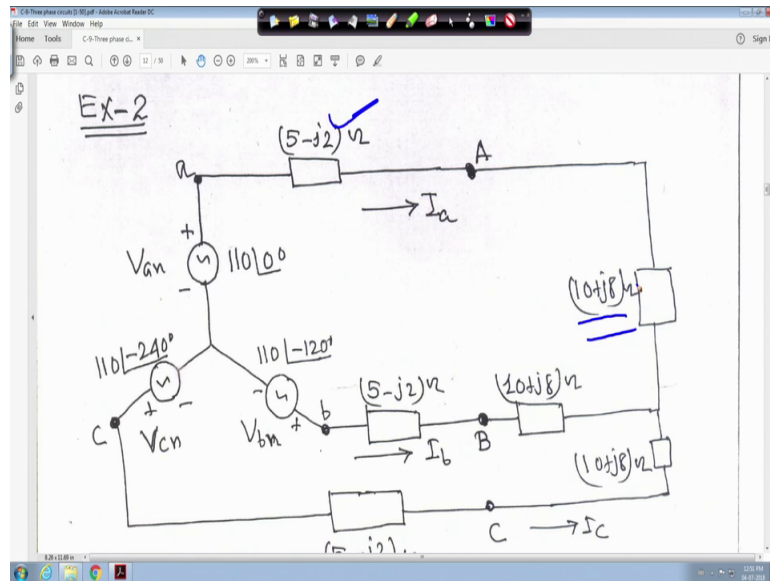


**Fundamentals of Electrical Engineering**  
**Prof. Debapriya Das**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 48**  
**3 Phase Circuits (Contd.)**

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Ok, so will take a small example of that. Suppose this is the example is taken this example right. So, in the line  $5 - j2$  ohm one impedance is connected, this is star connected, this is star connected, load  $10 + j8$   $10 + j8$  ohm and this is your star connected source  $110$  angle  $0$   $110$  angle  $-120$   $110$  minus  $240$ . And in the line each line  $5 - j2$  ohm,  $5 - j2$  ohm and  $5 - j2$  ohm impedance connected and current  $I_a$   $I_b$   $I_c$  already given right. So, now,  $Z_{star}$  if, that means, what you can do is that, you should not be confused with this what you can do is this all are same. This  $5 - j2$  ohm,  $10 + j8$  you add together make it as a  $Z_{star}$  right that is the idea.

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Handwritten derivation on a whiteboard:

$$Z_Y = (5-j2) + (10+j8) = (15+j6) \Omega$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{110 \angle 0^\circ}{(15+j6)} = 6.81 \angle -21.8^\circ \text{ Amp}$$

$$I_b = I_a \angle -120^\circ = 6.81 \angle -21.8^\circ - 120^\circ$$

$$\therefore I_b = 6.81 \angle -141.8^\circ$$

$$I_c = I_a \angle -240^\circ = 6.81 \angle -21.8^\circ - 240^\circ$$

So, in this case your  $Z_i$  sorry  $Z_y$  is equal to 5 minus  $j$  2 plus 10 plus  $j$  8 is 15 plus  $j$  6 ohm. Therefore,  $I_a$  is equal to compute same relationship although, line it is given you have clubbed together right. Therefore,  $I_a$  is equal to  $V_{an}$  up on  $Z_y$  is equal to 110 angle 0 by 15 plus  $j$  6 is equal to 6.81 angle minus 21.8 degree ampere. Similarly, you have derived this equation  $I_b$  is equal to  $I_a$  angle minus 120 degree, so, it will be 6.81, So, what you call  $I_a$  is 6.81 minus 21.8 degree. So, it will be minus 21.8 degree, minus 120 degree, so minus 141.8 degree right.

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Handwritten derivation on a whiteboard:

$$I_a = \frac{V_{an}}{Z_Y} = \frac{110 \angle 0^\circ}{(15+j6)} = 6.81 \angle -21.8^\circ \text{ Amp}$$

$$I_b = I_a \angle -120^\circ = 6.81 \angle -21.8^\circ - 120^\circ$$

$$\therefore I_b = 6.81 \angle -141.8^\circ$$

$$I_c = I_a \angle -240^\circ = 6.81 \angle -21.8^\circ - 240^\circ$$

$$\therefore I_c = 6.81 \angle 98.2^\circ \text{ Amp}$$

Similarly  $I_c$  is equal to  $I_a$  angle minus 240 degree, so 6.81 angle minus 21.8, this we have got here 21.8 degree minus 240 degree. So, this one will be  $I_c$  is equal to 6.81 but actually, there should not be any doubt or anything. Actually,  $I_a$  is equal to this one; that means, 6.81 angle minus 21.8 degree into your this angle minus 120 degree right because, into 1 in to 1 this one.

So, this 2 are added, so that is why your this thing is there right, similarly here right. So,  $I_c$  is equal to you will get 6.81, your 98.2 degree, why this is minus 240 minus 21.8 degree is there, just you other way you add 360 degree with this, just the way you show  $v_b$   $v_a$  that  $v_c$  lagging minus 240 degree. So, you add 360 degree other way should lagging then (Refer Time: 2:50) so, other way it is leading So, you add 360 degree with these, so 360 minus 240, minus 21.8, you will get 98.2 degree ampere.

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Handwritten derivation on a whiteboard:

~~Let~~  
 Let line voltage =  $V_L$   
 $\therefore V_L = \sqrt{3} V_p$   
 $\therefore V_{ab} = \sqrt{3} V_p \angle 30^\circ = V_L \angle 30^\circ$   
 $V_{bc} = \sqrt{3} V_p \angle -90^\circ = V_L \angle -90^\circ$   
 $V_{ca} = \sqrt{3} V_p \angle -210^\circ = V_L \angle -210^\circ$

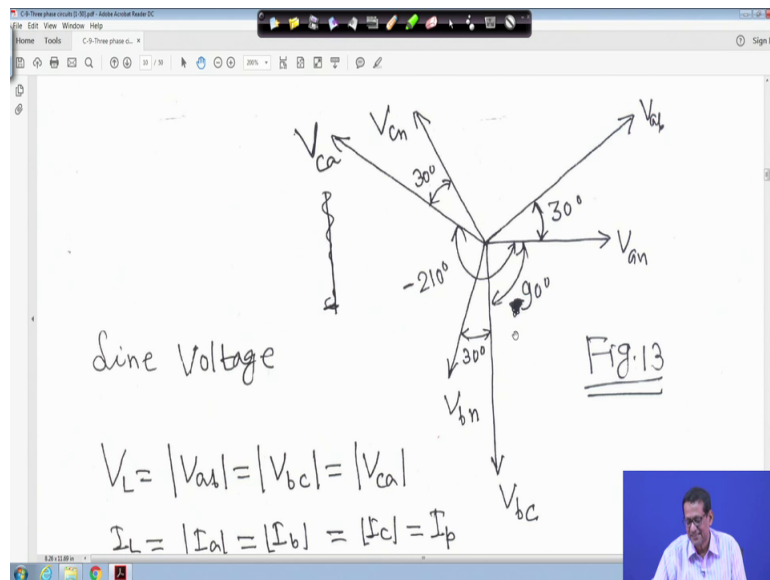
You do it yourself, right other phasor diagram. So line voltage is equal to  $V_L$  and we know  $V_L$  is equal to root 3 in to phase voltage right magnitude.

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$$\begin{aligned} \therefore V_{ab} &= \sqrt{3} V_p \angle 30^\circ = V_L \angle 30^\circ \\ V_{bc} &= \sqrt{3} V_p \angle -90^\circ = V_L \angle -90^\circ \\ V_{ca} &= \sqrt{3} V_p \angle -210^\circ = V_L \angle -210^\circ \end{aligned}$$
$$\therefore \frac{V_{ab}}{V_{bc}} = \frac{V_L \angle 30^\circ}{V_L \angle -90^\circ}$$

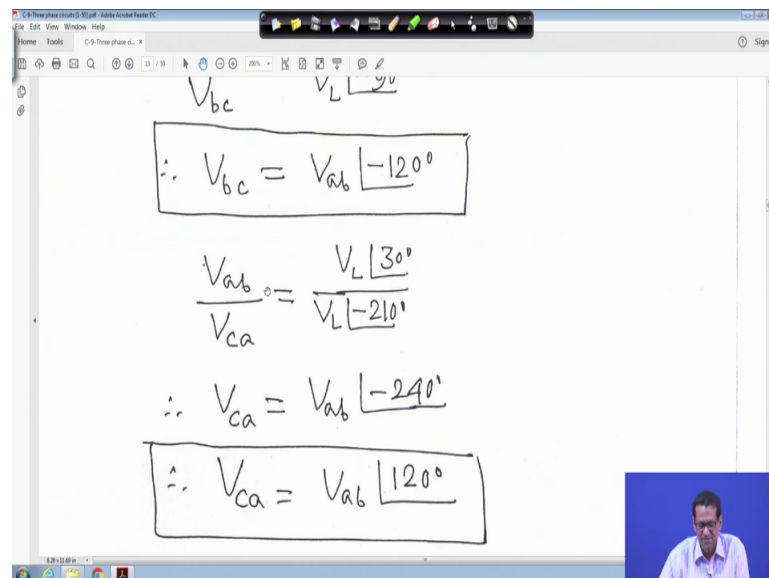
Therefore we know  $V_{ab}$  is equal to therefore, this also you derived,  $V_{ab}$  is equal to  $\sqrt{3} V_p \angle 30^\circ$  and  $V_L$  is equal to your writing  $\sqrt{3} V_p$ . So,  $V_L \angle 30^\circ$ .  $V_{bc}$  is  $\sqrt{3} V_p \angle -90^\circ$  that also we have done. So, that will be  $V_L \angle -90^\circ$ , your  $\sqrt{3} V_p$  is equal to  $V_L$ . And  $V_{ca}$  is equal to  $\sqrt{3} V_p \angle -210^\circ$  that is  $V_L \angle -210^\circ$ . This all these things in the phasor diagram we have made it, right, all these things are there that your what you call  $V_{ab}$  this one,  $V_{bc}$  this one and  $V_{ca}$  is this one with respect to your  $V_{an}$  right. That is your the from this am not going to the phasor diagram.

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I mean from this phasor diagram, again I have to go on top right just hold on here, here from this phasor diagram. With respect to that this is angle 30 degree, with respect to  $V_{an}$  it is ninety degree and with respect to  $V_{an}$  this is your minus 210 degree right. So, so, that is your what you call that your  $V_{ab}$ ,  $V_{bc}$  and  $V_{ca}$  right.

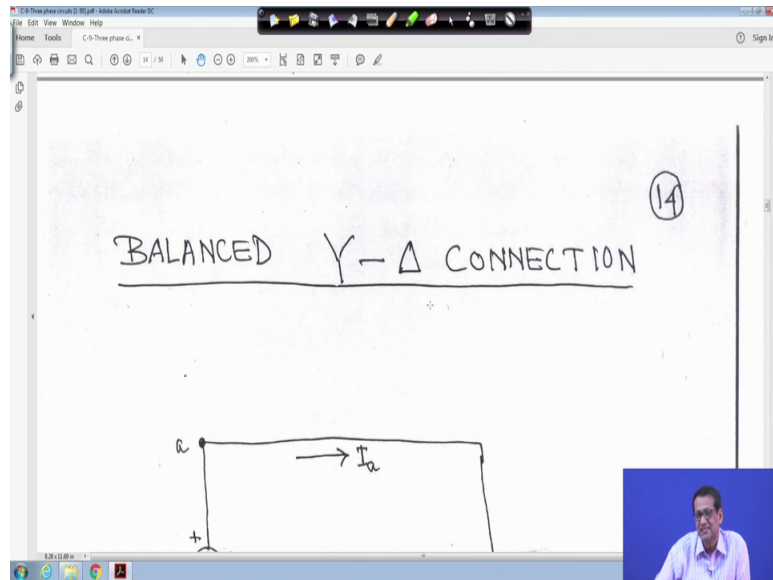
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Now, if you make divide  $V_{ab}$  by  $V_{bc}$ , if you make  $V_{ab}$  by  $V_{bc}$ , it is  $V_L$  angle 30 by  $V_L$  angle minus 90 therefore,  $V_{bc}$  is equal to, it is it will go up it will be 120 degree, so  $V_{bc}$  is equal to  $V_{ab}$  minus 120 degree right. Similarly  $V_{ab}$  upon  $V_{ca}$  if you make, you

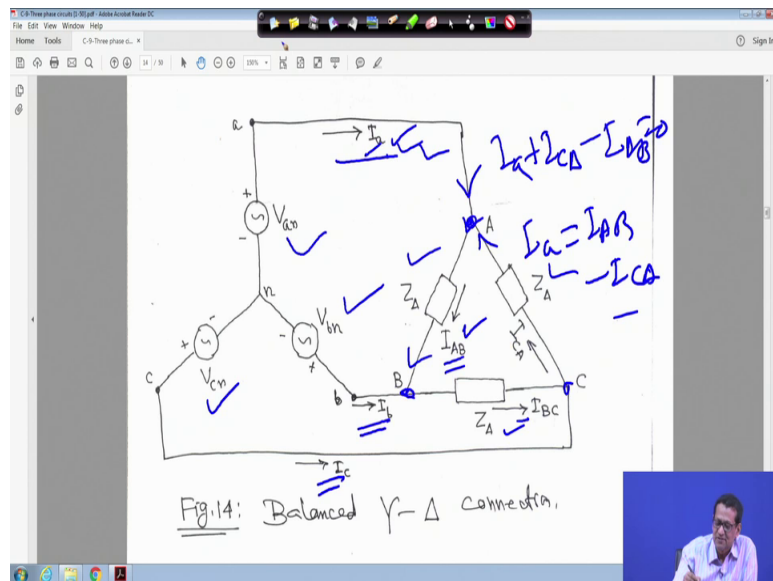
will get  $V_L$  angle  $30^\circ$ , while  $V_L$  angle minus  $210^\circ$  degree, you will get  $V_{ca}$  is equal to  $V_{ab}$  angle minus  $250.40^\circ$  degree or you add  $360^\circ$  with that other way  $V_{ca}$  is equal to  $V_{ab}$  angle  $120^\circ$  degree right, this is the relationship.

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Now, balanced star delta connection next will come to source star, but load is your delta connection.

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So, this is my source star and this is your my delta connection right. Now, question is that just if, you reduce the this thing it will be first for your explanation. So, if you look into this or this circuit that your this is my  $V_{an}$ , this is my  $V_{bn}$  and this is my  $V_{cn}$  and this is my delta connector load  $Z_{\Delta}$ ,  $Z_{\Delta}$ ,  $Z_{\Delta}$  So, this is a line current  $I_a$  right this is line current  $I_b$  and this is line current  $I_c$  right and in the load phase current this is  $I_{AB}$ ,  $I_{AB}$ , this is  $I_{BC}$  and  $I_{CA}$  right, this is my your what you call my your phase current right and this is my line current, this is my line phase current. Now, if you apply for example, we will come to that, but am telling you if you apply KCL let this current is coming right in coming.

So, this current is  $I_a$ , this  $I_{ca}$  current it also coming here, so plus  $I_{ca}$  and this current is your what we call it is leaving, so it is minus  $I_{ab}$  is equal to 0; that means, my  $I_a$  is equal to  $I_{ab}$  minus  $I_{ca}$  right. So, similarly you have to apply KCL here, you have to apply KCL here. So, in this case line current is not is equal to phase current because, this is the line current after that this is the phase current and this is the line current so, we have to find out some relationships between the line current and this is the line current and phase current this is the line current right.

So, this is delta, so in this case in this case, so let me let me let me eh just increase the zoom. So, so this is the circuit I told you right.

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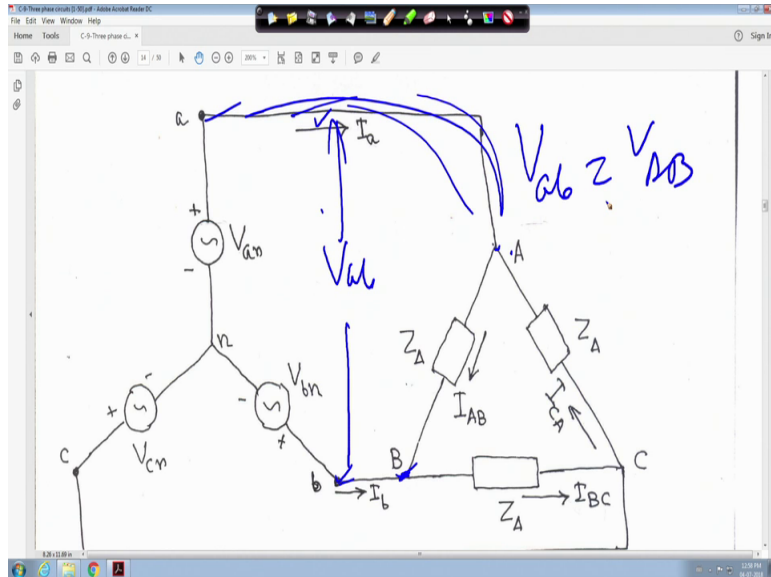
Handwritten equations on a whiteboard:

$$V_{an} = V_p \angle 0^\circ; \quad V_{bn} = V_p \angle -120^\circ; \quad V_{cn} = V_p \angle 120^\circ$$

Also

|  |                               |
|--|-------------------------------|
| $V_{ab} = V_{AB} = \sqrt{3} V_p \angle 30^\circ$   | $I_{AB} = \frac{V_{AB}}{Z_A}$ |
| $V_{bc} = V_{BC} = \sqrt{3} V_p \angle -90^\circ$  | $I_{BC} = \frac{V_{BC}}{Z_A}$ |
| $V_{ca} = V_{CA} = \sqrt{3} V_p \angle -210^\circ$ | $I_{CA} = \frac{V_{CA}}{Z_A}$ |

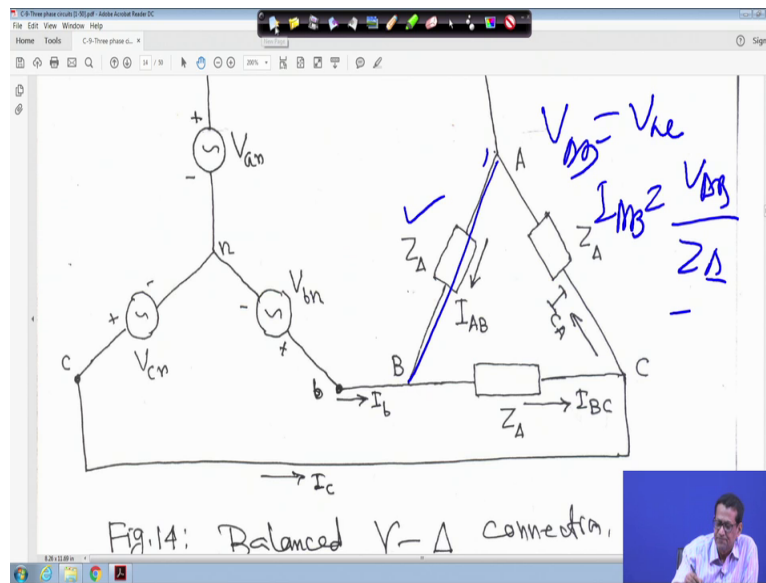
Now, for balanced star delta, so  $V_{an}$  again star case eh angle 0 degree,  $V_{bn}$  angle minus 20 degree and  $V_{cn}$  gives you angle 120 degree. And we also know  $V_{ab}$  is equal to  $V_p$  root 3 angle 30 degree. If you look into the circuit, if you look into the circuit right is your what we call that your  $V_{ab}$  is equal to this one, just let me go up that here it is  $V_{ab}$   $V_{small\ ab}$  is equal to  $V_{capital\ AB}$ . (Refer Slide Time: 07:25)



It is very simple because if this is your this is your b and this is your a right, so if it is if this is your  $V_{ab}$  right, this is your  $V_{ab}$  and this same because no anyway nothing is connected here and this. is B, so nothing is connected here So,  $V_{ab}$  is equal to  $V_{AB}$  right, similarly for B cc a right. So, let me clear it, so that means, my  $V_{ab}$  is equal to  $V_{capital\ AB}$  root 3 angle 30 degree, same thing  $V_{bc}$  also same thing,  $V_p$  angle minus 90 degree,  $V_{ca}$  also  $V_{ca}$  root 3  $V_p$  angle minus 210 degree.

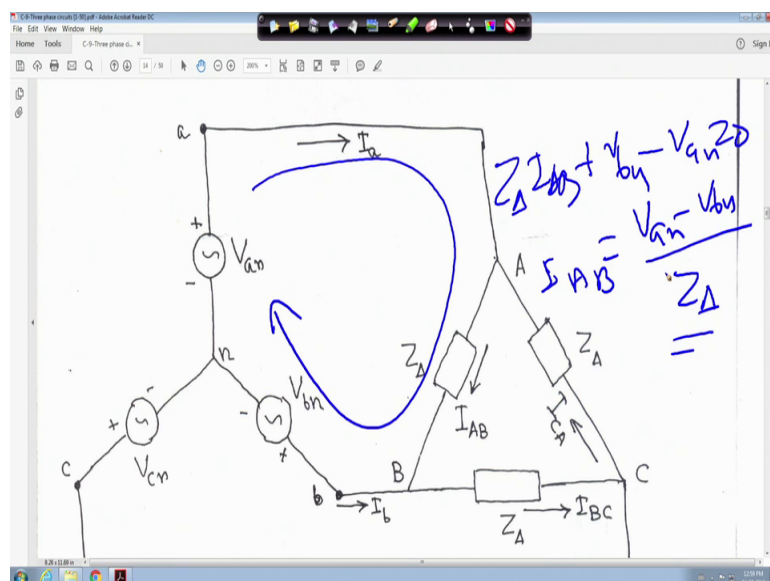


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Now,  $I_{AB}$  is equal to  $V_{AB}$  upon  $Z_{\Delta}$  because, this is my circuit; that means, this voltage your this voltage right that your  $V_{AB}$  that is my  $V_{AB}$ , this voltage A to B and is equal to  $V_{ab}$  right, same voltage impedance and this is my  $Z_{\Delta}$   $I_{AB}$ . So,  $I_{AB}$  is equal to  $V_{AB}$  upon  $Z_{\Delta}$  right, for this phase right. So, same thing here, we are writing same thing we are writing  $I_{AB}$  is equal  $V_{AB}$  upon  $Z_{\Delta}$ ,  $I_{BC}$  is equal to  $V_{BC}$  upon  $Z_{\Delta}$ ,  $I_{CA}$  is equal to  $V_{CA}$  upon  $Z_{\Delta}$  all right.

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So, now another way to get this phase current is to apply KVL, for example, applying KVL around loop, so, aABbna right.

I mean if you do so, if you apply your what you call KVL then, what you call we do is what we can do is we apply KVL in this loop, we apply KVL in this loop in this case, what will happen am coming like this term. So, Z delta into I AB then, encounter is plus terminal first, plus V bn then, minus V an is equal to 0 right. That means, my I AB is equal to V an minus V bn upon Z delta and we got now V an minus V bn is equal to root 3 angle 30 degree you got it now, same way we will get it. So, that is why IAB you can write like this, similarly for the other otherwise also we can do this. This is for one similar other two we can do it right.

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from the phase currents by applying KCL at nodes, A, B and C. Thus,

$$\begin{aligned}
 I_a &= I_{AB} - I_{CA} \\
 I_b &= I_{BC} - I_{AB} \\
 I_c &= I_{CA} - I_{BC}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \frac{I_{AB}}{I_{CA}} = \frac{V_{AB}}{V_{CA}} \times \frac{Z_A}{Z_C} \\ \therefore \frac{I_{AB}}{I_{CA}} = \frac{V_{ab}}{V_{ca}} \\ \therefore \frac{I_{AB}}{I_{CA}} = \frac{V_{ab}}{V_{ab} \angle -240^\circ} \end{array}$$

So, that means, here that is why you are writing this one IAB is equal to V an minus Z delta and that is actually nothing, but V ab we got it the phasor diagram by Z delta that is V capital AB upon Z delta. Similarly for bcc we will get it. So, line currents are obtained from the phase current by applying KCL at nodes IBC. I showed you at node A if you apply I a will be I AB minus I CA. I b will be I BC minus I AB.

I showed you one, here is all these all these your here at node A node B node C you apply KCL and you will get this equation, your right you will get this equation. Similarly I c is equal to ICA minus IBC. Now IAB upon ICA, IAB is equal to V AB upon Z delta and I CA is equal to V CA upon Z delta. If you do so I AB or I CA will be area

upon integer by Z delta into Z delta upon CA. So, Z delta Z delta will be cancelled. So, I AB upon I CA actually is V AB upon V CA, it is proportional to their your I AB phase current and I CA it is just a same thing at the line voltage V ab upon V ca right.

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The image shows a whiteboard with the following handwritten equations:

$$\left. \begin{aligned} I_a &= I_{AB} - I_{CA} \\ I_b &= I_{BC} - I_{AB} \\ I_c &= I_{CA} - I_{BC} \end{aligned} \right\} \begin{aligned} \frac{I_{AB}}{I_{CA}} &= \frac{V_{AB}}{Z_A} \times \frac{Z_A}{V_{CA}} \\ \therefore \frac{I_{AB}}{I_{CA}} &= \frac{V_{ab}}{V_{ca}} \\ \therefore \frac{I_{AB}}{I_{CA}} &= \frac{V_{ab}}{V_{ab} \angle -240^\circ} \\ \therefore I_{CA} &= I_{AB} \angle -240^\circ \\ \text{Also} \\ I_{BC} &= I_{AB} \angle -120^\circ \end{aligned}$$

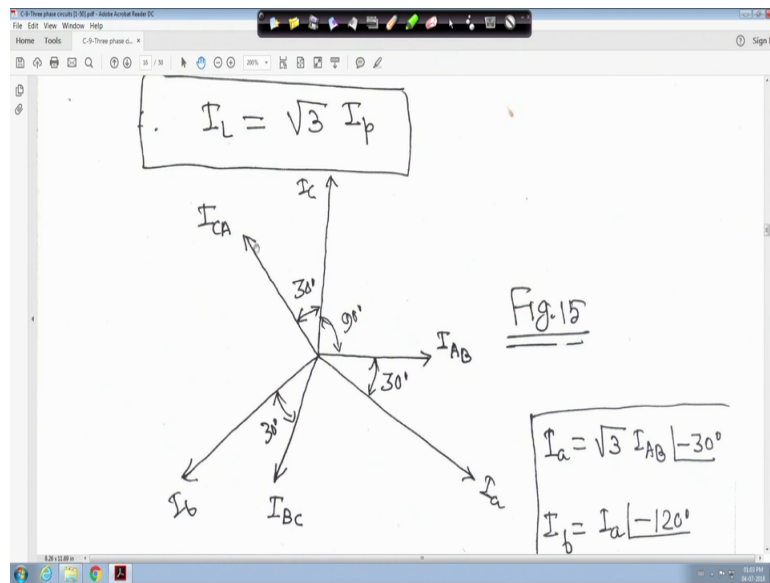
So, I AB upon I CA is equal to V ab you write and V ca is equal to you know V ab angle minus 240 degree you substitute. So, so V ab V ab will be cancelled therefore, I CA will be equal to I AB angle minus 24 240 degree; that means, also IBC similarly you will get I AB angle minus 120 degree. You please do it for in take your what you call I AB upon I BC, you will get your same relationship minus 120 degree. This is very simple nothing is there, just you just you put in there and step by step you will do you will get it right. Therefore, you will get I BC also I AB minus 120 degree it is balanced.

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The image shows a handwritten derivation on a whiteboard. The first line is  $\therefore I_a = I_{AB} - I_{CA} = I_{AB} - I_{AB} \angle -240^\circ$ . The second line is  $\therefore I_a = \sqrt{3} I_{AB} \angle -30^\circ$ . Below this, it says "Let" followed by  $I_L = |I_a| = |I_b| = |I_c| = \text{Line current}$ . The final line is  $I_p = |I_{AB}| = |I_{BC}| = |I_{CA}| = \text{phase current.}$

So, in this case therefore, we know  $I_a$  is equal to  $I_{AB}$  minus  $I_{CA}$ . So,  $I_{AB}$  keep, but  $I_{CA}$  is equal to  $I_{AB}$  angle minus 240 degree. You put here  $I_{CA}$  is equal to minus  $I_{AB}$  angle your minus 240 degree. If you simplify you will get  $I_a$  is equal to root 3  $I_{AB}$  angle minus 30 degree. That means, line current is equal to root 3 times the phase current and angle is minus 30 degree right and magnitude wise line current is equal to  $I_a I_b I_c$  that is line current magnitude. And phase current  $I_p$  is equal to mod  $I_{AB}$ , mod  $I_{BC}$ , mod  $I_{CA}$  is equal to phase current right. So, in this case this is my phase current, so  $I_L$  actually line current is equal to root 3 time phase current for delta connection right shown.

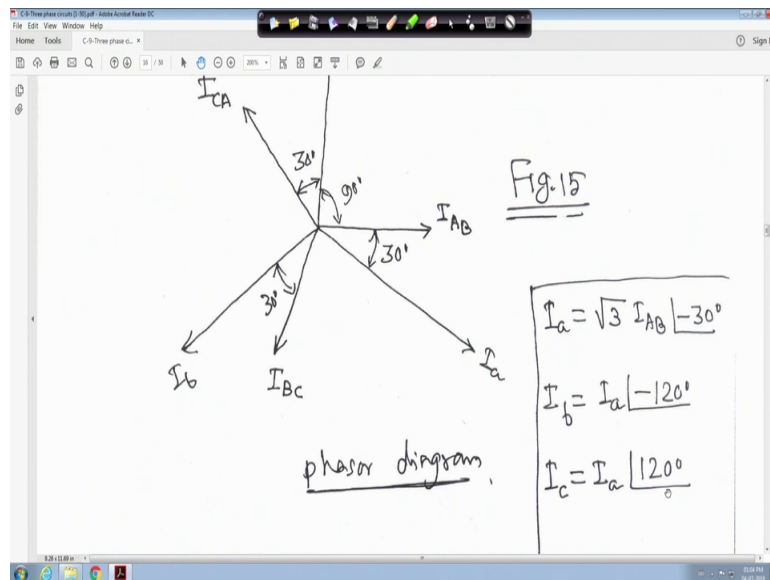
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So, line current earlier we saw star connection your line will your what you call your line voltage is equal to root 3 line phase voltage and for delta connector, volt line current is equal to your what you call going to that that is, your root 3 times phase current. So,  $I_L$  is equal to root 3 times phase current right.

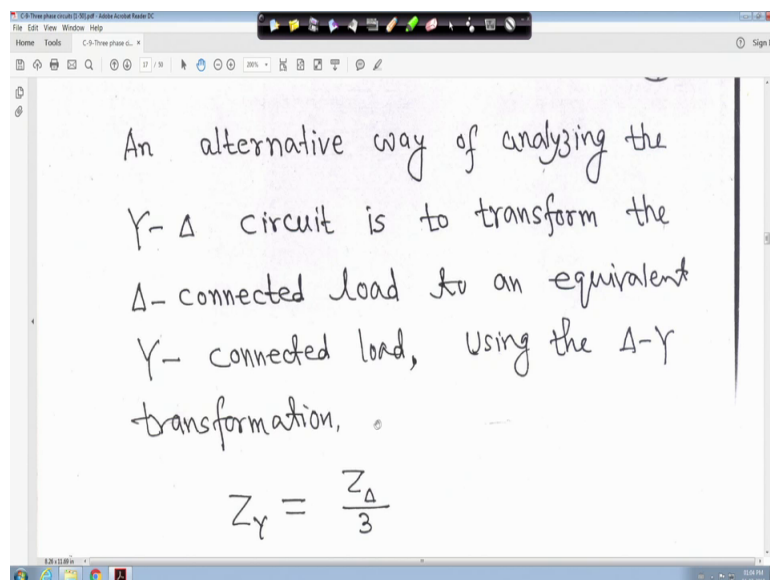
So, this is your just from one relationship we will get it, rest you take that is your 120 degree apart right. So, even if you do it just see the balance so, you will see that it will be just 120 degree apart right. So, if you draw the phasor diagram, this is my  $I_{AB}$ , this is my  $I_{BC}$  this is my  $I_{CA}$ , this is my phase current and this is when  $I_a$  is lagging  $I_a$  is lagging from this phase current angle minus 30 degree, that is why this is  $I_a$  30 degree then,  $I_b$  30 then  $I_c$ . Similarly you draw just 120 degree part between among your what we call between your line current as well as when you draw the phase current these are all this phase current 120 degree apart right. So, this is figure 15, so this is my phasor diagram right. Therefore,  $I_a$  is equal to root 3 angle  $I_{AB}$  minus 30 degree,  $I_b$  is equal to  $I_a$  angle minus 120 degree and  $I_c$  is equal to  $I_a$  angle 120 degree right.

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So, this is actually our relationship that is line to voltage. So, there should not be any confusion any confusion right.

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So, so an alternative way of analyzing star delta circuit is to transform the delta connected load to an equivalent star connected load, using that delta transformation. This not much done because, it is same as dc circuit right I never wanted to make these things much more lengthy.

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After this transformation, we now have a Y-Y system as in Fig. 10

The three-phase Y-Δ system in Fig. 14 can be replaced by the single phase equivalent as shown in Fig. 16

$Z_Y = \frac{Z_\Delta}{3}$

So, Z star is equal to Z delta by 3. If you do so then, after this transformation you have a star system as in figure 10, so this is your what you call that you have a if, you converge this delta to star it will be a star system, like your figure 10 you obtain star source your star lodes. So, here Z y is equal to Z delta by 3.

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single phase

Fig. 16

$Z_Y = \frac{Z_\Delta}{3}$

Fig. 16

This allow us to calculate only the line currents.

Now, if you see this equivalent circuit like a single phase, so this is  $V_{an}$ , this is  $I_a$  and  $Z_y$  is equal to  $Z_{\Delta}$  by 3 right. A simple a see as if it is a single phase balanced, so represented by only phase a right. So, this allows to calculate only the line currents right,

so star delta transformation at delta star transformation both are possible, if you will you can convert it to delta to star star to delta.

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EX-3: A balanced abc-sequence Y-connected source with  $V_{an} = 100\angle 10^\circ$  Volt is connected to a  $\Delta$ -connected balanced load  $(8+j4)\ \Omega$  per phase. Calculate the phase and line currents.

Soln.  
 $Z_{\Delta} = 8+j4 = 8.944\angle 26.57^\circ\ \Omega$

So, take an small example a balanced abc sequence that is star connected source with  $V_{an}$  is equal to 100 angle 10 degree. Voltage connected to a delta connected balanced load that is 8 plus j 4 ohm right per phase. Calculate the phase and line currents look,  $Z_{\Delta}$  is given 8 plus j 4, that is 8.944 angle 26.57 degree.

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$Z_A = 8+j4 = 8.944\angle 26.57^\circ$

If the phase Voltage  $V_{an} = 100\angle 10^\circ$  Volt,  
then the line Voltage is  
 $V_{ab} = V_{AB} = \sqrt{3} V_{an} \angle 30^\circ$   
 $\therefore V_{AB} = \sqrt{3} \times 100 \angle 10^\circ + 30^\circ = 173.2 \angle 40^\circ$  Volt  
 $I_{AB} = \frac{V_{AB}}{Z_A} = \frac{173.2 \angle 40^\circ}{8.944 \angle 26.57^\circ} = 19.36 \angle 13.43^\circ$  Amp



If the phase voltage  $V_{an}$  is 100 angle 10 degree volt right, in the line voltage will be  $V_{ab}$  is equal to  $\sqrt{3} V_{an}$  just now we have seen it is  $\sqrt{3} V_{an}$  angle 30 degree.

(Refer Slide Time: 15:33)

$$\begin{aligned} \therefore V_{AB} &= \sqrt{3} \times 100 \angle 20^\circ = 173.2 \angle 40^\circ \\ I_{AB} &= \frac{V_{AB}}{Z_A} = \frac{173.2 \angle 40^\circ}{8.944 \angle 26.57^\circ} = 19.36 \angle 13.43^\circ \text{ Amp} \\ I_{BC} &= I_{AB} \angle -120^\circ = 19.36 \angle 13.43^\circ - 120^\circ = 19.36 \angle -106.57^\circ \text{ Amp} \\ I_{CA} &= I_{AB} \angle -240^\circ = I_{AB} \angle 120^\circ \\ \therefore I_{CA} &= 19.36 \angle 13.43^\circ + 120^\circ = 19.36 \angle 133.43^\circ \text{ Amp} \end{aligned}$$

And  $V_{AB}$  is equal to  $\sqrt{3}$  then, it is 100 angle your  $V_{an}$  is equal to 100 angle 10 degree, so  $\sqrt{3} \times 100$  angle 10 plus 30. So, 173.2 angle 40 degree volt right. Therefore,  $I_{AB}$  that is the phase current  $V_{AB}$  upon  $Z_{\Delta}$ , so you substitute all this values you will get 19.36 angle 13.43 degree ampere, the rest are shifting one 20 degree right. So,  $I_{BC}$  will be  $I_{AB}$  angle minus 120 degree it will be 19.36. So, 13.43 minus 120, so, it will be angle is equal to current dimension minus once under 6.5 degree ampere.

Similarly  $I_C$  is equal to  $I_{AB}$  angle minus 240 degree is equal to  $I_{AB}$  angle 120 degree plus it is minus 240, so making it as 120 degree. So, we that is 19.36, 13.43 plus 120 so 19.36 angle 133.43 degree, actually this things are very simple just little bit you go through this book and just listen what have been discussed here and nothing to be worried about this thing it seems very simple 3 phase right. So, this is ampere.

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The line currents are

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$
$$\therefore I_a = \sqrt{3} \times 19.36 \angle 13.43^\circ - 30^\circ \text{ Amp}$$
$$\therefore I_a = 33.53 \angle -16.57^\circ \text{ Amp}$$
$$I_b = I_a \angle -120^\circ = 33.53 \angle -16.57^\circ - 120^\circ$$

So, line current we know  $I_a$  is equal to just now we have derived  $\sqrt{3} I_{AB}$  minus angle minus 30 degree. So, substitute your all these things,  $I_{AB}$  you substitute, so  $I_{AB}$  is 19.36 angle was 13.43 degree minus 30 degree. So, it is  $I_a$  is equal to this one right.

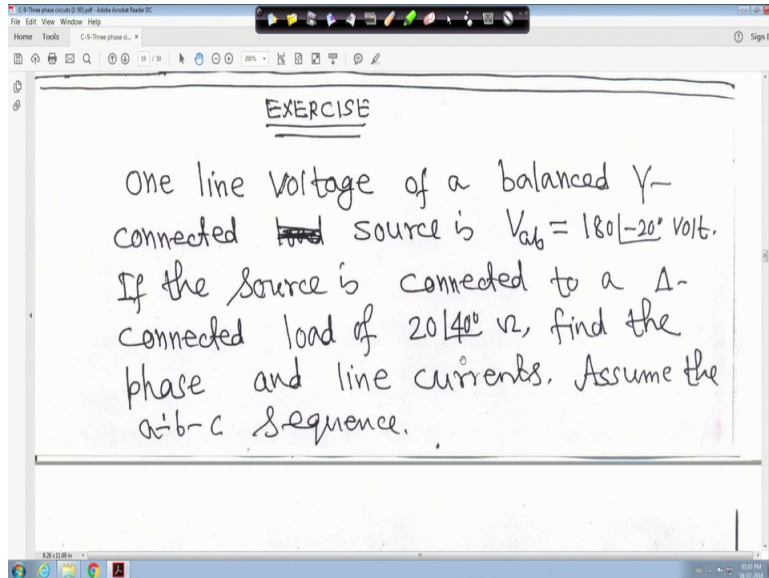
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$$I_b = I_a \angle -120^\circ = 33.53 \angle -16.57^\circ - 120^\circ$$
$$\therefore I_b = 33.53 \angle -136.57^\circ \text{ Amp.}$$
$$I_c = I_a \angle 120^\circ = 33.53 \angle -16.57^\circ + 120^\circ$$
$$\therefore I_c = 33.53 \angle 103.43^\circ \text{ Amp.}$$

Similarly  $I_b$  is equal to  $I_a$  angle minus 120 degree, you substitute  $I_a$  value, so  $I_b$  will be this much. Currents are same magnitude balance 33. Angle is just your, what you call if you take the difference of between these two angles you find the minus 120 degree right, it is a ampere. Similarly  $I_c$  is equal to  $I_a$  angle 120 degree, you will get  $I_c$  is equal

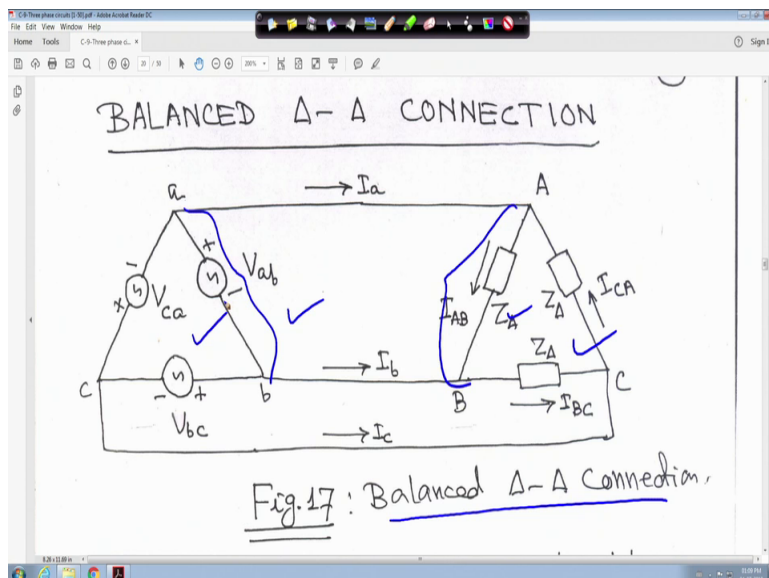
to 33.53 angle, 103.43 degree ampere right. So, this is your what you call this your that example you know exercise.

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So, this is your, what you call this exercise is given to you solve it. That single one line means single line one line voltage, one line voltage I mean one is star connected source is given this right. If the source is connected to a delta connected load of this much of impedance, 20 angle 40 degree we have to find out the phase and line current assuming abc sequence. This is an exercise for you that answer is not given right. So, we will do it.

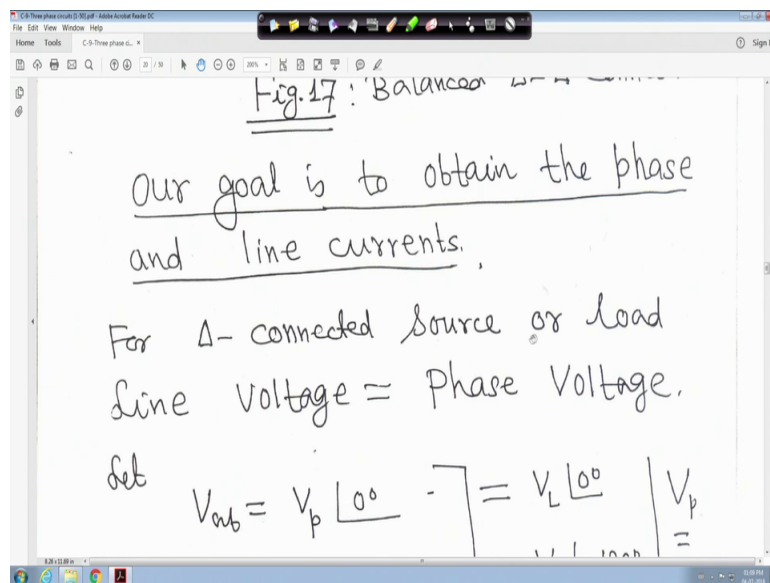
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Now, next is balanced delta connection right. So, in that case of balance delta delta connection, your in the case of delta load angle your delta source. So, in that case your line voltage and phase voltage remains same right. So, if it is a delta delta delta connection this is whatever you are  $V_{ab}$  will be in phase across this right. Similarly  $V_{bc}$  same because your from a delta connection your line voltage and phase voltage remains same right. So, here it is both delta both delta right. So, this is your balanced delta load is delta and source is also delta and series of impedance remains same right So,  $Z_{\Delta}$   $Z_{\Delta}$  is given right.

So, and this is the line current  $I_a$ ,  $I_b$ ,  $I_c$ , this is the line current but both are delta and these are the phase current  $I_{ab}$   $I_{bc}$   $I_{ca}$ . So, for delta delta your what you call line voltage is equal to phase voltage we see that we see that.

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So, our goal is to obtain the phase and line currents right. So, for delta connected source or load line voltage is equal to phase voltage because, both are delta delta, there is no question neutral no an bn cn these all line to line. So, line voltage is equal to phase voltage right.

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For  $\Delta$  connection,

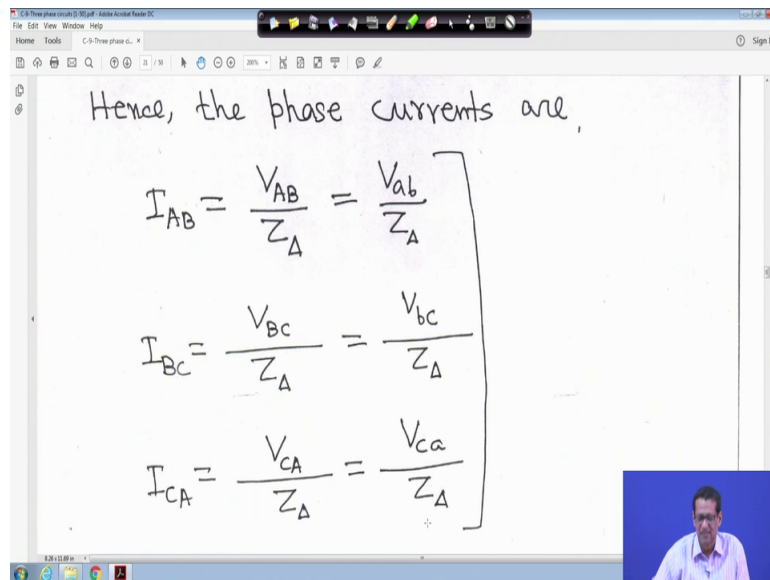
$$\text{Line Voltage} = \text{Phase Voltage.}$$

Let

$$\left. \begin{aligned} V_{ab} &= V_p \angle 0^\circ \\ V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle 120^\circ \end{aligned} \right\} = \begin{aligned} &= V_L \angle 0^\circ \\ &= V_L \angle -120^\circ \\ &= V_L \angle 120^\circ \end{aligned} \left| \begin{array}{l} V_p \\ = \\ V_L \end{array} \right.$$
$$\boxed{V_{ab} = V_{AB}, \quad V_{bc} = V_{BC}, \quad V_{ca} = V_{CA}}$$

So, it is line voltage is equal to phase voltage. So,  $V_{ab}$  you know  $V_p$  angle  $0^\circ$ ,  $V_{bc}$   $V_p$  angle minus  $120^\circ$  degree and  $V_{ca}$   $V_p$  angle  $120^\circ$  degree, that line voltage and phase voltage both are same. So,  $V_L$  is equal to here it is written here,  $V_p$  is equal to here I have written here  $V_p$  is equal to  $V_L$  right therefore,  $V_L$  angle  $0^\circ$  degree this one is equal to  $V_L$  angle minus  $120^\circ$  degree and this angle  $120^\circ$  degree here, both are same. So,  $V_{ab}$  is equal to  $V_{AB}$ ,  $V_{bc}$  is equal to  $V_{BC}$ ,  $V_{ca}$  is equal to  $V_{CA}$ . here it is same thing here it is capital A, capital B, capital A, capital B nothing is there in the line, so  $V_{ab}$  is equal to  $V_{ABC}$  similarly for  $V_{bc}$  angle  $V_L$  right.

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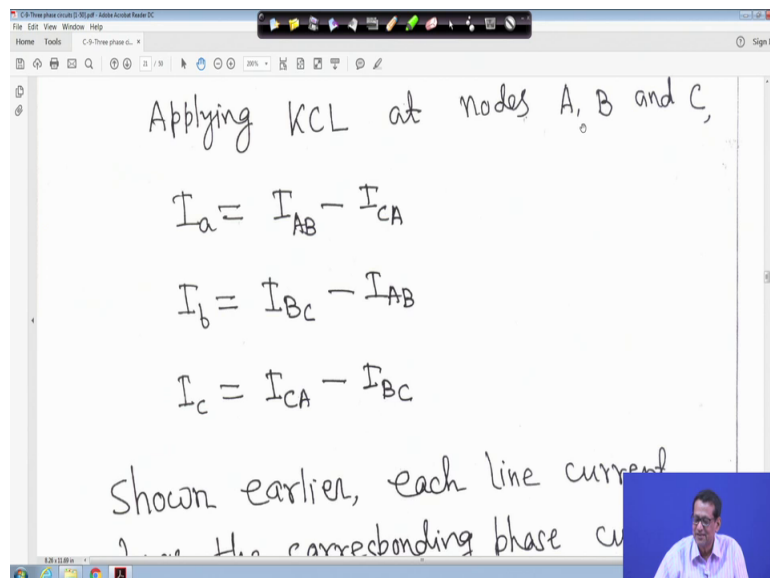


Hence, the phase currents are,

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{ab}}{Z_{\Delta}}$$
$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = \frac{V_{bc}}{Z_{\Delta}}$$
$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}} = \frac{V_{ca}}{Z_{\Delta}}$$

So, hence the phase currents are simply  $I_{AB}$  is equal to  $V_{AB}$  upon  $Z_{\Delta}$ , if you come to the circuit if you come to this circuit  $I_{AB}$  is equal to  $V_{ab}$  upon  $Z_{\Delta}$  right here. So similarly your,  $I_{BC}$  and  $I_{CA}$  this is  $I_{BC}$ ,  $V_{BC}$  upon  $Z_{\Delta}$   $I_{CA}$  is equal to  $V_{CA}$  upon  $Z_{\Delta}$  right.

(Refer Slide Time: 20:27)



Applying KCL at nodes A, B and C,

$$I_a = I_{AB} - I_{CA}$$
$$I_b = I_{BC} - I_{AB}$$
$$I_c = I_{CA} - I_{BC}$$

Shown earlier, each line current is equal to the corresponding phase current.

Therefore, applying KCL at nodes ABC, similar same as before, if you apply at node ABC KVL your KCL you will get  $I_a$  is equal to  $I_{AB}$  minus  $I_{CA}$ ,  $I_b$  is equal to  $I_{BC}$  it is just when you do this one just draw that circuit diagram first then, we will just listen to

this video lecture right. I then you do of your own and  $I_b$  is equal to  $I_{BC}$  minus  $I_{AB}$  and  $I_c$  is equal to  $I_{CA}$  minus  $I_{BC}$  right.

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$I_c = I_{CA} - I_{BC}$

Shown earlier, each line current lags the corresponding phase current by  $30^\circ$ . The magnitude  $I_L$  of the line current is  $\sqrt{3}$  times the magnitude  $I_p$  of the phase current.

$$I_L = \sqrt{3} I_p.$$

So, it is already shown earlier right, each line current lags the corresponding phase current by 30 degree. This already we have shown it previous example. So, the magnitude  $I_L$  of the line current it is root 3 times the magnitude of the phase current. Because, it is line current and then it is delta connected load, So, line current is equal to root 3 times phase current that we have done it and  $I_L$  is equal to root 3 into  $I_p$  right.

(Refer Slide Time: 21:19)

Ex-4

A balanced  $\Delta$ -connected load having an impedance  $(20 - j15) \Omega$  is connected to a  $\Delta$ -connected source having  $V_{ab} = 330 \angle 0^\circ$  Volt. Calculate the phase currents of the load and the line currents.

So, take another small example; a balanced delta connected load having an impedance  $20 - j15$  ohm is connected to a delta connected source the voltage is  $330$  angle  $0$  degree that is  $V_{ab}$  is given. Calculate that phase currents of the load angle and the line currents right.

(Refer Slide Time: 21:37)

The image shows a handwritten solution on a whiteboard. The text is as follows:

Soln.

$$Z_{\Delta} = (20 - j15) = 25 \angle -36.87^{\circ} \Omega.$$

Since  $V_{AB} = V_{ab}$ , the phase currents are,

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{330 \angle 0^{\circ}}{25 \angle -36.87^{\circ}} = 13.2 \angle 36.87^{\circ} \text{ Amp.}$$

$$I_{BC} = I_{AB} \angle -120^{\circ} = 13.2 \angle -83.13^{\circ} \text{ Amp.}$$

$$13.2 \angle 156.87^{\circ} \text{ Amp.}$$

So, in this case  $Z_{\Delta}$  is equal to  $20 - j15$ , So  $25$  angle minus  $36.87$  degree ohm. Since  $V_{AB}$  is equal to  $V_{ab}$  the phase currents are  $I_{AB}$  is equal to  $V_{AB}$  upon  $Z_{\Delta}$ . So, you know  $V_{AB}$ , so put  $Z$ , you will get this is the current  $13.2$ ,  $36.87$  degree ampere right.



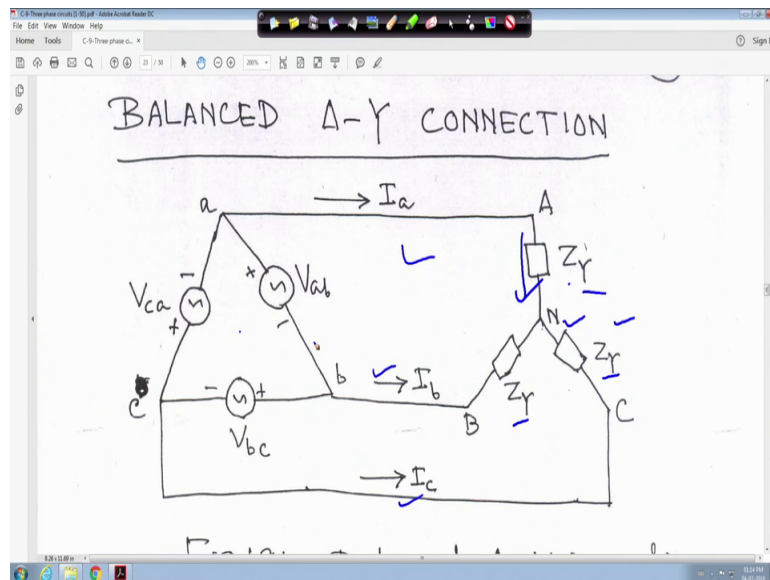
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Since  $V_{AB} = V_{ab}$ , the phase currents are,

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{33 \angle 0^{\circ}}{25 \angle -36.87^{\circ}} = 13.2 \angle 36.87^{\circ} \text{ Amp.}$$
$$I_{BC} = I_{AB} \angle -120^{\circ} = 13.2 \angle -83.13^{\circ} \text{ Amp.}$$
$$I_{CA} = I_{AB} \angle 120^{\circ} = 13.2 \angle 156.87^{\circ} \text{ Amp.}$$
$$I_{\alpha} = (\sqrt{3} I_{AB}) \angle -30^{\circ} = 22.86 \angle 6.87^{\circ} \text{ Amp.}$$

Similarly, I BC you know that relationship I B angle minus 120 degree. This I AB you substitute here you substitute here, you will get this one 13.2 angle minus 83.13 ampere. Similarly I CA you will get I AB 120 degree, so you will get 13.2, 156.87 degree ampere. And we know I line current to root 3 current and phase current angle minus these all we have derived, little bit you have to keep it in your mind right little bit practice is necessary. So, if you do so you will get value of I a, I b, I c, magnitude same angles also substituted angle dependence will be again 120 degree right. Magnitude are 22.86 degree 8 6 ampere and these are the angle right, so this is ampere ampere. Another one is balanced delta Y connection.

(Refer Slide Time: 22:45)



So, in this case your what we call that source is delta, but load is star. So, when loading this star in this case, line current is equal to phase current because same current is your what we call, the same current  $i$  is going entering here right. So, line current is equal to your phase current right so, but e your what you call this is voltage source, your what we call and this is at your delta source is delta.

So, in here also same philosophy will be there that, your phase voltage line voltage is root 3 times phase voltage. And this current  $I_a, I_b, I_c$  right and this is capital N and this is  $Z_y, Z_y, Z_y$ , source is delta and load is star. But here line current is equal to phase current right so, but here line your, what we call here, phase voltage is equal to line voltage by root 3 because, line voltage is equal to root 3 times phase voltage right. Race 2 remains same, so this is the circuit diagram and all polarity as instantaneous polarity I told you how to mark it.

(Refer Slide Time: 23:55)

17/10, Balanced 4-wire connection,

Let

$$\left. \begin{aligned} V_{ab} &= V_p \angle 0^\circ \\ V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle 120^\circ \end{aligned} \right\} = \begin{aligned} &V_L \angle 0^\circ \\ &V_L \angle -120^\circ \\ &V_L \angle 120^\circ \end{aligned}$$

Apply KVL to loop aANBba

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$   $V_L = 0$

So, that means,  $V_{ab}$  is equal to same thing we have done  $V_p$  angle 0 degree,  $V_{bc}$   $V_p$  angle minus 120 degree and  $V_{ca}$  is equal to  $V_p$  angle 120 degree. So,  $V_L$  angle 0 degree,  $V_L$  angle minus 120 degree  $V_L$  angle 120[0]

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$$V_{ca} = V_p \angle 120^\circ \quad \left\} = V_L \angle 120^\circ$$

Apply KVL to loop aANBba

$$I_a \cdot Z_Y - I_b \cdot Z_Y - V_{ab} = 0$$

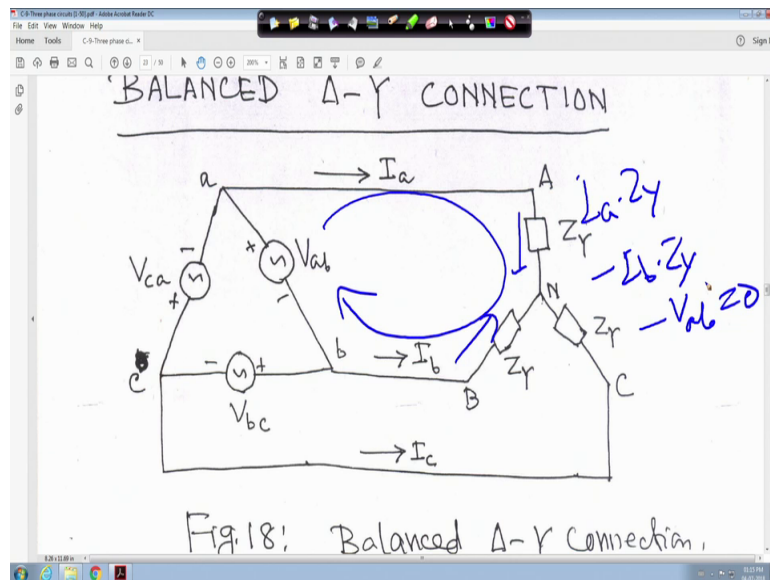
$$\therefore I_a - I_b = \frac{V_{ab}}{Z_Y}$$

$$\therefore I_a - I_b = \frac{V_p \angle 0^\circ}{Z_Y} = \frac{V_L \angle 0^\circ}{Z_Y}$$

$V_p = V_L$

Now, apply KVL to loop this one right, so what we will get.

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So, if you apply your KVL here, if you apply your KVL here, it will be  $I_a Z_Y$  minus  $I_b Z_Y$  minus  $V_{ab}$  is equal to 0. So, it will be  $I_a$  minus  $I_b$  is equal to  $V_{ab} / Z_Y$ . So, this  $I_a$  minus  $I_b$  is equal to  $V_{ab} / Z_Y$ . So, here it is given that  $V_{ab}$  is equal to  $V_L \angle 0^\circ$  is equal to your line voltage we write  $V_L \angle 0^\circ$ . So, this is upon  $Z_Y$   $I_a$  minus  $I_b$ .

This way we can write that, that equation we have written here that equation we have written here;  $I_a Z_Y$  minus  $I_b Z_Y$  minus  $V_{ab}$  is equal to 0, that means,  $I_a$  minus  $I_b$  is equal to  $V_{ab} / Z_Y$ . So, this  $I_a$  minus  $I_b$  is equal to  $V_{ab} / Z_Y$ . So, here it is given that  $V_{ab}$  is equal to  $V_L \angle 0^\circ$  is equal to your line voltage we write  $V_L \angle 0^\circ$ . So, this is upon  $Z_Y$   $I_a$  minus  $I_b$ .

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But  $I_b$  lags  $I_a$  by  $120^\circ$ ,

$$\therefore I_b = I_a \angle -120^\circ$$

$$\therefore I_a - I_a \angle -120^\circ = \frac{V_p \angle 0^\circ}{Z_Y} = \frac{V_L \angle 0^\circ}{Z_Y}$$

$$\therefore I_a = \frac{(V_p/\sqrt{3}) \angle -30^\circ}{Z_Y} = \frac{(V_L/\sqrt{3}) \angle -30^\circ}{Z_Y}$$

But  $I_a$  lags  $I_b$  by 120 degree, we know that, but  $I_b$  lags 120 degree, simply substitute  $I_b$  is equal to  $I_a$  minus 120 degree, you put it here and you simplify, if you simplify you will get,  $I_a$  is equal to  $V_p$  upon root 3 divided by angle minus 30 degree by  $Z_Y$ .

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$$\therefore I_a = \frac{(V_p/\sqrt{3}) \angle -30^\circ}{Z_Y} = \frac{(V_L/\sqrt{3}) \angle -30^\circ}{Z_Y}$$

$$I_b = I_a \angle -120^\circ$$

$$\therefore I_b = \frac{(V_p/\sqrt{3}) \angle -150^\circ}{Z_Y} = \frac{(V_L/\sqrt{3}) \angle -150^\circ}{Z_Y}$$

So, line voltage your  $V_p$  as your what you call it is that that can be written as,  $V_L$  upon root 3 angle minus 30 degree upon  $Z_Y$ . Here we have written here we have written that your  $V_{ab}$  is equal to  $V_L$  angle zero,  $V_L$  angle minus 120 degree,  $V_L$  angle minus your sorry plus 120 degree right. So, similarly here also you write  $V_L$  by root 3, angle minus

30 upon  $Z_y$  right  $Z^*$  or  $Z_y$ , that means, similarly your  $I_b$  is equal to  $I_a$  angle minus 120, these  $I$  you substitute here, this  $I$  you substitute here you will get  $V_L$  root 3 angle minus 150 by  $Z_y$ .

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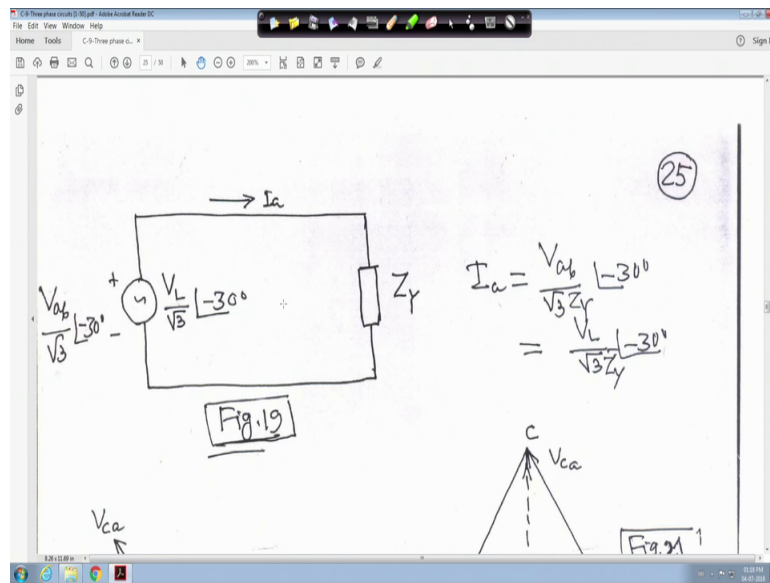
The image shows a digital whiteboard with handwritten equations. The first equation is 
$$\therefore I_b = \frac{(V_p/\sqrt{3}) \angle -150^\circ}{Z_Y} = \frac{(V_L/\sqrt{3}) \angle -150^\circ}{Z_Y}$$
 The second equation is 
$$I_c = I_a \angle 120^\circ$$
 The third equation is 
$$\therefore I_c = \frac{(V_p/\sqrt{3}) \angle 90^\circ}{Z_Y} = \frac{(V_L/\sqrt{3}) \angle 90^\circ}{Z_Y}$$

Similarly,  $I_c$  is equal to one angle  $I$  your 20 degree you put  $I_a$  is equal to here you will get  $V_p$  upon root 3 angle  $Z$  divided by  $Z_y$  and angle 90 degree.

So,  $V_L$  upon root 3 angle 90 degree upon  $Z_y$  right. So, this all the thing is that you have to remember nothing to be this thing very simple it is right because, if phase difference is that 120 degree lagging or leading right and just you have to simplify. And here we have repeat phase voltage your what you call for delta source, phase voltage and line voltage remain same for delta source and in if it is a delta source is phase are lined same. So, phase voltage is equal to line voltage for the delta source right.

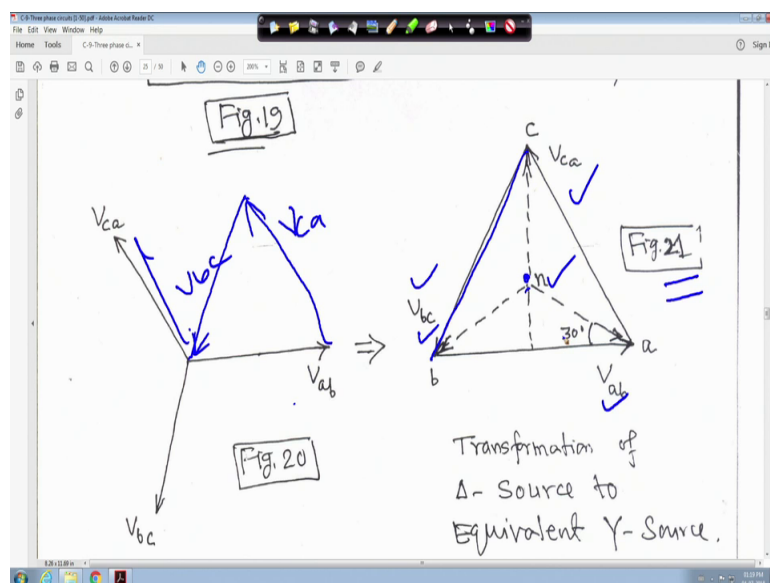
So, source is delta, line voltage is equal to phase voltage, that is why here it is taken line voltage is equal to phase voltage. That is why all these thing is replaced by phase voltage because your source is delta right, so line voltage is equal to phase voltage. This certain things you have to keep it in mind right So, I mean if you try to represent this by your what you call whatever you have got say for phase a for phase a right if you see that  $I_a$  is equal to  $V_L$  upon root 3 angle minus 30 upon  $Z_y$ .

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So, if you represent it by your  $V_{ab}$  upon root 3 angle minus 30, this one is nothing, but  $V_L$  upon root 3 minus 30 degree and this is your  $Z_Y$ , this is the current flowing  $I_a$ . So,  $I_a$  will be simply  $V_L$  upon root 3  $Z_Y$  angle minus 30. You are this is equal to this one and because  $V_{ab}$  is nothing, but the  $V_L$ . So, this is represented by only one phase right. So, this we also can represent.

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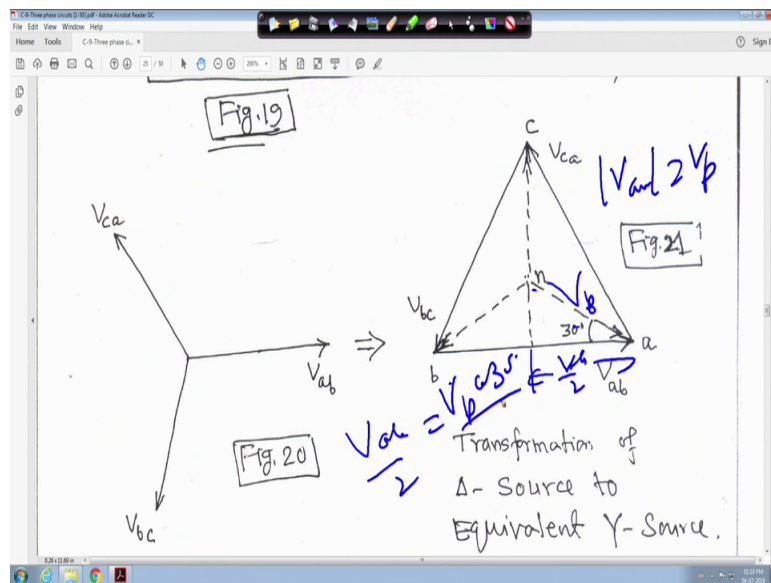
So, similarly if you draw the phasor diagram, this is your  $V_{ab}$ , this is  $V_{bc}$ , this is  $v_{ca}$ . So, if you try to find out if you try to find out this thing, your what you call if you try to

make this delta, so, this is my V ab this all 120 degree, 120 this is my V ca, so this my V bc no, so just let me clear it.

This diagram actually it will be like this, this is my V bc, so this is my V bc this is drawn for here, this is V bc and this is my V ca, so this is my V ca this one, so this is my V ca and this is my V bc and this b. That is why this is my V bc whatever is here that is my V ab and this is my V ca just you make competitive this triangle and if, you join all it will be same actually this is my same some your something is marked this as a m right.

So, this is figure 20 one and if you do so, if you do so, this angle is 30 degree right and your V an, your what you call V an is nothing, but your V p V bn also V p we solved that here what we call all these magnitude.

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So, that means, this one this V ab right, V ab this is actually this portion actually V ab by 2 it is half it is equilateral triangle. So, V ab by 2 is equal to this is V an, this is actually V p right because, this is the magnitude V an, so this is V an is equal to V p right. So, it is V p then, cos 30 degree right here is equal to is equal to V p cos 30 degree right.



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$$|V_{an}| \cos 30^\circ = \frac{|V_{ab}|}{2} = \frac{V_p}{2} = \frac{V_L}{2}$$

$$\therefore |V_{an}| = \frac{V_L}{\sqrt{3}}$$

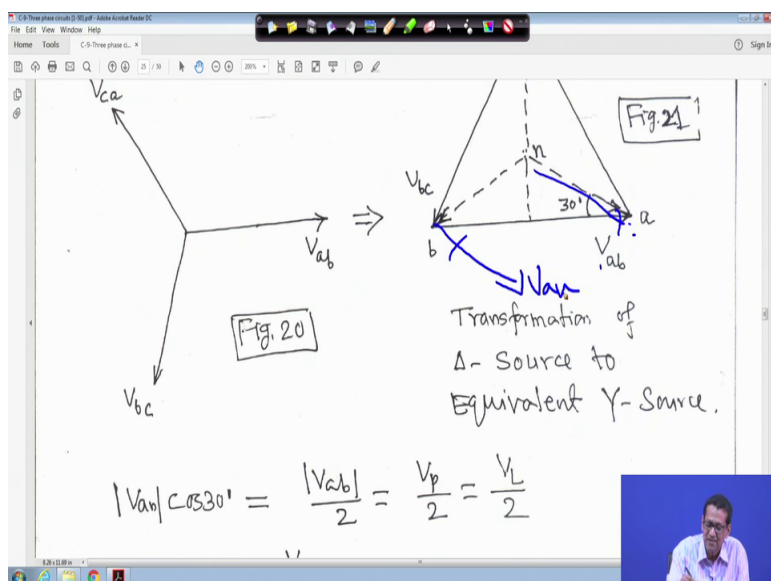
$$\therefore V_{an} = \frac{V_L}{\sqrt{3}} \angle -30^\circ = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$

$$V_{bn} = \frac{V_L}{\sqrt{3}} \angle -150^\circ = \frac{V_p}{\sqrt{3}} \angle -150^\circ$$

$$V_{cn} = \frac{V_L}{\sqrt{3}} \angle 90^\circ = \frac{V_p}{\sqrt{3}} \angle 90^\circ$$

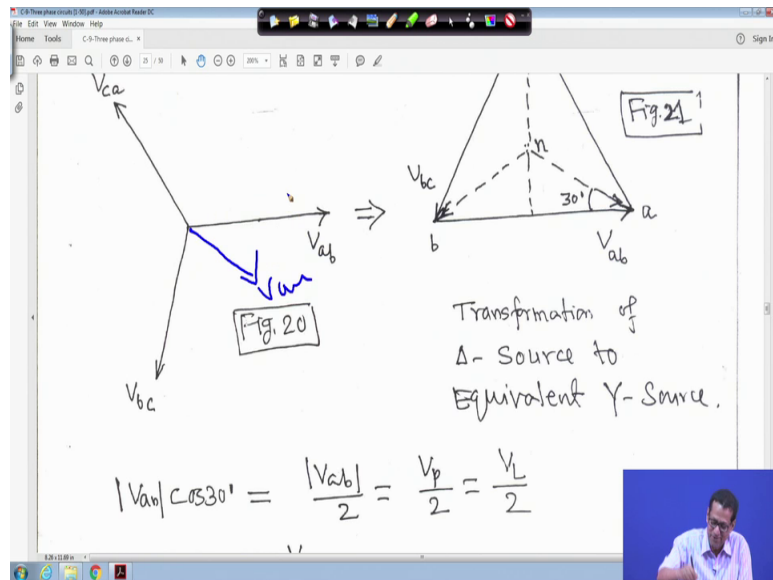
That means, that means, this one actually your  $V_{an} \cos 30^\circ$  is equal to  $V$  angle to magnitude only right. So,  $V_p$  by 2 is equal to  $V_L$  by 2. From this diagram only right magnitude u 2 so; that means, your what we call mod val is equal to  $V_L$  upon root 3 cos here what we call  $\cos 30^\circ$  is equal to 3 by 2. So, what you call that your phase voltage is equal to  $V_L$  by root 3 is equal to  $V_L$  line voltage by root 3. Otherwise line voltage is equal to root 3 times phase voltage. Therefore, we can write  $V_{an}$  is equal to  $V_L$  upon root 3 angle minus 30 degree right. So, eh so that means, your this is that means, this is my  $V_{an}$  right.

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So, it is in this direction  $V_{an}$ , that means, this is my say just as a reference am putting, this is  $V_{an}$ , if i make it right because this is this direction, so  $V_{an}$  will lag from  $V_b$  by minus 30 degree So, that is why that is why your  $V_{an}$  is equal to  $V_L$  upon root 3 angle minus 50. So,  $V_p$  upon root 3 angle minus 30 degree.

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Similarly for  $V_{bn}$  and similarly for  $V_{cn}$ , that means, if you plot it here, so mean this is my your what you call  $V_{an}$ ,  $V_{bn}$ ,  $V_{cn}$  if you plot it here, so in this case it will be your what you call  $V_{an}$  right. So, this is my  $V_{an}$  same was before same as before right. Earlier it was shifting 30 degree, but you wrote it like 30 degree this  $V_{an}$  right, same as before So, this is then  $V_{bn}$  is equal to  $V_p$  upon root 3 angle minus 150 50 and  $V_{cn}$  will be an  $V_{pn}$  root 3 angle 90 degree right.

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$$V_{AN} = I_a Z_p = \frac{V_L}{\sqrt{3}} \angle -30^\circ = \frac{V_p}{\sqrt{3}} \angle -30^\circ \quad (26)$$
$$V_{BN} = V_{AN} \angle -120^\circ$$
$$V_{CN} = V_{AN} \angle 120^\circ$$

So, this is your, what we call and  $V_{AN}$  is equal to your  $I_a$  into  $Z_p$ . That is  $V_L$  upon root 3 angle minus 30, is equal to  $V_p$  upon root 3 angle minus 30 degree. Because, line source are delta correction source line voltage is equal to phase voltage. So, we know  $V_{BN}$  is equal to  $V_{AN}$  angle because angle minus 120 degree because, it lags by 120 degree, just relationship we have to use. And if you put  $V_{AN}$  here and this  $V_{CN}$  is equal to  $V_{AN}$  120 degree. So, this is your what we call if you substitute  $V_{AN}$  is equal to  $V_p$  by root 3 here it will be  $V_p$  by root 3 angle minus 150 degree and if you make it here your  $V_{an}$  is equal to  $V_p$  upon root 3 angle minus 30 it will be  $V_p$  upon root 3 angle 90 degree right. So, with this thank you very much we will be back again.