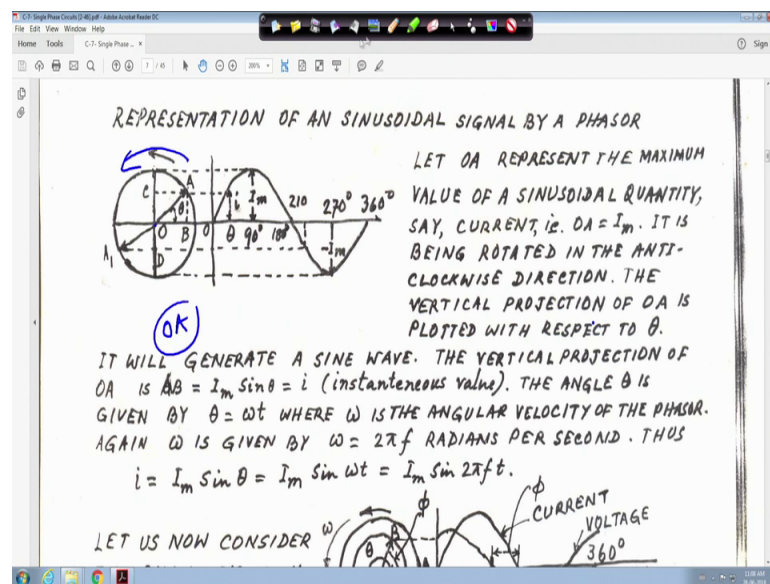


Fundamentals of Electrical Engineering
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Lecture - 39
Single Phase AC Circuits (Contd.)

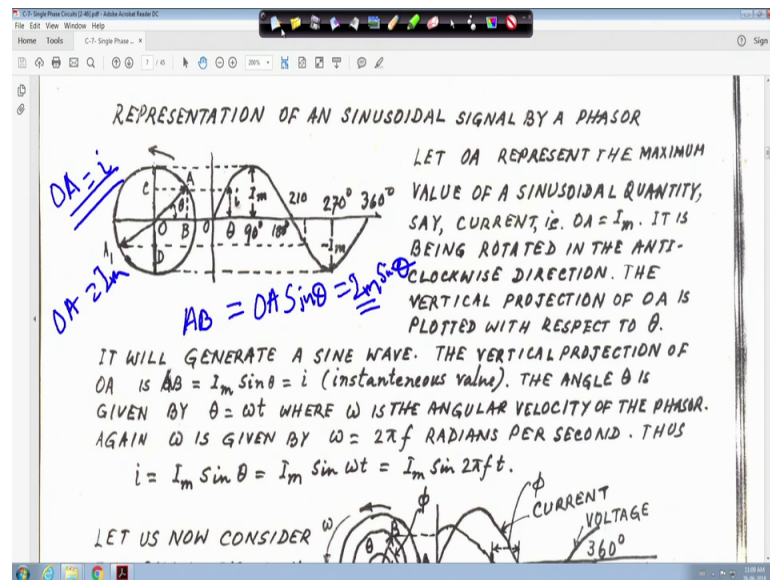
So, we are back again. So now we will see the representation of an what you call sinusoidal signal by phasor, right.

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So, here actually first thing is that you know our understanding should be clear, such that you know for solving numericals or for getting ideas things will be totally clear, right. So, let us see how we can make it. So, representation of an sinusoidal signal by phasor if you look into that all, right up is there will come to that, but just listen that Suppose, you have you some what you call that O A, O A is that radius suppose O A, suppose this is center of the circle O and this is a O A represent that your maximum value of a sinusoidal quantity. It may be current, it may be voltage, right. So, in the and it is at this position it is rotating in the your what you call in the anticlockwise direction, right. So, in that case if you take suppose that vertical projection so that is A B.

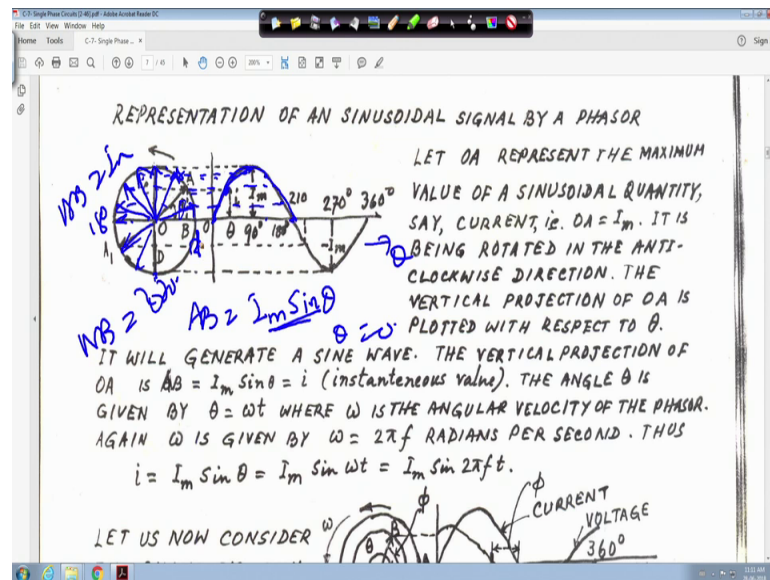
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Let me clear it, say A B this is a vertical projection A B is equal to your O A. It is your sin theta vertical projection; that means A B is equal to right.

So, O A is the maximum value of what you call of a sinusoidal quantities. For example, suppose O A is equal to suppose; suppose this signal O A is equal to i, right. This is the current signal write i, I will say it is maximum value say O A is equal to I m that is the maximum value of the current the peak value of the current right; that means, this one is equal to your I m sin theta, right. And this A B is the maximum value that is I m that is O A O a rather O a, right. And A B is equal to I m sin theta, right?

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Now, let me clear it; that means, my A B that is the vertical projection of this is equal to $I_m \sin \theta$.

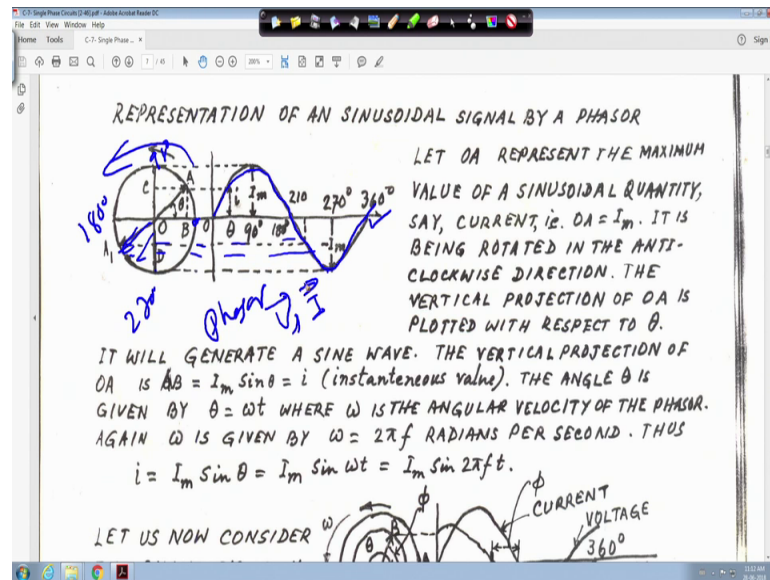
Sorry, now when theta is equal to 0 right when theta is equal to 0, then A B is equal to 0, then A B is equal to your what you call is equal 0. So, make it a projection when theta is equal to 0. So, this is the origin here it is starting. Now if you take another value of another value of theta say, this angle is theta; this angle is theta for which A B is equal to $I_m \sin \theta$. So, this way you take another point here. Similarly, for somewhere here, here you can take another point here for theta is increasing this is the theta, right.

Now this way when it will come to some this point say this O A when it will come this O A will come here. You will take another point here. Similarly, when it will come to this when theta is equal to 90 degree that is when theta is equal to 90 degree, then your A B is equal to I_m . So, take a projection here this is the maximum value. So, if you plot this, if you plot this it graph will come like this. Now when you are going beyond this that is theta you are what you call more than a 90 degree, then again you take the projection, if the point will come here theta is increasing this side this theta is increasing.

Similarly, when it will come here, right; so further when theta is more than 90 degree so, somewhere here; so this point will come here, similarly when it will come here, this point will come here. And finally, when it will come theta is 180 degree again it is 0. And if you take all these point and join it and plot it, it will be a sin curve, right. That is the

generation of the sin finally, that O A is rotating. So, it will come to some position here, some position here, this way it will move and it will come to the your what you call? That this is one 80 degree, then this is 270 degree, and when it will come to this one it is 360 degree, right.

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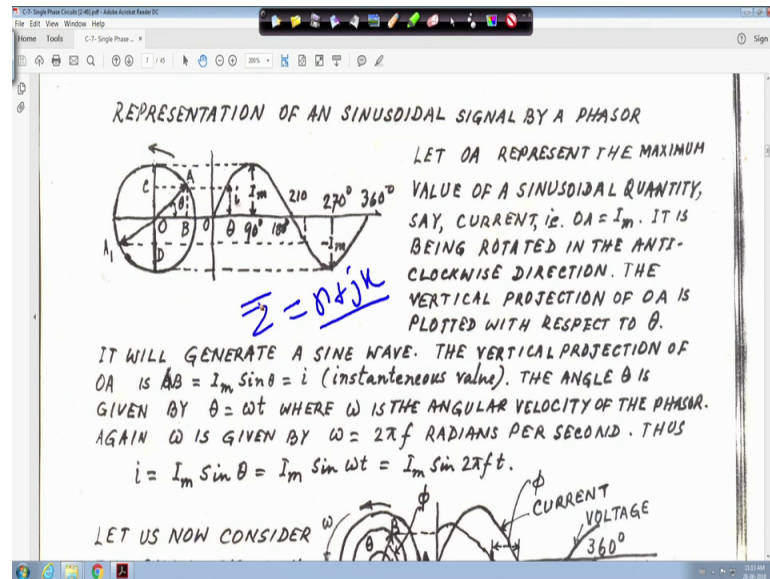
So, it is moving like this. So, let me clear it. So, this is my 90 degree, this is my one 80 degree, and this point is 270 degree. And finally, this point is 360 degree. So, when it is come to this side when your are coming to this side. So, again this value will become negative. So, for this point it will be somewhere here, from this point it will this way it will coming and finally, it up to 360 degree if you do, it will be a sinusoidal what you call generation signal generation, right. That is this that means, this way you take the maximum value of any voltage and current, and then you are moving in the clock anticlockwise direction.

And accordingly theta is increasing theta in between 0 to 360 degree, and if you plot like this and O A and your vertical projection of A B is equal to os sin theta; that is, A B is equal to I m sin theta, and if you take each point and plot it will give you sinusoidal your what you call sinusoidal signal, right. This how that sinusoidal representation of an sinusoidal signal by a phasor; that means, phasor actually phasor is a rotating vector, right. Because angular line between 0 and a 360 degree so, and it is moving your either

here we are taking anticlockwise either clockwise or anticlockwise whatsoever the phasor is a rotating vector right.

That means voltage when we write this way suppose V arrow, I arrow this is actually phasor quantity, right.

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But impedance, let me clear it, but impedance when you write z is equal to later we will see r plus jx, right. So, impedance I just put z bar, right, impedance is not a phasor quantity. We will come to that why it is not a phasor quantity, but voltage and current and phasor quantity, and phasor is a rotating vector, right. So, this is what? That sinusoidal generation and everything is given here omega is equal to 2 pi f. So, it is written I m sin 2 pi f t.

So, alright up here alright up is here for you, but whatever I told this expression in here, right. So, next is that your this one.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$. $\omega = 2\pi f$

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING $OB = V_m$.
 $u = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

Now, you we have to here also we have to understand certain things. Write up is here that is a different that is that you read later but just listen. Suppose I have a current and a voltage wave form, right. So, for this one what you call this one actually this one, this is a your what you call the voltage wave form.

Now this inner circle this one, right. This is the maximum value of say O B is the maximum value of the voltage magnitude at sometime if; that means, O B is equal to say for example, V_m this is the voltage z , right. Forget about scale another thing that is not an issue, thing is that we have to understand. And at that time the current this O A, let O A is equal to at that time current a position was this time, right. And this angle between this voltage and this current, it is phi angle is the phi, right. Now similar way so, I mean this phi this both are rotating in the your what you call in that your anticlockwise direction, right.

That is omega is given, right. So, omega is equal to 2 what you call 2 pi f into t omega sorry, omega is equal to 2 pi f. So, that that is your omega, omega is equal to 2 pi f, right. F is the frequency so; it is rotating in the anticlockwise direction. Now if you take the voltage wave from I mean and current waveform suppose current when current position O A current position was at A and voltage at V and this phi. And they are moving at your what you call in anticlockwise direction, and omega is given that your what you call that as frequency, right.

So, f is the frequency and ω is equal to $2\pi f$. So, in that case, as this is moving when; that means, angle between B and A whatever may be the position angle ϕ will remain constant. For example, suppose if B is somewhere here, and A is somewhere here. This angle ϕ always remain constant, because both are moving together it is a, right, and in the anticlockwise direction; so because the frequency is same. So, in that case what will happen, and if you make the projection of the voltage from this point, it is some point will be there when it is coming to this one, this is the your what you call this point, and when will come to more than 90 degree it is a peak value V_m finally, it will go and come to 0 and again it will move like this.

Similarly, when this your O A is the your what you call I_m we have taken. So, at this point, your this is the your what you call that θ is equal to 0, right. So, in that case what will happen? That your this point this point it will start, this point it will start, and in that way that \sin your what you call this current wave will be your what you call this sinusoidal one voltage also sinusoidal one, but at θ is equal to this time your t or θ is equal to 0, current waveform will start from there because position is here and voltage will start from here somewhere here, right. So this way, suppose this sinusoidal waveform voltage and current both are generated, but angle between these 2 are remain same.

For example, that just this thing here we have taken the 2 quantities on voltage and current, right. Represented by O B and O A everything is written here, here it is shown that the phasor O B is leading the current phasor diagram that what is the leading or that what does it mean? It mean is a leading or lagging, right. So, in that case what will happen? That if you take that v is equal to, first we have to understand leading and lagging and then will come to this. So, first let me clear it. Now question is that what is leading, and you have to see that out of this 2 wave form which one is reaching it is peak value. Forget about magnitude and another thing which one is reaching it is peak value earlier than the other one.

So, in that case this is the voltage wave form, it is reaching peak value earlier before the current. Because when it is reaching peak value current is here. So, first voltage actually reaching it is peak value before current because current is here. This is one thing or other way that which one is coming to the 0 first the voltage is coming to that what you call the 0 0 first compare to the current, right. So, 2 ways can be done if that out of this 2 wave

form in this case that voltage peak is your reaching first then the current peak; that means, voltage actually leading the current. Or other way the voltage actually coming to the 0 first, then the current; that means, voltage is what you call leading what you call leading the current and this angle phi.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS

$$i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t.$$

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .

$$v = V_m \sin (\theta + \phi) = V_m \sin (\omega t + \phi) = V_m \sin (2\pi f t + \phi).$$

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

Actually this is the angle phi, right. This is what between this voltage and current, this is angle pi or the peak differences; this angle this difference is phi, right. So, leading means you have a suppose voltage and current waveform. If voltage is reaching its peak before then the current; that means, voltage is leading the current right.

Or if voltage is reaching 0 before the current; that means, voltage is leading the your what you call current other way is, other way that current is your reaching peak later than the voltage; that means, current lags from the voltage. Or current reaching to 0 before what you call after your voltage becomes 0 or currents lags from the voltage. So, either this distance is phi or peak to peak this distance is phi. This is the concept of leading or your what you call leading or your what you call lagging concept, right.

So, in this case what will happen? That as voltage is leading by this angle what you call current of phi, right. So, if we take the let me clear, if we take, this is my current signal.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .
 $v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

That means my v is equal to, right $V_m \sin \omega t$, right. This is your this is your what you call this current signal $\sin \omega t$, right. And your theta is equal to ωt . So, $V_m \sin \sin \omega t$. Therefore, this is the your what you call that current signal, but in the case of voltage is leading by this angle phi sorry, this is sorry, sorry this is let me clear it.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .
 $v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

This is not voltage, this is sorry this is current, right. This current I_m is equal to I wrote by mistake voltage. So, current is equal to I is equal to $I_m \sin \omega t$. So, so this is the, this is that your wave form of the current, right. Now voltage case, voltage is leading this

current by phi so, voltage case we can write v is equal to $V_m \sin(\omega t + \phi)$, right plus phi, right, because theta is moving like this from this position to that position. So, it will be $\sin(\omega t + \phi)$, because voltage is leading the current. So, phi angle should be positive, right.

So, we can write V_m is equal to $\sin(\omega t + \phi)$.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY O.A. & O.B. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR O.B. IS LEADING THE CURRENT PHASOR O.A. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING O.B. = V_m .

$$v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$$

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

$i = I_m \sin(\omega t) \left(\frac{I_m}{\sqrt{2}} \right)$

That means, in phasor if you make it like this in phasor suppose we are writing I is equal to $I_m \sin \omega t$, right this way writing. So, that means, my rms value is actually I_m by root 2, this is the rms value of the current right; that means, when you write this your what you call? This is my rms value; that means, we can write let me clear it.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .
 $v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

$I = \frac{I_m}{\sqrt{2}} \angle 0^\circ$

We can write I is equal to say I_m by root 2, right. An angle 0 degree, because I is equal to your $I_m \sin \omega t$ no, no nothing is there with that. So, you can write this, whenever we write such thing we write I is equal to I_m by root 2 angle 0. And throughout this little bit initially I have made an arrow here. This will represent the phasor quantity is equal to we can write it is I angle 0 degree.

So, no arrow is here means this is the magnitude and this is your rms value; that means, when you write I is equal to I_m by root 2 it is rms value, right. We have seen before and it is angle is 0 degree. So, I is equal to say if we write I angle 0 sometimes we are writing small I or capital I it does not matter, this is your I right.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .

$v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

$v = \frac{V_m}{\sqrt{2}} \sin(\omega t + \phi) = V \sin(\omega t + \phi)$

$V = \frac{V_m}{\sqrt{2}}$

$\phi = \phi$

Now, in the case of voltage, voltage we are writing v is equal to $V_m \sin \omega t + \phi$, right. So, this one we can write in phasor notation say \vec{v} arrow ok. Arrow means different way it can be represented, but putting arrow \vec{v} arrow is equal to rms value is V_m by root 2, right. Then angle should be your what you call angle should be ϕ because it is leading by this angle ϕ . So, angle should be ϕ . So, that mean this one actually can be written as $V \angle \phi$. V is the rms value, but no arrow is here it means rms value magnitude. So, v is equal to V_m by root 2, the rms value, right. Whenever we will solve numericals unless and until it is specified we will take the rms value.

So, this is your actually $V \angle \phi$, right.

(Refer Slide Time: 15:27)

GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .
 $v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

Now question is, let me clear it that means my, I is equal to if it is I, right angle 0 degree. And my v is equal to my V angle phi, than if we plot the phasor diagram (Refer Time: 15:35) of vector diagram, but phasor is a rotating vector that is the only difference you have to see. So, if you draw this so, if we make it suppose this is my, I this is my reference line angle in 0 when this is my, I and this is V angle phi. So, it will be my v, right. And it will be my, I and this angle should be what you call this angle should be phi right; that means, v actually leading the current I by angle phi.

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GIVEN $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR.
 AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS
 $i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t$.

LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS LEADING THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIANS PER SECOND. THUS WE CAN WRITE TAKING OB = V_m .
 $v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$.

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

Or other way we can write if you take the v as the reference, let me clear it, otherwise this is my v and this is my, I right.

So, in this case it is also phi. So, same thing say if when you are drawing when we are making this one I and this is my V and this angle is phi, right. So, what we are doing bringing? Your just rotate it this one by angle phi. So, v will come to this and I will move. So, again I is lagging here also I lags V by phi phi degree by an angle phi or v leads I by phi. Same thing here also, v is leading I by phi or I lags from V by phi, right. So, this is the idea of what you call leading and lagging. So, that means, if you have 2 waveforms, you have to see which one is reaching it is first. Peak it reaching it is peak first, that quantity is leading the other quantity.

Or which one is reaching to 0 before the other one that quality leading or vice versa, right. So, this is what all, right up is here I hope you have understood this, right. So, things are very simple looking at the figure this is notes only little bit closely written for next 3, 4, 5 pages. After that I have made it this thing, but everything is everything is written here leading lagging whatever I say everything is written here, right. So, this is the concept for leading or lagging right.

(Refer Slide Time: 17:28)

$$v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi)$$

ϕ IS KNOWN AS THE PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS $v = V \angle \phi$

$$v = V_m \sin \theta = V_m \sin \omega t = V_m \sin 2\pi f t$$

$$i = I_m \sin(\theta - \phi) = I_m \sin(\omega t - \phi) = I_m \sin(2\pi f t - \phi)$$

AND CAN STATE THAT THE CURRENT IS LAGGING BEHIND THE VOLTAGE BY AN ANGLE ϕ .

THE PHASOR DIAGRAMS ARE DRAWN WITH THE RMS VALUES OF SIGNALS. THE PHASOR DIAGRAM CAN BE DRAWN AS EITHER IN (a) OR IN (b).

(a) V is the reference phasor along the positive x-axis. I lags V by an angle ϕ .

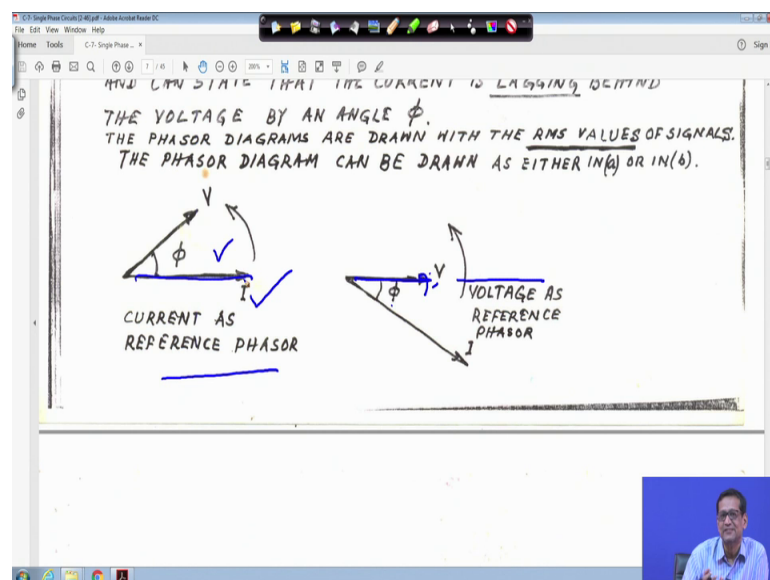
(b) V is the reference phasor along the positive x-axis. I leads V by an angle ϕ .

So, that is why that is why here we are making another thing that I mean whatever I told there in this diagram; that means, if we make that it was v is equal to V angle phi and I is equal to I angle 0, right.

So, this will v is leading your what you call leading your I or I lags from V , this one also you can write it, if we take v is equal to $V \sin \omega t$ reference then I will be your $I \sin(\omega t - \phi)$, right. This thing and these 2 things are same. Here also v if you put same thing it is $I \sin \omega t$ degree, I lagging from V by ϕ degree or v leads I by ϕ degree, here also if v take reference v is leading I by ϕ degree or I lags from v by ϕ . So, same way it can be written. So, that is why here it is taken $V \sin \omega t$, just for your understanding, in that case, current will be your $I \sin(\omega t - \phi)$ instead of plus ϕ , right

So, this is the idea of your what you call that leading or lagging concept.

(Refer Slide Time: 18:39)



So same thing here; the current as reference phasor I gave you all these thing, there I took small, but here it is taken capital does not matter, right. So, this is V , this is I and this is voltage as reference phasor that here same thing that v is leading I by ϕ or I lags from V by ϕ and here also v is leading ϕ or v or what you call I lags from V by ϕ angle. This is current as a reference phasor here is voltage that is (Refer Time: 19:02) it is because voltage here is taken as reference phasor.

And here current is taken as a reference phasor, but meaning is same when you will do the numerical also everything will remain same. There will be no change, right. So, this is the concept of lagging and leading of voltage or current, right and generation of sinusoidal waveform.

(Refer Slide Time: 19:25)

ADDITION AND SUBTRACTION OF PHASOR QUANTITIES:

$\vec{V}_A = \vec{V}_1 + \vec{V}_2$

PHASORS AS COMPLEX NUMBERS.

THERE ARE THREE EQUIVALENT NOTATIONS OF A PHASOR

POLAR FORM: $\vec{V} = V \angle \theta$

RECTANGULAR FORM: $\vec{V} = V \cos \theta + j V \sin \theta$

EXPONENTIAL FORM: $\vec{V} = V e^{j\theta}$

(NOTE: $e^{j\theta} = \cos \theta + j \sin \theta = 1 \angle \theta$)

THE OPERATOR j PRODUCES 90° COUNTER CLOCKWISE ROTATION

Next now, this is from what you call from your vector your analysis from your (Refer Time: 19:29) physics you know, you know all these things suppose your addition and subtraction of 2 phasor quantities, the way you do vector same way you do here also. Suppose this is my V_1 , I want to add this is my V_2 , it is the angle θ_1 and this is by V_2 , and angle it is given θ_2 . So, V_A is equal to $V_1 + V_2$ arrow is given to represent it is phasor.

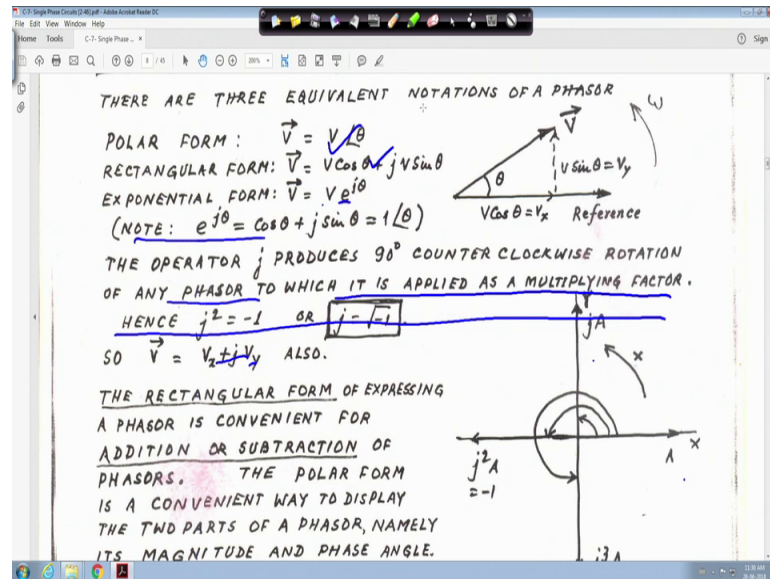
So, and your what you call the way you do in the your what you call that vector analysis in your (Refer Time: 19:59) physics same thing same thing, right. So, similarly and this line is a horizontal line is a reference, here also it is reference, here it is subtraction, right. So, this is what you call my resultant is same, this is V_A stands a stands for addition, and here V_S stands for your subtraction one arrow is missing, but anyway it is V_S , right. So, this is your V_1 and this is your V_2 , right.

So, if you want to find out what will be your V_S what will be your V_S . So, you can easily find out that V_S is equal to your V_1 minus what you call V_2 , right. So, in this case, 2 way, 1 way it is made if it is V_2 , this side will be minus V_2 , right? That means, now if you go for phasor sum or now you do vector sum. So, V_S will be is equal to your V_1 minus V_2 . In other way in other way, if you do this is V_2 , and this is also this one also you can take V_2 this parallel to this, right. Therefore, V_S equal to V_1 this direction

and arrow is here. So, it will be minus $V \sin \theta$ either this way or this way. The way we do same way we can do, right in vector same way we have to done.

But just cannot be algebraic sum, right. It will be phasor sum, we will see that later when we will take the numerical, right.

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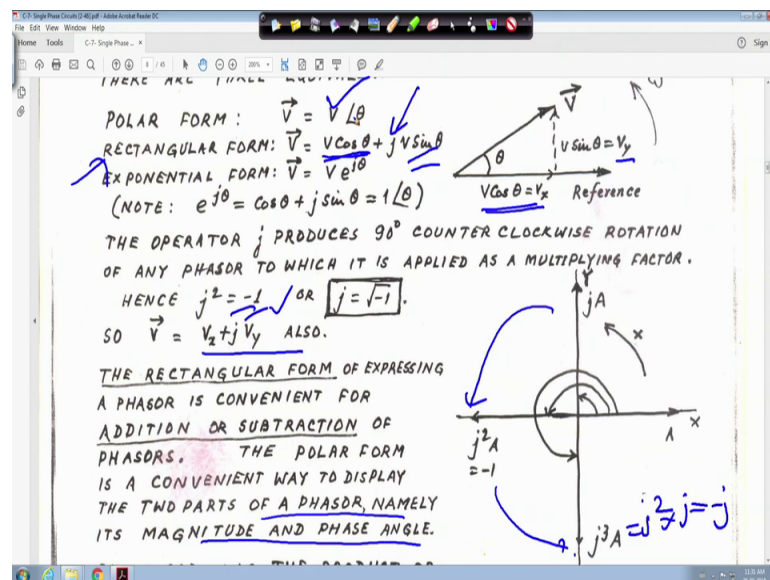
Now phasor as complex number; so there are 3 equivalent notation of a phasor, you know the complex number you have started in your high (Refer Time: 21:30) mathematics, right. So, one way one thing is that polar form, I mean first thing is the polar form if you write v is equal to $V \angle \theta$. Here V is the magnitude and θ is the angle, we put we can write the $V \angle \theta$ right.

Now, in rectangular form, if you write it will be $v \cos \theta + j v \sin \theta$, right. And, another thing exponential form that V is your arrow you have given, that $V e^{j\theta}$ to the power $j \theta$. V is the magnitude $e^{j\theta}$ is equal to $\cos \theta + j \sin \theta$, right. And it is written here $e^{j\theta} = \cos \theta + j \sin \theta$, then one angle θ that is angle θ basically. So now, another thing is that we will come to that later. So, another thing is that the operator j produces 90-degree counter clockwise rotation, right of any phasor to which it is applied as a multiplying factor.

Actually hence j square is equal to minus 1. So, operator j produces 90-degree counter clockwise rotation of any phasor to which it is applied as a multiplying factor that is; that means, j square is equal to minus 1. So, this is your x . This is real axis, this is imaginary axis, right. So, in that case if this is j this is we call it is imaginary this is j , when you come to this just hold on little bit let me move upward, right; so in this case, in this just hold on. So, in this case, this is j now when you are moving further it is j square, j square is equal to minus 1.

When you will come to this j cube, j cube is equal to minus j . Because this one actually j square into j .

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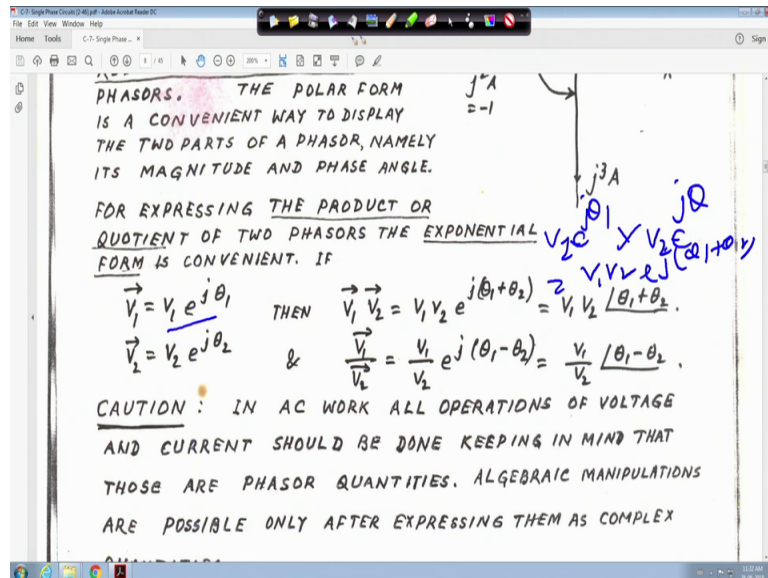


So, j square is equal to minus 1 is your complex quantity. So, that is minus j and when you will finally, you come to this point j to the power 4. So, it will be something like one, right. So, it is moving what you call your counter your what you call anticlockwise direction. So, this is this one, and this $v \cos \theta$ if you take this is x component is $v \cos \theta$. So, $v \cos \theta$ here is equal say v_x , and $v \sin \theta$ is this one. So, it is this one you take v_y is equal to $v \sin \theta$, and this is the resultant one v . So, basically v will be your v your $v \cos \theta$ plus $j v \sin \theta$, right.

So, I mean this is your what you call; that means, v is equal to every time arrow and arrow is there, I am not telling it is a understandable is equal to v_x plus $j v_y$ also, right. So, the rectangular form of expressing a phasor is convenient for addition or subtraction,

right. I mean when you make the rectangular form, I mean this form, right. This is a rectangular form from this form, because for that addition and subtraction will be helpful, right. That polar form; that means, this one polar form is convenient to a to display 2 parts of a phasor namely it is magnitude and phase angle, because this v is the magnitude an angle I your theta right, magnitude an angle.

(Refer Slide Time: 24:48)



So, let me clear it, right. And from your (Refer Time: 24:48) mathematics, right. You know the operation of complex number, right. So, here suppose V 1 is taken is equal to V 1 e to the power j theta 1 it is V 1 arrow V 2 arrow it is V 2 e to the power j theta 2, then if you multiply this it will be V 1 your I am writing here it if you multiply this it is V 1 e to the power j theta 1 into V 2 e to the power j theta 2, right. So, it actually it is V 1, V 2 and then e to the power j theta 1 plus theta 2, right.

So, that that is actually that is actually what is written here. This one can be written as V 1 angle theta 1 plus theta 2, right.

(Refer Slide Time: 25:35)

PHASORS. THE POLAR FORM IS A CONVENIENT WAY TO DISPLAY THE TWO PARTS OF A PHASOR, NAMELY ITS MAGNITUDE AND PHASE ANGLE.

FOR EXPRESSING THE PRODUCT OR QUOTIENT OF TWO PHASORS THE EXPONENTIAL FORM IS CONVENIENT. IF

$$\vec{V}_1 = V_1 e^{j\theta_1} \quad \text{THEN} \quad \vec{V}_1 \vec{V}_2 = V_1 V_2 e^{j(\theta_1 + \theta_2)} = V_1 V_2 \angle \theta_1 + \theta_2$$

$$\vec{V}_2 = V_2 e^{j\theta_2} \quad \& \quad \frac{\vec{V}_1}{\vec{V}_2} = \frac{V_1}{V_2} e^{j(\theta_1 - \theta_2)} = \frac{V_1}{V_2} \angle \theta_1 - \theta_2$$

CAUTION: IN AC WORK ALL OPERATIONS OF VOLTAGE AND CURRENT SHOULD BE DONE KEEPING IN MIND THAT THOSE ARE PHASOR QUANTITIES. ALGEBRAIC MANIPULATIONS ARE POSSIBLE ONLY AFTER EXPRESSING THEM AS COMPLEX

That means any angle theta can be written as e to the power j theta, right. So, here let me clear it. So, here angle theta 1 plus theta 2 is equal to e to the power j theta 1 plus theta 2, right. So, that is here this one is equal to is written as this one right; that means, let me clear it.

(Refer Slide Time: 25:58)

PHASORS. THE POLAR FORM IS A CONVENIENT WAY TO DISPLAY THE TWO PARTS OF A PHASOR, NAMELY ITS MAGNITUDE AND PHASE ANGLE.

FOR EXPRESSING THE PRODUCT OR QUOTIENT OF TWO PHASORS THE EXPONENTIAL FORM IS CONVENIENT. IF

$$\vec{V}_1 = V_1 e^{j\theta_1} \quad \text{THEN} \quad \vec{V}_1 \vec{V}_2 = V_1 V_2 e^{j(\theta_1 + \theta_2)} = V_1 V_2 \left[\cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2) \right] \angle \theta_1 + \theta_2$$

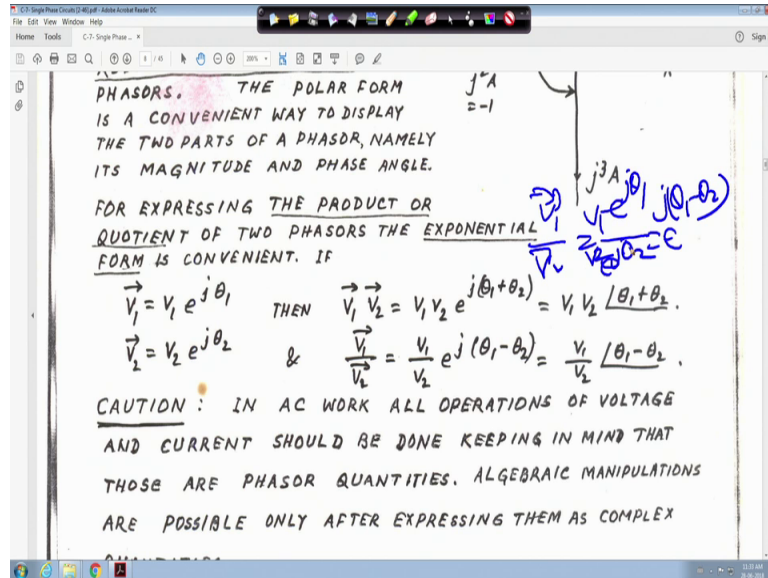
$$\vec{V}_2 = V_2 e^{j\theta_2} \quad \& \quad \frac{\vec{V}_1}{\vec{V}_2} = \frac{V_1}{V_2} e^{j(\theta_1 - \theta_2)} = \frac{V_1}{V_2} \angle \theta_1 - \theta_2$$

CAUTION: IN AC WORK ALL OPERATIONS OF VOLTAGE AND CURRENT SHOULD BE DONE KEEPING IN MIND THAT THOSE ARE PHASOR QUANTITIES. ALGEBRAIC MANIPULATIONS ARE POSSIBLE ONLY AFTER EXPRESSING THEM AS COMPLEX

That means this one is equal to V 1 V 2 in bracket I write cos theta one just for your understanding plus j sin theta 1 plus theta 2, right.

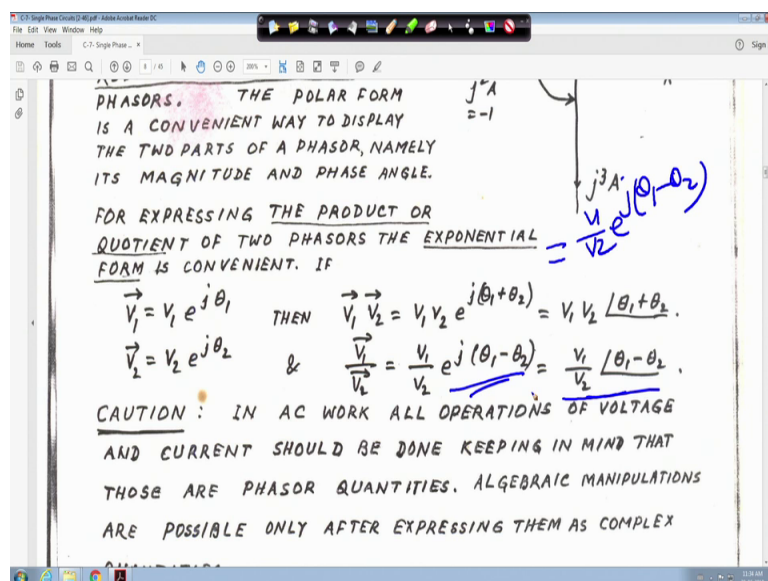
So, this one you can write. So, this is actually the way we write V 1 and V 2 are the magnitude, right. And theta one theta 2 are the angles of V 1 and V 2 respectively, right.

(Refer Slide Time: 26:25)



So, similarly let me clear it, similarly V 1 by V 2. So, here I am making it V 1 by V 2. That is e to the power j theta 1 by e to the power j theta 2. So, it is division so, e to the power j theta 1 minus theta 2, right. That is your and it is your V 1 here it is V 2, right. So, that means, it is just hold on that means this one.

(Refer Slide Time: 26:48)



And this one is equal to V_1 by V_2 e to the power j theta 1 minus theta 2. That is what is written here, right. What is written here? And again angle theta 1 minus theta 2; that means, this one it will be V_1 upon V_2 then in bracket cosine of theta 1 minus theta 2 plus j into sin of theta 1 minus theta 2. So, let me clear it.

(Refer Slide Time: 27:16)

The screenshot shows a presentation slide with the following content:

$$\vec{V}_1 = V_1 e^{j\theta_1} \quad \text{THEN} \quad \vec{V}_1 \vec{V}_2 = V_1 V_2 e^{j(\theta_1 - \theta_2)} = V_1 V_2 \angle \theta_1 - \theta_2$$

$$\vec{V}_2 = V_2 e^{j\theta_2} \quad \& \quad \frac{\vec{V}_1}{\vec{V}_2} = \frac{V_1}{V_2} e^{j(\theta_1 - \theta_2)} = \frac{V_1}{V_2} \angle \theta_1 - \theta_2$$

CAUTION: IN AC WORK ALL OPERATIONS OF VOLTAGE AND CURRENT SHOULD BE DONE KEEPING IN MIND THAT THOSE ARE PHASOR QUANTITIES. ALGEBRAIC MANIPULATIONS ARE POSSIBLE ONLY AFTER EXPRESSING THEM AS COMPLEX QUANTITIES.

So now some caution I have written some caution is written here. So, in AC, right work all operation of voltage and current should be done keeping in mind that those are phasor quantities. So, algebraic manipulations are possible only after expressing them as complex quantities right.

So, when you will do the calculation be careful about that so, that is all. And next is that for example, that add the following current as wave as phasor, right.

(Refer Slide Time: 27:43)

Ex-2 Add the following currents as waves and as phasors:

$$i_1 = 5 \sin \omega t ; i_2 = 10 \sin(\omega t + 60^\circ)$$

As Waves: $i = i_1 + i_2 = 5 \sin \omega t + 10 \sin(\omega t + 60^\circ)$

$$= 5 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ$$

$$= 10 \sin \omega t + 8.66 \cos \omega t. \quad \sqrt{10^2 + 8.66^2} = 13.23$$

$$= \left[\frac{10}{13.23} \sin \omega t + \frac{8.66}{13.23} \cos \omega t \right] \times 13.23$$

$$= 13.23 [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$$

$$= 13.23 \sin(\omega t + \alpha) = 13.23 \sin(\omega t + 40.9^\circ)$$

As PHASORS: $\Sigma \alpha = \frac{1}{\sqrt{2}} (5 + 10 \cos 60^\circ) = \frac{1}{\sqrt{2}} 10$

Diagram: A right-angled triangle with hypotenuse 13.23, side 10, and side 8.66. The angle α is opposite the side 8.66. Below the triangle, it is noted that $\alpha = 40.9^\circ$.

Small video inset of a presenter in the bottom right corner.

So, in this case i is equal to you have taken $5 \sin \omega t$. And i_2 is equal to $10 \sin \omega t + 60^\circ$. Now if you add ok, right $i_1 + i_2$ it will be $5 \sin \omega t + 10 \sin \omega t + 60^\circ$. So, $5 \sin \omega t$ is there it is $10 \sin \omega t$ your $\sin \alpha \cos \nu \cos \alpha \sin \nu$ you are explained so, it will be this term. So, after simplification you will get it is $10 \sin \omega t + 8.66 \cos \omega t$.

Now, you what you will do? This is this actually this one this 10 and 8.6 you find out $\sqrt{10^2 + 8.66^2}$, right that will give 13.23 say, right. So, that then divide this what you call this quantity by 13.23 and multiply.

(Refer Slide Time: 28:40)

As Waves: $l = l_1 + l_2 = 5 \sin \omega t + 10 \sin(\omega t + 60^\circ)$

$= 5 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ$

$= 10 \sin \omega t + 8.66 \cos \omega t$

$= \left[\frac{10}{13.23} \sin \omega t + \frac{8.66}{13.23} \cos \omega t \right] \times 13.23$

$= 13.23 [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$

$= 13.23 \sin(\omega t + \alpha) = 13.23 \sin(\omega t + 40.9^\circ)$

As PHASORS:

$\Sigma x = \frac{1}{\sqrt{2}}(5 + 10 \cos 60^\circ) = \frac{1}{\sqrt{2}}10$

$\Sigma y = \frac{1}{\sqrt{2}}(0 + 10 \sin 60^\circ) = \frac{1}{\sqrt{2}}8.66$

$SUM = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \frac{1}{\sqrt{2}}\sqrt{10^2 + (8.66)^2} = 13.23$

$\alpha = \tan^{-1} \frac{\Sigma y}{\Sigma x} = \tan^{-1} \frac{8.66}{10} = 40.9^\circ$

So, if you do so, then this one, if you do so you 10 by 13.23 sin omega t plus 8.66 13.23 cos omega t in to 13.23, right. So, that means, this one is 10 and this one is 8.6. So, this is 13.2 10 this is a alpha; that means, cos alpha is equal to assume that your what you call this 10 by 13.23.

That is why it is written cos alpha. So, I am writing for you cos alpha is equal to 10 by 13.23, right. And sin alpha here I am writing is equal to 8.66 by 13.23. So, that is why here this one is replaced by cos alpha and this one is replaced by sin alpha, right. And this is multiplied by this 13.23. So, it is sin a cos b plus cos a sin b. So, that is 13.23 sin omega t plus alpha, that is 13.23 sin it is 40.9 degree, because if cos alpha is equal to 10 by 13.23 alpha will be 40.9 degree. So, let me clear it. So, this is alpha is equal to 40.9 degree, this is one way.

Now, this is the peak value, this is the peak value say, right. If this is the peak value just let me clear it. So, now this is the peak value. Now this one if you think that your rms value this is the peak value.

(Refer Slide Time: 30:06)

Ex-2 Add the following currents as waves and as phasors:

$$i_1 = 5 \sin \omega t; \quad i_2 = 10 \sin(\omega t + 60^\circ)$$

As Waves: $i = i_1 + i_2 = 5 \sin \omega t + 10 \sin(\omega t + 60^\circ)$

$$= 5 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ$$

$$= 10 \sin \omega t + 8.66 \cos \omega t$$

$$= \left[\frac{10}{13.23} \sin \omega t + \frac{8.66}{13.23} \cos \omega t \right] \times 13.23$$

$$= 13.23 [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$$

$$= 13.23 \sin(\omega t + \alpha) = 13.23 \sin(\omega t + 40.9^\circ)$$

$$\alpha = 40.9^\circ$$

AS PHASORS: $\sum \alpha = \frac{1}{2}(5 + 10 \cos 60^\circ) = \frac{1}{2}10$

A right-angled triangle is shown with a hypotenuse of 13.23, a base of 10, and a height of 8.66. The angle α is at the bottom-left corner.

So, when you do it this thing in rms value. So i_1 , I am writing for your understanding i_1 rms should be is equal to 5 by root 2 ampere, right. It rms value similarly for this one you do that i_2 should be is equal to it is rms value, for your 10 by root 2 only magnitude you are putting, this is the ampere, right. So, it is 10 by root 2 ampere 5 by root 2 ampere and 10 by root 2 ampere.

Now, in this case this $\sin \omega t$ means there is no angle associated with that basically it is ωt plus 0 degree right; that means, I can write, let me clear it, that means this one I can write, say i_1 is equal to when you will put rms value 5 by root 2 angle 0 degree, right.

(Refer Slide Time: 30:40)

Ex-2 Add the following currents as waves and as phasors:

$$i_1 = 5 \sin \omega t; \quad i_2 = 10 \sin(\omega t + 60^\circ)$$

As Waves: $i = i_1 + i_2 = 5 \sin \omega t + 10 \sin(\omega t + 60^\circ)$

$$= 5 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ$$

$$= 10 \sin \omega t + 8.66 \cos \omega t$$

$$= \left[\frac{10}{13.23} \sin \omega t + \frac{8.66}{13.23} \cos \omega t \right] \times 13.23$$

$$= 13.23 [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$$

$$= 13.23 \sin(\omega t + \alpha) = 13.23 \sin(\omega t + 40.9^\circ)$$

As PHASORS: $\Sigma x = \frac{1}{\sqrt{2}}(5 + 10 \cos 60^\circ) = \frac{1}{\sqrt{2}}10$

And similarly here angle 60 degree is attached; that means, I can write i_2 is equal to 10 by root 2 angle 60 degree, because 60 degree is there right. That means, this i_2 actually leading i_1 by 60 degree because angle is positive 60 degree right; that means so, this is your how when we will solve the unless and until is stated.

If anything term attached with this sin term; that means, this is a peak value, you have to divided it by root 2 to take the rms value, because on all our calculations will be based on later we will see only on rms value, right.

(Refer Slide Time: 31:39)

$$= \left[\frac{13.23}{13.23} \sin \omega t + \frac{13.23}{13.23} \cos \omega t \right] \times 13.23$$

$$= 13.23 [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$$

$$= 13.23 \sin(\omega t + \alpha) = 13.23 \sin(\omega t + 40.9^\circ)$$

As PHASORS: $\Sigma x = \frac{1}{\sqrt{2}}(5 + 10 \cos 60^\circ) = \frac{1}{\sqrt{2}}10$

$$\Sigma y = \frac{1}{\sqrt{2}}(0 + 10 \sin 60^\circ) = \frac{1}{\sqrt{2}}8.66$$

$$\text{SUM} = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \frac{1}{\sqrt{2}} \sqrt{10^2 + (8.66)^2} = 13.23 \frac{1}{\sqrt{2}}$$

$$\alpha = \tan^{-1} \frac{\Sigma y}{\Sigma x} = \tan^{-1} \frac{8.66}{10} = 40.9^\circ$$

So $i = 13.23 \sin(\omega t + 40.9^\circ)$

WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS

Ans: $8.66 \sin(\omega t - 90^\circ)$

So, in this case, that your this one your this diagram, that is why it is at your i this 5 by root 2 angle 0, that is why it is the reference one, another one I told you 10 by root 2, this one, this is 10 by root 2 and this 60 degree this 60 degree, right. So, another we are getting is. So, it is taken as rms value, and this is resultant one, it is 13.23 this is the 13.23 by root 2.

Because here also it is 13-point root 3 and it is rms value it is rms value actually 13.23 by root 2, right. This is the magnitude, and it is angle is 40.9 degree. So, this angel is 40.9 degree. So, this resultant current actually leading this 1 by your 40.9 degree, and I told you i 2 is leading this angle by your this thing what you call this i 1 by 60 degree, right. This is actually your phasor diagram, this way it is a phasor diagram for this one. Now if you take a projection on the x axis, right. So, it will be this is reference so, 5 by root 2.

Actually everywhere root 2 will come that is why this root 2 is taken outside, right. Plus, this 10 by root 2 root 2 is outside and cosine of your 60-degree cosine of 60 degree. This one and this one this is the x for the (Refer Time: 32:45) projection on x axis, that is 10 by root 2, if you do it. Similarly, for y axis projection it will be 1 by root 2. So, it is it is reference thing. So, it is a projectional y axis is nothing but 0, plus it is 10 sin 60 because this angle is y axis projection is 10 sin 60 that is 8.66 by root 2.

Now, sum if you take resultant will be root over sigma x square plus root over sigma y square. So, that is 1 upon root 2 it will be root over 10 square plus 8.66 square, it will become 13.23 by root 2. Whatever it is written here 13-point root 3 by 2, 2 and alpha will be tan inverse your what you call tan inverse your sigma y upon sigma x. That is tan inverse 8.66 by root 2 by 10 by root 2, right. And that is equal to 40.9 degree.

So, whatever we have got here, right. So, I is equal to similar way the 13.23 sin this is the maximum value 13.23 sin omega t plus 40.9 degree. This is phasor diagram is drawn, that is why rms value is taken, right. For phasor diagram you should take rms value not the maximum value both ways it is solved.

Thank you very much, we will be back again.