

Fundamentals of Electrical Engineering
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Lecture – 42
Single Phase AC Circuits (Contd.)

So, with that numerical only right let us we have come back again.

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Ex-3. Given $v(t) = 100 \sin 40t$; $R = 10 \Omega$, $L = 0.2 \text{ H}$,
 $C = 0.0014 \text{ F}$.

i) Determine $i(t)$, $v_R(t)$, $v_L(t)$, $v_C(t)$,
 ii) Calculate the power loss,
 iii) show the phasor diagram

Sol. Here $\bar{Z} = R + j(\omega L - \frac{1}{\omega C}) = 10 + j[40 \times 0.2 - \frac{1}{40 \times 0.0014}] \Omega$
 $= (10 - j10) \Omega = 14.14 \angle -45^\circ$

So $i(t) = \frac{100}{14.14 \angle -45^\circ} \sin 40t = 7.1 \sin(40t + 45^\circ)$

$v_R(t) = i(t) \cdot R = 71 \sin(40t + 45^\circ)$
 $v_L(t) = i(t) \cdot (jX_L) = 56.8 \sin(40t + 135^\circ)$

So, Z is equal to you know this is a series R L C circuit it is R plus j L omega minus 1 upon omega C, omega I told you it is 40 because sin omega t. So, omega is 40 so a L omega a L is given 0.2 Henry. So, a L omega minus 1 upon omega C omega is 40 and C is given 0.0014 farad. So, it is actually becoming 10 minus j 10 ohm. So, 14.14 angle minus 40 degree because it is 10 minus j 10. So, your what you call it is root over 10 square plus 10 square, the magnitude and that is equal to 10 root 2. So, 10 root 2 will be 14.14 and angle will be tan inverse your what you call minus 10 by 10 because it is 10 minus j 10.

So, it is tan inverse your minus 1 that is minus 45 degree, that is why this angle is minus 45 degree right. So, let me clear it so this is my impedance Z right this is my Z. Now, therefore, i t is equal to it is look first as it is mentioned in terms of time. So, you have to use the sin omega t. So, i t is equal to your this is 100 sin 40 t. So, 100 sin 40 t divided by the Z 14.14 angle minus 45 because voltage by current voltage by impedance right.

So, it is becoming 7.1 if you divide 100 by your, what actually this will become approximately 7.07 because it is 100 divided by $10\sqrt{2}$. So, it is becoming 10 by root 2 that is 10 into 0.707 because $1/\sqrt{2}$ that is actually 0.707. So, it is 7.07 I have approximated as 7.1 right and this angle is your what you call this angle is your angle minus 45 degree. So, basically it is $\sin(40t + 45^\circ)$.

How we made it right, so let me clear it. So, whenever we whenever you make your phasor quantity for example, this is my this is my your what you call my voltage this is the maximum voltage right. If you take the rms value so, it will be $100/\sqrt{2}$ right, $100/\sqrt{2}$ that is it is $50\sqrt{2}$ volt right an impedance is $10\sqrt{2}$ actually I told you $10 - j10$ means $10\sqrt{2}$. Therefore, my current will be your what you call $50\sqrt{2}/10\sqrt{2}$ is equal to 5 ampere this is my rms value of the current right; if this is my rms value if you multiply by $\sqrt{2}$ it will be 7.07.

Right now, let me clear it; that means, and that means, this one this i I means when you write in the phasor term right. So, i actually will i will be is equal to it is $50\sqrt{2}$ it is $\sin(40t)$. So, $\sin(\omega t + \theta)$ no θ is here 0. So, it will be $50\sqrt{2}$ angle 0 degree and this one 14.14 we can write $10\sqrt{2}$ angle minus 45 degree; that means, this one actually $\sqrt{2}/\sqrt{2}$ will be cancel it will be 5 angle 45 degree because angle 0 degree by angle minus 45 degree it will go up plus \sin . So, it will be 0 degree plus 45-degree angle is equal to angle 45 degree right.

So, that is why this current will be actually rms value it actually 5 angle 45 degree now that let me clear it. So, current i is equal to 5 angle 45 degree right. Now, it is peak value will be 5 into $\sqrt{2}$ that is 7.07 this is the peak value and if you want to put it in your, what you call in the t th term. So, it is angle 45 degree means it will be $\sin(\omega t + 45^\circ)$ right. So, 7.07 I have approximated this one as 7.1 this value is equal to 7.07 right; so, but approximated as 7.1 right.

So, that is why we are what you call we are making it is $7.07 \sin(40t + 45^\circ)$. So, it has been made; hope you have understood this one right while you will go through this video (Refer Time: 04:49) see that every if you have any question anything you put the question in the forum. We will answer all your questions right. So, if you have anything, but I told you tell just see how things are right. Then next is your next is your just hold on.

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$$Z = (10 - j10) \Omega = 14.14 \angle -45^\circ$$

So
$$i(t) = \frac{100}{14.14 \angle -45^\circ} \sin 40t = 7.1 \sin(40t + 45^\circ)$$

$$v_R(t) = i(t) \cdot R = 71 \sin(40t + 45^\circ)$$

$$v_L(t) = i(t) \cdot (jX_L) = 56.8 \sin(40t + 135^\circ)$$

$$v_C(t) = i(t) \cdot (-jX_C) = 127.8 \sin(40t - 45^\circ)$$

In terms of phasors

$$\vec{I} = \frac{\vec{V}}{Z} = \frac{(100/\sqrt{2}) \angle 0^\circ}{14.14 \angle -45^\circ} = 5 \angle 45^\circ$$

$$\vec{V}_R = R \vec{I} = 50 \angle 45^\circ, \quad \vec{V}_L = jX_L \vec{I} = 40 \angle 135^\circ$$

$$\vec{V}_C = -jX_C \vec{I} = 90 \angle -45^\circ$$

NOTE: THE VALUES IN TERMS OF PHASOR QUANTITIES ARE

So, next is that your $v_R(t)$ and your what you call and $v_L(t)$ and $v_C(t)$. So, $v_R(t)$ is $i(t)$ into R so, it is your; that means, R is equal to 10 ohm. So, it was 7.1 A so it is $71 \sin 40t$ plus, actually it should have been 7 point your what you call your 70.7 because 7.07 so approximated as 71 right. So, there should not be any confusion now, similarly your V_L is equal to $i(t)$ into jX_L right. Now $i(t)$ is equal to your what you call we are our $i(t)$ is equal to 7.1, we have taken approximate value then \sin then $40t$ plus 45 degree right. Now, what we are doing is if you just and multiplied by jX_L . X_L is equal to 1 upon 1 omega you calculate my question is a j means angle 90 degree, it is angle 90 degree.

So, that is plus angle with 90 degree and 45 degree if you add it will be 56.8 actually $40t$ plus 135 degree right. So, same as same way I have showed you if you to take rms value phasor and if you just multiply this one. So, this is j means it is angle 90 degree plus 90 degree that is why it is $\sin 40t$ plus your 135 degree. Similarly, $v_C(t)$ is equal to $i(t)$ into minus jX_C . So, minus j x minus j means it is angle minus 90 degree.

That means when multiplied by $i(t)$. So, here it is 45 degree so, 45 minus 90 that is it is minus 45 degree right that is my $v_C(t)$. So, all these all these things this is your plus by just let me clear it. This is your plus minus instantaneous polarity at this voltage is V right. So, all these things are shown here it is 10 ohm it $j8$ ohm and it is your minus your jX_C . So, minus $j8$ ohm and it is 50 volt it is 40 volt and it is your what you call 90 volt.

These all are actually your rms value because V is equal to given 100 your $100 \sin 40 t$ this is given right. So, 100 by $\sqrt{2}$ is equal to 70.7 your approximately it is 71 volt it is 70 this is rms value your if you when you are calculating v or t this is the peak value v_m is equal to your 71 approximately it is 70.7 . So, if you take the rms value 70.7 by $\sqrt{2}$ it will be your what you call 50 volt that is why this one is 50 volt right similarly for V_L t this is 56 . Let me clear it so $V_L t$ is equal to 56.8 this is the maximum value.

So, 56 by 8 by $\sqrt{2}$ that will become your 40 volt that is why this is 40 volt this is rms value. Similarly, for this capacitor it is a voltage across this V_C , it is the peak value 127.8 divided by $\sqrt{2}$. That is actually 90 volt right and this V_I told you this is 71 means 70.7 volt right. And this is the current I told you I is equal to rms value of the current is 5 ampere because it is 7.1 so, that is actually 7.1 means.

It is approximate value 7 point it is a maximum value if you divide by $\sqrt{2}$ it is 5 ampere. So, that is why this is 5 ampere this all this diagram is completely based on this rms value. So, let me clear it so all these things are rms value rms value these are all rms value and this is that your what you call circuit. This example this can be example I have taken. Simultaneously, I can show you the peak value and the rms value peak value of the rms value such that our understanding will be very much clear right. So now, question is in terms of phasor now I told you I have already told you in terms of phasor, it will be just Z bar you may this thing right complex quantity.

So, it is 100 by $\sqrt{2}$ angle 0 degree because it is $100 \sin 40 t$ the voltage. So, that is $\sin \omega t$ plus 0 degree. So, it is 0 and divided by this Z so this is the current 5 angle 45 angle 45 degree and this is V_R $R I$ is equal to this. Your what you call 50 angle 45 degree these are all magnitude 50 volt 40 volt this thing now it is $R I$. So, it is angle is 45 degree now V_L is j into X_L . So, your $j L X_L$ into I I is equal to 5 45 -degree angle 45 degree and $j X_L$ means it is angle 90 degree so 90 plus 45 so 135 degree.

And multiplied by $X_L X_L$ is computed here it is your what you call your $j X_L$ into I . So, whatever value comes it is your 8 ohm it is given. So, it is 40 volt right this is all volt right 40 volt similarly V_C is equal to minus $j X$ into I because of minus j angle minus 90 degree. So, minus 90 degree and it is 45 degree plus. So, it is minus 45 -degree volt right so this is your what you call this is your V_R This is your V_L and this is your V_C . So, this is this already I all I have explained you that how one should take. So, that is why

this example is taken such that maximum value and rms value all the concept our concept will be clear.

Right

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$\vec{V}_R = R\vec{I} = 50\angle 45^\circ$, $\vec{V}_L = jX_L\vec{I} = 40\angle 135^\circ$
 $\vec{V}_C = -jX_C\vec{I} = 90\angle 45^\circ$

NOTE: THE VALUES IN TERMS OF PHASOR QUANTITIES ARE ALL RMS VALUES WHEREAS AS TIME FUNCTIONS OR IN INSTANTANEOUS VALUES ARE EXPRESSED WITH MAXIMUM VALUES.

ii) Power loss $= I^2 R = (5)^2 \times 10 \text{ W} = 250 \text{ W}$. No power loss in inductance or capacitance.
 Power factor $\cos \theta = \cos 45^\circ = 0.707$ leading.

iii) Phasor diagram: CIRCUIT IS BEHAVING LIKE R-C CIRCUIT.

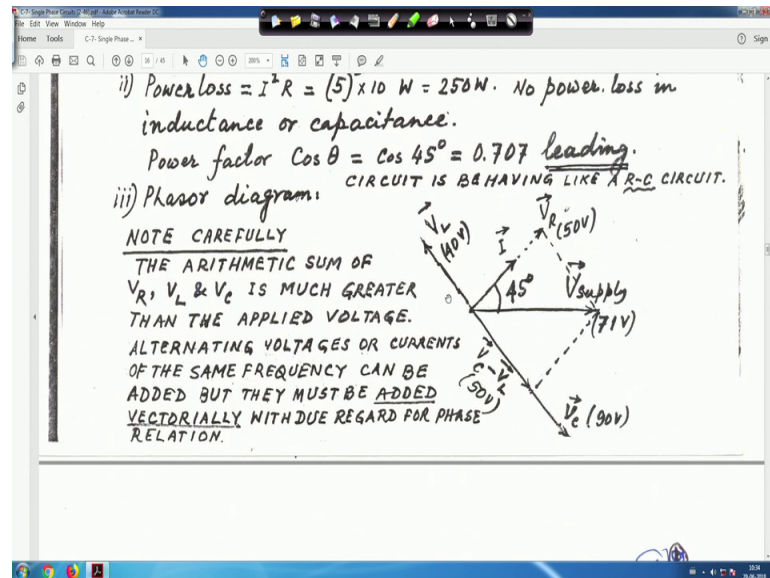
NOTE CAREFULLY
 THE ARITHMETIC SUM OF V_R, V_L & V_C IS MUCH GREATER THAN THE APPLIED VOLTAGE.

Next is, now question is that note, that in terms of phasor quantities are all rms value right. So, whereas, as time function or instantaneous values as explained it is maximum value when you are putting in time function, it should be maximum values. And will be putting in the phasor form or quantity this will be rms value right, that is that is that is your that is this note written here that is the note is written here.

Now, in the circuit power loss is simply $I^2 R$ so basically it will be 250 watt that is your $I^2 R$ loss the same way you can do it that power factor the. What you call another thing is that, you can make it your, you just calculate it; that $VI \cos \theta$ also will give you the same value right. So, if you if you I mean if you find out that your what you call this power loss dissipated in the register R is $I^2 R$ I is the 5 ampere is a rms value into $10R$ is the 10 250 watt use the relationship $VI \cos \theta$ and you will and $\cos \theta$ power factor later we will see there it is.

It is leading power factor so that is your 0.707 so if you calculate $VI \cos \theta$ you will get the same value 250 watt right so; that means, if you draw the phasor diagram.

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This is your reference you take that is your supply voltage it is 70.7. So, approximately it is 70 this is a supply voltage if you come to V_R V_R is equal to actually it was given V_R it was given 50 angle 45 degree that is with respect to this one so this is my V_R . Similarly, I also this one is equal to 5 angle 45 degree and V_R is equal to R has no angle it is simply multiplied by the ten. So, same along the same line this I is plotted and this V_R is plotted because your V_R is equal to 50 angle 45 degree and I is equal to your 5 angle 45 degree.

So, this is the with respect to this is your apparent the supply voltage that is your 70.7 that is approximately 71 volt next is your V_L , V_L is equal to your 40 angle one 35 degree that we have seen. So, this is these angle this total angle is 135 degree. With respect to, the apparent v supply that is why V_C or V_L this is your V_L . Similarly, V_C is actually is equal to 90 angle minus 45 degree we have seen right if you come to this that we have seen all right.

These all V_R V_C V_L all are there right so in this case you are in this case this is V_C (Refer Time: 13:28) this angle these angle is 45 degree right. So, if you look into that that current I actually leading the voltage by 45 degree right; that means, power factor is leading current here is leading current is leading because the angle is positive right. So, current is leading and this is 45 degree and this is V_L and this is V_C angle between this your V_L and V_C you know just opposite so it is 180 degree who make you ready.

So, resultant actually your what took as the current is leading. So, V_C minus V_L is will be your, what you call in that case as current is leading. So, V_C will be greater than V_L look at that V_C is 90 volt and V_L is 40 volt. So, V_C is greater than V_L that is why current is leading right we have to understand each and everything, right that is that is why this current is 45 degree. So, you know the you know the your what you call this current is 5 angle you know that current is 5 angle 45 degree right. And voltage is equal to say 70.7 that is your 71 angle 0 degree right. So, you can you can you can easily you are what you call you can easily find out what will be your, what will be your this thing power right.

So, that is why you are what you call that VI it is $\cos \theta$. So, V is equal to your 71 right this is rms value I is equal to 5 and $\cos \theta$ $\cos 45$ degree that is $1/\sqrt{2}$ if this 71 by $\sqrt{2}$ approximately 50. That is your 250 watt right this is the power $VI \cos \theta$ earlier I showed you this $I^2 R$ also 250 watt. So, it has to be same right so that so this; that means, all calculations are correct if something goes wrong if it is not matching means somewhere your calculation is wrong right. So, we have to be careful about the calculation because complex number is involved in every step.

So, this is how this is how that your phasor diagram. So, you can you have understood everything that how we draw the phasor diagram. And I told you that your angle and if you if you if you make it your what you call of 1st this is your 1st quadrant this is your 2nd quadrant if you say this is your third and this is 4th quadrant right. So, voltage this voltage is in the second quadrant and this voltage V_C is in the your what you call your in that 1st quadrant and this one is first.

So, it can be any value because phasor is a rotating vector right where $j\omega L$ impedance is not a phasor quantity it is a complex quantity right so, that I told you so, this one example will help you lot to understand each and everything right. So, all this write up is here.

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VOLT AMPERE, ACTIVE AND REACTIVE
POWER

Product of r.m.s. values of current
and applied voltage is called
APPARENT POWER OR VOLT-AMPERE
(VA). A larger unit is kVA

So, you will read this now volt ampere active and reactive power right. So, volt ampere means VA active and reactive power so now, this is your this thing.

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Product of r.m.s. values of current
and applied voltage is called
APPARENT POWER OR VOLT-AMPERE
(VA). A larger unit is kVA or
MYA. (Apparent power = VI)

ACTIVE POWER

Now, product of rms values of current and applied voltage is called apparent power; that means, that means your V into I is called apparent power right this is voltage this is current. So, volt is V Current is I ampere so, we call it is volt ampere right or you call volt ampere right or is in short we call VA a larger unit will be kilo volt ampere KVA or a

mega volt ampere (Refer Time: 16:54) and that this is also called apparent power. So, apparent power is equal to V into I right so no $\cos \theta$ is involved it is V into I right.

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ACTIVE POWER

$$P = VI \cos \theta \text{ (Watt)}$$
$$\therefore \cos \theta = \frac{P}{VI} = \frac{\text{Active Power}}{\text{Volt-Ampere}}$$

REACTIVE POWER

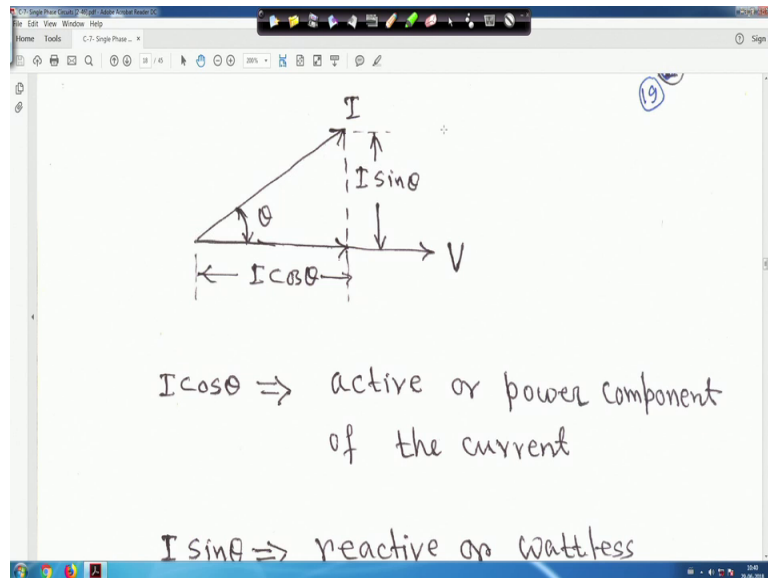
$$Q = VI \sin \theta \text{ (VAR)}$$

Reactive Power Factor = $\sin \theta$

So now active power we have seen now that $VI \cos \theta$ it is watt. So, active power that is your $VI \cos \theta$ this is $VI \cos \theta$; that means, $\cos \theta$ is equal to P upon VI right. So, that is; that means, P is the active power divided by the volt ampere this is your what you call $\cos \theta$ and reactive power is equal to Q is equal to $VI \sin \theta$ we called volt ampere we call VAR VAR V for volt a capital A small r is stand for actually your ampere, but we call in short VAR just to differentiate both right.

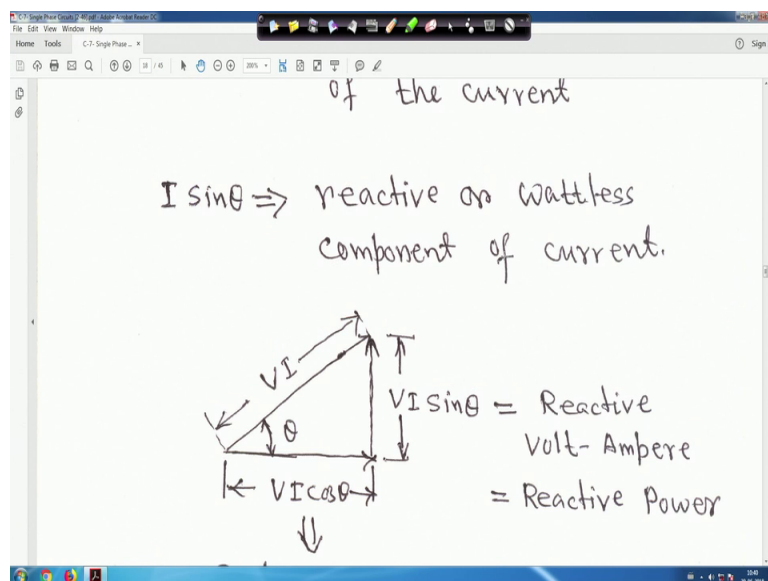
So, Q is equal to $VI \sin \theta$ we call VAR right reactive power so let me clear it. So now, reactive power then $\sin \theta$ is equal to then your Q upon VI right; so, reactive volt ampere by volt ampere that is Q upon VI right.

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So, therefore if this current is I if the current is I suppose this current is leading this angle is θ right. Then, your projection on this x axis it is $I \cos \theta$ and this is $I \sin \theta$ and this is your voltage V . Right therefore, $I \cos \theta$ active or power component of the current because power we know $VI \cos \theta$ right. So, this $I \cos \theta$ actually that is why, we call active or power component of the current right. So similarly, this reactive power $VI \sin \theta$ so this is actually reactive or wattless component of the your reactive power so if $VI \sin \theta$ this is $\sin \theta$ so.

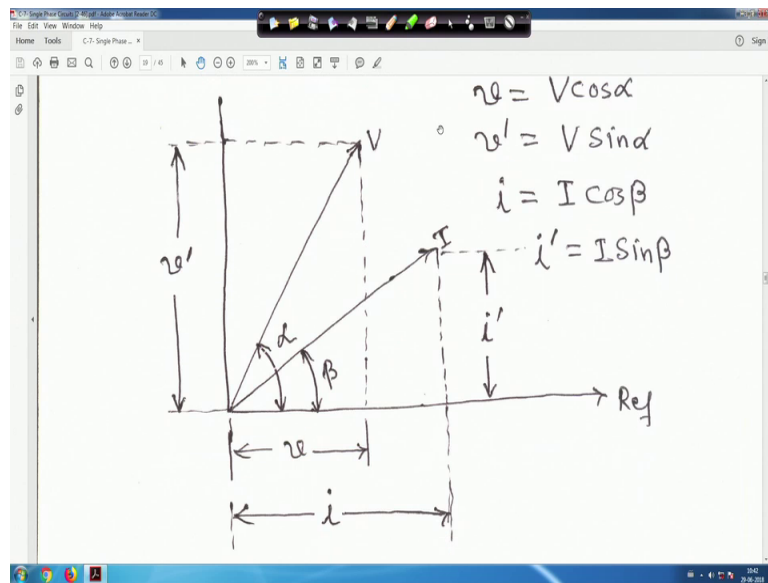
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This is your a reactive or wattless component of the current right. Therefore, this one; that means, this triangle is there it is I it is I cos theta, I sin theta for all these things for all these arm you multiply by V. So, this will be VI this will be VI cos theta this will be VI sin theta if you multiply by V. So, just see this diagram is like this.

Next is your if you multiply by V it will be VI it will be VI cos theta it will be VI sin theta; that means, reactive volt ampere is equal to this is your reactive volt ampere reactive power this is we call real power right this we call real power or a power. Therefore, volt ampere is equal to VI volt is equal to real power square plus reactive power square right. So, this is I mean if your real power is P suppose if it is P we have taken if it is Q then this one will be P square plus Q square right. So, this is your volt ampere VI so let me clear it.

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So, calculation of power using complex notation now onwards; so, everywhere we are using arrow and arrow now we will not do we hope it is everything will be understandable. So, phasor and other thing magnitude everything I will tell so we will not concentrate or we will not your, what you call again and again. We will not put arrow and other thing, but understandable; so, calculation of power using complex notation right.

So, if you look into this phasor diagram say this is my this is my V this is my V and this is my this is my reference line this is my reference line and this angle is alpha say. So,

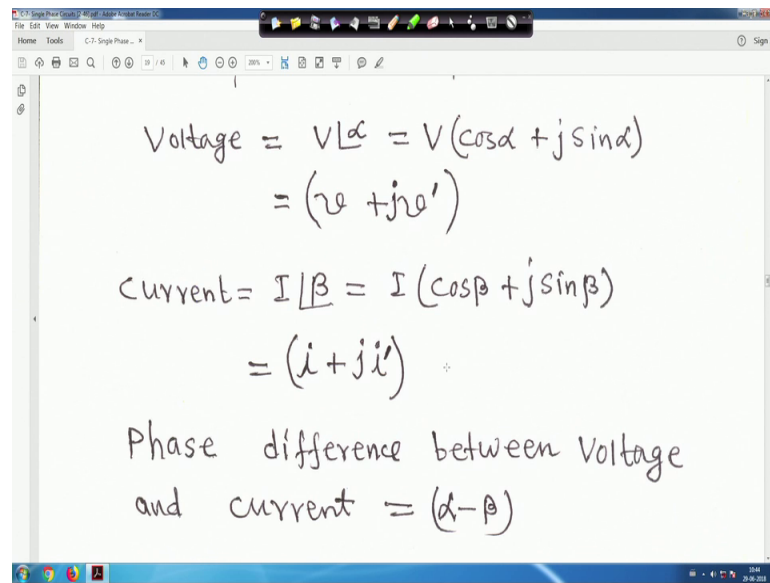
this side your what you call this is this your horizontal projection V is equal to $V \cos \alpha$ right. And this vertical projection this one V dash is equal to $V \sin \alpha$. Similarly, this is the current this is the current this horizontal projection your what you call I is equal to it is written at the bottom just hold on just hold on. So, it is it is your what you call this I this horizontal projection I is equal to your $I \cos \beta$ and vertical one is I dash is equal to $I \sin \beta$ right.

And angle between voltage and current this is the angle between voltage and current is equal to $\alpha - \beta$ right. So that means, this is actually the angle between the voltage and current is called the power factor angle; that means, for this kind of thing power factor will be $\cos \alpha - \beta$ right. So, because the your what you call whenever you call power factor angle means it is the angle between the voltage and the current right.

So, that is why it is $\alpha - \beta$ difference in $\alpha - \beta$ so power factor angle is $\cos \alpha - \beta$ right let me clear it. Now, this voltage this voltage this voltage we can write just let me move little bit up right. So, this voltage your you can I am writing it later I will show you this voltage you can right it is $V \angle \alpha$ right; that means, $V \cos \alpha + j V \sin \alpha$ right. So, V is the rms value everything is rms now so; that means, $V \cos \alpha + j V \sin \alpha$ right this is V . Similarly, let me clear it similarly for current it is $I \angle \beta$ that is $I \cos \beta + j I \sin \beta$ that is your $I \cos \beta + j I \sin \beta$ right this is your $I \angle \beta$.

So, next is that means, let me clear it that means all these things V is equal to V V is equal to capital $V \cos \alpha$ V dash is equal to capital $V \sin \alpha$ I is equal to your capital $I \cos \beta$ I dash is equal to capital $I \sin \beta$.

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The screenshot shows a digital whiteboard with the following handwritten text:

$$\text{Voltage} = V \angle \alpha = V(\cos \alpha + j \sin \alpha)$$
$$= (v + jv')$$
$$\text{Current} = I \angle \beta = I(\cos \beta + j \sin \beta)$$
$$= (i + ji')$$

Phase difference between Voltage
and current = $(\alpha - \beta)$

So, all these expression whatever I have written here $V \cos \alpha$ you put v small V and $j \sin \alpha$ you put v dash you multiply and $V \sin \alpha$ is equal to v dash. Similarly, current I told you it is $I \cos \beta$ plus $j I \sin \beta$. So, $I \cos \beta$ is equal to i we from that phasor diagram and $j i$ dash plus $j i$ dash when you will read this video lecture first you draw the phasor diagram on the note book after that look into that one by one it is very easy right.

And the phase difference between voltage and current I told you α minus β here the power factor your $\cos \alpha$ minus β right. Now, therefore, power factor is equal to I told you $\cos \alpha$ minus β therefore, active power will be $VI \cos \alpha$ minus β right.

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The image shows a digital whiteboard with the following handwritten text:

$$\therefore \text{Active Power} = P = VI \cos(\alpha - \beta)$$
$$\therefore P = VI (\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$
$$\therefore P = (V_i + V'_i)$$
$$\text{Reactive Power} = Q = VI \sin(\alpha - \beta)$$

So, that means, it is active power P is equal to $VI \cos \alpha$ minus V ; so, $\cos \alpha \cos \beta$ plus $\sin \alpha \sin \beta$ right. So, $V \cos \alpha$ this one if you multiply by VI so, $V \cos \alpha$ is equal to put small V and $I \cos \beta$ you put VI right and because your $I \cos \beta$ you put I because, we have taken the vertical and horizontal projection that phasor diagram everything is written. Similarly, $V \sin \alpha$ is equal to is your V dash and $I \sin \beta$ is equal to I dash; that means, P is equal to V into I plus V dash into I dash. Here, it is all these things are given $V V$ dash $I I$ dash in terms of that we put that one right.

So, P is equal to VI plus V dash I dash. So, this is your this is our real power right active power now reactive power will be Q is equal to $VI \sin \alpha$ minus β . So, it is $VI \sin \alpha \cos \beta$ minus $\cos \alpha \sin \beta$ formula. So, $\sin \alpha \cos \beta$ minus $\cos \alpha \sin \beta$ so $V \sin \alpha$ is equal to V dash and your $I \cos \beta$ is equal to I . So, it is V dash I minus $V \cos \alpha$ is equal to V minus $I \sin \beta$ is equal to I dash. So, Q is equal to V dash i minus VI dash right now current is equal to I angle β ; that means, current conjugate will be I angle.

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$$\therefore Q = (v'i - v i')$$

Now

$$\text{Current} = I \angle \beta$$
$$\therefore (\text{Current})^* = I \angle -\beta$$
$$\therefore P + jQ = (\text{Voltage}) \times (\text{Current})^*$$

Minus beta from your complex number right that is that I beta is equal to I e to the power j beta right; that means, if we take it is conjugate then (Refer Time: 25:14) it will be I e to the power minus j beta we are taking the conjugate. That is why this angle conjugate is minus beta. So, P plus jQ is equal to voltage into your what you call your current conjugate. Now, to get this why the conjugate right this we will find in book and other thing, but why is the conjugate we take? Actually we take the conjugate. This one your what you call P plus jQ is equal to voltage into current conjugate or you can write P minus jQ is equal to voltage conjugate just opposite into your simply current right.

Either way you can use right either P plus jQ voltage into current conjugate or P minus jQ is equal to voltage conjugate into current, you will get the same result. Now, question is why the conjugate conjugate is actually to capture the power factor angle between the voltage and current. That is why we take conjugate for example, for example, before proceeding further for example, suppose you have been given say voltage is equal to 10 volt is rms say it is 30 degree right. And I is given say I is equal to given this is volt and I is given say 5 angle minus 15 degree ampere right.

Now, question is there: what is the angle between the voltage and current. So, in that case suppose if I take a reference line this is my reference line. So, V is equal to 10 angle 30 degree. So, with respect to this reference it will be your this side this is my 10 volt and this is my 30 degree and with respect to reference, it is minus 15-degree current. So, it

will be on the negative side this is my current I is equal to 5 ampere and this is my voltage and this angle is 15 degree then question is this is the voltage this is the current. So, what is the angle between the voltage and current it is actually 30 plus 15 that is your 45 degree right.

Therefore, then how we will get the current if we do not take the conjugate right. So, in general if you put like this suppose, we have taken $P + jQ$ is equal to voltage into current conjugate. So, if it is I is equal to 5 angle minus 15 degree then I conjugate will be is equal to 5 angle 15 degree. It is it is your because it is minus 15 degree it is conjugate will be plus 15 degree. So, voltage is 10 30 degree so it is 10 angle 30 degree into 5 angle your 15 degree because current conjugate so that is actually 50 angle 45 degree right.

That means this one will be your $50 \cos 45$ so 50 by root 2 plus j your 50 by root 2 because $\sin 45$ is also 1 by root 2; that means, my P will become in this case for this example whatever I take it will be 50 by root 2 watt and Q will be 50 by root 2 var right because $P + jQ$ is equal to 50 by root 2 plus j 50 by root 2. So, real and imaginary part you separate now you have studied in the complex number. So, that is why this conjugate is must right otherwise you cannot capture the angle between the voltage and current. Otherwise, if you add this we are getting 45 degree, but mathematically how we get it unless and until we take the conjugate that is why let me clear it.

That is why we your, what you call we consider conjugate right to capture the power factor angle that is why $P + jQ$ is equal to voltage into current conjugate. I hope you are understanding now, clear right so just 1-minute so this is your $P + jQ$.

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The image shows a handwritten derivation on a whiteboard. The first line is $\therefore P + jQ = (v + jv')(i - ji')$. The second line is $\therefore P + jQ = (vi + v'i') + j(v'i - vi')$. The third and fourth lines are enclosed in a box and show $\therefore P = (vi + v'i')$ and $Q = (v'i - vi')$. Below the box, it says "EXAMPLE -4".

Now that means, we calculate $P + jQ$ is equal to now voltage is equal to you have this is horizontal component plus vertical component $V + jV'$ into $i + jI'$ look at that phasor diagram because, you have 2 component one is real component, one is real component and another is your what you call complex your this thing. However, it, but horizontal one is vertical. So, this can be written as V can be written as $V + jV'$.

Similarly, i can be written as $i + jI'$ and it is conjugate; that means, $V + jV'$ into $i - jI'$. You multiply if you multiply and simplify you will find $P + jQ$ is equal to this one and your what you call and this imaginary part is this one complex part is this one; that means, your; that means, if you separate real and imaginary part. So, P will become $VI + V'I'$ and Q will be $V'I - VI'$. So, here we got to $V'I - VI'$ using the relationship and P we got $VI + V'I'$.

So, similarly if you do this conjugate same result you will get right. So, conjugate is required to capture the power factor angle right. I hope up to this single phase circuit I hope you have understood this look that actually AC circuit it your, what you call your it involves complex number. So, only few 3 4 more examples I can show you because it, but you take any book any good book I suggest you take and solve some problems right because complex number involves your (Refer Time: 31:54).

You need lot of computation time right. So, anyway so these are the thing your and your what you call as it is a basic course. So, if you have any doubt anything you put the question in the form right we try to answer all this question now next is you take a simple thing.

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EXAMPLE - 4.

$$V = (100 + j200) \text{ Volt}$$
$$I = (10 + j5) \text{ Amp}$$
$$\therefore P + jQ = VI^* = (100 + j200)(10 - j5)$$
$$= (2000 + j1500)$$
$$\therefore P = 2000 \text{ Watt} = 2 \text{ kW}$$
$$Q = 1500 \text{ VAR} = 1.5 \text{ kVAR}$$

Suppose, it is given that V is equal to your what you call 100 plus j 2 volt and current is given 10 plus j 5 ampere everything you taken in rms right. So, VI P plus jQ is equal to just for your understanding we have taken VI conjugate. So, it is 100 plus j 2 and I is equal to 10 plus j 5 and it will be 10 minus j 5 the conjugate right. Therefore, if you multiply it will become 2000 plus j 15 100 right so; that means, separate real and imaginary part. So, P is equal to 2000 watt that is 2 kilo watt divided by 1000 and just let me move little bit up and.

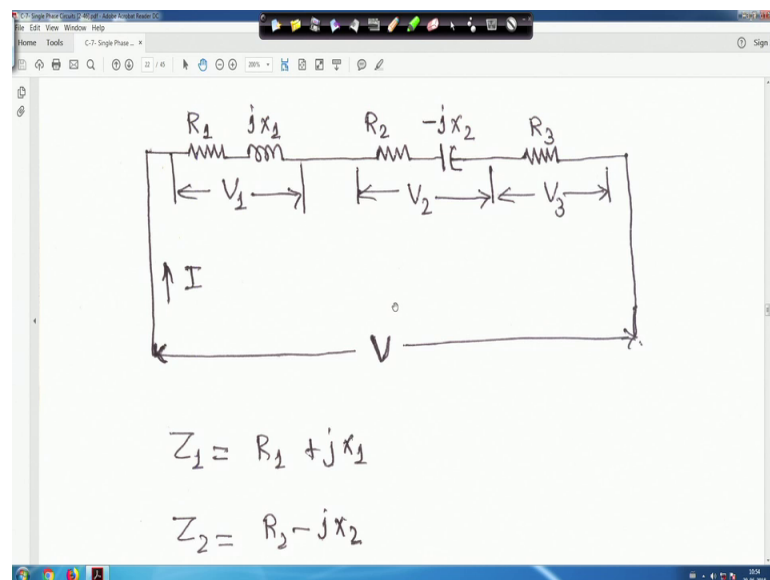
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A screenshot of a digital whiteboard showing the calculation of complex power. The equations are written in black ink on a white background. The first equation is $\therefore P + jQ = VI^* = (100 + j200)(10 - j5)$. The second equation is $= (2000 + j1500)$. The third equation is $\therefore P = 2000 \text{ Watt} = 2 \text{ kW}$. The fourth equation is $Q = 1500 \text{ VAR} = 1.5 \text{ kVAR}$. The equations are grouped by a large right-facing curly bracket on the right side.

$$\therefore P + jQ = VI^* = (100 + j200)(10 - j5)$$
$$= (2000 + j1500)$$
$$\therefore P = 2000 \text{ Watt} = 2 \text{ kW}$$
$$Q = 1500 \text{ VAR} = 1.5 \text{ kVAR}$$

Q is equal to 1500 var is equal to 1.5 kilo var divided by 1000 separate real and imaginary part right.

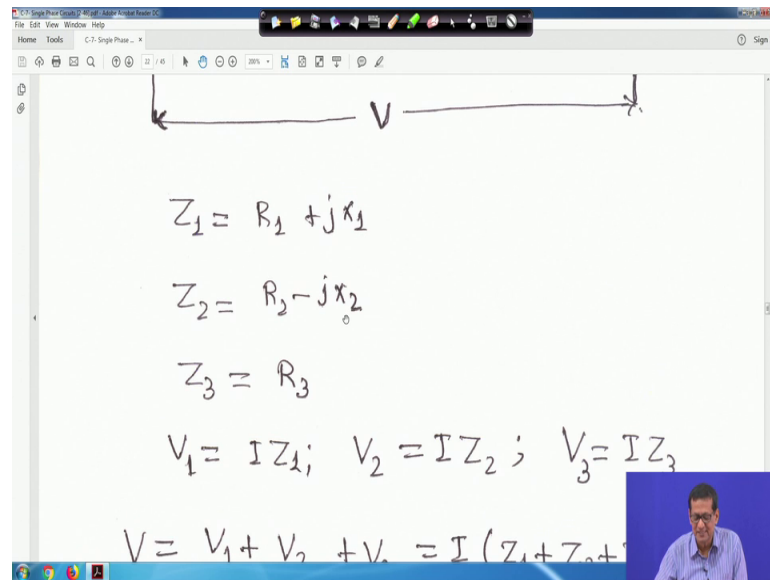
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Now, single phase circuit analysis suppose, you have a circuit like this right you have a circuit like this say it is it is inductive circuit this part is inductive R_1 plus jX_1 this is capacitive R_2 minus jX_2 and another thing is that R_3 . Voltage across this it is V_1 it is V_2 it is V_3 and total is V Right supply voltage is V and current flowing through this is I current flowing through this is your, I this is the I right. Everything is rms this is rms

now, everything that unless and until it is stated in $\sin \omega t$ all are rms right. So, this is as this is your inductive part this is capacitive part and this is right. And this is this is just let me move little bit up. So, when you will solve this one first you draw the circuit then you look what has been done.

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So, Z_1 we are taking R_1 plus jX_1 Z_2 , we are taking R_2 minus jX_2 and Z_3 is equal to say R_3 . So, this is Z_1 this is Z_2 this is your Z_3 this is your this is your Z_1 , this is your Z_2 and this is your say Z_3 right all that is R , but will write, but everywhere we will not put bar. Now, we will put simply Z and I and V understandable these all are phasor your what you call I and V are phasor quantity and this is complex quantity right. So, so V_1 across the voltage I_1 into your 1 into Z_1 V_2 your I_2 your I into Z_2 and V_3 is equal to I into Z_3 .

So, here V_2 is equal to I into Z_2 and V_3 is equal to I into Z_3 . So, V is equal to do not putting I again and again phasor quantity understandable it is not like a dc algebraic sum do not do it. So, it is a phasor sum like vector vector sum you have to do it so V_1 plus V_2 plus V_3 is equal to I into. Z_1 plus Z_2 plus Z_3 is equal to you substitute all these values V_1 V_2 V_3 Z_1 is equal to R_1 plus jX_1 Z_2 is equal to your R_2 minus jX_2 and Z_3 is equal to R_3 .

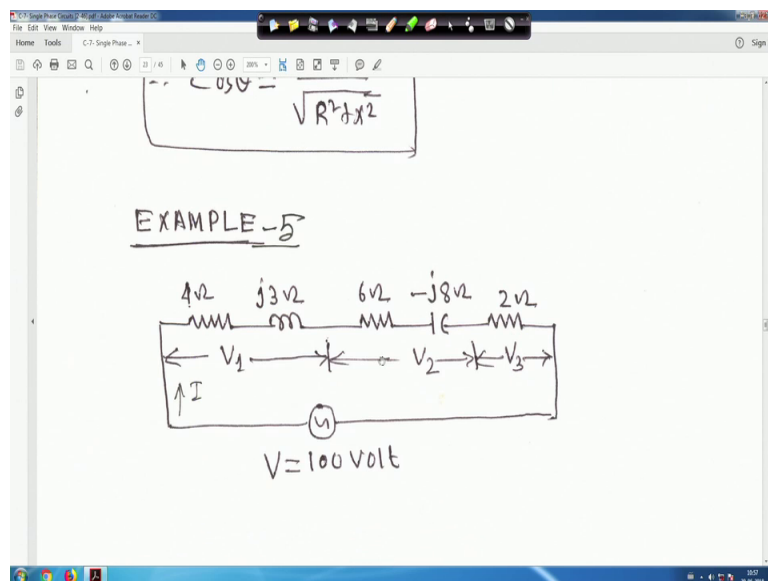
You will get I into R_1 plus R_2 plus R_3 plus j into X_1 minus X_2 or in general V is equal to I into Z is equal to I into R plus jX . So, R is equal to like your series resistance

adding same as dc circuit R will be R 1 plus R 2 plus R 3 and X will be X 1 minus X 2. Because, one is inductive another is capacitive. So, X 1 minus X 2 right and V is equal to I into Z that is I into R plus jX and R and X are defined here right.

Therefore, Z is equal to now we know root over your R square plus X square the magnitude because it is your R plus jX. So, it is magnitude is root over R square plus X square and angle theta right. Therefore, theta is equal to your tan inverse X upon R. So, that is X is equal to X1 minus X2. Tan inverse x 1 minus x 2 upon R 1 plus R 2 plus R 3 therefore, I is equal to V angle 0 by root over R square plus X square angle theta right. So, this is the equivalent total impedance is equal to V angle 0 minus theta divided by root over R square plus x square.

Therefore, it is V upon root over R square plus X square angle minus theta. This is I and power factor is cos theta, right is the angle between the voltage and current because voltage angle is 0 and current angle is minus theta; that means, current is same if it is theta is positive then current is lagging if theta is negative then current is leading right. So, tan theta is equal to X by R this one you can write sin theta upon X is equal to cos theta upon R is equal to 1 upon root over R square plus X square right. Because, it is sin theta by cos theta is equal to X by R. Therefore, sin theta upon X is equal to cos theta upon R is equal to 1 upon root over R square plus X square; so, this is your cos theta R upon root over X square plus X square right.

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So, with this one we will take one small example suppose it is $4 + j3$ and this is $6 - j8 + 2$ V V_1 V_2 V_3 and voltage is 100 volt is given right and this is V_1 V_2 V_3 . So, I is equal to V by Z so V is equal to if it is V is 100 volt is given means V is equal to 100 volts given. Means you you will always take this one that it is 100 angle 0 degree that one reference you have to assume. So, it is rms value all are rms value. So, 100 angle (Refer Time: 37:01) and this is the current is flowing right. So, 100 angle 0 degree so let me clear it so just hold on so this one your I is equal to V by Z 100 angle 0 by $Z_1 + Z_2 + Z_3$. Simply, you solve it you will get 7.69 angle 22.6-degree ampere. Simply, you solve it similarly that V_1 will be I into Z_1 .

So, that is I you know you know the I you multiply by $Z_1 + j3$ you will get 38.47 angle 59.5 degree. Similarly, V_2 is equal to $I Z_2$ this is $I Z_2$ is $6 - j2$ multiply and simplify you will get 76.92 angle minus 30.5 degree. Similarly, V_3 is equal to your I . Your what you call into your Z_3 is R only so if you multiply this will be your R your what you call R is equal to 2 ohm.

So, it will be 15.38 angle 22.6-degree volt right for checking if you made if for checking if you add $V_1 + V_2 + V_3$ it will become 100 plus j volt. I mean if you look into the circuit if you look into the circuit polarity will be anyway this 1 plus minus. You mark the polarity many places intentionally I have not marked it here just to show you in a your what you call, in this lecture class right plus minus. So, it is moving like this right. So, everywhere you can if you move like this.

And mark the polarity plus minus plus minus right. So, this is your suppose this is one part this is your capacity part and this is plus this one. So, you can make it plus minus right plus minus plus minus and plus minus if you make it and apply your KVL. So, it will $V_1 + V_2 + V_3$ is equal to V , but remember this is your phasor sum do not add the magnitude a magnitude one never you will get the result it is phasor sum right. So, all this angle you have to voltage and angle you have to consider right.

So; that means, that is why if you check it if you check it then you will get this one is equal to your 100 volt right for a checking. So, as you are not putting arrow and arrow, but it is a phasor sum you add it and P_1 is equal to $I^2 R_1$ you will get 237 watt P_2 is equal to $I^2 R_2$ right R_2 that is your R_2 is 6 R_1 is 4. So, it is 355 watt and P_3 is equal to.

I square R 3 it is 118-watt total power if you add P 1 plus P 2 plus P 3 it will become 710 watt and if you use V and V is equal to 100 angle 0, I is equal to 77.69 power factor angle is 22.6 degree. So, $\cos 22.6$ degree is 0.9232 it is leading because angle is positive current angle is positive right because voltage it is the angle is 0.

So, P is equal to VI cos theta, if you make you will get same as 710 watt right. So, here also 710 watt you will get the same result. Either way you can do it right and this R 1 R 2 R 3 is this one from the circuit. This is your, this is R 1 is 4-ohm R 2 is 6-ohm R 3 is equal to 2 ohm right. So, with these we calculated this power this power calculated. Similarly, using VI cos theta we got it and your job is you please draw the phasor diagram phasor diagram I have not drawn for you. So, please draw phasor showing V showing V I V 1 V 2 and V 3 everything right so draw the phasor diagram.

Thank you very much we will be back again.