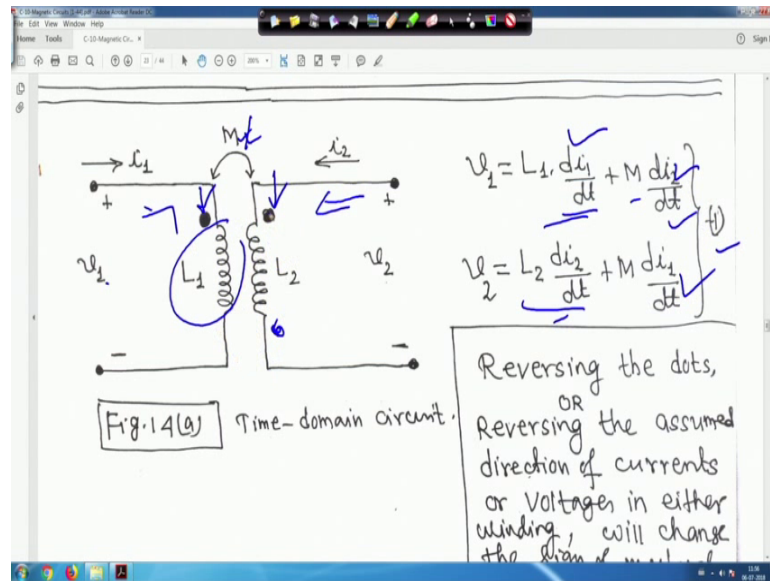
So, question is that if both dots are here, it will be plus if either of this either it is here or here or it is here. So, it is plus your plus into plus is equal to your plus right so, it is like this. So, if it is either is here or is here it is plus into minus. So, it will be minus sign right and if it is minus minus it will be plus sign. So, that is why it is plus sign this way you can easily remember like a right. So, this is one this rule will not tell to anywhere right this is just to if you want keep it your mind, then you will find things are very simple right.

You see both dot I mean this kind of thing here it is minus; here it is minus right here current leave the terminal reference dot is this one it is coming flux right. Similarly, if you look here; if you look here it is here it is your what you call here it is plus and here you must plus minus here it is plus here it is minus so, plus minus minus right. Similarly, if you put the dot is here and dot is here so, plus minus it will be your again it will be your what you call it will be minus right.

So, this way you can remember right similarly if you come to your other 2 figure, here if you look this is your if you mark this one this is my plus this is plus minus. So, plus minus look minus as coming to reverse the dot, this way also you can like this then it will also it will become your what you call it will be minus right.

So, let me clear it same is for the figure 1 so, same is for your this figure 1; this is plus if you mark this one the current is entering you mark this one as plus and this is minus so, plus plus o, it is plus. So, this way you can write plus plus plus minus minus plus plus, minus minus minus plus minus this way you can do it please if you remember right so, with this that how to get the sign right.

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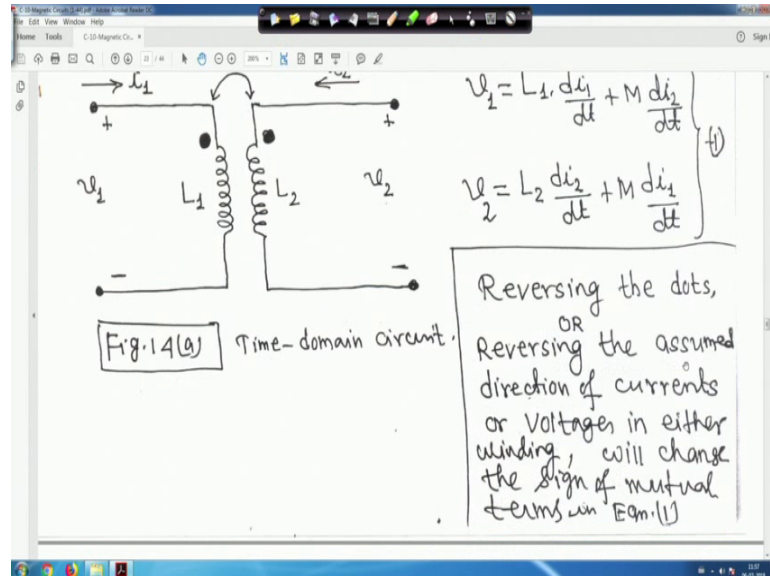
So, dot convention now this one now next one will take this is your  $L_1$   $L_2$  that inductance of coil self inductance of coil  $L_1$   $L_2$  and mutual inductance  $M$  is given this side current is  $i_1$  this side is current is  $i_2$ . So, in this case what will happen that both the cases current is here current is here, but look into that the convention is current is entering into the dot and this current also entering into that dot. Both the currents entering into the dot; that means, sign will be plus that is why  $v_1$  is to write it will be  $L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$  this is  $v_1$ .

So, this is for your  $v_1$  you write  $L_1 \frac{di_1}{dt}$  it is written then, but due to this  $i_2$  that some you are voltage will be induced in the coil your what you call coil 1. So, that is the mutual inductance  $M$  into  $\frac{di_2}{dt}$  plus  $M \frac{di_1}{dt}$ . Similarly, for  $v_2$  will be  $L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$  this together this is the equation 1 so, this way you can write.

Now, if you either of the either of this dot if it is changed after some if you put some this dot is here or let me clear it or this dot is here, this will be there in that case your what will happen the sign will change the same sign will change right. For example, if this your this dot is here suppose this is not there. So, it will minus I told you plus minus minus right something like this right. Because, current actually leaving this dot if dot is here means this current is entering and another is leaving you have to take the minus

sign, in that case it will be your what you call minus. So, here also it will be minus so, it will be now plus minus right plus minus.

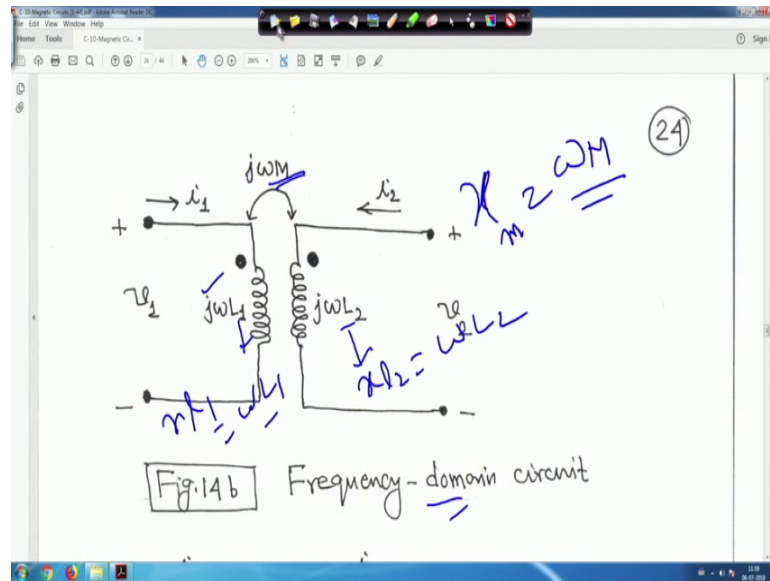
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So, that is why reversing the dots or reversing the assumed direction of current right or voltages in either winding will change the sign of mutual your terms in equation 1. Even though either of the I told you either that dot or you reverse the you have to see that current is leaving right the dot.

For example, if you just instead of this directions suppose current direction is like this so; that means, what it is this is the current the current leaving the dot right and it is entering the dot it will be minus right you have see. If both current enters the dot it will be plus sign if both current will leave that dot there also it will be plus sign if one current entering the your dot and leaving the dot they it will be negative sign right. So, this is everything is written here for you.

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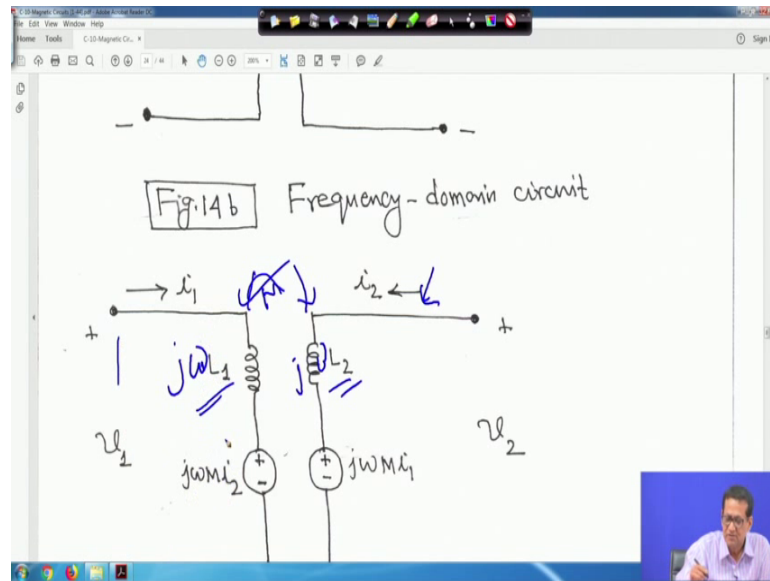


Now, if you that is this one in both in time domain. Now, if we want that this will be your frequency domain the reactance mutual reactance will be  $\omega$  into  $L$ ,  $L \omega$  we have studied. So, it will be  $j$  into  $\omega M$  and this is my your  $I$  current  $i_1$  this is  $i_2$  and this is  $j \omega L_1$  right and this is your  $j \omega L_2$ , this is  $X_{L1}$   $X_{L2}$  right  $X_{L1}$  is  $\omega L_1$   $X_{L2}$  is  $\omega L_2$  and  $X_M$  is equal to  $\omega M$  that is the mutual reactance right.

So, here sometimes you write; sometimes you write  $X_M$  is equal to  $\omega M$  this is your mutual reactance. And here this one is equal to your  $X_{L2}$  is equal to  $\omega L_2$  and this one your  $X_{L1}$  is equal to  $\omega$  reactance of coil 1 reactance of coil 2 and  $j$  is the because it inductive coil and this is  $j$  is there because it is inductive coil and this is your mutual reactance right.

So, this is in the frequency domain, here will solve all the problem in whatever little bit in frequency domain. So, now, this one now because of this coil the sorry because of this current a mutual voltage will be induce in this.

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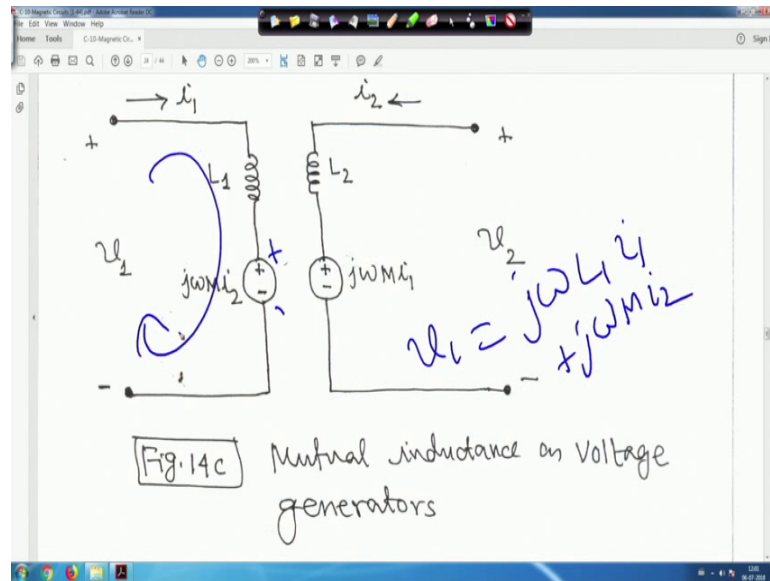


So, in that case you can write this one  $i_1$ ,  $L_1$  is there this is the inductor this is  $L_1$  or you can write; or you can write if is a reactance you can put  $\omega L_1 j$   $\omega L_1$ , this is also you can write  $j \omega L_2$  this is the reactance of coil 1 and because of there is voltage will induce it. So, it will be  $j \omega M i_2$  plus minus and  $j \omega L_1$ . Because of this current  $i_2$  a voltage will induce and this is your mutual inductance was  $M$ . So, it will be  $j \omega M$  plus minus and it will be  $j \omega M$  your what you call  $i_1$ .

Now, question is just hold on let me so, this is understandable right instead of  $L_1$  I make reactance and this current  $i$  so, voltage is here easily you can find out what will be the current right. So, just let me clear it so, here this in this equation also in this equation also, same thing you will get what you will do you just remember at this stage you  $d$   $d$   $t$  you replace by your  $j \omega$ . If you replace  $d$  by  $d$   $t$  this  $d$  by  $d$   $t$   $j \omega$  and you will find this will become  $j \omega L_1 i_1$  plus your  $M$ ,  $j \omega M i_2$  right you replace  $d$  by  $d$   $t$  by  $j \omega$  in this equation you will get the same thing right.

So, according to that in the your what you call in that your frequency domain the polarity of that induce voltage in other coil is taken. If you put here if you put your  $d$  by  $d$   $t$  you replace by your  $j \omega$  this right; then this one will become  $L_1 j \omega$  into  $i_1$ ; that means, basically it will become  $j \omega L_1 i_1$  here also it will become  $j \omega M i_2$  right. .

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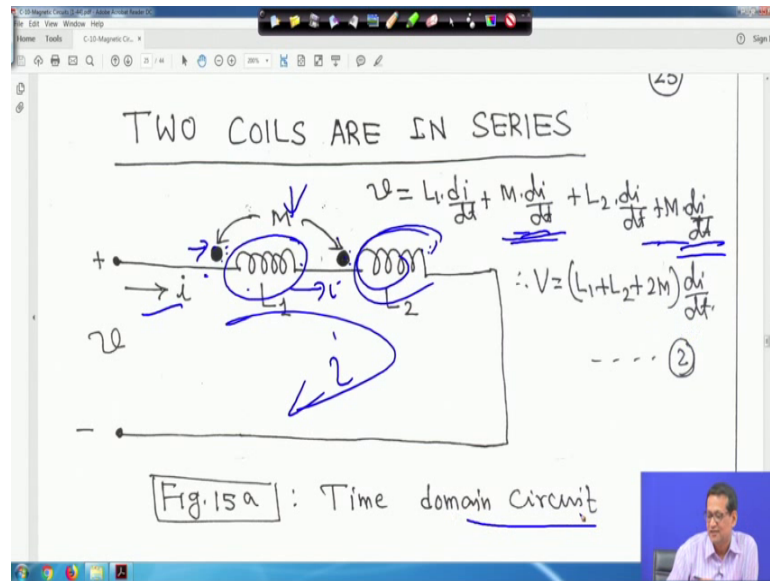


So, that means, this is your what you call this equation is written. Now, then  $i_1$  you can easily write a  $i_1$  is equal to  $v_1$  divided by  $j\omega L_1$  plus  $j\omega M i_2$  your what you call this you can write this equation your  $v_1$  is equal to your  $i_1 j\omega L_1$  plus  $j\omega M i_2$  right. So, I mean you I mean if you write equations 1 later we will see how to solve it for example, if you write  $v_1$  is equal to your  $j\omega L_1$  right then  $i_1$  and this is your induce voltage your due to the mutual coupling this voltage source is added here.

So, plus your  $j\omega M i_2$  right is equal to  $v_1$  a  $v_1$  you put  $v_1$  here your applying  $v_1$  here right later will see how to solve it will take some problem will solve right. So, mutual inductance and voltage generator so, this way you can replace in frequency domain you can written here mutual inductance and your voltage generator as if it is make you generating some voltage right.



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So, now 2 coils are in series; if 2 coils are in series how it looks like look in this case only how to write the equation in this case the current I because it is series 2 coils are they look. Here in the take this 1st coil is dot current is entering into the dot in the 2nd coil also same current i is entering into the dot; that means, sign should be plus this is L 1 L 2 their inductances. So, clear i1 entering the dot all this is one coil is dot is marked here.

So, current is entering into the dot and this is a another coil is dot is entering marked here. So, current actually entering into the dot of the 1st coil same current I entering the dot of the your 2nd coil so, sign should be plus right. So, that is why first if you write the v; v is equal to it will be L 1 d I and their mutual inductor between them is M right. So, v is equal to L 1 d i upon d t plus M. d i d t now current is same it is a series circuits so, current is same right plus L 2 d i 2 by d t plus M d i by d t.

Because 2 coils are there so, repetition will be there you have see circuit twice it will come. Once because of the when your writing your for this coil it will be it was M d i is the d t will come for this coil because of this coil and for M d i d t will come here because of this coil. So, repetition will be there right this should not this one should not make any error for this that is why twice it is there you have plus it your coil considering each coil independently right and that is why twice it is there.

So, once because of this coil same current is flowing voltage will be induced there in  $M \frac{di}{dt}$ . Similarly, because of this coil that current  $I$  is flowing a voltage will be induced in just 2 coil  $M \frac{di}{dt}$ . So, that is why twice it has come so,  $v$  is equal to  $L_1$  plus  $L_2$  plus  $M \frac{di}{dt}$  this is the time domain circuit right.

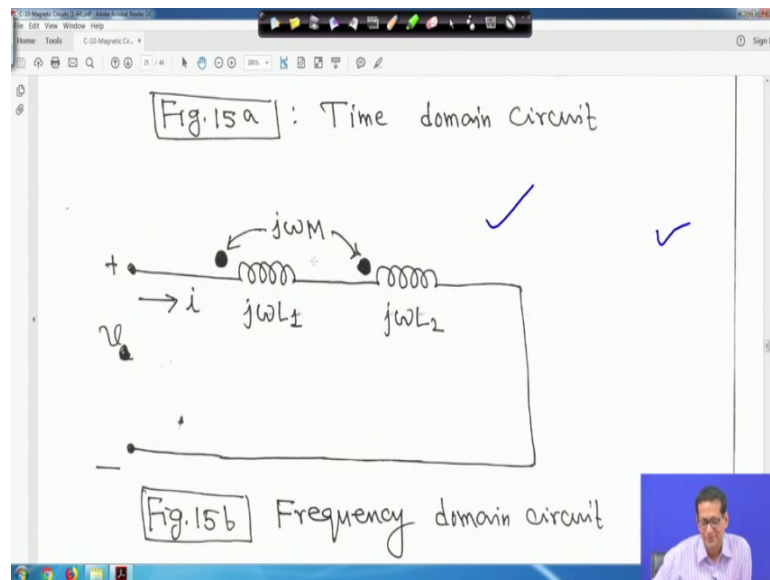
Same thing will happen suppose another thing can happen let me clear it same thing suppose dot is not here suppose  $I$  put that dot here of this coil. So, in that case what will happen the current  $I$  actually entering into this dot, but this  $I$  leaving the dot right this is not there; this is not there suppose dot is here you have made the dot. So, current entering in one coil, but other coil current leaves the dot.

So, in that case sign of this one should be plus while sign of this one should be minus it will be minus right it will be minus  $M$  sorry not this one it should let me clear it, that sign of this one will be minus and sign of this only will be minus. If you put instead of dot here if you put dot because, here current is entering then current is leaving; that means, in that case general instead of plus it will be minus, it will be minus then  $L_1$  plus  $L_2$  minus  $2M$ . In general in general actually this should have been  $L_1$  plus  $L_2$  plus minus  $2M$ . Because, if you put the dot here if currently this thing similarly if it happened that is dot is here and this dot is not there and this dot is here so, in that case what will happen current actually leaving the dot right.

But, in this coil current entering the dot again it will be minus again it will be minus again it will be minus right so that means, you are what you call this one sign would let me once it is once it is entering another is leaving it will be minus. Now, similarly if you take if you take another suppose dot is here and dot is here, this dot is not there this dot is not there.

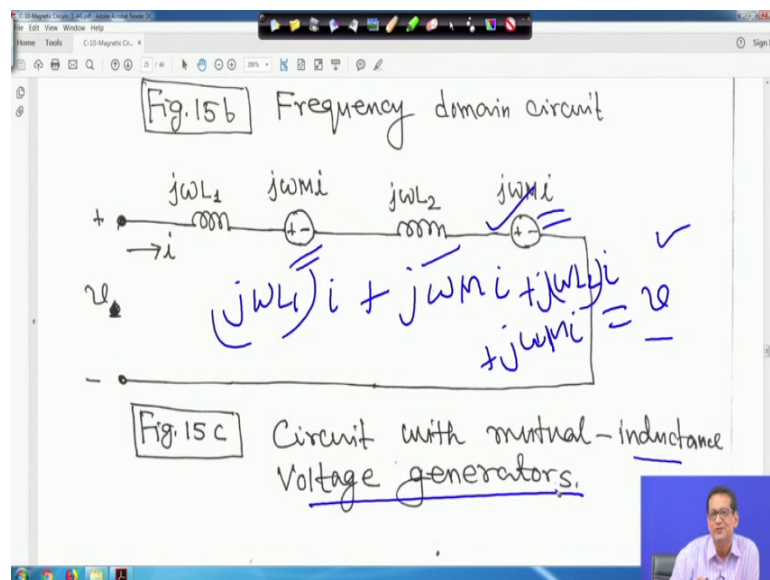
So, here also current leaving the dot, here also current leaving the dot current both the coils current leaving the dot; in that case this sign should be plus right sorry not this one sign should be plus this one and this one the sign should be plus. So, that is why if both the if the current leaves both the dot or enters both the dot plus sign, if the current either of this coil once it is entering the dot another is leaving the dot it will be minus sign right.

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So, this is time domain, now if you come to your frequency domain. So, mutual inductance will be  $j\omega M$  and this is  $j\omega L_1$   $j\omega L_2$  right.

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So, now because of this if like your way or your what you call that is a circuit with mutual inductance voltage generator, I told you twice it will be it will be repeated right. So,  $j\omega L_1$   $j\omega M$   $i$  and  $j\omega L_2$   $j\omega M$  and  $i$  right. So, this is I told you it will be repeated so, this way you can represent the circuit in the frequency domain.

So that means, if you now try to find out  $i$ , then  $i$  that if you now not apply  $k v l$  here. So, at this written here  $j$  then it will be  $j \omega L_1$  into  $i$  plus  $j \omega M$  your sorry  $j \omega L_2$  plus  $j \omega M$  this will be your voltage source it will then in I mean writing for you.

So, if you make it here it will be  $j \omega L_1$  into  $i$  then this is plus  $j \omega M$   $i$  right plus  $j \omega L_2$  your  $i$  right and then plus  $j \omega M$  right  $j \omega M$   $i$  is equal to  $v$  right. So, you can find out what is  $i$  right because this has this is a voltage source  $j \omega$  of  $i$  and this  $j \omega$  of  $i$  right. So, that is why it is inductance voltage generator this way you have to you consider it right. So, that is why what I have written here right this is what I have written here.

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from Fig. 15(c),

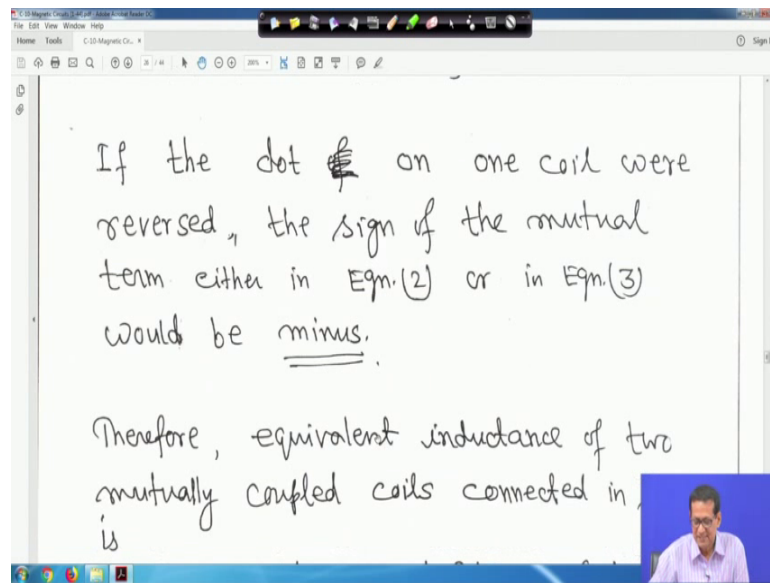
$$i = \frac{v}{j\omega(L_1 + L_2 + 2M)}$$

$$i(j\omega L_1 + j\omega L_2) + j\omega(2M)i = v \quad \dots (3)$$

$$\therefore i = \frac{v}{j\omega(L_1 + L_2 + 2M)} \quad \dots (4)$$

So, ultimately it will become  $i$  is equal to  $V$  upon  $j \omega L_1$  plus  $L_2$  plus  $2 M$  right. If it is the way we have taken and all versions I have showed you how to take it right.

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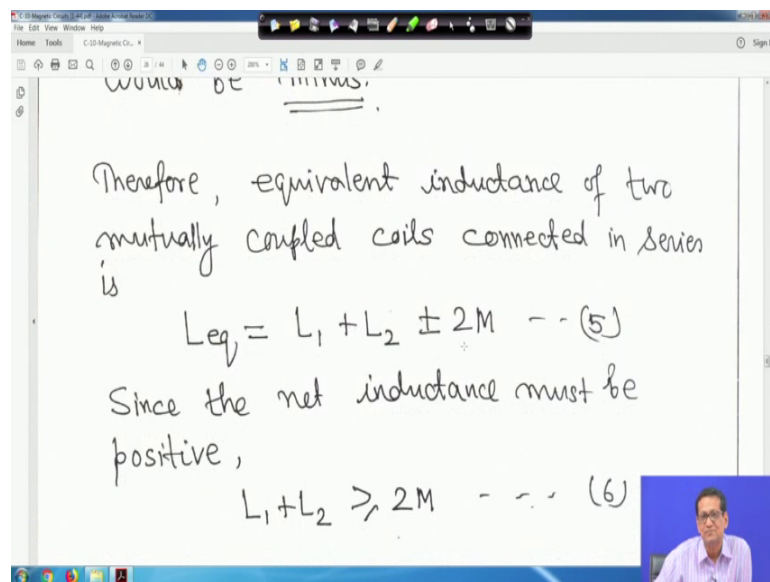
If the dot ~~is~~ on one coil were reversed, the sign of the mutual term either in Eqn.(2) or in Eqn.(3) would be minus.

Therefore, equivalent inductance of two mutually coupled coils connected in series is

*(A small video inset of a man speaking is visible in the bottom right corner of the whiteboard screenshot.)*

If that dot on one coil reverse the sign of the mutual, they you are the term either in equation 2 or in equation 3 will be minus I explain everything to you.

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would be minus.

Therefore, equivalent inductance of two mutually coupled coils connected in series is

$$L_{eq} = L_1 + L_2 \pm 2M \quad \dots (5)$$

Since the net inductance must be positive,

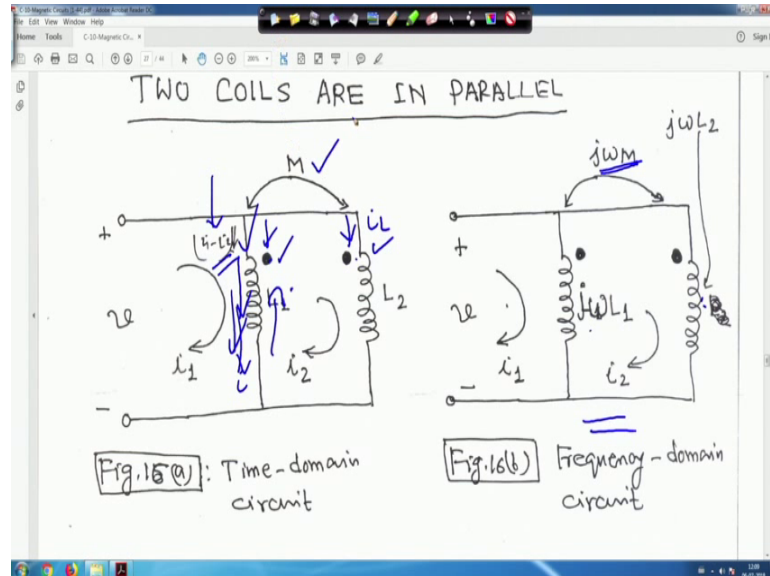
$$L_1 + L_2 \geq 2M \quad \dots (6)$$

*(A small video inset of a man speaking is visible in the bottom right corner of the whiteboard screenshot.)*

That means the equivalent inductance of the 2 mutually coupled coils connected in series it will be  $L_1 + L_2 \pm 2M$  right. I mean if you the if you just one of the coil if you just change this position right. So, it will be in  $L_1 + L_2 - 2M$  right so; that means, in general it will be  $L_1 + L_2 \pm 2M$  right. Since the net inductance

must be positive so, it will be  $L_1 + L_2 \geq 2M$  this is equation 6 right it has to be greater than equal to  $2M$ .

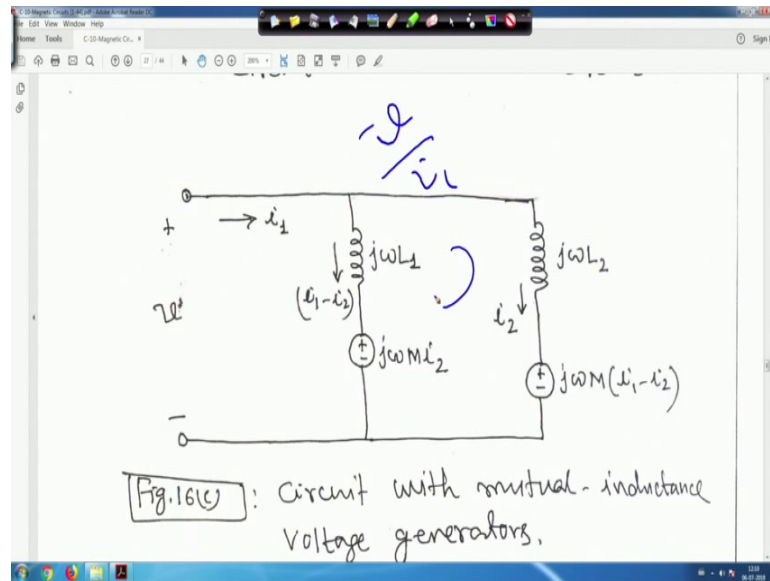
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Now, when 2 coils are in parallel suppose these 2 coils are in parallel that is series now this is a parallel what you have done it here that, current is this cyclic current is taken  $i_1$  and  $i_2$ . So, in the direction of the in the direction of this one it is  $i_1$  minus  $i_2$  it is marked here and here the current is cyclic. So, this is current is  $i_2$ ; that means,  $i_1$  minus  $i_2$  entering into the dot  $i_2$  the current  $i_2$  also entering into the dot right.

So, whenever this is in this is what you coil and this is actually frequency domain that is this is 2 dot and this is a mutual reactance  $j\omega M$  which is written here, this is  $j\omega L_1$  and this is directly  $j\omega L_2$  rest are same right. So, as the 2 currents are entering into the dot  $i_1$  minus  $i_2$  entering into the because this direction is  $i_1$  this direction is  $i_2$ , you have taken the resultant in this directions so,  $i_1$  minus  $i_2$  right. So,  $i_1$  minus  $i_2$  entering into the dot,  $i_2$  also entering into the dot so, that means, this way you will go for your dot conventioning while you write the equation plus sign will come and this the mutual inductance  $M$ .

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So, if you write like this, this is this equation so, that like your that it is same way you are putting that mutual inductance voltage generator in frequency domain we are writing it. So,  $i_1 - i_2$  is showing,  $j\omega L_1$  this will be  $j\omega M i_2$  right. So, plus will be the plus minus will be there because both  $i_1$   $i_2$  entering the dot  $i_1$   $i_2$  entering the dot; that means, along with your this dot it will be additive right little bit be careful about this. If I change the dot from here to here  $L_2$  suppose (Refer Time: 33:01) top if I put the bottom than polarity here will changed right you have to be careful about that.

So, similarly here also due to this  $i_1 - i_2$  voltage will be induced here so,  $j\omega M$  into  $i_1 - i_2$  right. So, you now what you do is, that 2 meshes are there. So, what you can do it you try to find out what will be  $v$  upon  $i_1$  what will be your  $v$  upon your  $i_1$  right ah. So, here you apply k v l here to apply k v l and solve it for  $i_1$  and  $i_2$ . So, I i will giving you the final solution so, some time will be safe right.

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Fig.16(c): Circuit with mutual-inductance voltage generators.  
Equivalent impedance of the parallel combination is given by

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$$Z_{eq} = \frac{v}{i_1} = \frac{j\omega(L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)}$$

Equivalent inductance,  
 $L_{eq} = \frac{(L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)}$

So, in this case if you solve it will be your what you call it will be actually small  $i_1$ . Here it is capital  $I_1$  actually it should have been by mistake I have a capital  $N$   $v$  upon. If you solve it, it will be your  $j\omega$  into  $L_1 L_2$  minus solve for your  $Z_{eq}$   $v$  upon  $i_1$  if you solve it will be  $j\omega L_1 L_2$  minus  $M^2$  divided by  $L_1$  plus  $L_2$  minus  $2M$   $i$  suggest you please derives this from the previous circuit. That this from here you please derive this from here right you derive this from here we apply  $k v l$  and solve it for  $v$  by  $i_1$ .



So, it is small  $i_1$  right small  $i_1$  you will get this. Now, look and that whether  $M$  is negative or positive,  $M$  square will be always positive right and it is  $L_1$  plus  $L_2$  minus  $2m$ . So, equivalent inductance will be  $L_1 L_2 - M^2$  divided by  $L_1 + L_2 - 2m$  right in this title.

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$$L_{eq} = \frac{(L_1 L_2 - M^2)}{(L_1 + L_2 - 2m)} \quad \dots \dots (8)$$

The sign of  $M$  in the denominator of Eqn. (8) changes with the dot polarity, but the numerator does not change since  $M$  is squared.

The sign of  $M$  in the denominator of equation 8 this we a changes with the dot polarity, but the numerator does not change since a you are what you call  $M$  square is positive. That means, if you put the dot if you inter change the dot right then sign will change sign of  $M$  will change right; that means, these equation; that means, these equation it becomes plus also these denominator right.

So, you have whatever dot you have taken according is just combiners it will inter change the dot and solve it will become plus right. That means this your  $M$  is negative or positive does the your it will be here it will change, but here  $M$  square is always positive right.

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In Eqn.(6), we have established that the ~~expression in the~~

$$L_1 + L_2 \geq 2M$$

Since overall inductance must be positive

$$L_1 L_2 - M^2 \geq 0 \quad \dots (9)$$

From Eqn.(6),

$$M \leq \left( \frac{L_1 + L_2}{2} \right) \quad \dots (10)$$

So, that is what is written in right. So, equation 6 we have established that  $L_1 + L_2 \geq 2M$ ; that means,  $M$  and here for since overall inductor that is for series circuit; you have seen series circuit  $L_1 + L_2 \geq 2M$  and the overall inductance must be positive, if  $L_1 L_2 - M^2 \geq 0$  right.

Here you have established that  $L_1 + L_2 \geq 2M$  that is always positive here also for making positive  $L_1 L_2 - M^2 \geq 0$  greater than equal to 0 or  $M \leq$  from this equation; from this equation  $M \leq$  equal to  $\frac{L_1 + L_2}{2}$  and from this equation  $M \leq$  your equal to root over your  $L_1 L_2$  right.

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The whiteboard content includes the following text and equations:

- Top equation:  $M \leq \left(\frac{L_1 + L_2}{2}\right)$  with a circled 4 next to it.
- Below that:  $\sqrt{3 \times 5}$  and  $\sqrt{15}$  written in blue.
- Text: "From eqn. (9),"
- Equation:  $M \leq \sqrt{L_1 L_2}$  with a circled 4 next to it.
- Handwritten notes: "3, 5" and " $\frac{3+5}{2} = 4$  (29)".

So, this is arithmetic mean and this is geometric mean right. So, geometric mean is always less than arithmetic mean; that means, this value this is the way. So, this is the value you have to taken for example, suppose you have you have a two number say 3 and 5 it is arithmetic mean is 3 by 5 by 2 is equal to 4 right.

But, if you take it geometric mean, it is 3 into 5 it will be root 15 right. So, this is geometric mean it is less than your what you call less than 4. So, geometric mean is less than the arithmetic mean except they are equal. If you take both are same 4 4 then both will be same right.

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Eqm (10) states that the mutual inductance must be less than the arithmetic mean of  $L_1$  and  $L_2$ .

While, Eqm (11) states that the mutual inductance must be less than the geometric mean of  $L_1$  and  $L_2$ .

But  
(geometric mean of  $L_1$  and  $L_2$ )  $<$   $L_1$

So, that means, equations 10 state that a mutual inductance must be less than the arithmetic mean of  $L_1$   $L_2$ , while equation eleven states that mutual inductance must be less than the geometric mean of  $L_1$  and  $L_2$ . But, geometric mean of  $L_1$   $L_2$  less than the arithmetic mean of  $L_1$   $L_2$  if they are not equal.

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inductance must be less than the geometric mean of  $L_1$  and  $L_2$ .

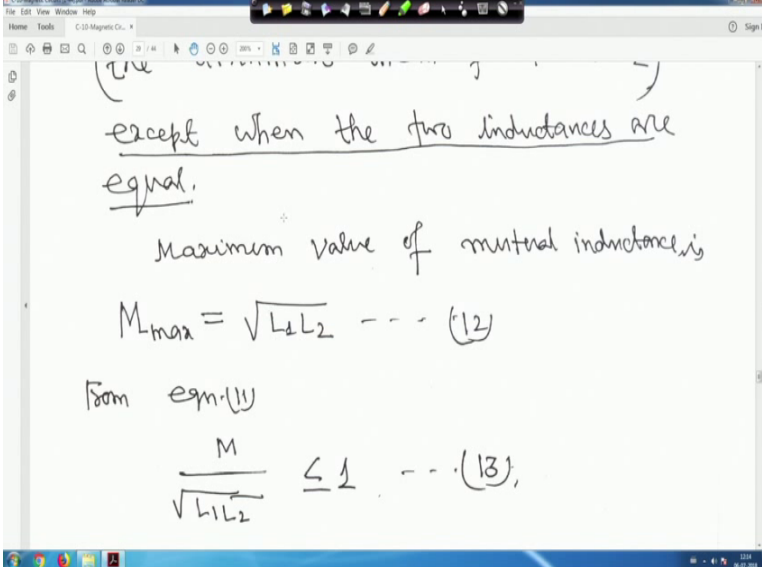
But  
(geometric mean of  $L_1$  and  $L_2$ )  $<$  Less than  
(the arithmetic mean of  $L_1$  and  $L_2$ )  
except when the two inductances are equal.

$$\sqrt{L_1 L_2} \leq \frac{L_1 + L_2}{2}$$

$L_1 = L_2$

For example; that means, root over  $L_1 L_2$  your less than equal to  $L_1$  plus  $L_2$  divided by 2 right. So, if  $L_1$  mean it will be equal only when  $L_1$  is equal to  $L_2$  otherwise it is less than right.

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except when the two inductances are equal.

Maximum value of mutual inductance is

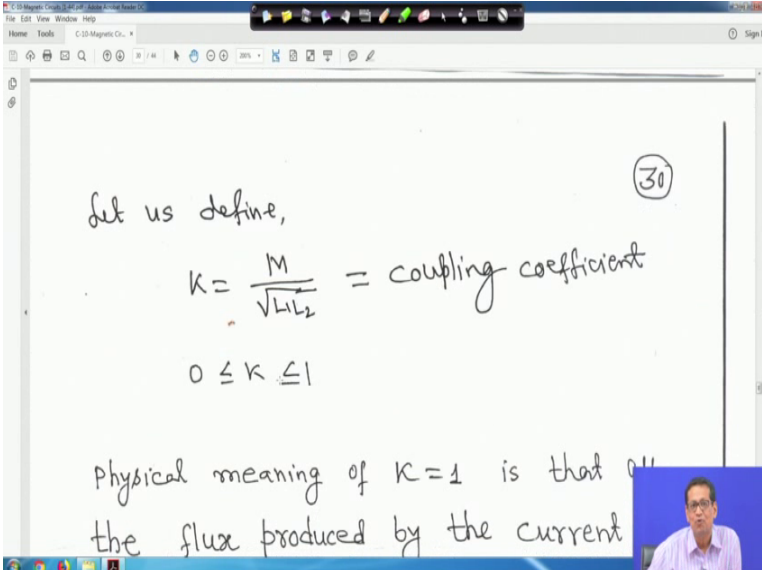
$$M_{\max} = \sqrt{L_1 L_2} \quad \dots (12)$$

From eqn. (11)

$$\frac{M}{\sqrt{L_1 L_2}} \leq 1 \quad \dots (13)$$

So, therefore, the maximum value of mutual inductance can be taken as  $M_{\max}$  is equal to root over  $L_1 L_2$ , but not  $L_1 L_2$  by 2 right. Therefore, some equation 11, we define that will get  $M$  by root over  $L_1$  upon  $L_2$  less than equal to 1 this is equation 13.

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let us define,

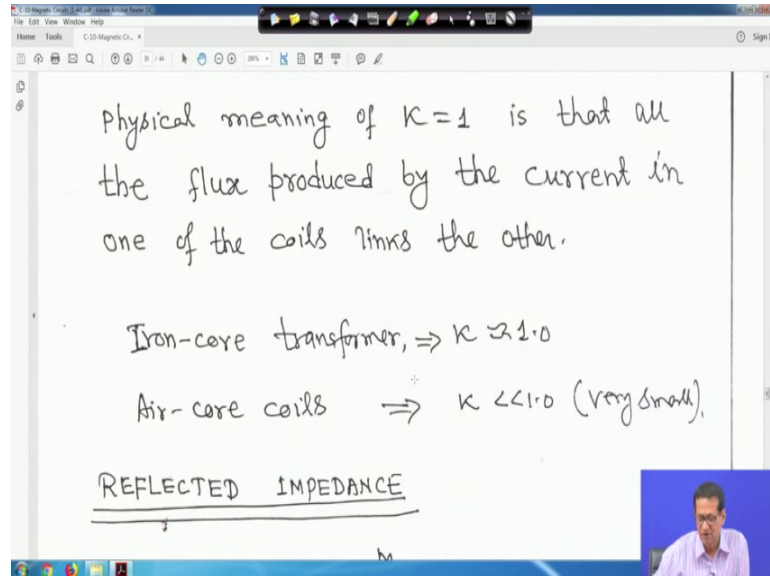
$$k = \frac{M}{\sqrt{L_1 L_2}} = \text{coupling coefficient}$$

$$0 \leq k \leq 1$$

Physical meaning of  $k=1$  is that all the flux produced by the current

Therefore, we define  $K$  is equal to say this term  $k$  is equal to  $M$  by root over  $L_1 L_2$ , this is called coupling coefficient and  $K$  line between 0 and 1 right.

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Physical meaning of  $K=1$  is that all the flux produced by the current in one of the coils links the other.

Iron-core transformer,  $\Rightarrow K \approx 1.0$

Air-core coils  $\Rightarrow K \ll 1.0$  (very small).

REFLECTED IMPEDANCE

Physical meaning of  $K=1$  is that, all the flux produced by the current in one of the coil links the other right. So, if it is  $K$  is equal to 1 iron core transformer  $K$  is approximately equal to 1 and air core coil  $K$  will much much less than equal to 1 that is very very small right so, with that.

Thank you very much will be back again.