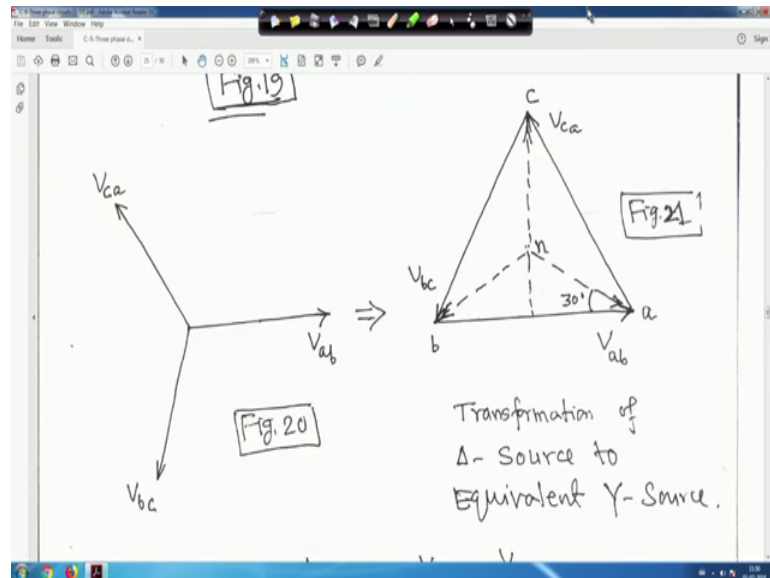


**Fundamentals of Electrical Engineering**  
**Prof. Debapriya Das**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 49**  
**Three phase circuits (Contd.)**

(Refer Slide Time: 00:22)



So, just now we have seen that transformation of your delta source to equivalent your what you call this equivalent to your; equivalent to your star source right. So, I showed you that how one can do it. So, this is actually this is your what you call if for if it is your  $V_{ab}$ ,  $V_{bc}$ ,  $V_{ca}$ , you form the delta I told you right that  $V_{bc}$  so this direction and  $V_{ca}$  this direction, then you make equivalent star, and this point you mark as  $n$  right. And accordingly, I showed you that how the relationships are right.

(Refer Slide Time: 00:58)

$$|V_{an}| \angle -30^\circ = \frac{|V_{ab}|}{2} = \frac{V_p}{2} = \frac{V_L}{2}$$
$$\therefore |V_{an}| = \frac{V_L}{\sqrt{3}}$$
$$\therefore V_{an} = \frac{V_L}{\sqrt{3}} \angle -30^\circ = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$
$$V_{bn} = \frac{V_L}{\sqrt{3}} \angle -150^\circ = \frac{V_p}{\sqrt{3}} \angle -150^\circ$$
$$V_{cn} = \frac{V_L}{\sqrt{3}} \angle 90^\circ = \frac{V_p}{\sqrt{3}} \angle 90^\circ$$

So, now so this is all this relationships and you one thing is (Refer Time: 01:00) this thing that note that your that angle between this what you call voltages, it will be difference should be 120 degree, this all we have seen right.  $V_{an}$  is equal to  $V_L$  upon root 3 is equal to  $V_p$  upon root 3 angle minus in the delta case line voltage is equal to phase voltage right.

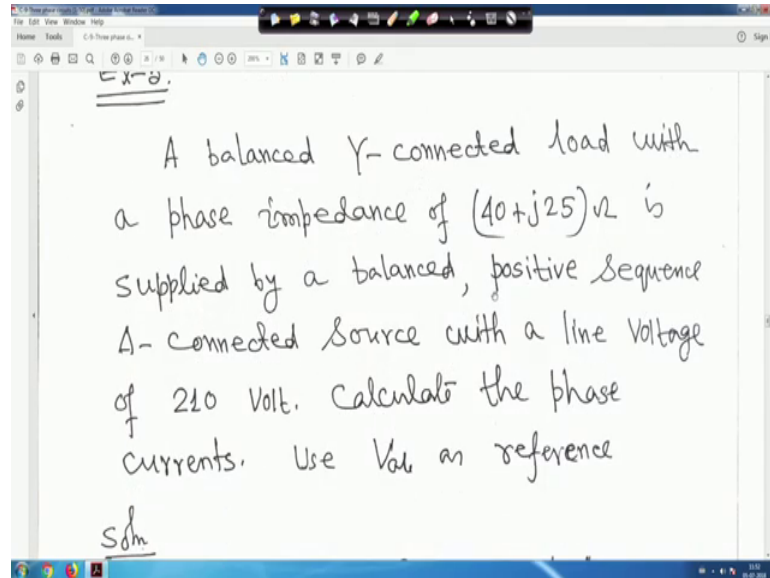
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$$V_{AN} = I_a Z_Y = \frac{V_L}{\sqrt{3}} \angle -30^\circ = \frac{V_p}{\sqrt{3}} \angle -30^\circ \quad (26)$$
$$V_{BN} = V_{AN} \angle -120^\circ$$
$$V_{CN} = V_{AN} \angle 120^\circ$$

So, similarly  $V_{AN}$  will be  $I_a Z_Y$  is equal to  $V_L$  upon root 3 angle minus 30 degree is equal to  $V_p$  upon root 3 angle minus 30 degree. Similarly, for  $V_{BN}$ , and similarly for  $V_{CN}$ .

CN right. So, this is what we call that your this transformation of delta source to equivalent star source, just show you that how one can do it right.

(Refer Slide Time: 01:51)



So, next you take one small example that a balanced star connected load with a phase impedance  $40 + j 25$  ohm is supplied by a balanced, positive sequence delta connected source. (Refer Time: 02:02) say positive sequence delta connected source means, it will be you take a, b, c sequence right with a line voltage of 210 volt it is given right. Calculate the phase current, use  $V_{ab}$  as reference phasor right.

So,  $Z_y$  that star connected it is so,  $Z_{star}$  or  $Z_y$  is given  $40 + j 25$  is equal to  $47.17$  angle  $32$  degree ohm. And  $V_{ab}$  is the reference voltage. So, here it is with a line voltage of to your line voltage means, it is a line to line voltage right. Whenever you say line voltage, it is line to line voltage. Whenever we say phase voltage, it is phase to neutral right.

(Refer Slide Time: 02:43)

CURRENTS, use val v1 = 0-j120...

Soln

$$Z_Y = (40 + j25) = 47.17 | 32^\circ \Omega$$
$$V_{ab} = 210 | 0^\circ \text{ Volt.}$$
$$I_a = \frac{(V_{ab} | -30^\circ)}{Z_Y} = 2.57 | -62^\circ \text{ Amp}$$

So,  $V_{ab}$  is equal to 210 angle 0 degree, this is the reference we have to take. So, we know we have derived this we know,  $I_a$  is equal to  $V_{ab}$  upon root 3 angle minus 30 degree by  $Z_Y$ . Just now, we have derived this formula. So, just go through this right. So, if you  $V_{ab}$  is equal to 200 by 210, if you substitute  $V_{ab}$  and  $Z_Y$ , you will get 2.57 angle minus 62 degree ampere. This is your  $Z_Y$  is given, and  $V_{ab}$  is also given. So, you just simplify, it will get 2.57 angle minus 62 degree ampere.

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(27)

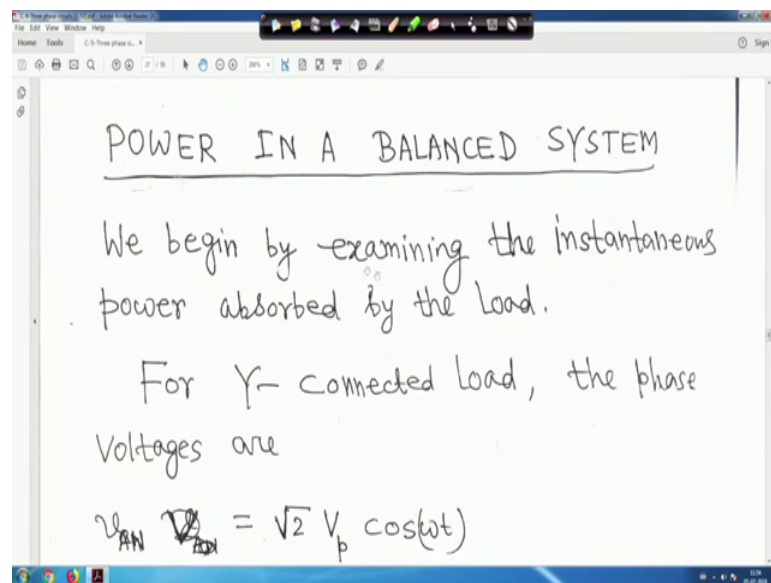
$$I_b = I_a | -120^\circ = 2.57 | -182^\circ \text{ Amp.}$$
$$I_c = I_a | 120^\circ = 2.57 | 58^\circ \text{ amp.}$$

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POWER IN A BALANCED SYSTEM

So, now we know  $I_b$  is equal to  $I_a$  angle minus 120 degree. So, this  $I_a$  value 2.57 angle minus 62 degree, your you substitute here. If you substitute here, so it will be minus 62 minus 120 angle will be minus 182 degree and this magnitude of the current will be mentioned 2.57 ampere. Similarly,  $I_c$  also is equal to  $I_a$  angle 120 degree. So, here it will be  $I_a$  is equal to 2.57 an angle minus 62 plus 120, so it will be 58 degree ampere right.

(Refer Slide Time: 03:55)



Next is a power in a balanced system in a balanced system. Now, we begin by examining the instantaneous power absorbed by the load right, so power in a balanced system so power, because we have to see the three-phase power also. For star connected load, the phase voltages are this is actually  $v_{AN}$ , we can say phase voltage  $v_{AN}$ , we are making it capital AN right star connected load. So, a I mean it is your it is your something like this.

Suppose, you have a suppose you have a we have seen already I have made it for you before right, this is a star connected load right, this is a star connected load. And this is your capital A say, this is capital B, and this is C, and this is capital N right, and that is why this is this is called  $v_{AN}$ . This is your  $v_{AN}$ , this is  $v_{BN}$ , and this is  $v_{CN}$  right.

And your phase voltage phase voltage is  $V_p$  that is your that is your magnitude, magnitude is the phase voltage that is rms value. And multiplied by root 2 means it is the peak value and represented by the cosine term sin cosine right cosine also you can

convert it to sin your what you call sin; sin 90 degree minus theta cos theta right. So, we are representing this as a cosine thing right say cosine (Refer Time: 05:08), So, now let me clear it.

(Refer Slide Time: 05:19)

Voltages are

$$v_{AN} = \sqrt{2} V_p \cos(\omega t)$$

$$v_{BN} = \sqrt{2} V_p \cos(\omega t - 120^\circ)$$

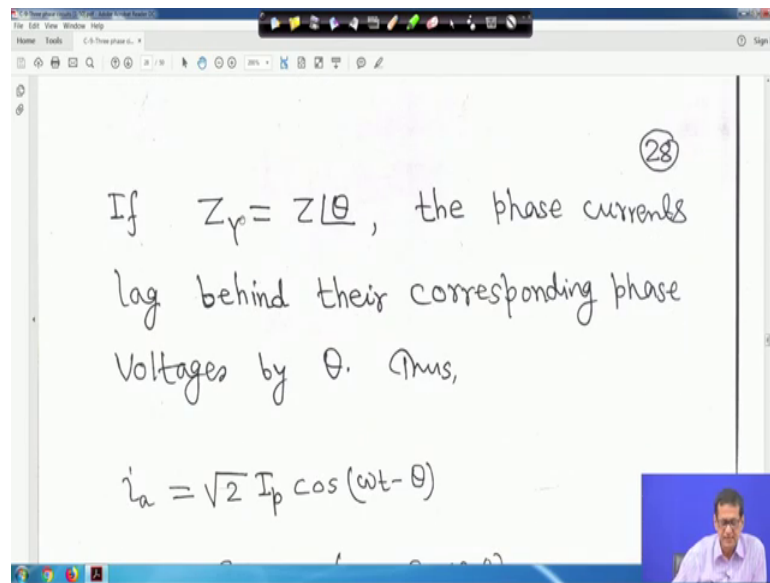
$$v_{CN} = \sqrt{2} V_p \cos(\omega t + 120^\circ)$$

$V_p \Rightarrow$  r.m.s. Value ( $V_p^{\max} = \sqrt{2} V_p$ )

So, in this case that means, the earlier we have seen that your  $v$  is equal to we have already seen know,  $v$  we have already seen for single-phase circuit  $v$  is equal to  $v_m$  let me clear it,  $v$  is equal to  $v_m$  that sin of  $\omega t$ . So,  $v_m$  is the peak value. And  $v_m$  is equal to actually root 2 into  $v$  rms value that we have seen for single-phase circuit. So, same thing is here also instead of sin, we are represent it by cosine, meaning is same right.

So, in this case, this peak value is equal to then root 2  $V$ . So,  $V_p$  is the your what you call that is the phase voltage, but it rms value, so multiplied by root 2 is peak value. So, next one will be it will be  $\omega t$  minus 120 degree understandable same as before three-phase system. And for  $v_{CN}$ , it will be root 2  $V_p$  again cos  $\omega t$  plus 120 degree right. So, this is a representation of  $v_{AN}$ ,  $v_{BN}$  and  $v_{CN}$  right. It is written here  $V_p^{\max}$  is equal to root 2  $V_p$  peak value right. So, this is 120 degree apart. And  $V_p$  I have told you, it is phase voltage phase value, but it is rms value right, so phase voltage rms value right. So, let me clear it.

(Refer Slide Time: 06:38)



28

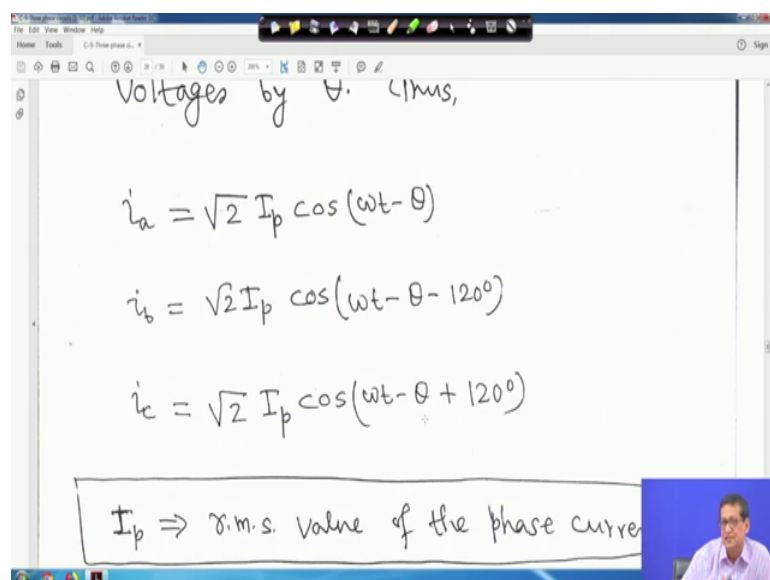
If  $Z_Y = Z \angle \theta$ , the phase currents lag behind their corresponding phase voltages by  $\theta$ . Thus,

$$i_a = \sqrt{2} I_p \cos(\omega t - \theta)$$

A small video inset in the bottom right corner shows a man speaking.

So, now angle the current sorry the impedance  $Z_Y$  is equal to  $Z \theta$  say. So, the phase current lag behind their corresponding phase voltage by  $\theta$ . So, this is my star impedance  $Z_Y$  what you call your impedance of the star connected load, so that is  $Z \theta$ , it is  $Z \angle \theta$ , it is balanced one. The phase current lag behind their corresponding phase voltage by  $\theta$ .

(Refer Slide Time: 07:02)



Voltages by  $\theta$ . Thus,

$$i_a = \sqrt{2} I_p \cos(\omega t - \theta)$$
$$i_b = \sqrt{2} I_p \cos(\omega t - \theta - 120^\circ)$$
$$i_c = \sqrt{2} I_p \cos(\omega t - \theta + 120^\circ)$$

$I_p \Rightarrow$  r.m.s. value of the phase current

A small video inset in the bottom right corner shows a man speaking.

Thus, we can say  $i_a$  also we can write that  $\sqrt{2} I_p \cos(\omega t - \theta)$ , this is (Refer Time: 07:09) current will lag by your voltage here, voltage here it is  $\omega t$

minus 120 degree,  $\omega t + 120$  degree, and current will lag from this voltage. So, current also can be represented as  $i_a$  is equal to  $\sqrt{2} I_p$ ,  $I_p$  is the your what you call rms value of the current that is your phase current right, so phase value, so multiplied by  $\sqrt{2}$ , so this is your peak value right, and it is star connected. So,  $i_a$  is equal to  $\sqrt{2} I_p \cos(\omega t - \theta)$  right.

So, your what it is written here  $I_p$  is the rms value of the phase current right. So,  $I_p$  is equal to  $\sqrt{2} I_p \cos(\omega t - \theta)$  (Refer Time: 07:50) minus 120 degree, because shifting by 120 degree. And  $i_c$  will be  $\sqrt{2} I_p \cos(\omega t - \theta + 120)$  degree. For voltages, it was  $\cos(\omega t)$ ,  $\cos(\omega t - 120)$  degree, (Refer Time: 08:02)  $\omega t + 120$  degree, because the current lags (Refer Time: 08:05) angle  $\theta$ , so minus  $\theta$ , minus  $\theta$ , minus  $\theta$  will be introduced here right, this is understandable.

(Refer Slide Time: 08:12)

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases, i.e.,

$$p = p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c$$

$$p = 2V_p I_p \left[ \cos(\omega t) \cos(\omega t - \theta) + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ) \right]$$

Now, the total instantaneous power in the load is the sum of the instantaneous power in the three phases right, so all the three phases right. So, if you do so,  $p$  is equal to  $p_a$  plus  $p_b$  plus  $p_c$  is equal to  $v_{AN}$  into  $i_a$  plus  $v_{BN}$  into  $i_b$  plus  $v_{CN}$  into  $i_c$  right. I mean the circuit wise if we draw like this, it will be something like this.

Suppose, this is your star connected, this is your star connected right load, so this is star connected. So, this is your end, this is your A, suppose this is your B, this is your C. This current is say  $i_a$  right, this current is say  $i_b$ , and this current is say  $i_c$  right. So, this here



then it will be  $v_{AN}$  for this phase into  $i_a$ . Similarly, for this phase, it will be  $v_{BN}$  into  $i_b$ . For this phase, it will be  $v_{CN}$  into your what you call the  $i_c$  this (Refer Time: 09:15) point is N right, so that is why we are making it  $v_{AN}$  into  $i_a$  plus  $v_{BN}$  into  $i_b$  plus  $v_{CN}$  into  $i_c$  right in I mean (Refer Time: 09:28) is phase.

And if you substitute your  $i_a$ ,  $i_b$ ,  $i_c$  and from here your  $v_{AN}$ ,  $v_{BN}$  and  $v_{CN}$ , if you substitute and simplify, please do this right. So, such that if you do this, it will be in this form  $p$  is equal to  $2 V_p I_p$ , then  $\cos \omega t \cos \omega t - \theta$  plus  $\cos \omega t - 120^\circ$  into  $\cos \omega t - \theta - 120^\circ$  plus  $\cos \omega t + 120^\circ$  into  $\cos \omega t - \theta + 120^\circ$ , you simplify this I have written the final one, but you simplify this right.

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(29)

Apply  $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

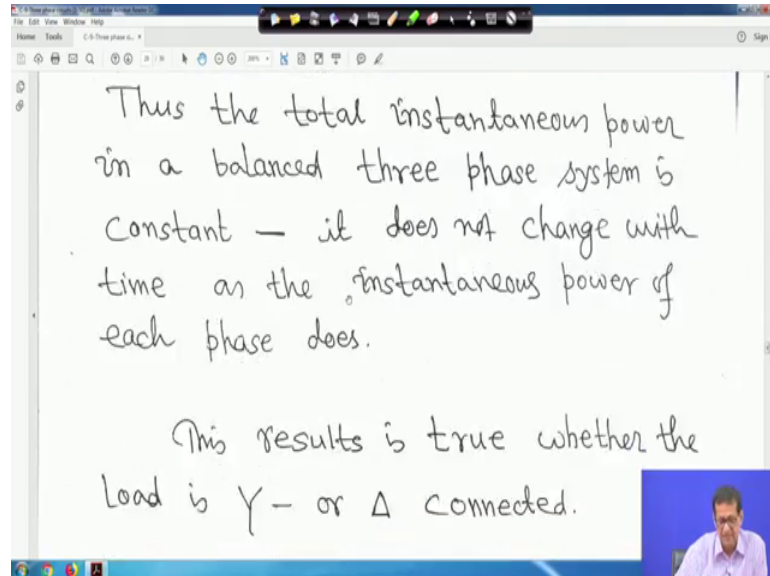
$\therefore p = 3 V_p I_p \cos \theta$

Thus the total instantaneous power

So, I mean (Refer Time: 10:07) mathematics trigonometric formula, you use this formula like this  $\cos A \cos B$  is equal to half  $\cos A + B$  plus  $\cos A - B$  and simplify. If you do so, it will become  $p$  is equal to  $3 V_p I_p \cos \theta$  right. So, thus the total instantaneous power in a balanced three-phase system is constant that means, it is time invariant, it is not a function of time right. So, it is time invariance. So, it is system is constant. It does not change with time as the instantaneous power of each phase does. For instantaneous power, for your what you call each phase it is that as the each for each phase does, (Refer Time: 10:47) for three-phase if you make it together, it will be  $3 V_p I_p$

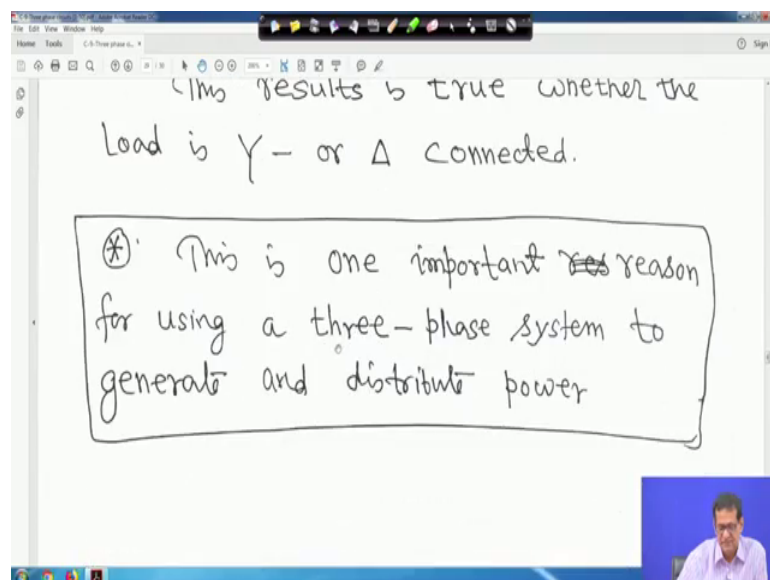
$p \cos \theta$ , so it is independent of time right. This is one of the main advantages of three-phase using three-phase system right.

(Refer Slide Time: 10:53)



So, the results is true whether the load is a star or delta whether star or delta, this this  $3 V p I p \cos \theta$  is true right.

(Refer Slide Time: 11:10)



So, this is one important reason for using three-phase system to generate and distribute power because of your, this is power (Refer Time: 11:16) what you call independent of the time.

(Refer Slide Time: 11:22)

(30)

Since the total instantaneous power is independent of time, the average power per phase  $P_p$  for either  $\Delta$ -connected load or the  $Y$ -connected load, is  $P/3$ .

$$\therefore P_p = V_p I_p \cos \theta$$

A small video inset of a man speaking is visible in the bottom right corner of the whiteboard interface.

So, that means, since the total instantaneous power is independent of time I told you, the average power per phase  $P_p$  for either delta connected load or the star connected load will be  $P/3$ , because it is the balanced system, it is a balanced system. Therefore,  $P_p$  is equal to then for phase power will be  $P/3$ ,  $P = 3 V_p I_p \cos \theta$ , so  $P_p$  will be  $V_p I_p \cos \theta$  for phase right.

(Refer Slide Time: 11:46)

load, is  $P/3$ .

$$\therefore P_p = V_p I_p \cos \theta$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin \theta.$$

The apparent power per phase is

A small video inset of a man speaking is visible in the bottom right corner of the whiteboard interface.

And the similarly, and the reactive power per phase will be  $Q_p$  is equal to  $V_p I_p \sin \theta$  right. So, the apparent power per phase will be  $S_p$  will be is equal to  $V_p I_p$  right, so no multiplied of  $\cos \theta \sin \theta$ , simply will be  $V_p I_p$ .

(Refer Slide Time: 12:04)

The screenshot shows a whiteboard with the following text and equations:

The apparent power per phase

$$S_p = V_p I_p$$

The complex power per phase is

$$P_p + jQ_p = V_p I_p^*$$

A small video inset in the bottom right corner shows a man speaking.

So, the complex power per phase will be  $P_p + jQ_p$  is equal to  $V_p I_p$  conjugate. This thing has been explained also for single-phase right, so meaning is very simple.

(Refer Slide Time: 12:16)

The screenshot shows a whiteboard with the following text and equations:

Total average power,

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta$$

$$\therefore P = \sqrt{3} V_L I_L \cos \theta$$

For a Y-connected load

A circled number (31) is written in the top right corner.

A small video inset in the bottom right corner shows a man speaking.

So, total average power will be then  $P_a + P_b + P_c$  is equal to  $3 P_p$ , so it will be  $3 V_p I_p \cos \theta$  right. But, this can be written as  $\sqrt{3} V_L I_L \cos \theta$  load. For a star

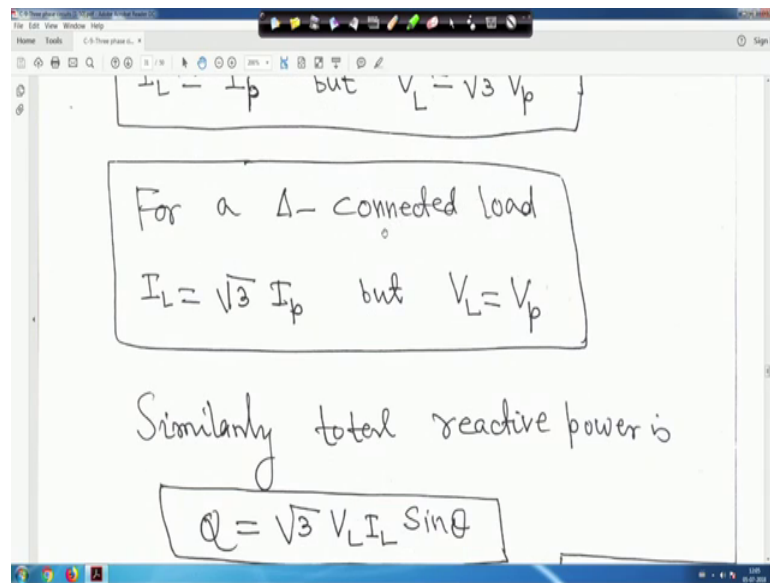
connected load line current is equal to phase current. When you will solve numericals, this thing should always remain in your mind right. So, for a star connected load, line I told you earlier line current is equal to phase current, but your line voltage right line voltage will be root 3 times phase voltage.

The meaning is one (Refer Time: 12:46) I am telling, suppose this is your star connected load right, suppose this is your star connected load, this is your star connected load right. So, in this case, this is my line current, this is my line current, same current is going to this phase, so this is my phase current. So, line current is equal to phase current, this way I am telling you. And this point this point is your N right, this point is your N. And this is your what you call say this is my v a, b and c.

So, my  $V_{an}$  actually this again we make small n for your understanding, so it is small n. So,  $V_{an}$  is this point, this is my phase voltage. But your what you call, but line to that is your what you call that  $V_p$  is equal to say all balanced it is, so it is  $V_{an}$  is equal to  $V_{bn}$  is equal to  $V_{cn}$  magnitude right. So,  $V_p$  is equal to  $V_{an}$  but, line to line voltage is this (Refer Time: 13:43) this one, so this is my  $V_{ab}$  right. So, magnitude of  $V_{ab}$  right is equal to then root 3 time your phase voltage that is root 3 time  $V_{an}$ . But,  $V_{an}$  is equal to  $V_p$ , so it will be root 3 time  $V_p$ . So, line voltage is equal to root 3 time phase voltage. So, let me clear it.

So, there is that is why for a star connected load that is why this one it can be written as your  $I_L$  is equal to  $I_p$  and  $V_L$  is equal to that means, it is actually it is  $3 V_p I_p \cos \theta$ , this can be written as root 3, then in bracket you write your root 3 your what you call  $V_p$  right into  $I_p$  then  $\cos \theta$ . So, root 3 will be there, I mean root 3  $V_p$  is equal to your line voltage, it is  $V_L$ , and this  $I_p$  is equal to  $I_L$  right, because phase current is equal to line current (Refer Time: 14:39) you star your star connected load. Then it will be  $I_L$ , then  $\cos \theta$ . So, this term is written like this, so it is root 3  $V_L I_L \cos \theta$  right. So, this is for a star connected load.

(Refer Slide Time: 14:54)



Similarly, for a delta connected load, just opposite. So, no need to explain again, similarly you can do it. Line current is equal to root 3 time phase current, but  $V_L$  is equal to  $V_p$  (Refer Time: 15:03) just let me make it for you. So, this is my delta connected load, this is my delta connected load right and this is phase a, phase b, phase c. Suppose, this is a, this is b, this is c right, no (Refer Time: 15:20) N point is involved here. So, this is balanced. So, this is my your so line current. And as per magnitude wise, this is my phase current. So, this is my  $I_p$  magnitude wise right.

And in that case, your then here line voltage is equal to phase voltage your what you call line voltage is equal to phase voltage. For example, if it is  $V_{an}$  that is the voltage across this one only, it is also delta connected load. So, line voltage  $V$  what you call this is my  $V_{ab}$ , this is my  $V_{ab}$  is equal to basically this is also voltage across this is phase voltage same as the line voltage. So, it is  $V_{ab}$  is equal to  $V_p$  say magnitude. Similarly, for  $V_{bc}$  (Refer Time: 16:01) all are  $V_p$  phase voltage. But, here the value here the phase current is here, and this is line current. So, line current will be is equal to root 3 times phase current, this we have seen earlier also.

So, similar way that means, similar way for delta connected load also, just  $I_L$  should be your root 3  $I_p$  and  $V_L$  should be is equal to  $V_p$ , line voltage is equal to phase voltage. This you have to keep it in your mind from the your what you call from numerical point of view right. Similarly, total reactive power is  $Q$  is equal to root 3  $V_L I_L \sin \theta$ .

(Refer Slide Time: 16:36)

Summary total reactive power is

$$Q = \sqrt{3} V_L I_L \sin \theta$$

Total complex power is,

$$3(P_p + jQ_p) = 3V_p I_p^* = 3V_p \left( \frac{V_p}{Z_p} \right)^*$$

$$= \frac{3V_p V_p^*}{Z_p^*} = \left( \frac{3V_p^2}{Z_p^*} \right)$$

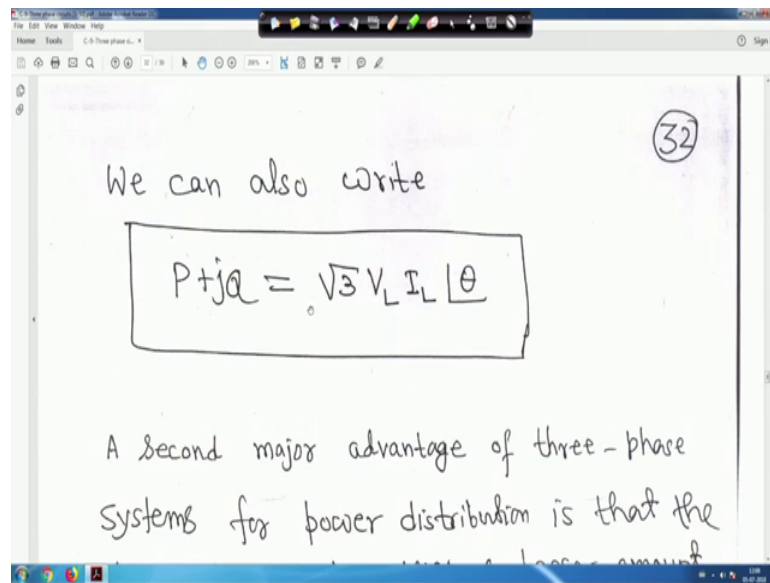
$Z_p = Z_l \theta$   
 = load impedance per phase  
 $Z_p = Z_Y$   
 or  
 $Z_p = Z_\Delta$

So, total complex power, it will be three-phase right. So, power phase power is  $P_p$  plus  $jQ_p$ , so multiplied by 3. So,  $3P_p$  plus  $j3Q_p$  is equal to this one.  $P_p$  plus  $jQ_p$  is equal to we can write  $V_p I_p$  conjugate we have seen, so it will be  $3V_p$ , and  $I_p$  will be is equal to  $V_p$  upon  $Z_p$  whole conjugate, so that means,  $3V_p V_p$  conjugate upon  $Z_p$  conjugate. So,  $V_p$  into  $V_p$  conjugate will be magnitude  $V_p$  square.

Because, if  $V_p$  suppose if  $V_p$  is equal to  $V_p$  angle theta for example say, then  $V_p$  conjugate will be  $V_p$  angle minus theta. So, if you multiply this, it will be  $V_p$  and  $V_p$   $V_p$  square magnitude and it is theta minus theta (Refer Time: 17:23) angle will be 0. So, it is just  $V_p$  square that is why it is  $V_p$  square  $3V_p$  square. And this is this whole thing is conjugate, so  $V_p$  conjugate is here divided by  $Z_p$  conjugate, so this is  $Z_p$  conjugate right.

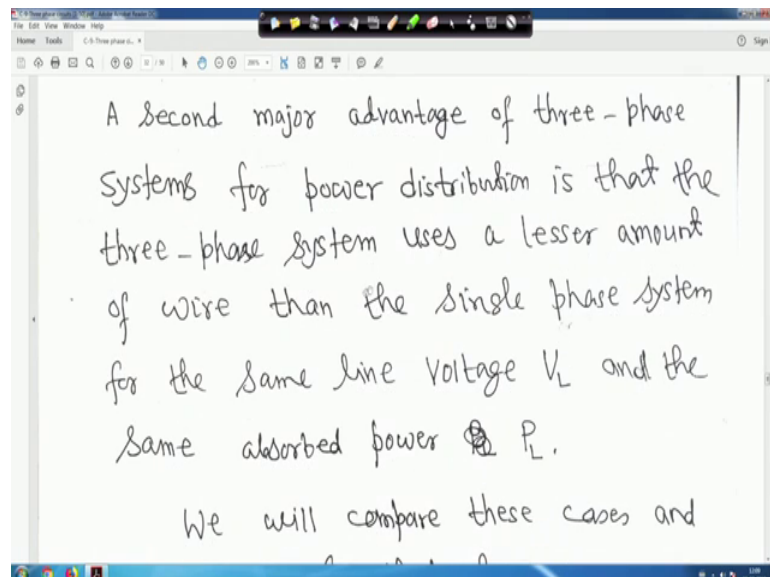
And  $Z_p$  is equal to your, what you call  $Z_p$  angle theta. So,  $Z_p$  is equal to  $Z_p$  angle theta, then  $Z_p$  conjugate will be  $Z_p$  angle minus theta, this is magnitude right. Sometimes I write like this (Refer Time: 17:50) it is understandable to you right rather than putting bar not (Refer Time: 17:54) but this is magnitude, this is angle understandable to you right. So, now right so, impedance per phase, so  $Z_p$   $Z_p$  phase is equal to  $Z_Y$  or  $Z_p$ , this is  $Z_p$  actually,  $Z_p$  is equal to  $Z_\Delta$ . So, this is  $3V_p$  square upon  $Z_p$  your conjugate right.

(Refer Slide Time: 18:18)



So, now we can write now, then  $P$  plus  $jQ$ , so we also can write  $P$  plus  $jQ$  is equal to root 3  $V_L I_L \angle \theta$ . Then (Refer Time: 18:24) if you make it, it will be root 3  $V_L I_L \cos \theta$  plus  $j$  root 3  $V_L I_L \sin \theta$ . Therefore,  $P$  is equal to root 3  $V_L I_L \cos \theta$  and  $Q$  is equal to root 3  $V_L I_L \sin \theta$ . So, we can write like this also right.

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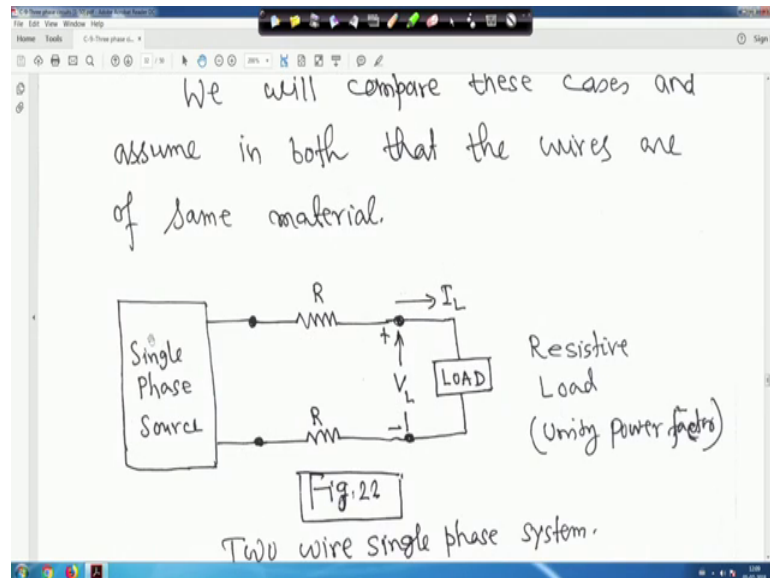


So, another thing is that that is second major advantage of three-phase system for power distribution is that the three-phase system uses a lesser amount of wire right. So, three-phase system your what uses a lesser amount of wire than the single-phase system for the



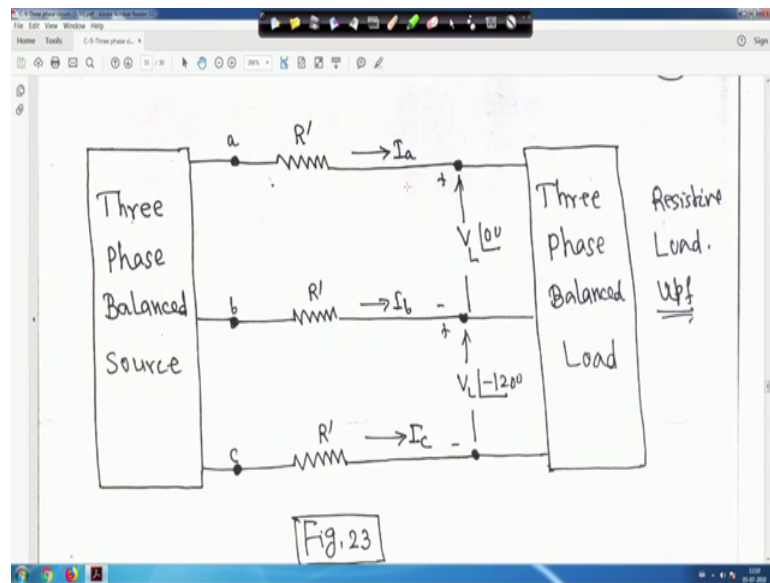
same line voltage  $V_L$  and the same absorbed power  $P_L$  right. So, this is your what you call that another major advantage. Now, how you will do this, you have to compare. For example, suppose we will compare these cases and assume in both that the wires are in the same material in both the cases single-phase or three-phase, we assume that wires are of made same material right.

(Refer Slide Time: 19:20)



Therefore, suppose if you take a single-phase source and suppose this is incoming path and this is return path, so resistance is  $R$  and  $R$ , so it will be two  $R$ . And this is the load connected for the sake of our calculations easy calculations are for the sake of explanation. This load is a purely resistive load right, this is a purely resistive load. If load is purely resistive means it is unity power factor load, because  $Z$  is equal to say  $R$  plus  $jX$ . If  $X$  is equal to 0, then it is purely resistive load. And across the load, the voltage is say  $V_L$  that is the load voltage; across the load, this voltage is  $V_L$  right. So, this is two wire single-phase system. This is incoming path; this is return path. And the resistance here is  $R$  same wire, so here is  $R$  and same length. So, this is figure 22 right.

(Refer Slide Time: 20:15)



Now, now this one your if for three-phase system, we assumed also same thing that your three-phase systems, so resistance is  $R'$ ,  $R'$ ,  $R'$ ; three lines is there phase a, b, c; this side is three-phase balanced source right, and this is three-phase balanced load. And we are also we are assuming it is a for each whether it is a star or delta does not matter, we assume a pure resistive load, so power factor of the load is unity right.

And here also what you call voltage across the your what you call that  $V_{ab}$  that is this side is  $V_{ab}$ , this is  $R'$  is there some voltage (Refer Time: 20:48) will be there. But, across that the phase (Refer Time: 20:50) what you call line to line voltage across this load, this is  $V_L \angle 0^\circ$ , and this is say b to c it is  $V_L \angle -120^\circ$ .

When you are writing that plus minus, plus minus, it is instantaneous polarity that means, my  $V_{ab}$  is equal to  $V_L \angle 0^\circ$ , and  $V_{bc}$  is equal to  $V_L \angle -120^\circ$  this is whatever written here. This is a to b, so this is basically line this line, this line, so basically it is your what you call if this we can write this we can write if we ignore this loss in this line right, then this  $V_{ab}$  is equal to  $V_L \angle 0^\circ$  is equal to  $V_{bc}$ . But, as the current is flowing, this is my  $I_a$  so there will be voltage drop here. So this one whatever  $V_L$  will be, it will be slightly less than a b right.

So, anyway, so in this case your, what you call in this case  $V_{ab}$  or  $V_{bc}$ , their magnitude not exactly equal to the your what you call the (Refer Time: 21:50) because there is some voltage drop will be there in this  $R'$  in the resistance right. But, anyway nothing to bother

about that, we will show that three-phase what you call takes less material material than the your single-phase.

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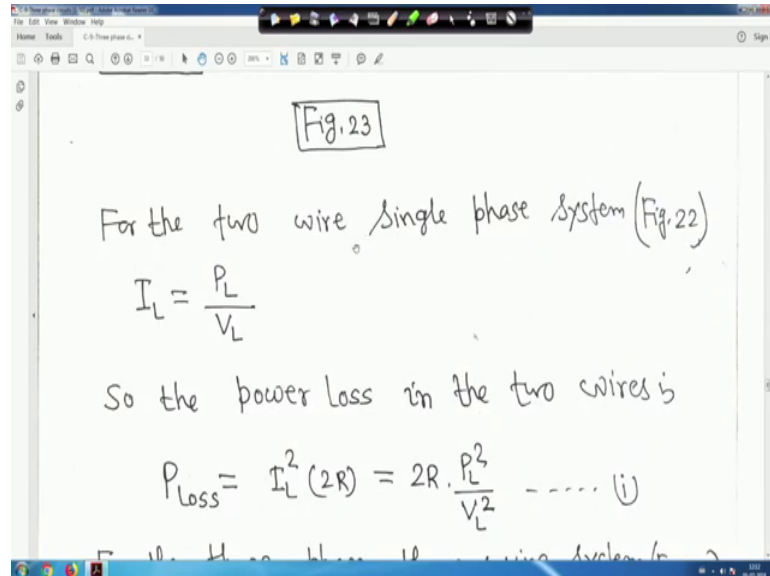


Fig. 23

For the two wire single phase system (Fig. 22)

$$I_L = \frac{P_L}{V_L}$$

So the power loss in the two wires is

$$P_{\text{Loss}} = I_L^2 (2R) = 2R \cdot \frac{P_L^2}{V_L^2} \quad \text{----- (i)}$$

So, now for the two wire single-phase system that is figure this from this figure, it will be  $I_L$  is equal to simply  $V_L$  right  $P_L$  upon  $V_L$ , so it is a resistive load. If power is the  $P_L$  resistive load, then  $I_L$  will be simply  $P_L$  upon say  $V_L$  right. So, what you call it is your AC signal phase system. So, if you take  $V_L$  angle 0 also, it will be  $V_L$  only right. And it is a resistive, so no  $Q$ , so  $I_L$  should be is equal to  $P_L$  upon  $V_L$ .

Then the power then the your what you call this current is  $I_L$ . Then power loss will be  $I_L$  square into  $2R$ , because here it is  $R$ , here it is  $R$  right. If you substitute this one, you will get  $P_L$  square into  $2R$  upon  $V_L$  that is what has been done right; that is what have been done here.

(Refer Slide Time: 23:07)

So the power loss in the two wires is

$$P_{\text{loss}} = I_L^2 (2R) = 2R \cdot \frac{P_L^2}{V_L^2} \text{ ----- (1)}$$

For the three phase - three wire system (Fig. 23)

$$I_L' = |I_a| = |I_b| = |I_c| = \frac{P_L}{\sqrt{3} V_L}$$

The power loss in the three wires is

$$P_{\text{loss}}' = (I_L')^2 (3R') = 3R' \cdot \frac{P_L^2}{3 V_L^2} = R' \frac{P_L^2}{V_L^2}$$

So,  $2R$  into  $P_L$  square, this is equation 1. Now, for the three-phase three wire system,  $I_L$  will be the magnitude  $I_a$  magnitude  $I_b$  magnitude  $I_c$  is equal to your say  $P_L$  upon root 3  $V_L$ . Because, your it is unity power factor load, so root 3  $V_L$ , we know that general root 3  $V_L$ , the current we have taken  $I_L$ . Because, for single-phase, we have taken  $I_L$  that is why it is  $I_L$  dash  $I_L$  dash right  $\cos \theta$  is equal to your  $P_L$  plus  $jQ_L$  that is the formula right, sorry is equal to  $P_L$  right.

So, now question is that  $\cos \theta$  is equal to unity, because resistive load right it is a resistive load. Therefore,  $v I_L$  dash will be  $P_L$  divided by root 3  $V_L$  right. So, now the the power loss in the three wires will be  $I_L$  dash square into 3  $R$  dash, because three lines are there, so 3  $R$  dash. So, it will be 3  $R$  dash into  $P_L$  square upon 3  $V_L$  square. So, 3, 3 will be cancel, it will be  $R$  dash into  $V_L$  square upon  $V_L$  square. So, this is equation 2 right.

(Refer Slide Time: 24:17)

(34)

Eqm (i) ÷ Eqm (ii)

$$\frac{P_{\text{Loss}}}{P'_{\text{Loss}}} = \frac{2R}{R'}$$

$$R = \frac{\rho l}{\pi r^2}$$

$$R' = \frac{\rho l}{\pi (r')^2}$$

$$\therefore \frac{P_{\text{Loss}}}{P'_{\text{Loss}}} = \frac{2(r')^2}{r^2} \dots (iii)$$

Now, if you divide equation 1 by equation 2, you will get  $P_{\text{Loss}} / P'_{\text{Loss}}$  is equal to  $2R / R'$ ; you will just do it, you will get it. Now, we know that  $R$  is equal to we use the same material that wire is made of the same material for both the cases, so  $R$  is equal to  $\rho l / \pi r^2$  that is the first case that is single-phase case  $\rho l / \pi r^2$ . We are assuming that a radiation of the conductor for the single-phase is  $R$  right, so it will be  $\rho l / \pi r^2$ .

Similarly, for the three-phase case,  $R'$  will be  $\rho l / \pi (r')^2$  right. So, they are of the same material and same length right, so  $\rho$  will remain same and your what you call only radius will be difference. Say for the single-phase, it is  $R$ , and the for the three-phase radius is  $R'$  right  $R'$ . So, it is  $R$  is equal to this,  $R$  is  $R'$  is equal to you substitute here, you substitute  $R$ , and this  $R'$  here you substitute. If you do and simplify, you will get  $P_{\text{Loss}} / P'_{\text{Loss}}$  is equal to  $2r'^2 / r^2$ . This is equation 3 right.

(Refer Slide Time: 25:23)

$$\therefore \frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2(p')^2}{r^2} \dots (iii)$$

If the same power loss is tolerated in both the systems, i.e.,  $P_{\text{loss}} = P'_{\text{loss}}$ , then

$$r^2 = 2(p')^2$$
$$\therefore \frac{r^2}{(p')^2} = 2 \dots (iv)$$

If now, if both the your what you call now if both if the same power loss is tolerated for both the system that is P loss is equal to P loss dash. If you do so, that means, if you do so P loss is equal to P loss dash equation, then it will become r square is equal to 2 r square right, so that means, my r square upon r dash square is equal to 2. This is equation 4 right.

(Refer Slide Time: 25:54)

$$\frac{(p')^2}{r^2} = 2 \dots (iv)$$

Now

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 l)}{3(\pi (p')^2 l)}$$
$$= \frac{2}{3} \times 2 = \frac{4}{3} = 1.333$$

Material for single-phase wire

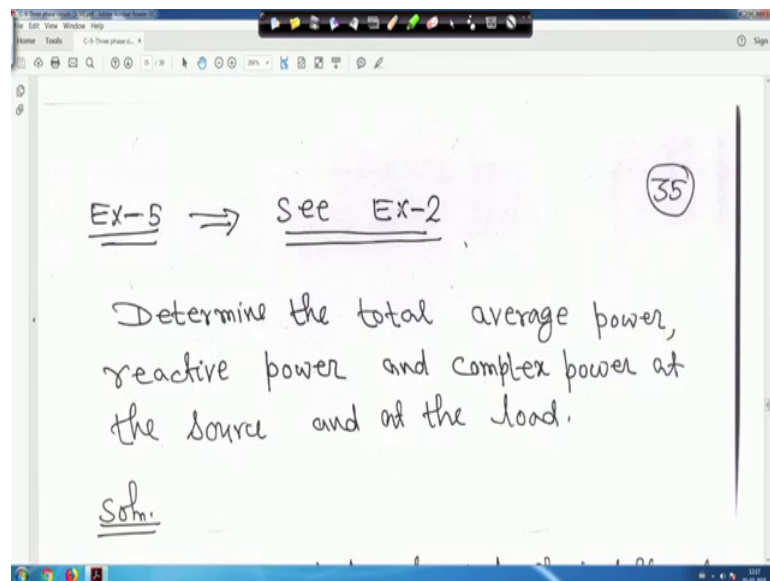
$$= (1.333) \times \text{Material for three-phase wire}$$

Now, that material for single-phase by material for three-phase, this is volume. So, conductor they are conductor can be represented that is a long wire represented by

cylinder right. So, if length is  $l$  and the single-phase case if  $r$  is the radius, so  $\pi r^2 l$  right. So, this is your what you call the volume, it is taking as cylinder right, because cross section is circular, divided by your material for the three-phase case, it is a two wire system. So,  $2 \pi r^2 l$  right that is for the single-phase case, this is the volume.

And for the material for three-phase case, it is a three wire. So,  $3 \pi r^2 l$ , this is the volume. So, if you if you simplify this, so  $\pi$   $\pi$  will be cancel,  $l$   $l$  will be cancel, it will be  $2$  by  $3$  into  $r^2$  by your  $r^2$ . So,  $r^2$  by  $r^2$  is equal to  $1$ , so it will be  $2$  by  $3$  is equal to  $1.333$  that mean, material for single-phase wire that is volume actually is equal to  $1.333$  into material for the three-phase. Then if you single-phase for the same power loss; for the same power loss right that volume of the conductor is  $33.3$  percent more than the three-phase right that is why three-phase three wire is the benefit what you call made beneficial. So, this is the idea.

(Refer Slide Time: 27:21)



So, now take one example, I mean for example, you please see the example 2, same example 2 you just see. You have to determine the total average power, reactive power and complex power at the source and at the load right. So, if you I mean I am not going to the example 2, but we will just go to that go back to that example 2.

(Refer Slide Time: 27:43)

Soln.

System is balanced and it is sufficient to consider one phase.

For phase a,

$$V_p = V_{an} = 110\angle 0^\circ \text{ Volt}$$
$$I_p = I_a = 6.81\angle -21.8^\circ \text{ Amp.}$$

At the source

So, in this case what happened, the system is balanced and it is sufficient to consider single-phase right. So, system is balanced, so single-phase analysis is sufficient. So, for phase a, phase voltage is equal to  $V_{an}$  at the 110 angle, this is given. And this  $I_p$  phase current also line current, we have also computed 6.81 angle minus 21.8. There we have compute all this things right.

(Refer Slide Time: 28:05)

$$V_p = V_{an} = 110\angle 0^\circ \text{ Volt}$$
$$I_p = I_a = 6.81\angle -21.8^\circ \text{ Amp.}$$

At the source

$$S_s = 3 V_p I_p^* = (2087 + j 834.6) \text{ VA}$$

Real power supplied is 2087 Watt.  
Reactive power supplied is 834.6 VAR.

Now, at the source, so  $S_s$  is equal to that is  $S$  suffix  $s$  small  $s$  stands for source is equal to  $3 V_p I_p^*$  conjugate, because three-phase system  $V_p I_p^*$  conjugate into 3. So, if you do



so, this is my  $V_p$ , and this is  $I_p$  conjugate right.  $I_p$  is 6.81, so  $I_p$  conjugate will be 6.81 angle 21.8 degree right. So, it will be  $3 V_p I_p$  conjugate will be 2087 plus j 834.6 volt ampere load.

When we are, you are making it, this is my watt, this is actually (Refer Time: 28:39) watt right, and this is my var right. So, basically when doing right in this form, watt plus your var this can be an volt ampere. For example, suppose if you write P is equal to say 30 watt and Q is equal to your 20 var right, it is ok. But, when you write in this form P plus jQ 30 plus j 20, it will be volt ampere that is why this together.

When you write this in form, I mean P plus jQ form, it will be volt ampere right, but this one is watt, and this one is var because, if you take its magnitude, it will be volt ampere that is why we write VA. So, in this case, the real power supplied is that two 2087 watt, and the reactive power is 834.6 var right. Now, you see that example 2, it is very simple data everything is given. So, at the load the complex power absorb is, so  $S_{load}$  is equal to  $3 I_p^2 Z_p$  if it is a balanced system, so  $I_p^2 Z_p$  per phase into 3 right. So, this we can write.

(Refer Slide Time: 29:45)

$$S_{LOAD} = 3 I_p^2 Z_p$$

$$Z_p = (10 + j8) \Omega$$

$$\therefore Z_p = 12.81 \angle 38.66^\circ$$

$$\therefore S_{LOAD} = 3 \times (6.81)^2 (10 + j8) \text{ VA}$$

$$\therefore S_{LOAD} = (1392 + j1113) \text{ VA}$$

Real power absorbed is 1392 Watt

Reactive power absorbed is 1113 VAR

So,  $Z_p$  is given 10 plus j 8 ohm that is 12.81 angle 38.66 degree right. Therefore, it is 3 into six point current magnitude is 6.81, so 3 into 6.81 square into 10 plus j 8 right. So, this is your volt ampere. So, this you can write  $S_{load}$  is equal to your 1398 plus j 1113 volt ampere right. So, in this case, the real power absorbed is 1392 watt, and reactive

power is 1113 var. And total if you make, it will be volt ampere. This is watt; this var I told you.

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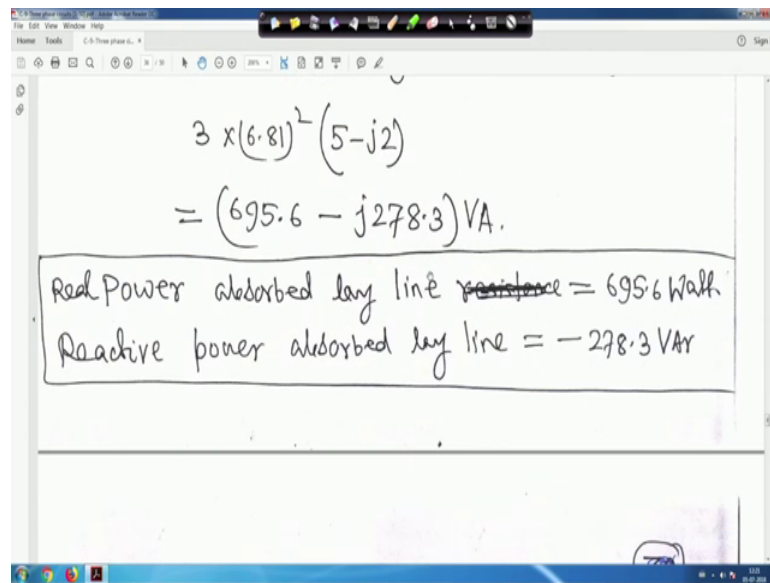
Real power absorbed is 2574 W  
Reactive power absorbed is 1113 VAR

The difference between the two complex powers is absorbed by the line impedance  $(5-j2)\Omega$

Power absorbed by the line is  
 $3 \times (6.81)^2 (5-j2)$

So, the difference between the two complex power is the power absorbed by the line impedance that is the power loss right. So, power absorbed by the line is it is 3 into current square right magnitude 6.81 square and line impedance 5 minus j 2, so it is coming 695.6 minus j 278.3. So, when it is minus means (Refer Time: 30:46) it is capacitive right, it is capacitive (Refer Time: 30:48) what you call impedance is capacitive.

(Refer Slide Time: 30:51)



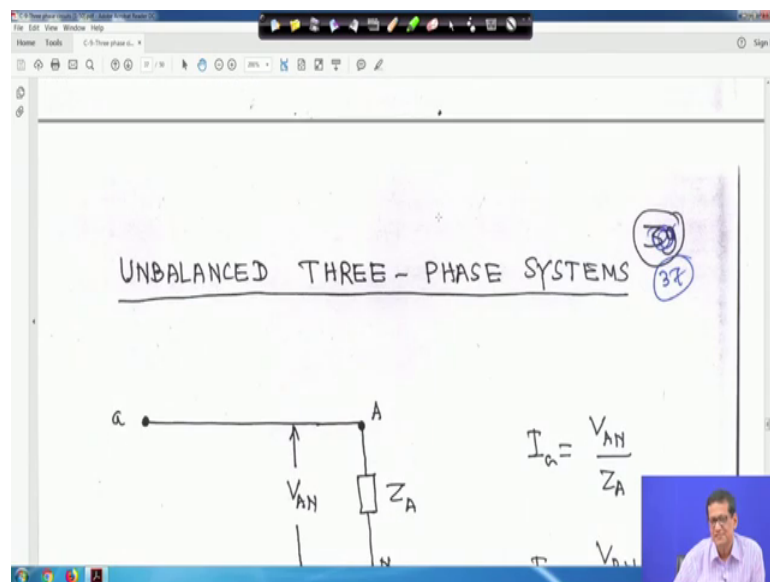
The whiteboard shows the following calculations:

$$3 \times (6.81)^2 (5 - j2)$$
$$= (695.6 - j278.3) \text{ VA.}$$

Real Power absorbed by line ~~resistance~~ = 695.6 Watt.  
Reactive power absorbed by line = -278.3 VAR

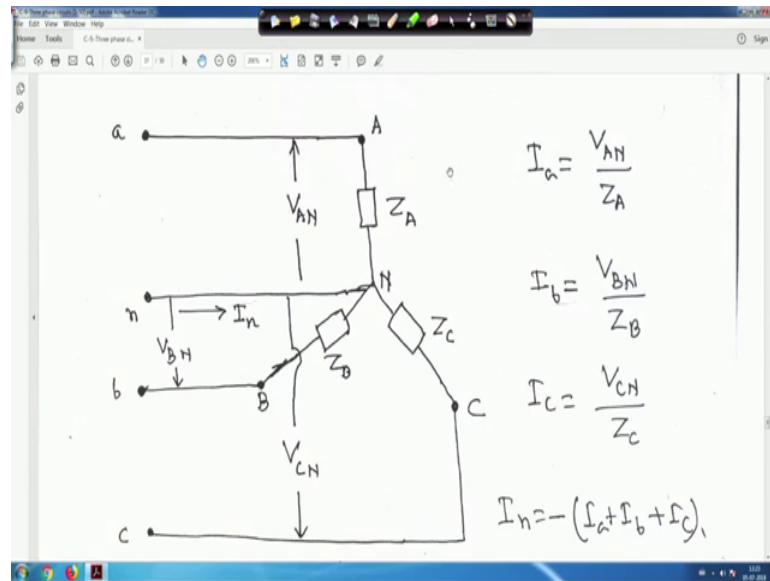
So, this real power absorbed by line is 695.6 watt that mean, this one. And reactive power absorbed by line is minus 278.3 var right.

(Refer Slide Time: 31:05)



Now, next is unbalanced three-phase system, just one or two or just half page I will tell you.

(Refer Slide Time: 31:10)



$$I_a = \frac{V_{AN}}{Z_A}$$

$$I_b = \frac{V_{BN}}{Z_B}$$

$$I_c = \frac{V_{CN}}{Z_C}$$

$$I_n = -(I_a + I_b + I_c)$$

Suppose, this load suppose is the angles are many thing  $Z_a$ ,  $Z_b$ ,  $Z_c$ , it may be different right, (Refer Time: 31:16) magnitude may be different, angle may be different. Similarly, supply voltage your  $V_{ab}$   $V_{bc}$   $V_{ca}$  right all these things may be different. I mean if one is different slightly, then it is unbalanced system. So, in the case of unbalanced system,  $I_a$  is equal to say current here, this current is your what you call this current, this is  $I_a$  current, so this current is  $I_a$  right. So, it is going to this phase  $I_a$ .

Similarly, for your this thing  $I_b$  right, and similarly for here it is  $I_c$  right, and this is that neutral point  $N$ . And this is  $Z_a$ ,  $Z_b$ ,  $Z_c$ , and this phase voltage  $V_{AN}$ ,  $V_{BN}$  and  $V_{CN}$ . So, unbalanced system phase voltage also may be unbalanced. So,  $I_a$  is equal to  $V_{AN}$  by  $Z_A$ .  $I_b$  is equal to  $V_{BN}$  by  $Z_B$ . And this is star connected, load and this is star connected load. So,  $I_c$  is equal to  $V_{CN}$  by  $Z_C$ .

Now, if you apply kcl here right, so it will be your  $i_n$  plus  $i_a$  plus  $i_b$  plus  $i_c$  is equal to 0 that means, my  $i_n$  is equal to your minus  $i_a$  minus  $i_b$  minus  $i_c$  that is what (Refer Time: 32:22) written here right. You take we put it in back, so it will be this thing minus of  $I_a$  plus  $I_b$  plus  $I_c$ . And in that case if it is unbalanced, this  $i_n$  I mean (Refer Time: 32:32) right. So, let me clear it, so that is all for little bit idea for unbalanced system right ok.

Thank you very much; we will back again.