Fundamentals of Electrical Engineering
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Lecture - 41
Single Phase AC Circuits (Contd.)

So, again we are back listen one thing that every time I am telling this the way things are being made right, only your job will be to solve more numericals from the book right. Although many things have been done right as far as theory is concerned more or less everything is covered right. So, next we will consider that series R-L C circuit. First we saw that is your purely resistive circuit, then we saw purely inductive circuit, then we saw purely capacitive circuit, step by step we are moving.

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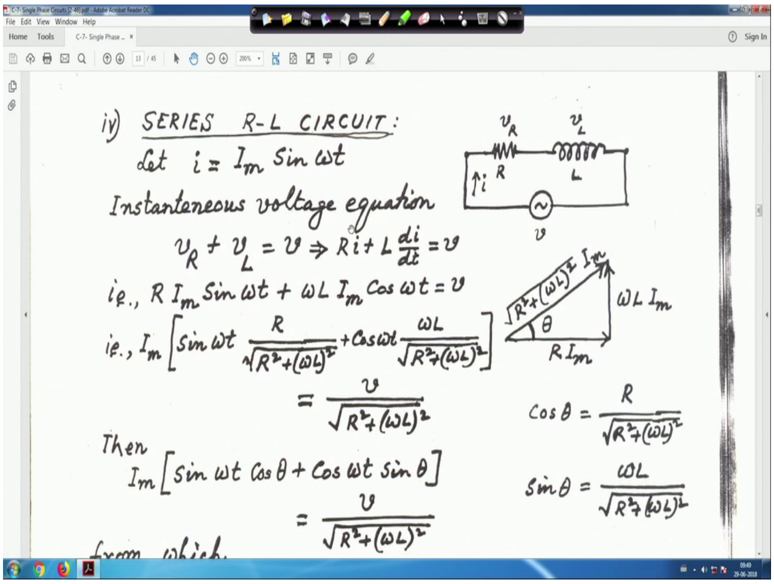

Now, we will consider series R-L your R-L circuit right. So, here you have to make instantaneous polarity plus, minus right. And current flowing through this is ideal simple series circuit right. And voltage across the resistance is v R, and across the inductance is v L right, our objective is a steady state analysis right. So, current the v, v is given. Now, we will assume let i is equal to I m sin omega t right that we will assume. So, just either voltage or current something, you have to assume right.

So, if you assume, i is equal to I m sin omega t before moving is that, whenever you make i is equal to I m sin omega t that means, that means in general your what you call, it will be rms value will be I m by root 2 I think if you write like this, and reference will be angle 0, because omega t. So, omega t plus 0 degree right, so that means it is I m by root 2, 0 degree. So, this is the rms value right.

Anyway, so next is instantaneous voltage that is v R plus v L is equal to v. So, v is equal to this v R is equal to current flowing through this is your what you call, this i, this i is the current right, so it is i R. And then, it is L into your di by dt that is your v L. v L is equal to is equal to v right. So, v is equal to v R that is i R plus L into di by dt. Now, i is equal to I m sin omega t right, therefore if you substitute here, it will be R I m sin omega t plus omega L I m cos omega t is equal to v right.

Then what you do, you take you take I m common right. So, I m is here, then this is your R, and this is your omega L. So, this is R this is your made R I m, this is say omega L I m. So, this is root over R square plus omega L square I m in general. If you take, if you take here I m common I m common, so in general here it is coefficient then is R right, it is R square you take, and here it is omega L square. So, numerator and denominator both side you divide by root over R square plus omega L square, R square plus omega L square is equal to v.

So, v by root over R square plus omega L square both side of this equation of this equation you divide by root over R square plus omega L square. And this I m you take common. And here from this angle is theta. So, cos theta is equal to R upon root over R square plus omega L square. And sin theta is equal to omega L by root over R square plus omega L square right. Therefore, now question is that this is your cos theta, this is your sin theta. So, this one this one you replace by cos theta. And this one you replace by sin theta. Then, I m common into sin omega t cos theta plus cos omega t sin theta, that is sin A cos B plus cos A sin B form.

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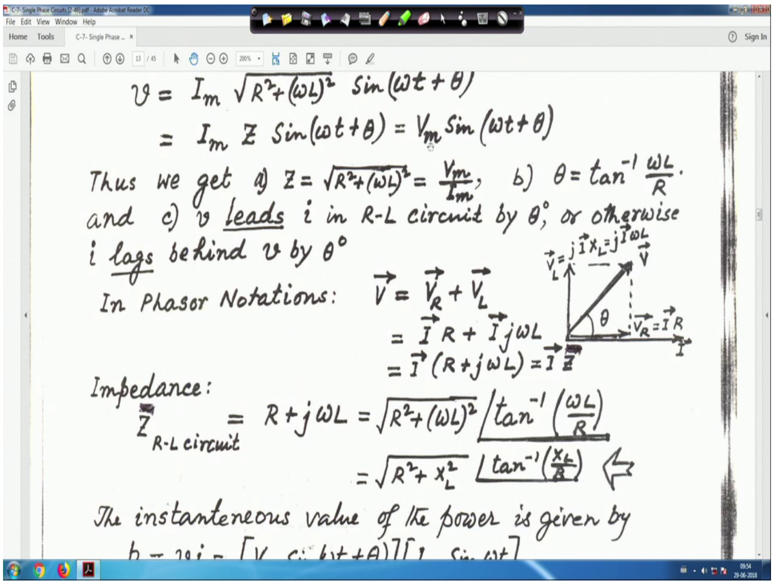

So, let me delete it, so that means from which you will get that your v is equal to v is equal to I m root over R square plus omega L square sin omega t plus theta. And this is given your sin theta, cos theta is given. Cos theta is equal to R upon root over R square plus omega L square. And sin theta is equal to omega L by root over R square plus omega L square.

So, this one we can write this one is equal to I m your what you call v is equal to I m into Z into sin omega t plus theta that is equal to V m sin omega t plus theta that means, this I m into this one root over R square plus omega L square this is the term. So, Z is equal to Z is equal to root over R square plus omega L square. This is actually your impedance of this R L circuit right. So your, I m into Z and V m is equal to your magnitude, these are all magnitude V m is equal to I m into z. This is also magnitude right. Therefore, Z is equal to root over R square plus omega L square is equal to V m by I m.

So, this Z is equal to I am writing for you Z is equal to right, when we write V m by I m right, basically it is equal to V m I hope you can see it V m by root 2 divided by I m by root 2 right, so that means RMS value, that means your let me clear it, that means Z is equal to the impedance magnitude is equal to V m by I m is equal to V by I, where V is the RMS value of the voltage, and I is the RMS value of the current.

When you solve the numericals, we will use V rms upon I rms understandable right. So, and that is why it is writing V m I V m upon I m, you can write V upon say I. If you say

V is the rms value, and I is the rms value, because both numerator and denominator you are dividing by root 2. And theta is equal to theta is equal to tan inverse omega L by R right. So, this is your just let me clear it. This is your if you looking, into this the diagram right here, tan theta is equal to your L omega by R, because I m, I m will be cancel.

So, L omega by R that means, theta is equal to tan inverse omega L by R right. So, let me clear it. Therefore, here it is given that your theta is equal to tan inverse omega L by R. So, this is your tan inverse omega L by R. In this case, what is happening that two thing that this is we choose actually, we took i is equal to i m sin omega t right, that we took.

So, if you take in terms of your, what you call in the phasor that, when you draw the phasor diagram, so generally we can write i is equal to I m by root 2 that rms value, and its angle will be 0 degree. So, this is your rms value. So, if we define this that it is capital I, then it will be I angle 0. I is the rms value right, so that is why this I is taken as a reference, this is capital I, this is capital I. So, this is your I the reference right.

Now, next is let me clear it, next is my V. So, V is equal to actually it is V m sin omega t. So, v actually is equal to V m sin omega sorry omega t plus theta omega t plus theta, this one theta. So, this can be written as in the terms of when you will draw the phasor diagram, V m when you take the rms value V m by root 2, and this one can be taken as theta, angle theta. So, this can be written as V angle theta. V is the rms value; V is equal to V m by root 2.

Similarly, I is equal to I m by root 2, the rms value right. So, V is the rms value. So, here an angle is theta, and theta is positive. So, this V is there, so this is my V. And this angle is theta, that means current I is lagging from this voltage V by an angle theta or V is leading the current I by an angle theta right. And, another thing is let me clear it, just hold on.

Let us go to the diagram. Another thing is that this diagram. So, here v R is equal to your i R right. This is what you call that voltage drop across the resistance; we can make this I R. If it is a rms value, and of course instantaneous polarity is there. And v L this is the inductive reactance. And inductive reactance means x L is equal to your j L omega right. j L omega that means, your voltage drop v L is equal to j I L omega, that is I x L. So, it is v L is equal to j into I L omega right.

Therefore, let because what you call for purely inductive circuit, we have seen it is j L omega right, that your what you call that your current lags 90 degree from the voltage right. For purely inductive circuit and it was basically on the imaginary axis right; so, j L omega. So, this is this is your R, and this is j L omega. So, let me clear it.

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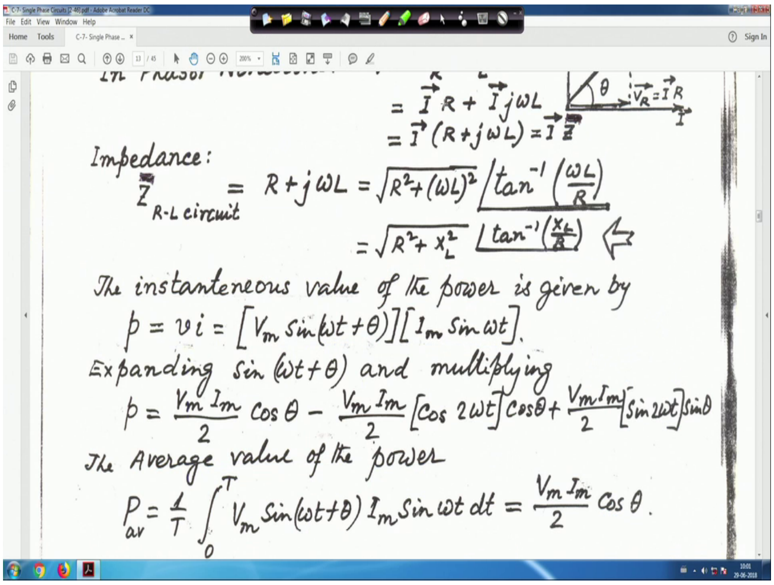

Therefore, this one is your V R is equal to I R. And this one is v L is equal to this is your v L is equal to j I X L; j means angle 90 degree right. If you take like this j I x L is equal to I x L. And j means angle 90 degree, because angle 90 degree means, cos 90 degree plus j sin 90 degree. So, cos 90 0, and j sin 90 1, so sin 90 1; so, it will become j I X L, this is j I X L. And this is the resultant V right.

Now, let us come back to your what you call to the diagram once again, that that means that impedance of an inductive circuit Z is equal to we can write R plus j L omega right, because this is this is purely resistive R, and this is your inductive circuit your what you call L. So, its reactance is your L omega. So, R plus j L omega that means, if you take like this, suppose this is a complex quantity put bar is equal to that is magnitude is R square plus L square omega square this is its magnitude, and its angle will be your tan inverse L omega by R right.

This is your Z bar is equal to your and this is the magnitude, that means this one that means that means, Z your what you call Z bar is equal to Z angle theta. So, theta we have defined that is tan inverse L omega by R. And Z is the magnitude of the impedance. So, any your what you call when inductive in inductive circuit, it is so R plus j L omega right, so this is the impedance of the your what you call the circuit. When we will solve the problem, we will know how things are right.

So, let me clear it, that means in general when you compute the current suppose this v is given, we got v is equal to v m sin omega t plus theta that you got, so that means, if you put in rms value, it will be v, v is equal to v m by root 2 right, and angle your theta right. So, and if you try to find out what is the current, so current should be is equal to here you can make it, if you want it is your phasor, so make it v arrow; i is equal to then this one v arrow by divided by that that your impedance Z, so that is equal to it is v angle your what you call theta sorry this one, your v is given here what you call angle theta, and divided by if you made this i, this Z, it will be R plus j L omega right.

So, in that case look how things will come, in that case your this is y i, this is my i, this is also phasor quantity. So, this is my i, so this is your V angle theta divide your divided by this R means, it is Z angle theta. Z is equal to root over R square plus L omega square, so that means, is equal to V by Z angle theta minus theta is equal to V by Z angle 0 degree right that means, the current whatever we have got, we assume know i is equal to I m sin omega t that is nothing but your i angle 0 degree.

So, this is also V by Z is i angle 0 degree, because I just show you the reverse calculation. When we will solve the circuit problem, at that time you will see into this I just came from the back right. So, this is i angle and it is coming know Z what you call this one is equal to i angle 0 degree. If I make it your in terms of phasor, because i is equal to I m by root 2 rms value, and angle 0 degree whatever it is coming here right.

So, let me clear it. Little bit understanding is required right. So, once it is done. So, this is what you call that your Z is equal to I told you that R plus j omega L. And this is your magnitude, we call X L is equal to L omega. And this is tan inverse X L upon R or L omega by R, I told you. Now, the instantaneous your what you call value of the power is given by power is equal to v into i. So, v you substitute V m sin omega t plus theta into I m sin omega t right. This one you just expand it in sin A cos B when plus your what you call cos A sin B, and you multiply.

If you do so, it will come V m I m upon 2 cos theta minus V m I m upon 2 cos 2 omega t into cos theta plus V m I m upon 2 sin 2 omega t into sin theta. Now, the average value of the power, average value of the power it is 1 upon T 0 to T V m sin omega t plus theta into I m sin omega t dt that means, this expression this expression right, this expression. So, directly we are putting it.

And if you put it, and try to integrate it integrate it, you will use this relationship omega is equal to 2 pi by T that means, my omega T will be is equal to 2 pi right. If you do, so the integrate it, it will become v m I m upon 2 cos theta. So, cos theta is the power factor right, that means, it is the angle between the voltage and the current that means, this one that means, this one this is your this theta this is the angle between the voltage and the current right.

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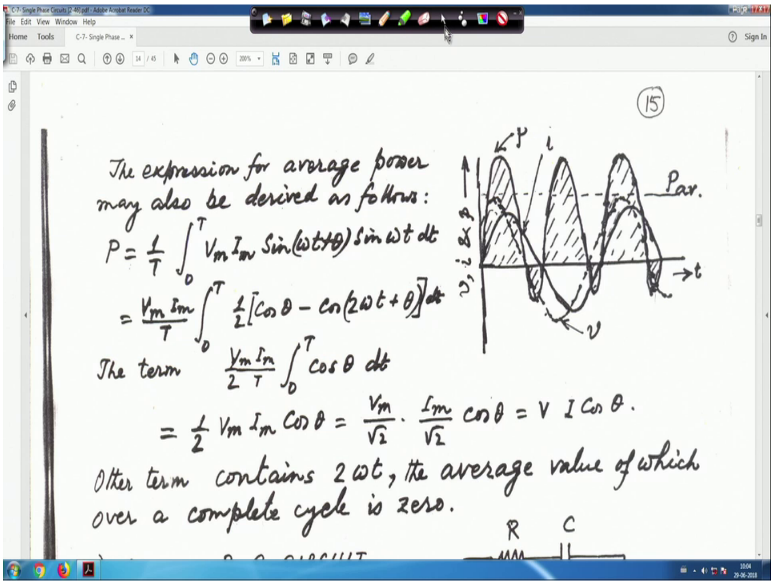

So, this is actually it is coming V m I m by 2 cos theta that means, this V m I m by 2 means, you can write this one V m by root 2 into I m by root 2 into cos theta right that means, this is my rms value. This voltage and this is rms current that means, it is VI cos theta, whereas V is the rms value of the voltage, and I is the rms value of the current multiplied by the cos theta. And cos theta is your power your power factor of the circuit.

Theta actually angle between the voltage and current right. So, this is actually V I cos theta, so that means, this one we can write V I cos theta right. And V and I both are the rms quantity right. Otherwise, if you take the peak value, it will be V m I m cos theta upon 2, but generally we will use always rms value in our numerical right. So, let me clear it.

Next is that your what you call that another way another way of doing it another way of doing this, and this is your what you call I am telling this, this is your this is your power right. This is your plot of the power, this is the current, this is this is current is marked, this is current right. And this is dash line is the voltage, and this is the V I and power v, i and p all these things are shown right. Just for the purpose of your that giving you some feeling that they are waveform right. So, this is the power. If you plot P is equal to V into I of this one, I mean this one if you plot, the plotting will be like this.

Now, another way of expression of this one is that this is 1 upon T 0 to t, this is easier one V m I m sin omega t plus theta into sin omega t. So, what you can do is that numerator and denominator you multiply by 2 that means, here you multiplied by 2 right, divided by 2. (Refer Time: 17:02) Then it is 2 sin A sin B, you expand this one 2 sin A sin B is equal to your, what you call cos A minus B minus cos A plus B right. And if you do, so and expand it, it will it will simplify, it will become like this. So, it is numerator and denominator you multiply by 2, and then you just 2 sin A sin B formula use for your class eleven trigonometry right. So, what you call, it will be 2 sin A sin B is equal to cos A minus B minus cos A plus B. So, if you do, so you will get this one.

Therefore, it will become V m I m upon T 0 to t cos theta dt right. So, if you integrate from this one this one, because these term actually will vanish. And finally it is again becoming your half V m I m cos theta is equal to V m by root 2, I told you I m by root 2 into cos theta is equal to V I cos theta. V is the rms value, and I is the rms value. Other terms contain 2 omega t, the average value of which over a complete cycle is zero right. If you do integration, you will see this will become 0 right. So, this is the idea.

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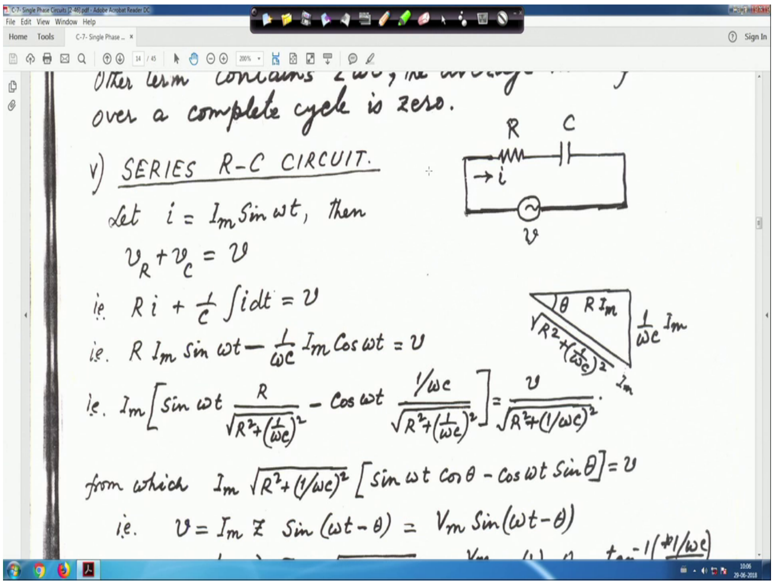

Next we will consider that your series R C circuit right. It is same like your inductive circuit, but here it is C. So, in this case we assume in this case, again we assume i is equal to I m sin omega t right. And it is instantaneous polarity. And this current i mean, it is flowing like this, it is series circuit R C circuit. So, v R same as before this voltage across resistance, let me clear it. Voltage across this one is v R, and voltage across this one is V C right. And this is your instantaneous polarity.

So, in this case R i and you know that i is equal to c into dv by dt from that you can write v c is equal to R i 1 upon C integration i dt is equal to v right. Then you write R i your i is equal to then your I m sin omega t. And this I m sin omega t you put it here i is equal to i an integrate, it will become minus 1 upon omega c I m cos omega t is equal to v right.

So, same as before, so this is I m you take common, and here it is given omega c I m multiplied, here also I m multiplied. So, voltage drop is multiplied this is R I m, and this angle is theta right. So, in this case, this is what you call this sin omega t the both side what you can do is you divide by R square root over R square plus 1 upon omega c square. If I divide both side, then right hand side will be v upon root over R square plus 1 upon omega c square right and that means, this one this is your sin omega, this is your sin omega t.

And if you look into the, if you look into this diagram right, so cos theta will be cos theta will be this is my base I m, I m will be cancel right, by root over R square plus 1 upon omega c square, this is cos theta. And then sin theta will be is equal to your 1 upon omega c by root over R square plus 1 upon omega c square right. So, this one that means, this one can be replaced your, this one can be replaced by your cos theta. And this one can be replaced by sin theta right.

So, accordingly this here it is cos theta and here this one is your sin theta. And is equal and this is equal to your what you call multiplied by your this one your I m into root over R square plus omega c square is equal to again, because this one is cross multiplication right.

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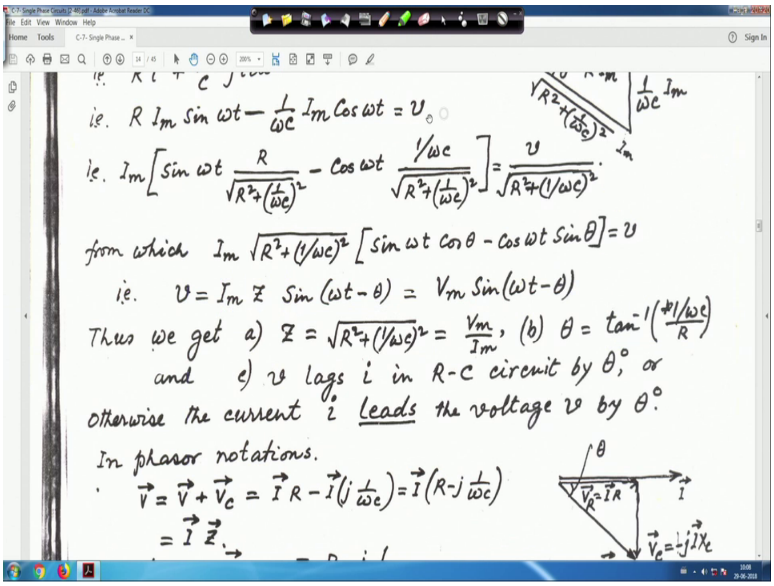

So, if you do, so same as like inductive circuit, here it is 1 upon omega c and minus sign is there. So, in this case also it is becoming v is equal to I m Z v is equal to I m Z sin omega t minus theta right. So, is equal to V m is equal to V m sin omega t minus theta. And v m is equal to I m z. So, V m is equal to actually magnitude I m Z right. So, sin omega t minus theta.

Thus we get Z is equal to root over R square plus 1 upon omega c square. And again is equal to v m by I m is equal to v by i where V and I is the rms value earlier I told you. This one also is equal to v upon I. v is the rms value and I is the rms value. Whenever will find solve a numerical will use most of the cases only rms value right. And theta is equal to tan inverse your 1 upon omega c by R, but one what you call that one minus sign is there right.

So, in the case of what you call two things you can do it that is tan inverse. And if you or this thing your minus 1 upon omega c R right. In the case of inductive circuit, it was your what you call theta was positive. It is tan inverse L omega by R. In the case of what you call in the case of capacity R C circuit, it will be theta is equal to what you call tan inverse minus 1 upon omega c. So, theta will be negative right. So, in this case, what is happening that just let me move little bit up.

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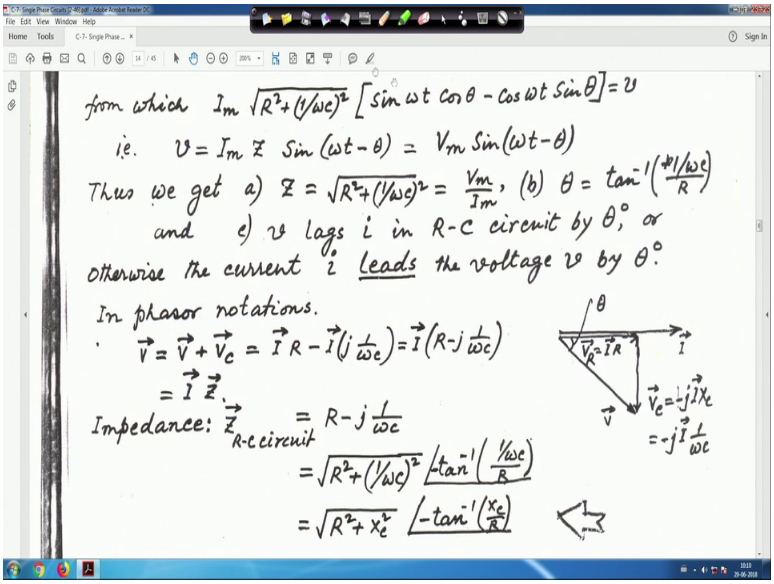

In this case, what is happening that your voltage actually lagging from the current by an angle theta or this current I this current I actually leading this voltage by an angle your what you call theta, because this angle this angle is theta, this angle is theta right. So, V c is equal to same as before j into your minus j into I X c that is X c is equal to 1 upon omega c, so that is minus your I into 1 upon omega c right. And this is my voltage V.

Therefore, V is equal to V plus V c that means, my I R minus j your actually this is my V R. And then your what you call plus V c, V c actually is equal to minus I into j into 1 upon omega c, because this is your minus j I 1 upon omega c, so that is equal to R minus j omega c that means, impedance for the R C circuit it is R minus j upon omega c right, so that means, this is equal to your what you call is equal to I into Z.

Therefore, impedance of the circuit is this one. And impedance that means, this is the magnitude. And this is tan inverse minus 1 upon omega c. So, this minus sign is taken out here it is minus tan inverse 1 upon omega c by R, but minus is here. So, minus theta we are writing that way right or we are writing root over R square plus X c square minus tan inverse X c by R, where X c is equal to 1 upon omega c right, that means. For that your series R C circuit Z is equal to this one R minus j 1 upon omega c right.

So, generally what happen that X c when we write like this, sometimes we write R plus your what you call j X c. In that case your X c actually your what you call if you write R plus j X c, then in that what will happen that you this X c value you have to take negative, that way you do not do. What we do let me delete it, what you do if you write R minus j X c, then you will take X c is equal to 1 upon omega c right. So, this is actually your capacity your reactant 1 upon omega c right. So, it is R minus this, so for the inductive case it is R plus j X L, and for capacitor capacity case it is R minus j X c and X c is equal to 1 upon omega c right.

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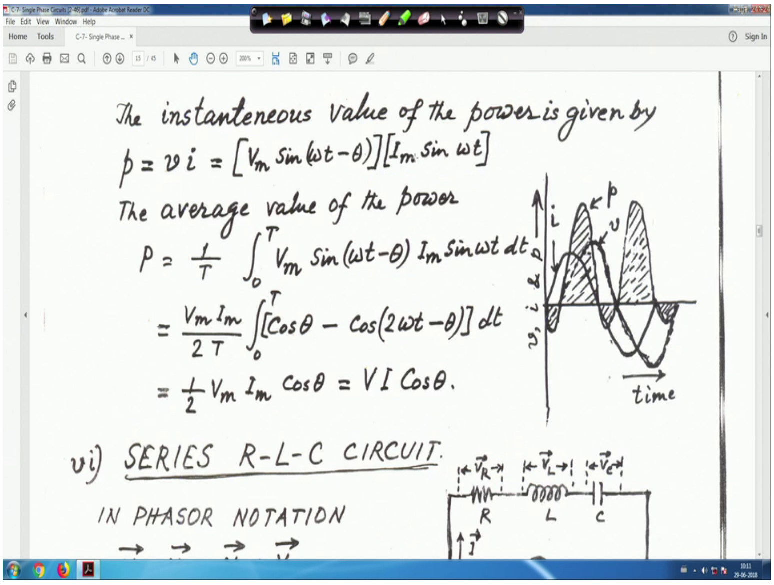

So, that means the same as before the instantaneous value of the power. So, it is your it is your v into i and you multiply this together you multiply. And then, it will be the average value of the power 1 upon T 0 to T same thing. So, here also numerator and denominator you multiply by 2. And it is 2 sin A sin B that is cos A minus B minus cos A plus B put that use that formula. And you will get again it is half V m I m cos theta is equal to V I cos theta. V is the rms value of the voltage and I is the rms value of the current.

So, once again i am writing for you V is equal to V m by root 2. And I is equal to I m by root 2 the rms value of the current right. And the diagram is shown here for the power voltage and current right. I just to give you a feeling that how actually this p this one you plot, this is the dash line, the dash portion shown, it is the power part right. So, just to give you a feeling that how it is.

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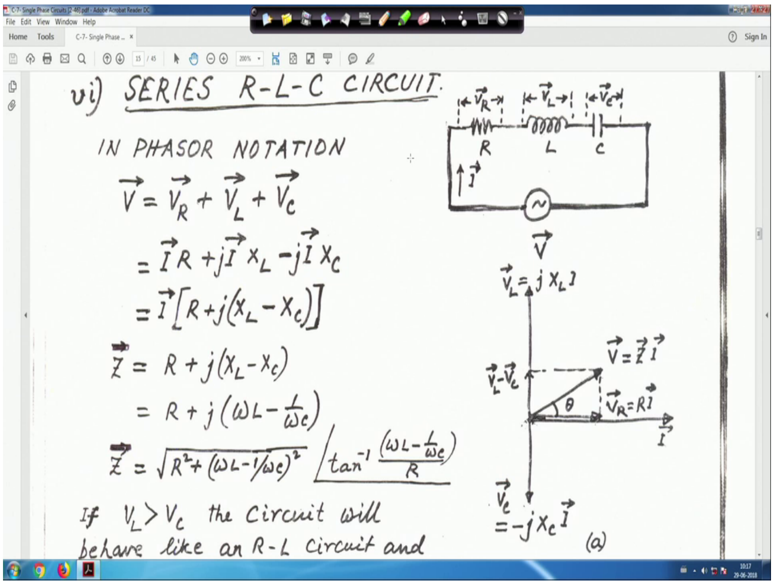

So, this is actually series R-L-C circuit series RC circuit. Now, we will see series R-L-C circuit right. So, now for series R-L-C circuit, we have three element three elements are in what you call are in series. So, it is voltage across a R is V R, voltage across L is V L, and voltage across this C. And you put instantaneous polarity plus, minus current flowing through this is I.

Now, one thing I will tell you that voltage and current, voltage and current this is I say, I told you in ac circuit phasor is a rotating vector. So, angle of the voltage this is first quadrant. This is second quadrant. This is third quadrant. This is fourth quadrant right. So, voltage or current right, the angle of the voltage and current it can be in anywhere, because phasor is rotating vector.

It may be in according to your circuit diagram network, angle everything right. Impedance what type of thing you have taken according to that the voltage or current phasor, it may be in first quadrant, it may be first or second or third or fourth. So, it may be in either all the your what you call all the four quadrant, it may be anywhere right, because it is a rotating vector, so that angle of the voltage and current.

But, if you look at the, if you look at the impedance, impedance when you write one or two places let me rectify you, one or two places while making the load some places it is written Z arrow. So, your impedance is not a your what you call is not a phasor quantity, because Z is equal to in general R plus j X you put right. Now, if you take this quadrant R is always positive, so R should be somewhere here on this axis.

Now, depending on the circuit is inductive type or capacitive type this, either it will be somewhere here X by if it is a inductive type, then it will be somewhere here right Z Z will be somewhere here, if it is capacitive type. So, it is your X will be negative. So, maybe it will be somewhere here, so this side. So, either in the first quadrant or in the fourth quadrant, but here it should not be there, because R is always positive. So, hence my conclusion is that this thing that Z actually is not it is a complex quantity, but it is not a phasor quantity right. Phasor is a rotating vector. So, angle can be anywhere for Z angle will be either first or in the fourth quadrant right.

So, next is that series R-L-C circuit. So, V is equal to V R plus V L plus V C, because this all these things all these things are given. So, V R is equal to I R. And V L is equal to it is not j I L X L, j I X L. And your V C is equal to minus j I X C everywhere it is shown this arrow phasor, but even it is not there also it is understandable, so that it is a phasor quantity ac quantity right.

So, if you take I common, so everywhere I will not take I arrow. So, this is nothing, this is your Z bar right. This is a simply complex number. So, I R plus j X L minus X C. Therefore, Z is equal to R plus j X L minus X C that means, it is R. X L is equal to L omega and X C is equal to 1 upon omega c. Therefore, its magnitude will be root over that is complex quantity. Magnitude will be root over R square plus omega L minus omega c square. And angle will be tan inverse L omega minus omega c upon R omega that is omega L minus 1 upon omega c by R, this will be the angle right.

Now, if it is given that if V L greater than V C, that is V L greater than V C or if X L greater than X C right, that means, circuit will behave like inductive circuit. If X L greater than X C means, that is your omega L minus omega c greater than 0, that means, it is positive that means, by the angle theta will be positive. That is why here it is V R is equal to R I, and this one is what you call this side is V L. This side my V L j X L I L. This is just for the sake of completeness, this is minus j X C I this side is negative side that is why minus, but when you will take the your what you call when you will take the difference V L minus V C, it will be actually j X L minus X C into I right.

So, if you do, so if you multiply this R plus I into j X L minus X C, so this is actually this, but at the time same time if it is V C is given minus j. Here it is V L, but do not take V L minus V C means; it will become plus do not do that. It is V L what you call minus V C is taken. So, here this side is negative side that is why we have taken, but when you take this difference, please do not do not do that at that time.

You make j into X L minus X C this is the phasor representation. So, this side no need to put your even if I do not put what you call the negative sign is does not matter, because this is negative side, but just to for the sake of your completeness. I have made this one right. So, this is your instantaneous polarity, and there will be resultant will be that is V L greater than V C. If X L minus X C is greater than 0, then naturally V L will be greater than V C, so that means, circuit behavior is like inductive type circuit, because theta is positive, because theta is equal to omega L minus 1 upon omega c by R.

So, omega L minus 1 upon omega c greater than 0 is positive; so that means, my theta will be positive; because theta this actually this angle actually theta, so, theta will be positive. So, phasor diagram will be like this right, so this is your resultant. This side because V L greater, because X L greater than X C means V L greater than V C. So, this is the resultant portion V L minus V C right. And this is theta, so that means, this circuit behave like inductive type.

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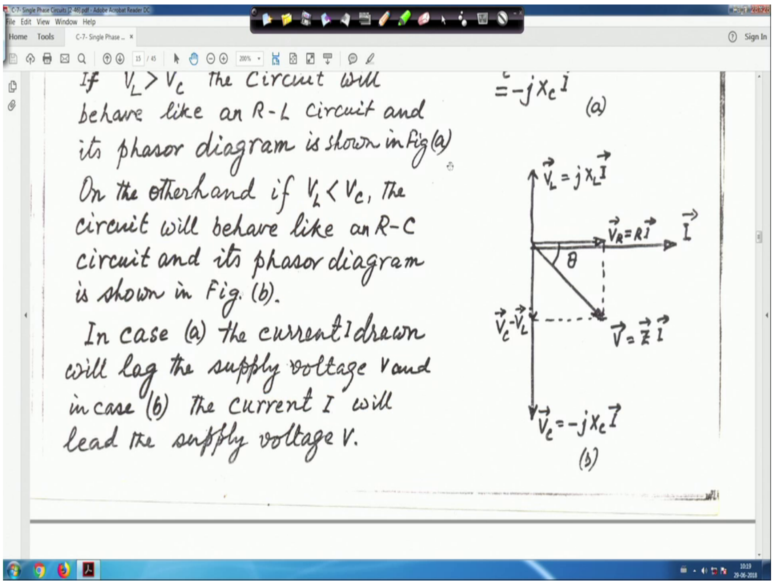

Similarly, similarly that if it is your V L less than V C right, that means your X L less than X C. Suppose if your V L less than V C, that means, my X L less than X C just opposite. So, in that case not giving you the same thing I explaining; in that case, theta will be negative. So, this is my I this is angle theta. So, this will be now V C minus V L. So, do not confuse with the sign this is to show this one right, but when you will take the difference of this do not ignore this one. It will be just j your V C minus V L will be what you call j your what you call it will be j X L minus X C into I right. So, so if you do not put here, does not matter.

So, in this case it will be V C minus V L, because V C greater than your V L. So, coming to this side, so that means, this is my theta. So, this time theta is negative that means what you call that circuit behavior is like a capacitive circuit. Because, in this case X L less than X C mean your L omega minus omega c, this is your less than 0. So, it is what you call that your what that means, my 1 upon omega c greater than L omega that means, my X C greater than X L right.

So, here it is written X L X L your X C greater than X L or X L less than X C. So, that is why circuit behavior is like your this thing. And this one should be your bar right it is not a phasor few places it is written there. So, Z is not a phasor quantity right, it is a complex quantity. So, anyway, so this is your when V L less than V C. So, circuit will be like your capacity what you call that your capacity circuit right.

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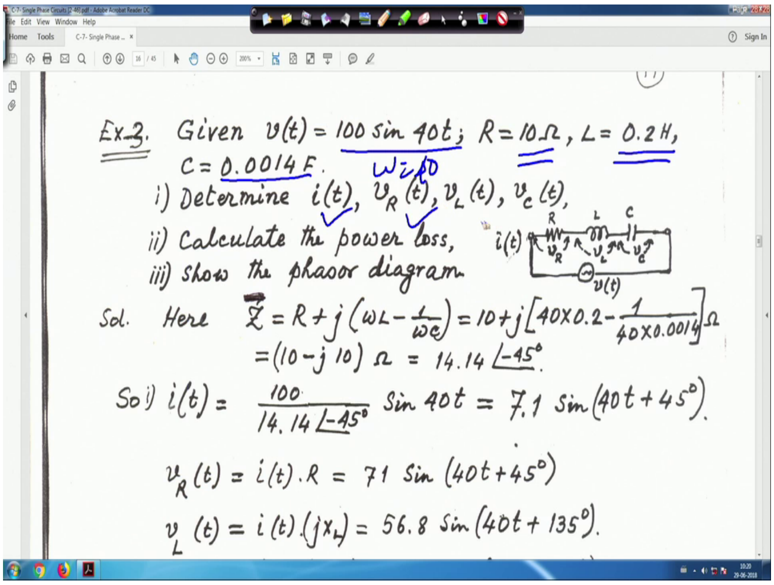

So, with this right, so with this your just your take this take this example that v t is equal to is given 100 sin 40 t that mean, 100 sin omega t that means, omega is equal to 40 radian per second right. And R is equal to 10 ohm, and L is equal to 0.2 henry. And capacitor is given C 0.0014 farad right. So, you have to find out I t, v R t, v L t, and v C t right. All these things you have to find out. Then you have to find out calculate the power loss, and you have to show the phasor diagram right.

So, the this is the circuit you have to take a instantaneous polarity plus, minus. This is given, and this is your v t already given 100 40 sin 100, sin 40 t. And this is your v R voltage across R. This is L that is voltage across L. And this is C voltage across capacitor C right. So, so you have to find out what you call that your i t, v R t, v L t v C t all these quantity.

Thank you very much, we will be back again.