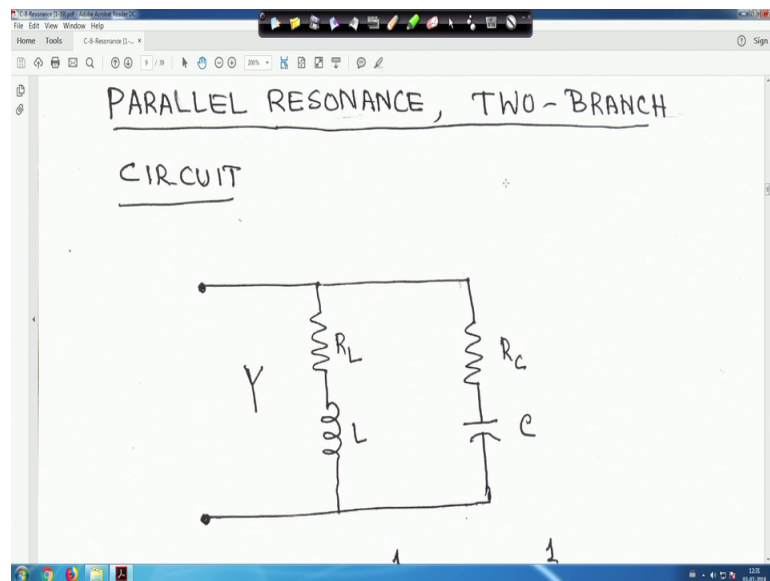


**Fundamentals of Electrical Engineering**  
**Prof. Debapriya Das**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 45**  
**Resonance and Maximum Power Transfer Theorem (Contd.)**

We are back again, now next is that parallel resonance that is two branch circuit.

(Refer Slide Time: 00:21)



We have taken a simple two branch circuit right. So, your this is  $R L$  over  $L$  that is one inductive branch and this is one capacitive branch right. So, it is  $R L L \omega$  and it is  $R C$  your  $X C$  right. So, this is you have to see the parallel resonance of the circuit. So,  $Y$  is the admittance right. So,  $Y$  is equal to your  $Y$  is equal to that admittance is equal to  $Y L$  plus  $Y C$  right.

(Refer Slide Time: 00:41)

$$Y = Y_L + Y_C = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$= \left( \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right) + j \left( \frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right)$$

So,  $Y_L$  is that admittance of this inductive circuit. So, it is 1 upon  $R_L$  plus  $jX_L$  and this is your plus 1 upon  $R_C$  minus  $jX_C$  right. So, now this one numerator and denominator this one you multiply by your  $R_L$ , this term we multiply by your  $R_L$  minus  $jX_L$ ; that means, this one will be  $R_L$  minus  $jX_L$  right divided by  $R_L$  plus  $X_L$  into  $R_L$  minus  $jX_L$  that is  $R_L$  square minus  $j$  square  $X_L$  square. So,  $j$  square is minus 1 so, it will be your  $R_L$  square plus your  $X_L$  square. So, just separate it will be  $R_L$  upon  $R_L$  plus  $X_L$  square and this one your what we call this will be your my here it is taken common.

So, here it is minus  $X_L$  upon  $R_L$  square plus  $X_L$  square imaginary side right. So, this is actually this two are real parts. So, real part is  $R_L$  upon your  $R_L$  square plus  $X_L$  square imaginary part is minus  $X_L$  upon  $R_L$  plus  $X_L$  square. So, here it is minus  $X_L$  upon  $R_L$  plus  $X_L$  square, similarly for  $R_C$  minus  $jX_C$  numerator and denominator you multiply by  $R_C$  plus  $jX_C$ . So, real part is here and imaginary part is here right. So, let me clear it.

(Refer Slide Time: 02:09)

Then

$$\frac{X_C}{(R^2 + X_C^2)} = \frac{X_L}{(R^2 + X_L^2)}$$

$X_L = L\omega = L\omega_0$   
 $X_C = \frac{1}{\omega} = \frac{1}{\omega_0 C}$   
 $\omega = \omega_0 = \frac{1}{\omega_0 C}$

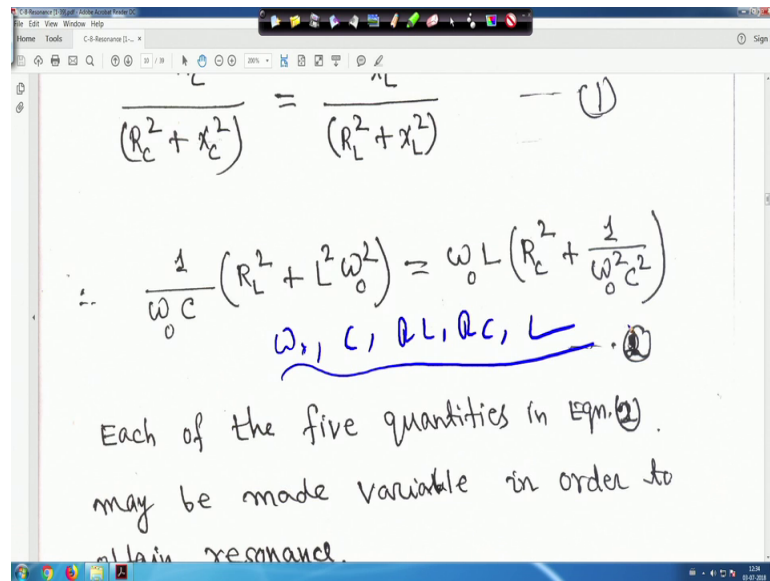
$$\therefore \frac{1}{\omega_0 C} (R^2 + L^2 \omega_0^2) = \omega_0 L (R^2 + \frac{1}{\omega_0^2 C^2})$$

Real of the five quantities in Eqn. (2)

So, this means; that means, circuit is at resonance when the complex admittance is a real number; that means, this one this imaginary part we set it to 0, this one we set is to 0. If you do so, it will become  $X_C$  upon  $R^2 + X_C^2$  is equal to  $X_L$  upon  $R^2 + X_L^2$  right. This is actually at  $\omega$  is equal to this is at  $\omega_0$ ,  $\omega$  is equal to  $\omega_0$  the resonance frequency, that is why that wherever we make it  $X_L$  is equal to  $L\omega_0$  that is your  $L\omega_0$  right and  $X_C$  is equal to  $1/\omega_0 C$  that is your  $1/\omega_0 C$  right.

So, this one you put it here that  $X_L$  is equal to your  $L\omega_0$  and  $X_C$  is equal to  $1/\omega_0 C$  that is  $\omega_0 C$  and just simplify right let me clear it.

(Refer Slide Time: 03:03)



$$\frac{L}{(R_L^2 + X_L^2)} = \frac{L}{(R_L^2 + X_L^2)} \quad \text{--- (1)}$$

$$\therefore \frac{1}{\omega_0 C} (R_L^2 + L^2 \omega_0^2) = \omega_0 L \left( R_L^2 + \frac{1}{\omega_0^2 C^2} \right)$$

$\omega_0, C, R_L, R_C, L$  . (2)

Each of the five quantities in Eqn. (2) may be made variable in order to obtain resonance.

So, this way if you just simplify that is alright cross multiply this one you cross multiply and then you put it. So, each of this five quantities in equation may be made variably in order to obtain resonance there are five variables right, there are five variables one is omega 0 then C then RL then RC then L. So, there are five variables right.

So, in this case that is each of the five quantities in equation 2 right. So, this is equation 2 this is equation 2 right, may be made variable in order to obtain resonance. Now, solving equation 1; that means, rather than I will say equation 1 or equation 2 rather than I will say equation 2 from equation 1 we got equation 2 so, you will get omega 0 is equal to 1 upon root over I mean what you do is you from this equation say from this equation 2 find out omega 0.

(Refer Slide Time: 03:51)

obtain resonance.

Solving eqn. (1) for  $\omega_0$ , we obtain

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

Thus the resonant frequency  $\omega_0$  of the two-branch parallel circuit differs

You do little bit of mathematical gymnastic right and then from that you simplify and then you do  $\omega_0$  is equal to  $1/\sqrt{LC}$  then root over  $R_L^2 - L/C$  by  $R_C^2 - L/C$ . From this equation 2 you will get thus expression I suggest you please do it I have given you the final expression. So, that one you get  $1/\sqrt{LC}$  the root over  $R_L^2 - L/C$  by then divided by  $R_C^2 - L/C$ .

Now, circuit will be are loop for series circuit that  $\omega_0$  this sub series circuit  $\omega_0$  was that your  $1/\sqrt{LC}$  only, but the parallel circuit right so for series RLC circuit, we have seen  $1/\sqrt{LC}$  for parallel circuit. One factor is multiplied and this is this factor right.

(Refer Slide Time: 04:49)

obtain resonance.

Solving eqn. (1) for  $\omega_0$ , we obtain

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - 4/LC}{R_C^2 - 4/LC}}$$

$R_L^2 - 4/LC > 0$   
 $R_C^2 - 4/LC > 0$

Thus the resonant frequency  $\omega_0$  of the two-branch parallel circuit differs

So, now resonance when resonance will occur that one condition is that your RL because your square root term you are under the root this term have to be positive. So, one condition is RL minus RC it has to be greater than 0 sorry RL square minus L by C it has to be greater than 0, another thing is that RC square minus L by C also greater than 0 this is one condition for resonance right.

(Refer Slide Time: 05:17)

obtain resonance.

Solving eqn. (1) for  $\omega_0$ , we obtain

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - 4/LC}{R_C^2 - 4/LC}}$$

$R_L^2 - 4/LC > 0$   
 $R_C^2 - 4/LC > 0$

Thus the resonant frequency  $\omega_0$  of the two-branch parallel circuit differs

So, we will get some value that because this has to be positive this has to be positive or this one has to be negative this one has to be negative, such that RL square minus L by C

less than 0 and your RC square minus L by C less than 0 if both are negative then it will be positive. Then you will get some value of resonance frequency right.

(Refer Slide Time: 05:39)

obtain resonance.

Solving eqn. (1) for  $\omega_0$ , we obtain

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

$R_L^2 = R_C^2 = L/C$

Thus the resonant frequency  $\omega_0$  of the two-branch parallel circuit differs

But if let me clear it, but if your what if it happens that your RL square is equal to RC square is equal to L by C; if this condition happen then it will resonate at every all frequencies right.

(Refer Slide Time: 05:55)

from that of the pure R, L and C in parallel by the factor

$$\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

Frequency must be a real positive number.

So, now let me clear it. So, this is your; that means, does the resonance frequency  $\omega_0$  of the two branch parallel circuit this is one condition right. It is your what you call this

factor has been multiplied, when the series RLC circuit is a 1 upon root over LC, but with this factor this has been multiplied that is I told you these are multiplied right.

(Refer Slide Time: 06:13)

positive number

Hence the circuit will have a resonant frequency  $\omega_0$  when

$$R_L^2 > \frac{L}{C} \quad \text{and} \quad R_C^2 > \frac{L}{C}$$

OR

$$R_L^2 < \frac{L}{C} \quad \text{and} \quad R_C^2 < \frac{L}{C}$$

When  $D^2 = D^2 = \frac{L}{C}$

So, now for resonant condition I told you that  $R_L^2$  minus  $L$  by  $C$  greater than 0 and  $R_C^2$  minus  $L$  by  $C$  greater than 0 both has to be positive; that means,  $R_L^2$  square greater than  $LC$  and  $R_C^2$  square sorry  $L$  by  $C$  and  $R_C^2$  square greater than  $L$  by  $C$ .

(Refer Slide Time: 06:27)

$$R_L^2 < \frac{L}{C} \quad \text{and} \quad R_C^2 < \frac{L}{C}$$

When  $R_L^2 = R_C^2 = \frac{L}{C}$

The circuit is resonant at all frequencies

Or  $R_L^2$  square minus  $L$  by  $C$  less than 0 that is  $R_L^2$  square less than  $L$  by  $C$  similarly  $R_C^2$  square minus  $L$  by  $C$  less than 0, that is  $R_C^2$  square less than  $L$  by  $C$ . So, this way



resonant will happen right, but when RL square I mean is equal to RC square is equal to L by C the circuit is resonant at all frequencies right that also I told you because, that factor tau will vanish right.

So, now question is that, if you solve equation 1 right for your what you call for L you will obtain something like this. I mean if this is equation 1 it is very interesting suppose from equation 1 right from equation 1.

(Refer Slide Time: 07:11)

The image shows a whiteboard with handwritten mathematical equations. The text 'Then' is written at the top left. The main equation is:

$$\frac{X_C}{(R_C^2 + X_C^2)} = \frac{X_L}{(R_L^2 + X_L^2)}$$

To the right of this equation, there are two boxed equations:

$$Z_C^2 = (R_C^2 + X_C^2) \quad (1)$$

$$Z_L^2 = (R_L^2 + X_L^2)$$

Below these, the following equation is derived:

$$\therefore \frac{1}{\omega_0 C} (R_L^2 + L^2 \omega_0^2) = \omega_0 L \left( R_C^2 + \frac{1}{\omega_0^2 C^2} \right)$$

At the bottom of the whiteboard, there is a partially visible line of text: "... quantities in Eqm. (1)".

So, your this one we call ZC square is equal to your RC square plus XC square this is my ZC square and ZL square is equal to R square plus XL square right.


(Refer Slide Time: 07:37)

Solving Eqn.(1) for L, we obtain

$$L = \frac{1}{2}C \left[ Z_c^2 \pm \sqrt{Z_c^4 - 4R_L^2 X_C^2} \right] \dots (3)$$

Where  $Z_c = (R_c^2 + X_c^2)$ ;  $Z_c^4 = (R_c^2 + X_c^2)^2$

Now if in Eqn.(3),



So, now let me clear it therefore, from this equation 1 from this equation 1, we will see this one that if you solve for L you will get this you solve it, I have done it for you, but this will solve it you will get L is equal to half into C half C bracket that ZC square plus minus root over ZC to the power 4 minus 4 RL square XC square this is equation 3 right, where ZC square is equal to this is square right. This is square, this is square Z C square is equal to RC square plus XC square therefore, ZC 4 is equal to RC square plus XC square whole square. So, from equation 1 you please solve this one, you please solve this one for your L. So, let me clear it.

(Refer Slide Time: 08:17)


Solving Eqn.(1) for L, we obtain

$$L = \frac{1}{2}C \left[ Z_c^2 \pm \sqrt{Z_c^4 - 4R_L^2 X_C^2} \right] \dots (3)$$

Where  $Z_c = (R_c^2 + X_c^2)$ ;  $Z_c^4 = (R_c^2 + X_c^2)^2$

Now if in Eqn.(3),

$Z_c^4 - 4R_L^2 X_C^2 = Z_0$



So, in this case what is happening that from this equation 3 that for resonance it has to be  $Z_C^4 - Z_C^2$  to the power 4 minus  $4R_L^2 X_C^2$  it has to be greater than 0 because it is square root so, it has to be positive right. So, in that case you will get two values of L right, two values of L for which circuit will your what we call resonance will happen right.

So, another this is one another thing is let me clear it another thing if  $Z_C^4$  to the power 4 minus  $4R_L^2 X_C^2$  with these term if it is equal to 0, you will have only single value of l right, but if  $Z_C^4$  to the power 4 if this term become negative, then there will be no resonance right. So, let me clear it so, that means whatever I told that  $Z_C^4$  to the power 4 has to be greater than  $4R_L^2 X_C^2$  the required root this under the square root term has to be positive right. So, we obtain two values of L for which the circuit is resonant right.

(Refer Slide Time: 09:03)

$$Z_C^4 = (R_C^2 + X_C^2)^2, \quad Z_C^4 = (R_C^2 + X_C^2)^2$$

Now if in Eqn. (3),

$$Z_C^4 > 4R_L^2 X_C^2,$$

We obtain two values of L for which the circuit is resonant.

If  $Z_C^4 = 4R_L^2 X_C^2,$

(Refer Slide Time: 09:09)


(12)

Solving Eqn.(1) for L, we obtain

$$L = \frac{1}{2}C \left[ Z_c^2 \pm \sqrt{Z_c^4 - 4R_L^2 X_C^2} \right] \dots (3)$$

Where  $Z_c = (R_c^2 + X_c^2)$ ,  $Z_c^4 = (R_c^2 + X_c^2)^2$

Now if in Eqn.(3),



So, that is if  $Z_c$  to the power for two values means because here plus minus is there plus minus is there. So, if  $Z_c$  your what you call your if  $Z_c$  to the power 4 is equal to  $4 R_L$  square  $X_C$  square.

(Refer Slide Time: 09:31)


The circuit is in resonance in

$$L = \frac{1}{2}C Z_c^2$$


---

When  $Z_c^4 < 4R_L^2 X_C^2$ ,

No value of L will make the circuit resonant.



The circuit is in resonance at L is equal to half C  $Z_c$  square, I mean if  $Z_c$  to the power 4 is equal to  $4 R_L$  square  $X_C$  square means this term will be 0 so; that means, L will be half C into  $Z_c$  square right. So; that means, the circuit is in resonance at L is equal to half C  $Z_c$  square and when  $Z_c$  to the power 4 is less than  $4 R_L$  square  $X_C$  square no value L will make the circuit resonant. So, if this condition is there; that means, under

root comes square root of is negative then this question is what there will be no value of L will make the circuit resonance right. So, this is for L.

(Refer Slide Time: 10:07)

Solving Eqn. (1) for C, we obtain

$$C = 2L \left[ \frac{1}{Z_L^2 \pm \sqrt{Z_L^4 - 4R_C^2 X_L^2}} \right] \quad (1)$$

Here if  $Z_L^4 > 4R_C^2 X_L^2$ , we obtain two values of C for which the circuit is resonant.

Similarly, from equation 1 solve for C this is I have written, but I suggest you to solve. So, again from equation 1 you solve for C you will get another expression  $2L$  into  $1$  upon  $Z_L$  square plus minus  $Z_L^4$  minus  $4R_C^2 X_L^2$  if the square root term is positive right, you will get two values of it for which circuit will resonant right. So, and if  $Z_L^4$  minus  $4R_C^2 X_L^2$  is equal to  $0$  you will have only one value of C right  $2L$  upon  $Z_L$  square at which circuit your what we call is resonance circuit right, but if this term becomes negative there is no value of C that circuit will become resonant.

So, this is the condition right. We obtained two values of C for which the circuit is resonance and if this one is equal to this one  $Z_L^4$  is equal to  $4R_C^2 X_L^2$  the circuit is resonance at C is equal to  $2L$  upon  $Z_L$  square right again this is for L this is for C.

(Refer Slide Time: 10:49)

If  $Z_L^2 = 4R_c X_L^2$ ,

The circuit is in resonance at

$$C = \frac{2L}{Z_L^2}$$

Solving Eqn. (1) for  $R_L$ , we obtain

$$R_L = \sqrt{\omega^2 L C R_c^2 - \omega^2 L^2 + L/C}$$

(Refer Slide Time: 11:07)

$$C = \frac{2L}{Z_L^2}$$

Solving Eqn. (1) for  $R_L$ , we obtain

$$R_L = \sqrt{\omega^2 L C R_c^2 - \omega^2 L^2 + L/C} \quad \text{--- (5)}$$

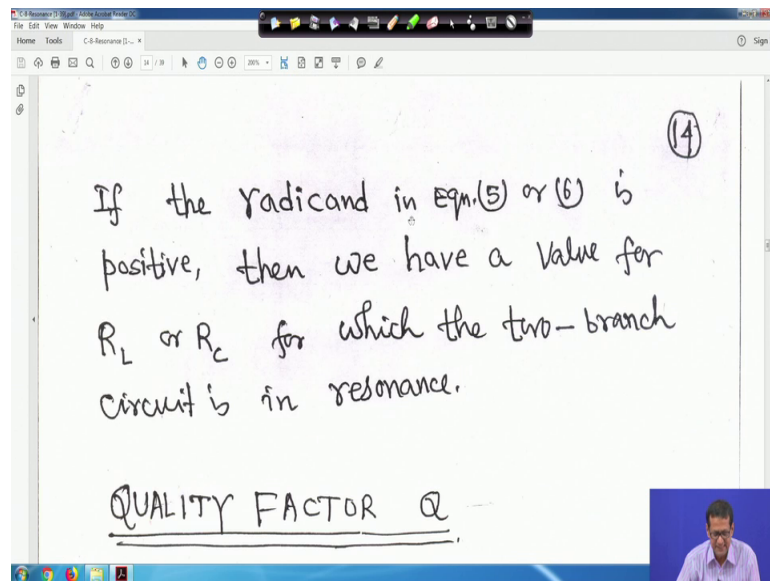
and solving for  $R_c$ ,

$$R_c = \sqrt{\frac{R_L^2}{\omega^2 L C} - \frac{1}{\omega^2 C^2} + L/C} \quad \text{--- (6)}$$

Now, again if you solve from equation 1 in RL if you solve for RL in terms of other quantities, what we are doing is each con variable we are trying to find out in terms of others right. So, if you so, solve equation 1 for RL you will get RL is equal to this expression, this we do it in simple and that your what we call if omega square LC RC square minus omega square L square plus L upon C if it is greater than 0 then circuit is at resonance right.

So, this is for RL and similarly if you solve for RC of the equation 1 only you will get RC is equal to square root of RL square upon omega square LC minus omega square C square plus L by C. So, this is actually your what we call greater than 0 has to be greater than 0 for which circuit is at resonance right. So, L C RL your what we call RC right all these things in terms of remaining variables we got it right, now question is that in this two in 5 and 6 I told you.

(Refer Slide Time: 12:05)



(14)

If the radicand in Eqn. (5) or (6) is positive, then we have a value for  $R_L$  or  $R_C$  for which the two-branch circuit is in resonance.

QUALITY FACTOR Q



But here it is written here whatever I said here it is written here right. So, this is your what you call that your resonance of your parallel circuit simple inductive and capacitive circuit.

(Refer Slide Time: 12:17)

QUALITY FACTOR Q

The quality factor of coils, capacitors and circuits is defined by

$$Q = 2\pi \left( \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}} \right)$$

$\rightarrow I$   

The image shows a whiteboard with the title "QUALITY FACTOR Q" underlined. Below it, a handwritten sentence states: "The quality factor of coils, capacitors and circuits is defined by". The formula  $Q = 2\pi \left( \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}} \right)$  is written in the center. At the bottom, there is a small diagram showing a resistor symbol followed by an inductor symbol, with an arrow labeled "I" pointing to the right above the resistor.

Next another term we define the quality factor it is called Q right. So, the quality factor of coils capacitors and circuit is defined by.

(Refer Slide Time: 12:27)

$Q = 2\pi \left( \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}} \right)$

Fig.1





$\rightarrow I$     
R  $X_L = j\omega L$

Fig.2

$\rightarrow I$     
R  $\frac{1}{j\omega C}$   $\frac{-j}{\omega C} = X_C$

The image shows a whiteboard with the same formula as the previous slide. Below the formula, two circuit diagrams are drawn. "Fig.1" shows a series circuit with a resistor R and an inductor with impedance  $X_L = j\omega L$ . "Fig.2" shows a series circuit with a resistor R and a capacitor with impedance  $\frac{1}{j\omega C}$ . To the right of Fig.2, the expression  $\frac{-j}{\omega C} = X_C$  is written.

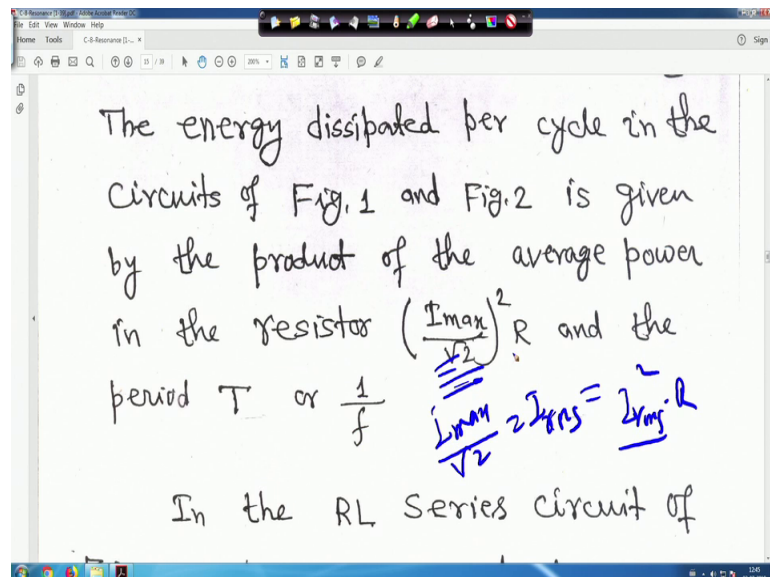
So, quality factor sometimes for a circuit is defined by 2 pi into maximum stored energy divided by energy dissipated per cycle right. So, for example, it is called quality factor right of the coils capacitors and circuit. So, in general Q is equal to 2 pi into maximum stored energy divided by energy dissipated per by your what you call per cycle. Now for



example, consider a simple your what you call your inductive circuit that is this is resistance R this is your XL is equal to JL omega and this is current flowing through I.

Similarly, in figure this is figure 1 say this is figure 2 say it is a capacitive circuit it is R current is flowing I and it is your what we call 1 upon j omega C that is 1 upon j omega C means it is minus your this one upon j omega C means it is minus j by omega C right. So, and this is your this is your this is your coil XC and this is your XL, XL is equal to JL omega and XC is equal to your minus j upon omega C right. So, separately we are considering.

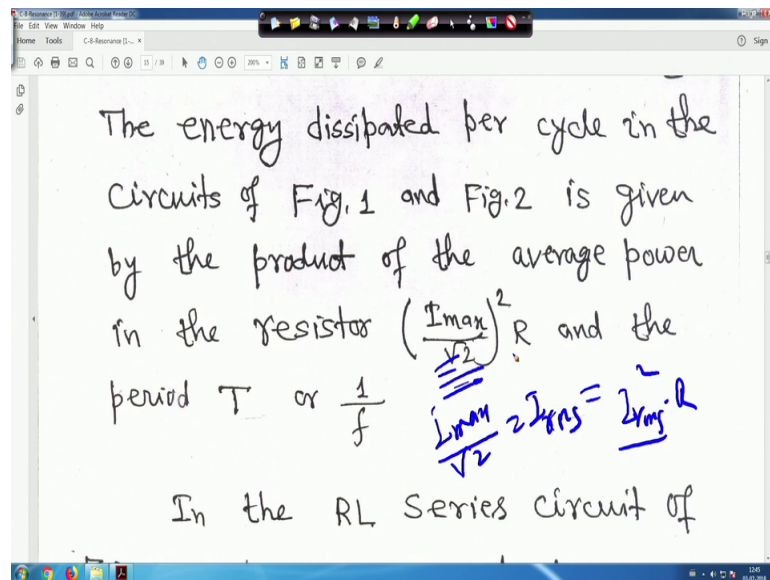
(Refer Slide Time: 13:39)



Now, now the energy dissipated per cycle in the circuit of figure 1 and figure 2 is given by the product of the average power of the resistor and a your what you call that is your I rms square r and the period T or 1 f; that means, that means this is we are writing I your what we call I max by root 2. Actually I max by root 2 is equal to basically your rms value I rms value right. So, it is rms value.

So; that means, power loss is equal to actually this is rms value. So, I rms square into R this is that your what you call that average power in the resistor that is I rms square into R. So, we are putting it I max by root 2 square into R right.

(Refer Slide Time: 14:27)



So, and it is your what you call that power your average power of the resistor and the period  $T$  or  $1$  or  $1$  upon  $f$ . That means if this is your what you call this is your average power say  $I_{max}$  by root  $2$  square in to  $R$  right and per cycle means is multiplied by this thing. So, this is your divided by  $f$  right. So, it is your energy dissipated per cycle in the circuit of figure 1 and figure 2 figure 1 is inductive circuit and figure 2 is a capacitive circuit right.

So, in this case in the case of RL series circuit figure 1 the maximum stored energy we have studied earlier half  $I_{max}$  square that you have studied right. Therefore, that  $Q$  is equal to  $2\pi$  it is maximum stored energy half  $I_{max}$  square; that means, where we have written here the  $q$  is equal to that  $Q$  is equal to maximum stored energy by energy dissipated per cycle.

So, in this case your this one is  $2\pi$  half  $I_{max}$  square divided by  $I_{max}$  square by  $2$  because it is  $I_{max}$  by root  $2$  square, so, whole square. So,  $I_{max}$  square by  $2$  in to  $R$  into  $1$  upon  $f$  right, because this is actually energy dissipated your per cycle right. So,  $f$  is the frequency that is your say frequency  $50$  hertz means; it is  $50$  cycles per second right.

(Refer Slide Time: 15:50)

In the RL series circuit of Fig.1, the maximum stored energy is  $\frac{1}{2} LI_{\max}^2$ . Then

$$Q = 2\pi \left( \frac{\frac{1}{2} LI_{\max}^2}{\left(\frac{I_{\max}^2}{2}\right) R \times \left(\frac{1}{f}\right)} \right) = 50 \text{ cycles/sec}$$

$f = 50 \text{ Hz}$

$$\therefore Q = \frac{2\pi f L}{R} = \frac{L\omega}{R}$$

Whenever we say whenever we say f is equal to f is equal to 50 hertz; that means, it is 50 cycles per second right. So, it is per cycle, so it is divided by f right. So, if you after simplification after simplification simply you will get Q is equal to 2 pi f L upon R that is your L omega by that is your L omega by R right.

(Refer Slide Time: 16:19)

$$\therefore Q = \frac{2\pi f L}{R} = \frac{L\omega}{R}$$

In the RC series circuit of Fig.2, the maximum stored energy is  $\frac{1}{2} CV_{\max}^2$  OR  $\frac{1}{2} I_{\max}^2 / \omega_c^2$

$V_{\max} = X_c I_{\max}$

$X_c = \frac{1}{\omega_c C}$

So, that is Q for inductive circuit right similarly in the RC series circuit of the figure 2 the maximum stored energy half CV max square or and another thing is that in that your what you call your what you call that your V max if you took XC is the if XC is the your

what you call the reactance of the capacitor magnitude will take therefore,  $V_{\max}$  right is equal to your  $X_C$  in to your  $I_{\max}$  right that is written here  $V_{\max}$  is equal to say  $I_{\max}$  into  $X_C$ .

(Refer Slide Time: 16:56)

$$Q = \frac{2\pi f L}{R} = \frac{L\omega}{R}$$

In the RC series circuit of Fig.2, the maximum stored energy is  $\frac{1}{2} C V_{\max}^2$  OR  $\frac{1}{2} \frac{I_{\max}^2}{\omega^2 C}$   $\left. \begin{matrix} W_{\max} \\ = I_{\max}^2 / \omega^2 C \end{matrix} \right\}$

Now, in that let me clear it so, this is actually half  $C V_{\max}^2$  capacitor your what you call energy stored is a capacitor or if you use this relationship you will get half  $I_{\max}^2$  square divided by omega square  $C$  right. So, in that case your what just you substitute and put  $X_C$  is equal to  $1 / \omega C$  and just simplify you will get this am not doing it is understandable right.

(Refer Slide Time: 17:23)

Then

$$Q = 2\pi \left( \frac{\frac{1}{2} I_{\text{max}}^2 / \omega^2 C}{\left( \frac{I_{\text{max}}^2}{2} \right) \cdot R \cdot \left( \frac{1}{f} \right)} \right)$$

$$\therefore Q = \frac{1}{\omega CR}$$

A series RLC circuit at resonance

So, this in that case for in the case of capacitor; so,  $2\pi$  half  $I_{\text{max}}$  square upon  $\omega$  square  $C$  divided by same thing  $I_{\text{max}}$  square by  $2$  into  $R$  into  $1$  upon  $f$  right. So, if you do, so  $Q$  will become one upon  $\omega C R$  right; that means, this for quality factor for inductive circuit is  $Q$  is equal to  $L \omega$  by  $R$  and for capacitive circuit it is  $Q$  is equal to  $1$  upon  $\omega C R$  right.

(Refer Slide Time: 17:47)

$$\therefore Q = \frac{1}{\omega CR}$$

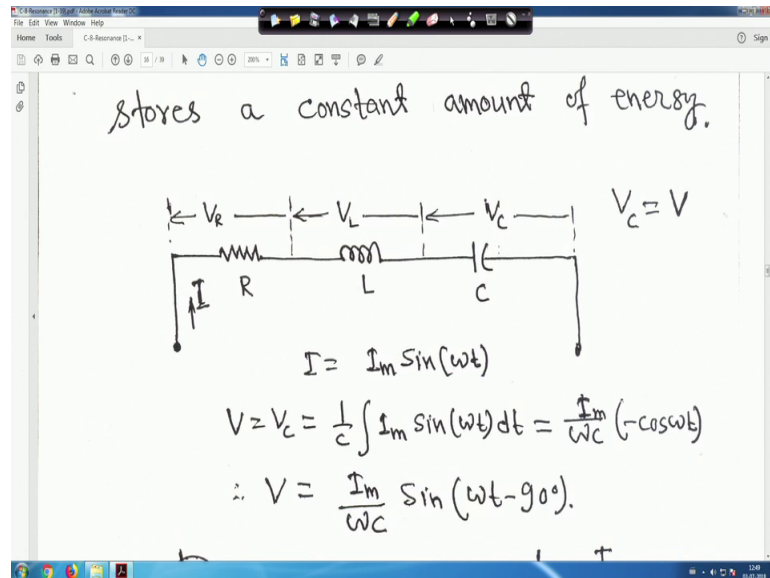
A series RLC circuit at resonance stores a constant amount of energy.

$V_R$     $V_L$     $V_C$     $V_C = V$

$I$     $R$     $L$     $C$

Therefore a series RLC your series RLC circuit at resonance stores a constant amount of energy right. Now let us consider a series RLC circuit at resonance stores a constant amount of energy.

(Refer Slide Time: 18:01)



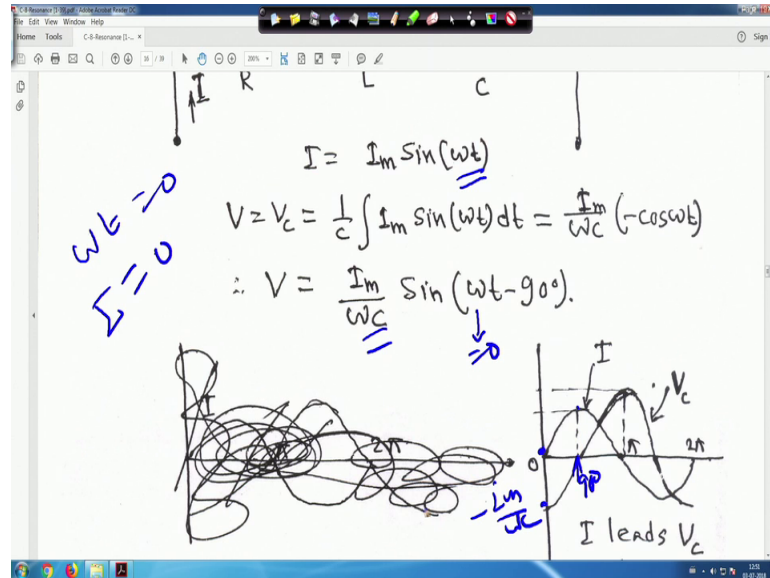
Now, suppose this is series RLC circuit say  $V_R$  is equal to  $I$  into  $R$  current going through this is  $I$  right and this is my  $I$ . So,  $V_L$  is equal to this voltage across inductor is  $V_L$  and voltage across capacitor is  $V_C$  and we are assuming that your what you call say for example,  $V_C$  is equal to  $v$  something am writing here right say  $V_C$  is equal to  $V$ .

Now say this  $I$  is equal to say  $I_m \sin \omega t$  right this is ac quantity. So, so  $I_m$  is equal to  $\sin \omega t$ , now voltage across the capacitor that is  $V_C$  is equal to  $V$   $V$  is equal to  $V_C$  just you have to integrate  $1$  upon  $C$   $I_m \sin \omega t dt$  if you integrate it will be  $I_m$  upon  $\omega C$  minus  $\cos \omega t$ .

So, we can write that your this  $\cos \omega t$  is  $\sin 90$  degree minus  $\omega t$ , but  $1$  minus  $\sin$  is here; that means, it will be  $I_m$  by  $\omega C$   $\sin \omega t$  minus  $90$  degree. That means, here your what we call that current  $I$  is equal to  $I_m \sin \omega t$  right and  $V$  voltage across the capacitor  $V$  actually it is  $I_m$  by  $\omega C$   $\sin \omega t$  minus  $90$  degree; that means, current actually is leading because it is  $\omega t$  plus  $0$  and leading this voltage  $V$  is equal to  $V_C$  and voltage across the capacitor.

So, we are so, that is why  $I_m$  by  $\omega C \sin \omega t$  minus 90 degree, now if you if this is  $\sin \omega t$  plot  $I_m$   $I$  is equal to  $I_m \sin \omega t$  plot.

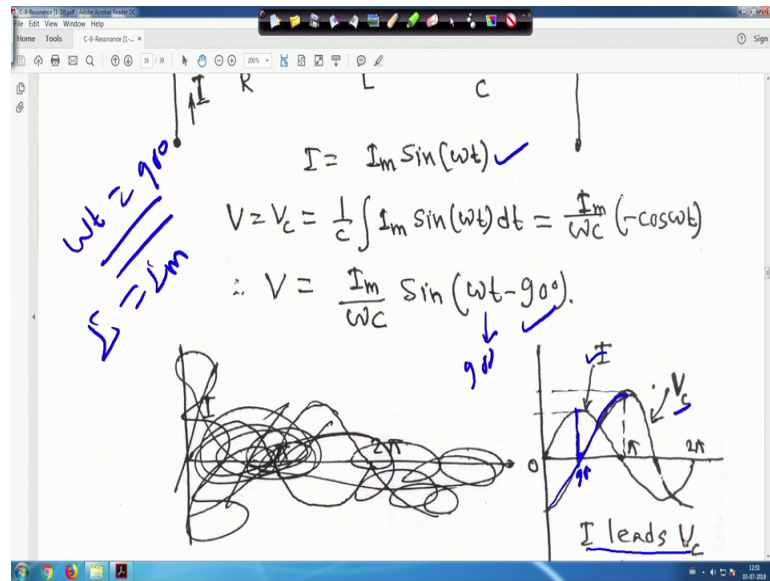
(Refer Slide Time: 19:29)



So, if in this case this is starting from 0 so, it is your  $I$  right. So,  $I$  is reaching the peak first right. So, and in this case when  $\omega t$  is equal to 0, when  $\omega t$  is equal to 0 say when  $\omega t$  is equal to 0 so,  $I$  is equal to 0. So, this is starting from here 0. Here when  $\omega t$  is equal to 0  $V$  is equal to  $I_m$  by  $\omega C \sin$  of minus 90 degree when  $\omega t$  is equal to 0 right. So, in that case  $\sin$  of minus 90 minus 1. So,  $V$  will be  $V$  is equal to  $V_C$  right that voltage across the capacitor. So, it is minus  $I_m \omega C$ .

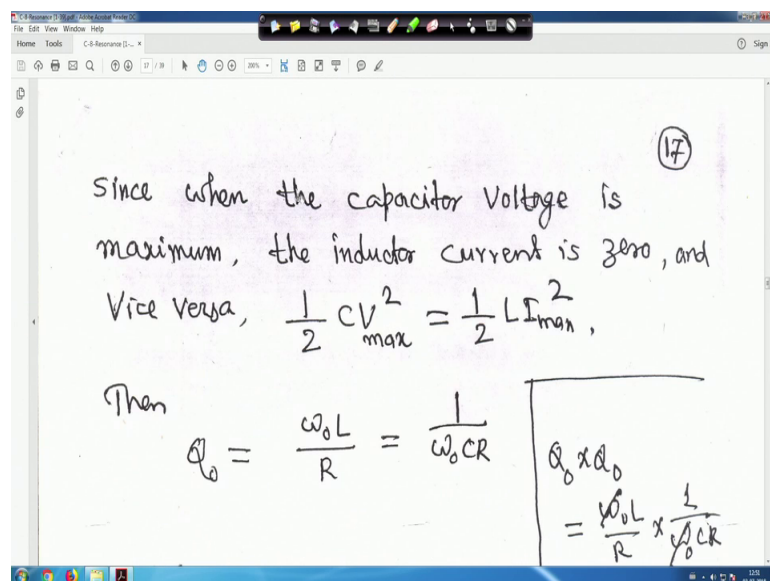
That is why this so, this point is your minus  $I_m$  by  $\omega C$  right. So, when current is when current is reaching its peak; that means, this angle is 90 degree. So, when  $\omega t$  let me clear it, let me clear it.

(Refer Slide Time: 20:21)



So, when  $\omega t$  is equal to 90 degree then  $I$  is equal to  $I_m$  right. So, this is my  $\omega t$  (Refer Time: 20:28) this is my your 90 degree  $I$  is equal to  $I_m$ . But when  $\omega t$  is equal to 90 degree if you put 90 degree then  $V$  is equal to 0. So, that is why  $V$  is equal to 0 because this is the curve for  $V$  so,  $V$  is equal to 0. So, current actually reaching it is peak faster than the voltage. So, this current is leading the voltage by 90 degree from this from this we can make it out easily right. Therefore it is written  $I$  leads  $V_c$  by an angle 90 degree.

(Refer Slide Time: 21:03)

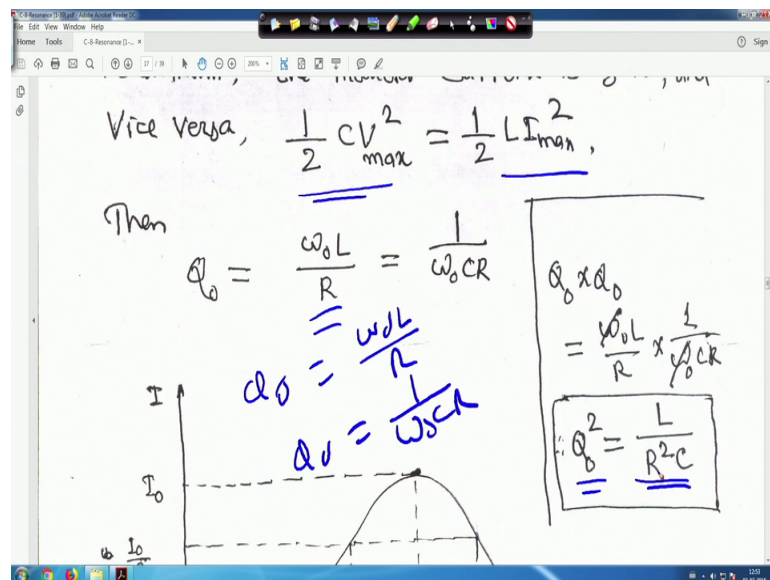




So, therefore, since when the now when the capacitor voltage is maximum the inductor current is 0 and vice versa when; that means, when capacitor voltage here it is this point 0 in this your what you call this inductor current is maximum or when inductor current is 0 this point capacitor voltage is maximum vice versa. So, from that your from that we can write that half CV max square is equal to half LI max square, from which we get right then Q 0 is equal to may be written as omega 0 upon L by IR that is one upon omega 0 CR right.

So, actually your what you call from if half CV max square is equal to half LI max square from that you then you Q 0 is equal to your L omega 0 by R is equal to 1 upon omega 0 CR right. So, from this your what we call what will do right.

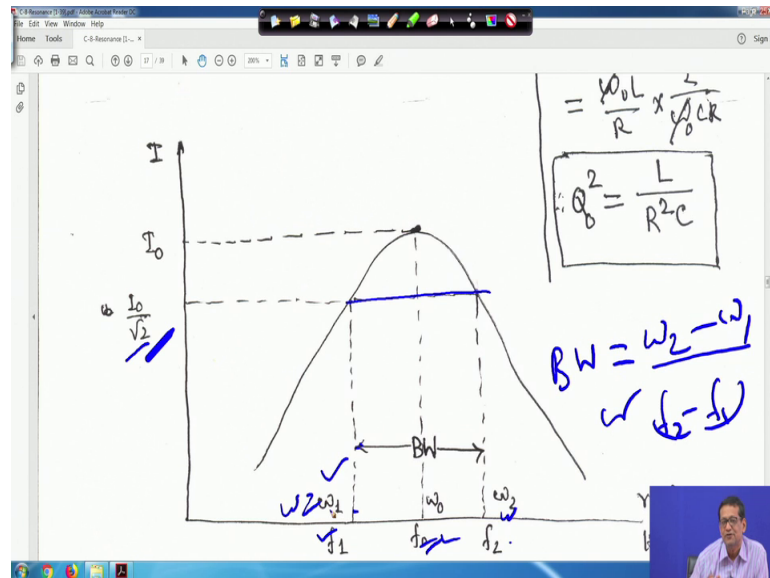
(Refer Slide Time: 21:59)



This is my equation this is the equation right. So, little bit little bit you try to do it yourself right. So, many things I have done I have been spending lot of time right little bit you do that, but one or two thing I am telling.

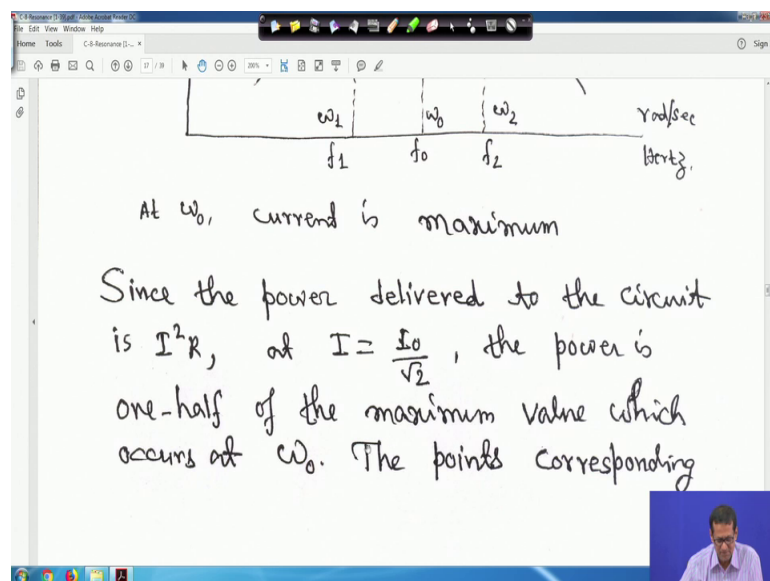
Suppose this is my Q 0 is equal to I mean you write separately say Q 0 is equal to omega 0 L by R and again you write Q 0 is equal to 1 upon omega 0 CR. Now Q 0 into Q 0 that is Q 0 into Q 0 then omega 0 l upon R into 1 upon omega 0 CR. So, omega 0 omega 0 will be cancelled right and finally, it will become L upon R square C the Q 0 square will become 1 upon R square C.

(Refer Slide Time: 22:57)



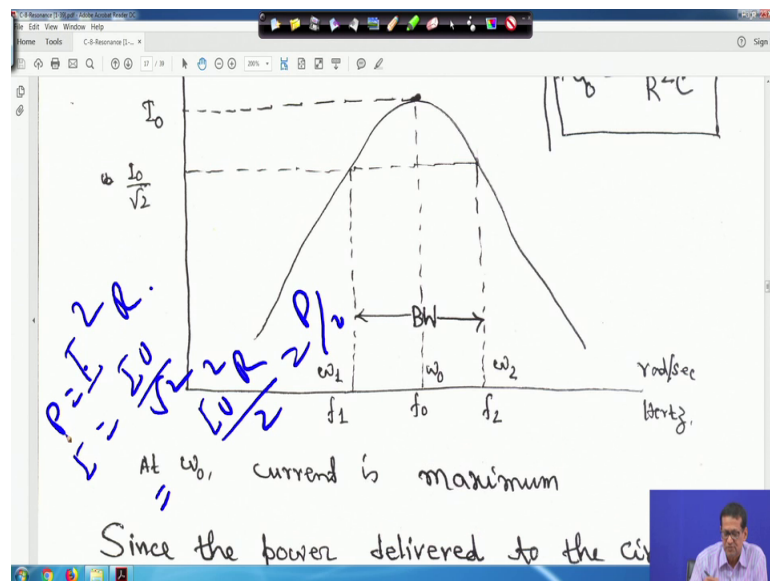
So, let me clear it, this is an exercise for you this is an exercise for you to do this little bit right. So, now if you now so, this is your  $Q_0$  right. Now question is that suppose before coming to this figure suppose at  $\omega_0$  current is maximum because for your what we call for resonance circuit, we have seen that your the resonant frequency series R LC circuit say that  $X_L$  is equal to  $X_C$ . So, in that case that your current is maximum and these current say it is  $I_0$   $I_0$  is the maximum current at resonance frequency when  $\omega$  is equal to  $\omega_0$  or  $f$  is equal to  $f_0$  right.

(Refer Slide Time: 23:25)



So, since the power delivered to the circuit is  $I^2 R$  therefore, at  $I$  is equal to  $I_0$  we know that power  $I^2 R$  is the power delivered to the circuit therefore, suppose at  $I$  is equal to  $I_0 / \sqrt{2}$  say the power is 1/2 of the maximum value which occurs at  $\omega_0$  right suppose your this one we know  $I^2 R$  right, now when  $I$  is equal to say  $I_0 / \sqrt{2}$  then if this is the power loss  $P$ ,  $P$  is equal to  $I^2 R$  is equal to  $(I_0 / \sqrt{2})^2 R$  that is it will be half  $P$  by 2 right. So, let me clear it. So, this is  $I_0$  this is your  $I_0$  and this is your  $I_0 / \sqrt{2}$  right I will come to this so.

(Refer Slide Time: 23:47)



So, power delivered to the circuit the power is 1/2 of the maximum value which occurs at  $\omega_0$  right therefore,; that means, whatever resonance at resonance power is  $I_0^2 R$  say  $I^2 R$  is equal to if  $I$  is equal to  $I_0$  then  $I_0^2 R$ , but at  $I$  is equal to  $I_0 / \sqrt{2}$  it will be half right. So, the points corresponding to  $\omega_1$  and  $\omega_2$  are called the half power points.

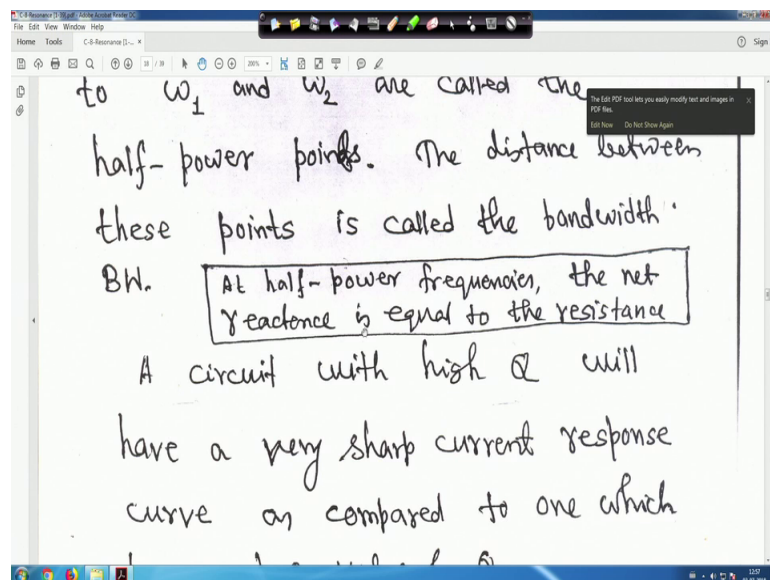
So, this is actually this  $I_0 / \sqrt{2}$  this is your  $I_0 / \sqrt{2}$  this is this is called half power point because  $I^2 R$  is  $I_0^2 R$ , but if it become  $I_0 / \sqrt{2}$  it will be  $I_0^2 R / 2$ . So,  $t$  by 2 so, this is called half power point. So, this is at frequency  $Z$  this is your  $\omega$  is equal to say  $\omega_1$  it is intersecting two point of this curve and this is your what we call  $\omega_1$   $f_1$   $f_0$   $f_2$  or  $\omega_1$   $\omega_0$   $\omega_2$ .

The difference between these this one this difference between this two that is your what we call this from this two I mean this width from here to here it is called your bandwidth

right. So, that is your  $\omega_2$  minus  $\omega_1$  or if it is in radian per second or if it is hertz it will be  $f_2$  minus  $f_1$  right and this is your resonance frequency this is called actually bandwidth time and this  $\omega$ , when it your  $\omega$  is equal to  $\omega_1$  means for series RLC circuit we have seen right, that your what you call that your  $X_L$  and  $X_C$   $X_C$  is your what we call greater than  $X_L$  or  $R^2$  square your what we call  $X_C$  than your  $X_L$  that we have seen.

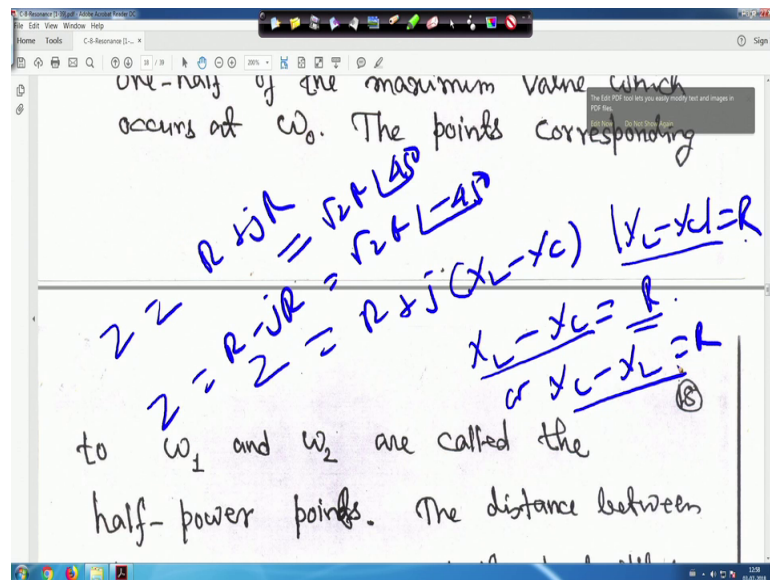
So, this two point call half power point and the and this distance which we call bandwidth right, and this is if it is  $\omega$  radian per second if it is frequency then it will be in hertz, so.

(Refer Slide Time: 26:17)



So, this is so, at half power frequency the net reactance is equal to the resistor this is written at half power frequency net resistance your what we call is equal to the resistor because here am making it for you.

(Refer Slide Time: 26:27)



Suppose  $Z$  is equal to your  $R$  plus  $j$  for series RLC circuit  $X_L$  minus  $X_C$  two things can happen; if one is  $X_L$  minus  $X_C$  equal your what we call it because it is half power frequency root 2 has to come right. So, it will be your  $X_L$  minus  $X_C$  may be  $R$  or  $X_C$  minus your what you call another thing will be  $X_C$  minus your  $X_L$  may be at two condition may happen right. So, in general if we take first condition that  $X_L$  minus  $X_C$  is equal to  $R$  ultimately mod  $X_L$  minus mod  $X_C$  is equal to actually  $R$  right.

Therefore  $Z$  is equal to one condition if you take it will be your what you call  $R$  plus  $jR$ ; that means, this one will be your what we call root 2  $R$  angle 45 degree. And, if  $Z$  is equal to  $R$  minus  $jR$  it will be root 2  $R$  angle minus 45 degree either of these because, mod  $X_L$  minus  $X_C$  is equal to  $r$  this is either one condition this is another condition right.

So, let me clear it therefore, your therefore, at half power frequencies the net reactance is equal to the resistance right. So; that means, a circuit with high  $Q$  will have a very sharp current response and response curve that is compared to one which has a low value of  $Q$ . So, we this your what you call several admittance verses angle other graphs are shown frequency right from that you can easily you can easily justify this one.

(Refer Slide Time: 28:07)

has a low value of  $Q$ .

~~at~~ We specify two frequencies  $\omega_1$  &  $\omega_2$   
at which  $\Rightarrow (|X_L - X_C| = R)$

Since at  $\omega_1$ , the circuit is capacitive ( $X_C > X_L$ ), therefore at  $\omega_1$ ,

$$X_C - X_L = R \quad \dots (i)$$

at  $\omega_2$

$$X_L - X_C = R \quad \dots (ii)$$

Similarly, that I told you it will be mod  $X_L$  minus  $X_C$  is equal to  $r$  since at  $\omega_1$  the circuit is I told you have to very very beginning of this topic, again I am not going to the figure when your  $\omega$  is equal to your what we call at  $\omega_1$  which is on the left hand side right of that curve. So, at that time  $X_C$  greater than  $X_L$  right  $X_C$  greater than  $X_L$ . So,  $X_C$  your greater than  $X_L$  therefore, at  $\omega_1$   $X_C$  minus  $X_L$  is equal to  $R$  right.

And similarly at  $\omega_2$  when is going to the other side  $X_C$  less than  $X_L$  or  $X_L$  greater than  $X_C$ . So, in that case  $X_L$  minus  $X_C$  is equal to  $R$  this is equation 1 this is equation 2. See previously I have explained this right whenever you I suggest whenever you go through a video lecture first draw those diagram and after that you just go through these that video lecture then it will be easy for you to make it on the notebook. So, if these two conditions hold therefore, the corresponding impedance I told you.

(Refer Slide Time: 29:13)

At  $\omega_1$  (19)

$$Z_1 = R + j(X_L - X_C) = R - jR$$

$$\therefore Z_1 = \sqrt{2} R \angle -45^\circ$$

At  $\omega_2$

$$Z_2 = \sqrt{2} R \angle 45^\circ$$

$V \angle 0^\circ$        $V \angle 45^\circ$

One will be at omega 1 Z 1 will be R plus j XL minus XC that will be R minus jR that will be Z 1 will be root 2 R angle minus 45 degree because, on root over R square plus R square and angle will be tan inverse minus 1 because it will be tan inverse your minus R by R. So, it will be minus 45 degree.

(Refer Slide Time: 29:33)

At  $\omega_2$

$$Z_2 = \sqrt{2} R \angle 45^\circ$$

At  $\omega_1$ ,  $I_1 = \frac{V \angle 0^\circ}{\sqrt{2} R \angle -45^\circ} = \frac{V}{\sqrt{2} R} \angle 45^\circ$

Similarly, at  $\omega_2$

$$I_2 = \frac{V}{\sqrt{2} R} \angle -45^\circ = \frac{I_0 \angle 45^\circ}{\sqrt{2}} = 0.707 I_0 \angle 45^\circ$$

Since  $\frac{V}{R} = I_0$ , the current at  $\omega_2$

Similarly, at omega 2 Z 2 will be root over 2 root 2 R and will be 45 degree right therefore, at omega 1 I 1 will be that V angle 0; this is the voltage reference right divided

by your impedance root 2 R angle minus 45 degree that will become V upon root 2 R and angle 45 degree right.

(Refer Slide Time: 29:55)

At  $\omega_1$ ,  $I_1 = \frac{V \angle 0^\circ}{\sqrt{2} R \angle -45^\circ} = \frac{V}{\sqrt{2} R} \angle 45^\circ$

Similarly, at  $\omega_2$

$$I_2 = \frac{V}{\sqrt{2} R} \angle -45^\circ$$

Since  $\frac{V}{R} = I_0$ , the current at resonance frequency  $\omega_0$ .

Similarly, at  $\omega_2$  your  $I_2$  is equal to  $V$  upon  $\sqrt{2} R$  angle minus 45 degree. Since  $V$  by  $R$  is equal to  $I_0$  because at resonance  $X_L$  minus  $X_C$  is equal to 0 and for which current is maximum if  $X_L$  minus  $X_C$  is 0. So,  $Z$  will be simply  $R$  therefore,  $I_0$  the maximum current right. So, at resonance that is  $V$  upon  $R$ . So, the current at resonance will be your frequency at  $\omega_0$  therefore, we can write  $I_1$  is equal to your  $I_0$  upon  $\sqrt{2}$  is there. So, and I mean this one  $I_1$  upon your this term, if you can write it will be basically this term will look at this term if you see it will basically  $V$  by  $R$  equal  $R$  upon  $I_0$  divided by  $\sqrt{2}$  angle 45 degree that is your  $0.707 I_0$  angle 45 degree right.



(Refer Slide Time: 30:53)

frequency  $\omega_0$ .

$$\therefore I_1 = 0.707 I_0 \angle 45^\circ$$

$$I_2 = 0.707 I_0 \angle -45^\circ$$

At  $\omega_1$ ,

$$X_C - X_L = R$$

So, this one therefore,  $I_1$  is equal to  $0.707$  angle  $I_0$  angle  $45$  degree similarly  $I_2$  is equal  $I_0.707$  angle minus  $45$  degree, but at  $\omega_1$  at  $\omega$  is equal to  $\omega_1$   $X_C$  minus  $X_L$  is equal to  $R$  right.

(Refer Slide Time: 31:15)

$$X_C - X_L = R$$

$$\omega = \omega_1 \quad L\omega_1$$

$$X_L = L\omega_1$$

$$X_C = \frac{1}{\omega_1 C} \quad (20)$$

$$\frac{1}{\omega_1 C} - L\omega_1 = R \quad \text{--- (iii)}$$

Therefore, that means, whenever that is at  $\omega$  is equal to  $\omega_1$  right. So,  $\omega$  is equal to  $\omega_1$ .

So, in that case your  $X_L$  will be  $L \omega_1$  and  $X_C$  will be  $\frac{1}{\omega_1 C}$ . So, here you put this, here you put this is say equation 3 right. So, let me clear it. So, similarly at

omega is equal to omega 2 similarly. So, XL will be 1 omega 2 and XC will be 1 upon omega 2 see this is actually whenever writing at omega 2 means it is omega is equal to omega 2.

(Refer Slide Time: 31:35)

$$\frac{1}{\omega_c} - L\omega = R \quad \text{--- (iii)}$$

At  $\omega_2$ ,  $\omega = \omega_2$

$$X_L - X_C = R$$

$$\therefore L\omega_2 - \frac{1}{\omega_2 c} = R \quad \text{--- (iv)}$$

Eqn (iii) - Eqn (iv)

So, this is equation 4 and this is equal to R actually right. So, now what you do subtract equation 4 from equation 3, I mean this equation you subtract from this equation if you do.

(Refer Slide Time: 32:03)

$$\therefore \left( \frac{1}{\omega_1} + \frac{1}{\omega_2} \right) \frac{1}{c} - (\omega_1 + \omega_2) L = 0$$

$$\therefore \frac{1}{\omega_1 \omega_2} = LC = \frac{1}{\omega_0^2}$$

$$\therefore \boxed{\omega_0 = \sqrt{\omega_1 \omega_2}} \quad \text{--- (v)}$$

Again Eqn (iii) + Eqn (iv)

$$1 - L\omega + \frac{1}{\omega} - 1 = 2R$$

$\omega_0 = \frac{1}{\sqrt{LC}}$   
 $LC = \frac{1}{\omega_0^2}$

So, if you do so, you will get  $\frac{1}{\omega_1} + \frac{1}{\omega_2} = LC$  or simply you will get  $\frac{1}{\omega_1 \omega_2} = LC$  is equal to  $\frac{1}{\omega_0^2}$ . Because, your because  $\omega_0$  is equal to resonance frequency is equal to  $\frac{1}{\sqrt{LC}}$ .

If you take the square of it so,  $LC$  will be is equal to  $\frac{1}{\omega_0^2}$ . So, that is what we are writing here therefore,  $\omega_0$  will be is equal to  $\frac{1}{\sqrt{LC}}$  into  $\omega_1$  into  $\omega_2$ . These are good your relationship and easy to remember that  $\omega_1$  is this side another side. So,  $\omega_2 - \omega_1$  is the bandwidth and  $\omega_0$  will be what your we call  $\frac{1}{\sqrt{LC}}$  very geometric meaning right.

(Refer Slide Time: 32:57)

The image shows a whiteboard with the following handwritten equations:

$$\therefore \left( \frac{1}{\omega_1} + \frac{1}{\omega_2} \right) \frac{1}{C} - (\omega_1 + \omega_2) L = 0$$

$$\therefore \frac{1}{\omega_1 \omega_2} = LC = \frac{1}{\omega_0^2}$$

$$\therefore \boxed{\omega_0 = \sqrt{\omega_1 \omega_2} \quad \dots (V)}$$

Again

Eqn (iii) + Eqn (iv)

$$\frac{1}{LC} - L\omega_1 + L\omega_2 - \frac{1}{LC} = 2R$$

Similarly, you add equation 3, equation 3 and equation 4 already equation 3 equation 4 we have got it. So, add these two equations if you do.

(Refer Slide Time: 33:07)

∴  $\omega_0 = \sqrt{\omega_1 \omega_2}$  ✓

Again  
Eqn (iii) + Eqn (iv)

$$\frac{1}{\omega_1 C} - L\omega_1 + L\omega_2 - \frac{1}{\omega_2 C} = 2R$$

$$\therefore \frac{\omega_2 - \omega_1}{\omega_1 \omega_2 C} + (\omega_2 - \omega_1)L = 2R$$

So, if you do so and simplify you add and simplify you will get omega 2 minus omega 1 upon omega 1 omega 2 C plus omega 2 minus omega 1 L is equal to 2 R right.

(Refer Slide Time: 33:19)

(21)

$$(\omega_2 - \omega_1) \left( \frac{1}{\omega_0^2 C} + L \right) = 2R$$

$$\therefore \frac{(\omega_2 - \omega_1) (1 + LC\omega_0^2)}{\omega_0^2 C} = 2R$$

$\omega_0 = \frac{1}{\sqrt{LC}}$   
 $\omega_0^2 = \frac{1}{LC}$   
 $LC\omega_0^2 = 1$

$$\therefore (\omega_2 - \omega_1) \left( \frac{1 + LC\omega_0^2}{\omega_0^2 C} \right) = 2R$$

Therefore your omega 2 upon simplification you take omega minus omega 1 common. So, you will get omega 2 minus omega 1 your 1 plus LC omega 0 square divided by your omega 0 square C. Now, LC omega 0 square is equal to 1 because omega 0 is equal to 1 upon root over LC you know. Now, therefore, square it omega 0 square is equal to 1

upon LC therefore, cross multiplication  $\omega_2 - \omega_1$   $\omega_0^2 LC$   $\omega_0^2$  is equal to 1. So, this term you put is equal to 1 such that 1 plus 1 you are getting 2 right. So, let me clear it.

(Refer Slide Time: 33:59)

The image shows a whiteboard with the following handwritten equations:

$$\omega_2 - \omega_1 = \omega_0^2 CR$$

$$\frac{\omega_2 - \omega_1}{\omega_0} = \omega_0 CR = \frac{1}{Q_0}$$

$$Q_0 = \frac{\omega_0}{\omega_2 - \omega_1} \quad \text{--- (vi)}$$

Additionally, there is a note on the right side of the whiteboard:  $Q_0 = \frac{\omega_0}{BW}$ .

So, if you put that  $\omega_2 - \omega_1$  is equal to  $2\omega_0$  both side will be cancelled it will be  $\omega_0^2 CR$  right or that it is  $\omega_0$  so, divide by  $\omega_0$ . So,  $\omega_2 - \omega_1$  upon  $\omega_0$  is equal to  $\omega_0 CR$  this we have seen is equal to  $Q_0$  we have seen one upon  $\omega_0 CR$ . So, it is 1 upon  $Q_0$  or  $Q_0$  is equal to  $\omega_0$  another expression of  $Q_0$  is equal to  $\omega_0$  divided by  $\omega_2 - \omega_1$  right. So, it is basically  $\omega_0$  by  $BW$  bandwidth. So, this is equation 6 right.

So, this are your what you call these are the things certain things it is very simple thing only thing is that that little bit of understanding and mathematical your what you call mathematical exercise there, but these are the very simple thing right. So; that means, we get this equation that  $\omega_2 - \omega_1$  this one or this thing what you call this  $Q_0$ , your  $Q_0$  also can be written as  $\omega_0$  by  $BW$  that is the bandwidth and  $BW$  is equal to  $\omega_2 - \omega_1$  right.

Thank you very much we will be back again.