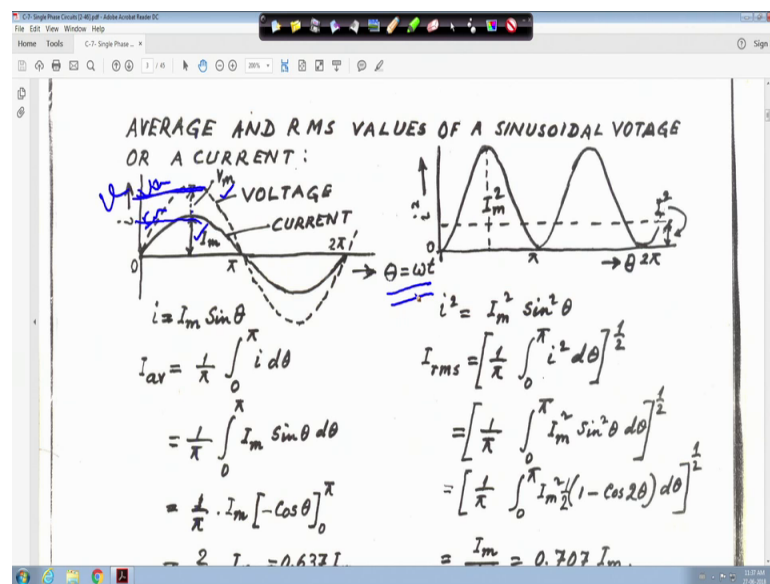


Fundamentals of Electrical Engineering
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 38
Single Phase AC Circuits (Contd.)

So now, average and RMS values of a sinusoidal voltage or a current right. So, let us first, go through the average value.

(Refer Slide Time: 00:24)



So, in this is I this is V also right. I forgot to put it here. It is V. Also, because this thick line this continuous line, it is basically, it is that current that current plot say, it is something like this and say this dash line. It is the voltage and the peak value for this one is your I m it is I m and peak value for this voltages is V m right. So, this is the peak value of the voltage. So, this is V m and this is your I m. Here it is marked and it is a V m. So, this is continuous line is current and your dash line is the voltage right.

And this is a sinusoidal waveform only sinusoidal waveform and theta is equal to omega t right. So, i is equal to expression is I m sin theta right. Because and this is 0 and we will make the average value over a half cycle. So, this can be written as your I average is equal to is your 1 to pi because, divide this by pi and 0 to pi i d theta right. So, it is a because, we will make only for this portion this portion over a half cycle. So, limit should be 0 to pi right and divided by the base that is pi, that is 1 upon pi i d theta right.

(Refer Slide Time: 02:00)

The slide contains the following handwritten content:

Graph: $i = I_m \sin \theta$, $\theta = \omega t$, $\theta \rightarrow \omega t$

Average Current Derivation:

$$I_{av} = \frac{1}{\pi} \int_0^{\pi} i \, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{1}{\pi} \cdot I_m [-\cos \theta]_0^{\pi}$$

$$= \frac{2}{\pi} I_m = 0.637 I_m$$

RMS Current Derivation:

$$I_{rms} = \left[\frac{1}{\pi} \int_0^{\pi} i^2 \, d\theta \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \theta \, d\theta \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi} \int_0^{\pi} I_m^2 \frac{(1 - \cos 2\theta)}{2} \, d\theta \right]^{\frac{1}{2}}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, $V_{av} = 0.637 V_m$, $V_{r.m.s.} = 0.707 V_m$.

Average Value of a sinusoidal current or voltage
 $= 0.637 \times \text{maximum value}$

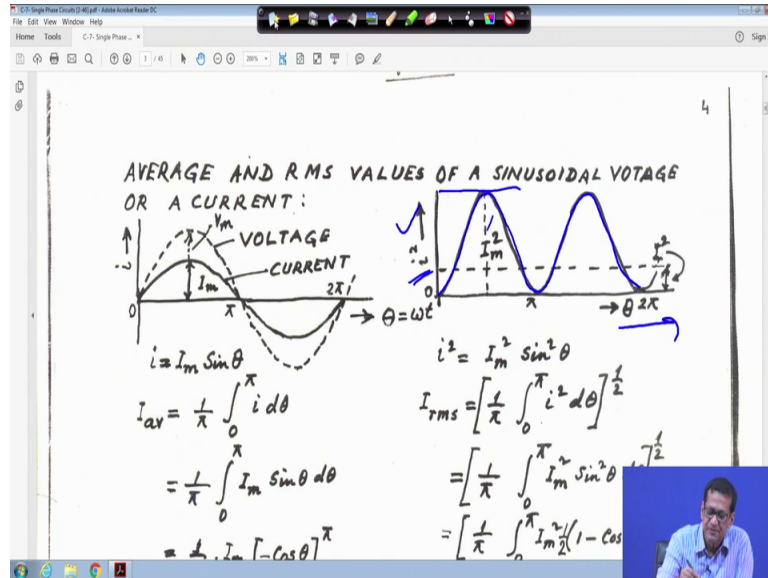
So, in this case, this side will come to that, this side is for your what you call for RMS value. So, it is basically i is equal to $I_m \sin \theta$. So, $\frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta$. So, if you integrate it, it will be $\frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta$. So, let me clear it. So, in this case, it is coming basically, it comes $\frac{2}{\pi}$ upon π after simplification into I_m is equal to $0.637 I_m$ right.

So, this is your what you call that your average value; that means, that means my I average this same thing, I average is this much $\frac{2}{\pi}$ upon π I_m is equal to $0.637 I_m$. Similarly, you will get V average. Also, because same way just I am writing one line. Same way, you can make say V average right. Say, V average is equal to $\frac{1}{\pi} \int_0^{\pi} V_m \sin \theta \, d\theta$ same thing right; same way, you integrate. And similarly, you will get V average is equal to same 0.637 into V_m same you will get because both are sinusoidal waveform only right. So, V_m V average will be 0.637 into V_m right.

V_m is the peak value of the voltage. Similarly, I_m is the peak value of the current or maximum value of the current and here it is maximum value of the voltage. This is your average value. Now, let me clear it. Now, when you come to your RMS value. This is suppose, only in the only in this case i square plot is given right. This is I_m and this is your i square i is equal to $I_m \sin \theta$. So, i square will be $I_m^2 \sin^2 \theta$ right. So, this is your I_m this is your i square plot this side is i square. It is marked

here. It is i square right and this peak is your what you call I_m square and if you plot it, it will be like this.

(Refer Slide Time: 03:34)



This area and this area, it is same it is symmetrical and this is π this is 2π . This side is θ right. So, that means, i square is equal to I_m square $\sin^2 \theta$. So, let me clear it. So, here I_{rms} is equal to it is because, we have seen know this under root square we have to take. So, it is 1 upon π 0 to π i square $d\theta$ to the power half because it is square root. So, to the power half right. So, it is your whole thing to the power half 1 upon π 0 to π i square $d\theta$.

Here is 1 upon π 0 to π i $d\theta$ for average value for RMS value 1 upon π 0 to π . Instead of i , it will be i square $d\theta$ to power half right, square root. Now, this will be your 1 upon π then, your 0 to π i is equal to I_m square $\sin^2 \theta$ right. So, it is i is equal to $I_m \sin \theta$. So, i square is equal to I_m square $\sin^2 \theta$ right is equal to your 1 upon π bracket 1 upon π 0 to π I_m square by 2 $1 - \cos 2\theta$ $d\theta$. Because, $\cos 2\theta$ is equal to $1 - 2 \sin^2 \theta$. Therefore, $\sin^2 \theta$ is equal to $1 - \cos 2\theta$ by 2 .

So, that is it made it here and you integrate and to the power half everywhere to the power half square root. If you integrate and simplify, it will simply become I_m by root 2 that is $0.707 I_m$ right. So, that is your I_{rms} we call root mean square value, I_{rms} is equal to I_m by root 2 right. So, let me clear it. Similarly, for the voltage V_{rms} also

0.707 V m the why I told you V average this thing. Here also, instead of just your I m square sin square theta, you make it V m square sin square theta d theta, V is equal to V m sin theta right. So, in that case, V rms also will be your 0.707 V m right. So, therefore, let me clear it.

(Refer Slide Time: 05:56)

$$= \frac{2}{\pi} I_m = 0.637 I_m \quad = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, $V_{av} = 0.637 V_m$ $V_{r.m.s.} = 0.707 V_m$

Average Value of a sinusoidal current or voltage
 $= 0.637 \times \text{maximum value}$

R.M.S. value of a sinusoidal current or voltage
 $= 0.707 \times \text{maximum value}$

Form Factor of a sine wave = $\frac{\text{R.M.S. value}}{\text{Average value}}$
 $= \frac{0.707 \times \text{maximum value}}{0.637 \times \text{maximum value}} = 1.11$

Peak or Crest Fact of a sine wave = $\frac{\text{Maximum Value}}{\text{R.M.S. value}}$

Therefore, the average value of a sinusoidal current or voltage will be 6-point sorry 0.637 into maximum value right. Average value of the sinusoidal current or voltage both are multiplied by 0.637, right. Similarly, the rms value of a sinusoidal current and voltage both are multiplied by 0.707 into maximum value. This is the maximum value; this the maximum value right.

Now, we define 1 or 2 things form factor of a sine wave right, form factor of a sine wave rms value by average value right. So, rms value is equal to point this rms value is equal to 0.707 into maximum value and the average value is equal to 0.637 into maximum value that equal to 1.11. So, for sinusoidal waveform the form factor is 1.11 right. You have you to keep it in your mind right sinusoidal wave form. Now, let me clear it. Similarly, the peak or crest factor right.

(Refer Slide Time: 06:56)

R.M.S. value of a sinusoidal current or voltage
 $= 0.707 \times \text{maximum value.}$

Form Factor of a sine wave $= \frac{\text{R.M.S. value}}{\text{Average value}}$
 $= \frac{0.707 \times \text{maximum value}}{0.637 \times \text{maximum value}} = \underline{\underline{1.11.}}$

Peak or Crest Factor of a sine wave $= \frac{\text{Maximum value}}{\text{RMS value}}$
 $= \frac{\text{maximum value}}{0.707 \times \text{maximum value}} = \underline{\underline{1.414}} = \underline{\underline{\sqrt{2}}}$

NOTE: RMS VALUE IS ALWAYS GREATER THAN AVERAGE EXCEPT FOR A RECTANGULAR WAVE IN WHICH CASE THE HEATING EFFECT REMAINS CONSTANT SO THAT THE AVERAGE & THE RMS VALUES ARE SAME.

This one it is crest factor right of a sine wave maximum value by RMS value. So, maximum value and RMS the maximum value keep it as maximum value and RMS value is equal to 0.707 into maximum value, that is actually 1.414 right. This is called peak factor; that means, later we will see that any AC voltage. Suppose, if it is 12 to 20 volt, that is basically RMS value unless and until it is not mentioned and if it say 224 AC, you have to assume this is RMS value. That means, it is peak value will be multiplied by this factor this is actually root 2. You have to multiply by this factor right.

So, that that is the idea that in AC anything we calculation anything we do that is on RMS value, but if it is ask the peak value, it will be multiplied by this your what you call root 2 right. So, that is your what you call peak factor. So, let me clear it.

(Refer Slide Time: 08:00)

The screenshot shows a whiteboard with the following content:

$$= \frac{0.707 \times \text{maximum value}}{0.637 \times \text{maximum value}} = 1.11.$$

Peak or Crest Factor of a sine wave = $\frac{\text{Maximum value}}{\text{RMS value}}$

$$= \frac{\text{maximum value}}{0.707 \times \text{maximum value}} = 1.414$$

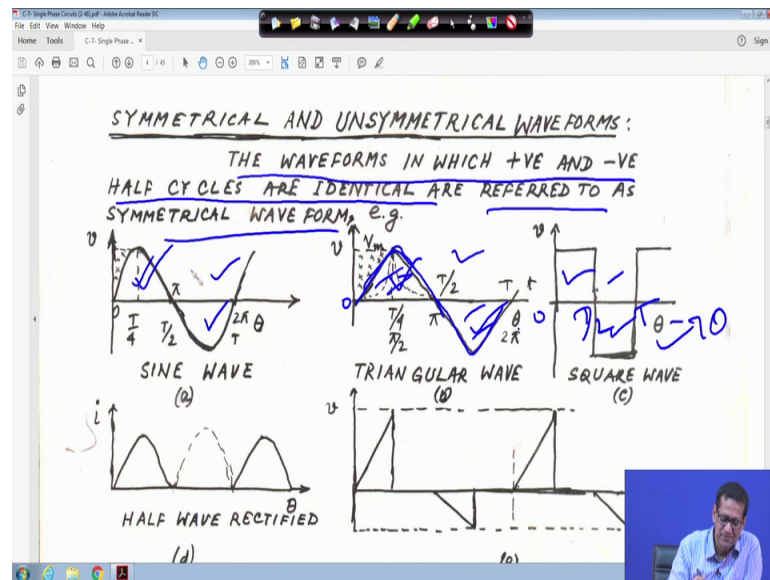
NOTE: RMS VALUE IS ALWAYS GREATER THAN AVERAGE EXCEPT FOR A RECTANGULAR WAVE, IN WHICH CASE THE HEATING EFFECT REMAINS CONSTANT SO THAT THE AVERAGE & THE RMS VALUES ARE SAME.

Therefore, the RMS value is always greater than this RMS value is always greater than average value for a your average value except for a rectangular wave. Particularly, for your square wave because different wave form you can generate right. In which case the heating effect remains constant.

So, that the average and the RMS values are the same right. So, RMS value is always greater than average except for a rectangular wave form right, particularly square wave. So, anyway so, that means, that that details little bit we will see on this right that how it is now symmetrical and unsymmetrical.

So, form factor case factor average value RMS value, I think it is understandable to you right. Anything is mentioned in the numerical unless and until it is specified. It is RMS value only if it is mentioned max maximum value peak value. You have to divided it by root 2 to get that your RMS value right. So now, next is that symmetrical and unsymmetrical wave forms.

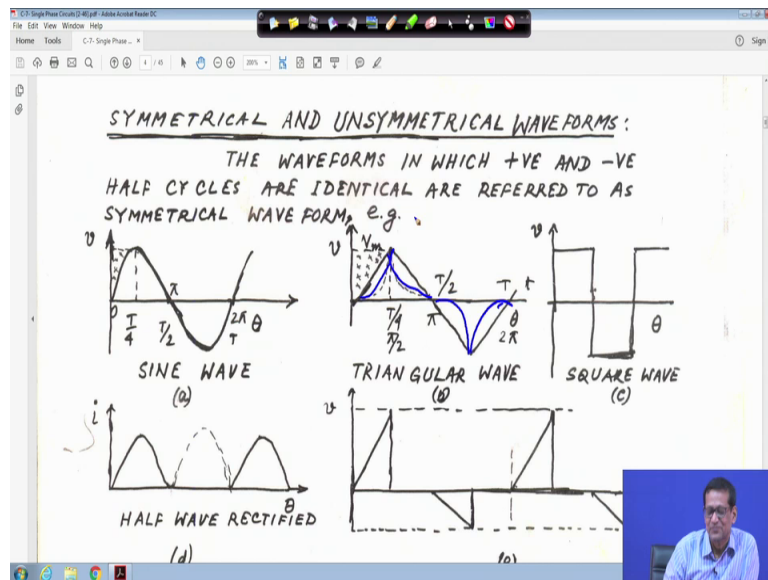
(Refer Slide Time: 08:58)



So, the wave forms the wave forms in which plus and minus half cycles are identical are referred to as the symmetrical waveform like this is sinusoidal one is plus and minus right your half cycles are identical. So, it is a symmetrical waveform this is a triangular wave form this is a triangular wave form. It is symmetrical because this side and this side is same look. This is this is 0 this is your what you call T by 4 T by 2 T. So, it is pi by 2 say if it is a pi by 2, it is pi. So, it is 3 pi by 4 and it is 2 pi right.

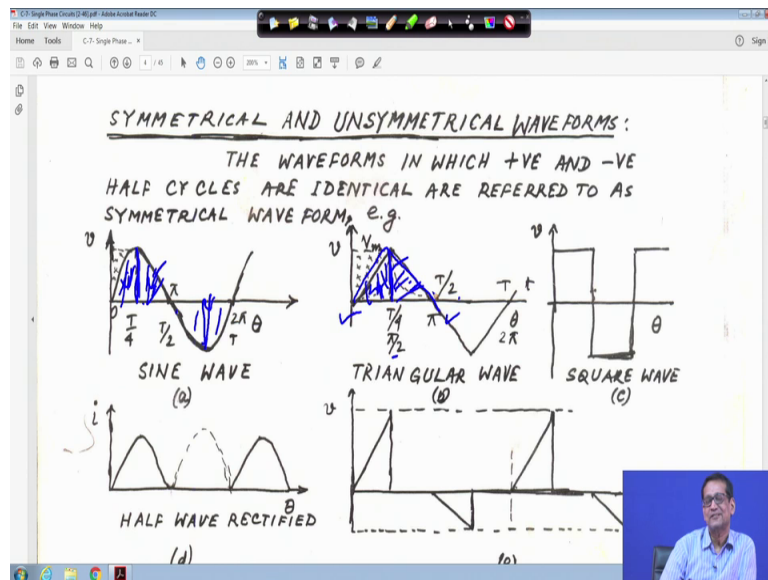
So, this wave this side is symmetrical and this is also symmetrical. So, this is symmetrical wave form it is triangular. Similarly, this square wave form. So, this one this one is symmetrical if it is 0, if it is T by 2, it is T. It is symmetrical and this is your theta it is theta right. So, this is say this 3, this 3 wave form are symmetrical wave form; that means, that plus and minus half cycles are identical are referred to as the symmetrical wave form right. Identical means, this side positive area negative area both will be same right. So and let me clear it.

(Refer Slide Time: 10:06)



Similarly, you may have other type of wave form. Suppose, if it is goes like this, if it is goes like this, if it is goes like this it is goes like this right some exponential thing this is also symmetrical right. So, anyway, that is why 1 dash line is shown here right and another thing is that for this one.

(Refer Slide Time: 10:25)



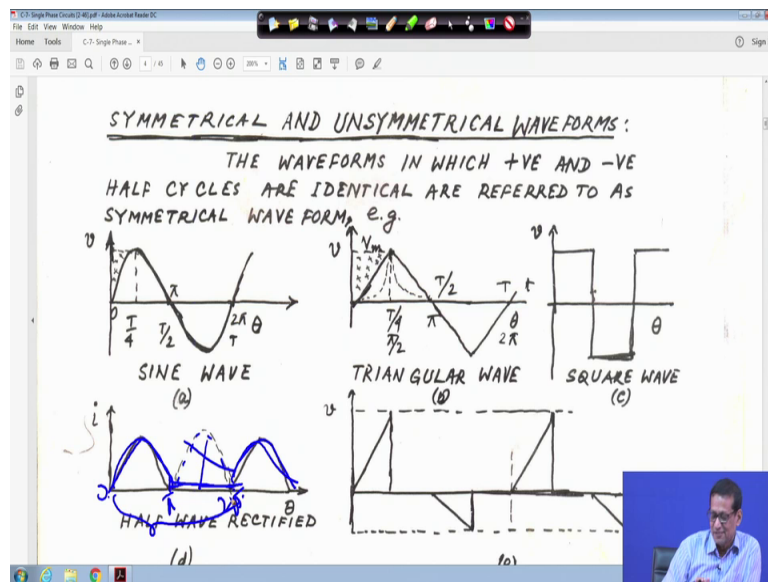
Suppose, I want to find out 0 to half cycle if you draw this line this side and this side it is same identical right, if you draw this in this thing this side and this side is same it is

symmetrical. So, instead of half cycle average value you can make it quarter cycle also 0 to T by 4. It will give you the same result right. It will give you the same result.

Similarly, here also it is a triangular wave form. If you try to find out what is the average value of this triangular wave form, it is a symmetrical wave form in between 0 to π right. Whatever result you get that right. So, same you will get in between 0 to π or 0 to T by 2 here also 0 to your what you call this is your 0 to π by 2 or 0 to 2 by 4, this side will get because this side area and this side area it is same.

Even quarter your; that means, in quarter cycle whatever or average value you will get you take half cycle. Also, you will get the same value because this side and this side, left hand side and right hand side are the same this is and this is this one and this one this side area and this side area. It is same symmetrical you will get the same result right. So, some problem some intuition you can do it things you can do it in very simple way rather than completing the half cycle. If you see the things are totally your what you call that is symmetrical right. So, let me clear it. Similarly, that unsymmetrical wave form if you look into, if you look into this.

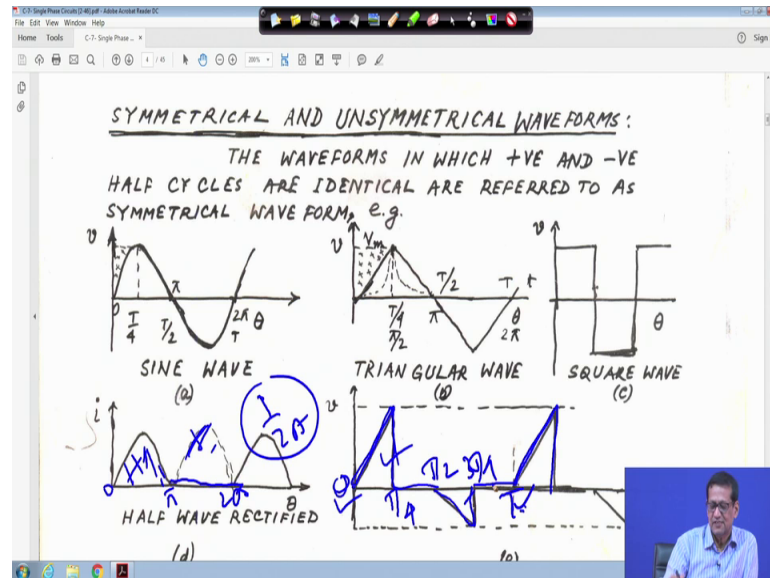
(Refer Slide Time: 11:48)



This is your i side this is 0. This is same as before. Say, this is π this is 2π like this right. So, wave form is there up to this after that nothing is there then it is like this. So, this portion dash portion nothing is there here right. But complete cycle is 0 to 2π right, but it is going your up to 0 to π after that, nothing is there till 0, but your total your

complete cycle is 2π . So, it is unsymmetrical wave form you have to consider that your what you call that total your, there you are taking that base is half cycle. Here you have to take base for complete cycle right.

(Refer Slide Time: 12:23)



So; that means, that means although it is 0. So, it is 0, it is π it is 2π . So, you have to find out this area to get this average value or RMS value, but you have to divide it by 2π you have to divide this by 2π .

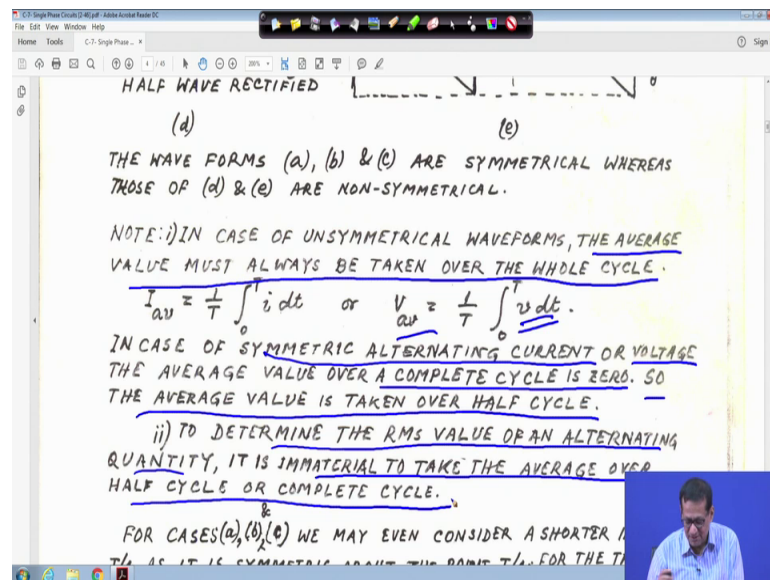
But you will integrate from 0 to π because, this dash line is shown that if it is there, but it is not there. So, it is it is going like this. So, for asymmetrical waveform right. When it is not symmetrical, you have to consider the complete cycle total base divided by whatever integration will be there 0 to π . We will see some example. Similarly, here it is also here also it is like this it is unsymmetrical. It is unsymmetrical wave from this side than this side it is not matching this side and this side it is not matching and this is your 0 and this is your T right. Again, because this wave form is started here and again this wave form starting here, this is T right. So, it is your what you call this one, it is your T by 4 this point is T by 2 this is $3T$ by 4 and this is T right.

So, this is unsymmetrical wave form. So, in that case you have to, but when you try to find out that your average RMS value, you have to complete. You have to consider the complete cycle. That is your sorry completed the your what you call that complete base

from 0 to T. The division should be divider should be T right, for unsymmetrical wave form.

You have to complete; you have to consider the whole cycle forgetting the average value or your RMS value right. So, this is the idea for this one your what you call the symmetrical and your what you call as the unsymmetrical wave form for unsymmetrical wave form. You have to consider that complete cycle to get that your average value or average value or your RMS value right. For symmetrical waveform, you have to consider half cycle or even quarter cycle looking at this. It will give you the same results, but for unsymmetrical wave form, you have to consider your, what you call that your complete cycle right to get that average value or RMS value right.

(Refer Slide Time: 14:23)



So, hope this is understandable to you. So, next is that suppose in the case of unsymmetrical wave form the your what you call the average value must always be taken over the whole cycle. I told you that average value your what you call must be taken over the whole cycle. So, I average should be $\frac{1}{T} \int_0^T i dt$. Similarly, for voltage V average. We have to consider from the whole cycle. So, $\frac{1}{T} \int_0^T v dt$ right.

And, in case of symmetrical, in case of symmetrical, alternating current or voltage right. The average value over a complete cycle is 0. So, the average value is taken over the half cycle because, if it is symmetrical, that I told you the negative and positive area and negative area will be same. So, in that case you are what you call the average value will

be 0. That is why, we will consider that half cycle for average value right. Now, second thing is to determine the RMS value of an alternating quantity. It is immaterial to take the average over half cycle or complete cycle. I told you RMS value means a square.

So, I told you that as soon as squaring it, if you take half cycle or full cycle, it will give you the same result right. Because, area for half cycle and another half cycle remain same. You have to take half cycle. Whatever result you will get, if full cycles, you will get the same result right. Now, just hold on.

(Refer Slide Time: 15:47)

THE AVERAGE VALUE OVER A COMPLETE CYCLE IS ZERO. SO THE AVERAGE VALUE IS TAKEN OVER HALF CYCLE.

ii) TO DETERMINE THE RMS VALUE OF AN ALTERNATING QUANTITY, IT IS IMMATERIAL TO TAKE THE AVERAGE OVER HALF CYCLE OR COMPLETE CYCLE.

FOR CASES (a), (b), (c) WE MAY EVEN CONSIDER A SHORTER INTERVAL $T/4$ AS IT IS SYMMETRIC ABOUT THE POINT $T/4$. FOR THE TRIANGULAR WAVE FORM Fig. (b)

$$V_{av} = \frac{1}{T/4} \int_0^{T/4} \frac{V_m}{T/4} \cdot t \, dt = \left(\frac{4}{T}\right)^2 V_m \frac{t^2}{2} \Big|_0^{T/4} = \frac{V_m}{2}$$

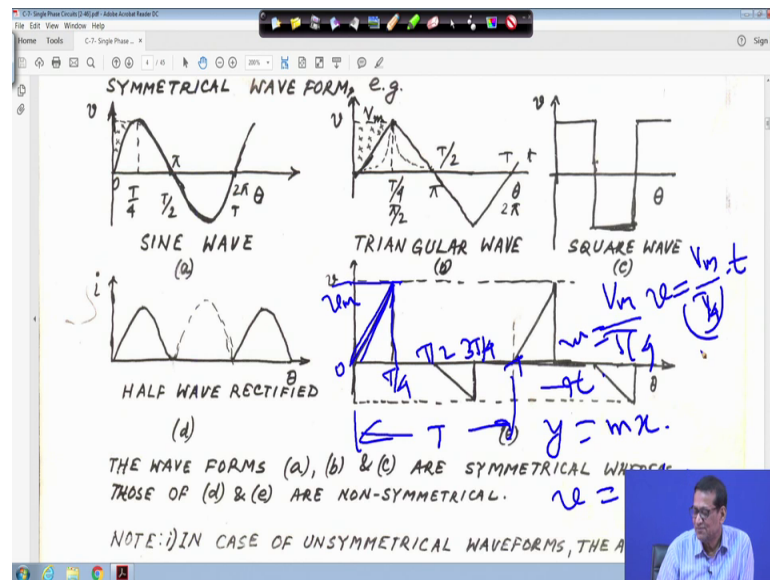
$$V_{rms} = \left[\frac{1}{T/4} \int_0^{T/4} \frac{V_m}{(T/4)^2} t^2 \, dt \right]^{1/2} = \frac{V_m}{\sqrt{3}}$$

FORM FACTOR = $2/\sqrt{3} = 1.155$

PEAK FACTOR = $\sqrt{3} = 1.732$

Now, therefore, for case for case a b c we may even consider a shorter interval right, that is your what you call the T by 4. As it is symmetrical about this your point T by 4 for the triangular wave form, look triangular waveform how we will do it right. Just try, just try to understand this.

(Refer Slide Time: 16:06)



Because, suppose we want to suppose this is your what you call this is 0 this is T by 4 this is T by 2 this is 3 T by 4 and this is T right. So, this total your this total your what you call this base length is T right for this one if you take this is my this is my V_m and this is your peak value this height is your this is my V_m right.

Then, the slope of the straight line the slope of the straight line will be V_m this is the height is V_m and base is T by 4 right and this is the slope and that equation is equal to this straight line is passing through the origin; that means, you are you know in general for straight line y is equal to $m \cdot x$ that you know, but y side is v . So, this is v should be is equal to m into T right and your m is equal to nothing but this is your slope v is equal to V_m upon T by 4; that means, somewhere I am writing.

So, I am writing somewhere here right let me see find out some space. So, here I am some writing that v is equal to right m is equal to your V_m divided by T by 4 right into the T this is the straight line passing through the origin because this time you say T right. So, this is the time this we will take say T instead of theta we will take T right. So, v is equal to $V_m \cdot T$ by 4 into T right. So, let me clear it. So, that means, if you come here we will come to that again.

(Refer Slide Time: 17:39)

QUANTITY, IT IS IMMATERIAL TO TAKE THE AVERAGE OVER HALF CYCLE OR COMPLETE CYCLE.

FOR CASES (a), (b) & (c) WE MAY EVEN CONSIDER A SHORTER INTERVAL $T/4$ AS IT IS SYMMETRIC ABOUT THE POINT $T/4$. FOR THE TRIANGULAR WAVE FORM Fig. (b)

$$V_{av} = \frac{1}{T/4} \int_0^{T/4} \frac{V_m}{T} t dt = \left(\frac{4}{T}\right)^2 V_m \frac{t^2}{2} \Big|_0^{T/4} = \frac{V_m}{2}$$

$$V_{rms} = \left[\frac{1}{T/4} \int_0^{T/4} \frac{V_m^2}{T^2} t^2 dt \right]^{1/2} = \frac{V_m}{\sqrt{3}}$$

FORM FACTOR = $2/\sqrt{3} = 1.155$
 PEAK FACTOR = $\sqrt{3} = 1.732$

$V = \frac{V_m \cdot t}{T/4}$

That means, if you come here that V average value. We are taking 1 upon T by 4 right for your what you call, then 0 to T by 4 V m by T 4 into dt right. So, this is your what you call that your V average that I told you 0 to T 4 and I told you this equation this equation I told you that V m by 4 t dt right. So, if you simplify this, it will be V m by 2 right. So, in your what you call that is your V average. Now, that is your just hold on because it is symmetrical waveform. Just hold on sorry.

(Refer Slide Time: 18:29)

SYMMETRICAL AND UNSYMMETRICAL WAVEFORMS:

THE WAVEFORMS IN WHICH +VE AND -VE HALF CYCLES ARE IDENTICAL ARE REFERRED TO AS SYMMETRICAL WAVE FORM, e.g.

(a) SINE WAVE
 (b) TRIANGULAR WAVE
 (c) SQUARE WAVE

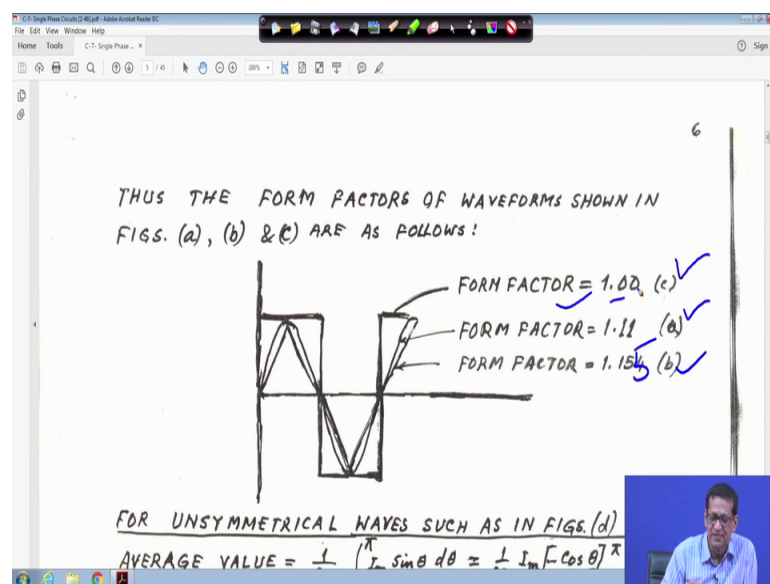
(d) HALF WAVE RECTIFIED
 (e)

This one previously I talked about this one this one, this one sorry this one this one actually T by 4 not this one right. I just I overlook this one this one. But, this is your T by 4 and in that case for symmetrical waveform because this side area and this side area is same.

So, I told you this equation will be v is equal to V_m divided by T by 4 into T this is also same thing for this one. We will come later right. So, it is T by 4. So, let me clear it. So, this is your average value this is your average value. So, if you 0 to T by 4 if you integrate, it will become your what you call V_m upon 2 right. So, this will become V_m upon 2. Similarly, that V_{rms} value, it will be your 1 upon T by 4 0 to T by 4. So, we got v is equal to V_m divided by T by 4 into t and you square it. If you square it, it will V_m square divided by T by 4 square into T square dt to the power half right and 0 to T by 4 you integrate, you will get V_m upon root 3 right. So, similarly form factor you can find out. It will become 2 by root 3 1.155 and peak factor will be 1.732.

So, let me clear it. So, that means, I mean here if you come again that your form factor is equal to RMS value by average value and your peak factor is equal to maximum value point points your what you call is equal to maximum value by RMS value. So, for this triangular wave form then you will get your peak factor 2 by root 3. Just simplify. You will get it 1.155 and peak factor root 3 is 1.732. And, this one, now next one is this is triangular waveform.

(Refer Slide Time: 20:16)



Now, does form factor of all wave forms, if you look into this. If you look into this that this one your, this one is for your square waveform. It will be 1 for sinusoidal waveform, it will be 1.11 and for this triangular waveform. You just do it it will be 1.154. Just now, we calculated know just now we calculated for the triangular one for the triangular one we calculated 1.155. So, here also it is 1 point actually it is 1.155 right. So, this is for figure c for figure a and figure b right. So, for form factor for your square waveform it is 1 right.

(Refer Slide Time: 21:08)

FOR UNSYMMETRICAL WAVES SUCH AS IN FIGS. (d) & (e)

$$\text{AVERAGE VALUE} = \frac{1}{2\pi} \int_0^\pi I_m \sin \theta d\theta = \frac{1}{2\pi} I_m [-\cos \theta]_0^\pi$$

$$= \frac{1}{2\pi} \cdot 2I_m = \frac{1}{\pi} \cdot I_m$$

$$\text{RMS VALUE} = \sqrt{\frac{1}{2\pi} \int_0^\pi I_m^2 \sin^2 \theta d\theta} = \frac{I_m}{2\sqrt{2}}$$

$$\text{FORM FACTOR} = \frac{I_m}{2\sqrt{2}} \times \frac{\pi}{I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

(d)

(e)

$$\text{AVERAGE VALUE} = V_{av} = \frac{1}{T} \int_0^T v dt$$

Now, next actually after unsymmetrical waveform suppose this for this case for this case if you look into this for unsymmetrical waveform you have to consider that whole base 2 pi right, but this is your integration should be 0 to pi. So, average value 1 upon 2 pi 0 to pi I m sin theta d theta will be 1 upon 2 pi I m minus cos theta between 0 to pi right. So, it is 1 upon 2 pi into integrate it, you will get 1 upon pi into I m right. Basically, I m by pi now for RMS value it will be 1 upon 2 pi 0 to your pi I m square sin square d theta here your what you call one half is missed. So, to the power half right.

So, if you integrate it then you will get I m upon 2 by root 2 right. So, this is your RMS value now form factor will be same way you can get it, it will be 1.11 right. So, although it is your half cycle other thing, but for sine waveform is sinusoidal your I m getting 1.11 right now for this triangular waveform your what you call at that time I was showing this one this triangular waveform that this for finding out.

(Refer Slide Time: 22:16)

The image shows a handwritten slide with the following content:

RMS VALUE = $\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \theta d\theta = \frac{I_m}{2\sqrt{2}}$

FORM FACTOR = $\frac{I_m}{2\sqrt{2}} \times \frac{\pi}{I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$ (d)

A graph (e) shows a triangular waveform with a peak of 10V and a trough of -5V. The period is T. The positive half-cycle is a triangle from (0,0) to (T/4, 10) to (T/2, 0). The negative half-cycle is a triangle from (T/2, 0) to (3T/4, -5) to (T, 0). The slope of the positive half-cycle is $V = \frac{10}{T/4} \cdot t$.

AVERAGE VALUE = $V_{av} = \frac{1}{T} \left[\int_0^T v dt \right]$

$$= \frac{1}{T} \left[\int_0^{T/4} \frac{10}{T/4} \cdot t dt + \int_{T/4}^{T/2} 0 dt + \int_{T/2}^{3T/4} \left(\frac{-5}{T/4}\right) t dt + \int_{3T/4}^T 0 dt \right]$$

RMS VALUE = V_{RMS}

$$= \sqrt{\frac{1}{T} \int_0^T v^2 dt} = \sqrt{\frac{1}{T} \left[\int_0^{T/4} \left(\frac{10}{T/4}\right)^2 t^2 dt + \int_{T/2}^{3T/4} \left(\frac{-5}{T/4}\right)^2 t^2 dt \right]}$$

Suppose this is given suppose this is unsymmetrical waveform suppose peak is given 10 volt right and T by 4 t by and this side it is minus 5 volt this is plus 10 volt right. So, and that means, the slope of the straight line slope of the straight line will be this 10 divided by T by 4 right.

So, and the another thing is that we have to come from 0 to T right from 0 to T you have to come. So, in this case, we have to complete the, you have to consider the whole cycle. That is why we have taken $\frac{1}{T} \int_0^T v dt$ this is a V average value. So, first we have to consider this area and then we have to consider this area. So, for this, this is the equation passing through a straight line. So, it will be your this equation this equation for this straight line will be your V is equal to your that your. This slope 10 divided by T by 4 into t right.

So, for this part it is 1 upon T, then 0 to T by 4 10 divided by T by 4 into t dt. Now, next one is look for your intuition you have to make it. Next one is that, this one this portion. Now, let me first clear it. Now, this portion if you try to find out, a straight line of this equation you have to make it y is equal to m x plus c.

(Refer Slide Time: 23:41)

$$RMS\ VALUE = \frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2 \theta d\theta = \frac{I_m}{2\sqrt{2}}$$

$$FORM\ FACTOR = \frac{I_m}{2\sqrt{2}} \times \frac{\pi}{I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$y = mt + c$$

$$v = mt + c$$

$$AVERAGE\ VALUE = V_{av} = \frac{1}{T} \left[\int_0^{T/4} v dt + \int_{T/4}^{3T/4} v dt + \int_{3T/4}^T v dt \right]$$

$$= \frac{1}{T} \left[\int_0^{T/4} \frac{10}{T/4} t dt + \int_{T/4}^{3T/4} \left(\frac{-5}{T/4} \right) t dt + \int_{3T/4}^T \left(\frac{-5}{T/4} \right) t dt \right]$$

$$= 0.625\ V.$$

$$RMS\ VALUE = V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2 dt} = \sqrt{\frac{1}{T} \left[\int_0^{T/4} \left(\frac{10}{T/4} t \right)^2 dt + \int_{T/4}^{3T/4} \left(\frac{-5}{T/4} t \right)^2 dt + \int_{3T/4}^T \left(\frac{-5}{T/4} t \right)^2 dt \right]}$$

This way you have to make it or other way, you have to make it V is equal to your slope m into t plus C right.

So, forget about this. So, in this way you have to get the equation, but remember from the symmetry our interest to get what is the area right how we will find out. So, what you can what you can do is, suppose if I super if I bring this one, I mean just for the mathematical computation, nothing else. If I bring this one, suppose if it is here and if it is here, the area will remain same as if another straight line is passing through the origin right.

So, in that case this slope will be minus 5 your what you call minus 5 divided by T by 4 just we are just we are for mathematical things because if you want to find out y is equal to m x plus C, I mean v is equal to m t plus c. Then you have to obtain m you have to obtain C right. Because this is one coordinate. This is another. You have to put and you have to solve for m and c no need.

Just from the symmetry you bring this bring this portion to here, as if it is passing although reality it is not, but as if it is passing through the origin right. So, in that case slope will be minus 5 into T by 4. Therefore, this is also and integration will be also 0 to T by 4. This duration also 3 T by 4 minus T by 2 will be again T by 4 same thing right.

So, this from here to here this duration your 3 T by 4 minus T by 2 will be again T by 4, but; that means, same integration 0 to T by 4 we are taking this portion we are taking this portion that is minus 5 T by 4 into t dt. Because this is straight line passing through the origin this is a slope minus 5 T by 4 into t dt right. So, therefore, your what you call you need not find out this equation again just say because just we are super imposing it here because our interest is only this area and the negative sign. So, I hope you have understood this right.

So, this is I have taken intentionally to make you think that it is understandable to you right. Ultimate ultimately, if you make this one and integrate, you will get the same result. So, why I will spend more time on this, directly I will do it. So, that is why I have written like this that minus 5 T by 4 into t dt. This a straight another straight line passing through the origin the slope is negative right.

(Refer Slide Time: 26:04)

The slide displays a graph of a periodic waveform $v(t)$ over one period T . The waveform consists of a positive half-cycle from $t=0$ to $t=T/4$ and a negative half-cycle from $t=T/4$ to $t=T$. The peak value is $5V$. The average value is calculated as follows:

$$\text{AVERAGE VALUE} = V_{av} = \frac{1}{T} \left[\int_0^T v \, dt \right]$$

$$= \frac{1}{T} \left[\int_0^{T/4} \left(\frac{10}{T/4} \right) t \, dt + \int_{T/4}^T \left(\frac{-5}{T/4} \right) t \, dt \right]$$

$$= 0.625 \, V.$$

The RMS value is calculated as follows:

$$\text{RMS VALUE} = V_{RMS}$$

$$= \sqrt{\frac{1}{T} \int_0^T v^2 \, dt} = \sqrt{\frac{1}{T} \left[\int_0^{T/4} \left(\frac{10}{T/4} \right)^2 t^2 \, dt + \int_{T/4}^T \left(\frac{-5}{T/4} \right)^2 t^2 \, dt \right]}$$

$$= 3.23 \, V.$$

So; that means, this RMS value average value will be if you integrate this it will become 0.625 volt this will be the average value right. Similarly, RMS value also same thing if you if you look into this, this RMS value also square root of this. So, it is 1 upon T 0 to T v square dt. So, this portion first this portion 1 under root , 1 upon T 0 to T T by 4 and this 10 upon T by 4 square into this whole thing you make it square right plus 0 to T 4. This whole thing you make it square minus 5 by T by 4 square t square dt right.

So, it will measure DC and A 2 is the moving iron ammeter. This another ammeter A 2 is there. It measures the AC right. Now, in this case 2 sources are there; one your DC voltage source is connected in series, another is AC source AC voltage source is connected one switch is there you close the switch right. So, this is the this is the series circuit.

It is given V_m is equal 100 volt that is the peak value is given 100 volt and V_0 your what you call and DC voltage V_0 is given this is 100-volt DC this is 120-volt DC right and AC peak value V_m . These 2 are connected in series. Initially, I have taken these kind of example. You will see this one example we will make our your understanding very clear right and then and A 1 is the moving coil ammeter and A 2 is the moving iron ammeter.

So, moving coil measures DC and moving iron measures AC right and the voltage acting in the circuit is the total voltage will be 100 plus 100 sin ωt . Because, peak value is given 100 volts right and it is; that means $v t$ is equal to V_0 plus $V_m \sin \omega t$.

So, V_0 is your 120. So, this is 120 right and your V_m is equal to 100 volts. So, 100 120 plus 100 sin ωt volt right. So, this is my your V_m now. So, the current $i t$ will be your 6 plus 5 sin ωt because this resistance R is given as 20 ohm it is 20-ohm R is given and this is my voltage 120 plus 100 sin ωt . So, it is 120 plus 100 sin ωt divided by 20 that will be 6 plus 5 sin ωt that is sorry it is your what you call 100. So, it is your 25. So, 6 plus 5 sin ωt .

So, this is your $i t$ is equal to 6 plus 5 sin ωt ampere right, now if you try to find out the average value it will 1 upon T from 0 to T 6 plus 5 sin ωt dt because this is $i t$ into dt and ω is equal to 2π by T . Your this one you should keep it in mind. This you know it also form your higher (Refer Time: 29:49) physics ω is equal to 2π by T right. So, that is written here right. By mistake initially written T by 2π . Actually, it is not a 2π by $2T$ right.

So, then what you do you just 0 to T 6 plus 5 sin ωt dt and using this relationship ω is equal to 2π by T right. You integrate this if you integrate this I am not showing it here sometime will be saved then you just integrate it I have this I can do it integration for you, but it is a simple thing from just simply integrate and use this relationship that ω is equal to 2π by T right. So, let me clear it.

(Refer Slide Time: 31:06)

EX.1. $\frac{120}{20} = 6A$

Circuit diagram: A series circuit containing a DC voltage source of 120V, an AC voltage source $V_m \sin \omega t$, a resistor $R = 20 \Omega$, and two ammeters A_1 and A_2 .

GIVEN: $V_m = 100 V$, $V_0 = 120 V$
 $R = 20 \Omega$
 $A_1 =$ Moving coil ammeter
 $A_2 =$ Moving iron ammeter.

WHAT WILL BE READINGS OF A_1 & A_2 ?

THE NET VOLTAGE ACTING IN THE CIRCUIT $= 120 + 100 \sin \omega t$ volts.
 SO CURRENT $i(t) = 6 + 5 \sin \omega t$ amps.

$I_{av} = \frac{1}{T} \int_0^T (6 + 5 \sin \omega t) dt$; $\omega = \frac{2\pi}{T}$
 $= 6 A = I_{dc}$
 $=$ READING OF AMMETER A_1

$I_{RMS} = I = \sqrt{\frac{1}{T} \int_0^T (6 + 5 \sin \omega t)^2 dt}$

A graph of current $i(t)$ vs time t is shown, illustrating the average value of 6A and the AC component.

So, you will use this you use this relationship ω is equal to 2π by T and integrate it you will get I average will be 6 ampere that is your I_{dc} right; that means, that you are you are what you call average value will be just 6 ampere. Because, if you look into that 124 DC is there and your R is 20 ohm. So, if you take 120 by 20 you will find it is 6 ampere right. It will 6 ampere. So, you are getting 6 ampere. So, I_{dc} will be 6 ampere. Then, it is moving coil ammeter the ammeter A_1 will read 6 ampere because you are you are what you call moving coil ammeter is this DC right.

So, let me clear it. So, similarly the RMS value reading of ammeter this is the reading of ammeter A_1 . Now, for you are just 1 minute. Now, for the diagram we will come later.

(Refer Slide Time: 31:59)

$$I_{av} = \frac{1}{T} \int_0^T (6 + 5 \sin \omega t) dt \quad \omega = \frac{2\pi}{T}$$

$$= 6 \text{ A} = I_{dc}$$

$$= \text{READING OF AMMETER } A_1$$

$$I_{rms} = I = \sqrt{\frac{1}{T} \int_0^T (6 + 5 \sin \omega t)^2 dt} \quad \omega = \frac{2\pi}{T}$$

$$= \sqrt{\frac{1}{T} \int_0^T (36 + 60 \sin \omega t + 25 \sin^2 \omega t) dt} = \sqrt{36 + \frac{25}{2}}$$

$$\approx 7 \text{ A} = \text{READING OF AMMETER } A_2$$

THUS IF DC IS SUPERIMPOSED ON AC SIGNAL, THE RESULTANT RMS VALUE = $\sqrt{I_{dc}^2 + I_{rms(ac)}^2} = \sqrt{6^2 + \left(\frac{5}{\sqrt{2}}\right)^2} \approx 7 \text{ A}$.

NOTE: i) MOVING COIL INSTRUMENT READS AVERAGE VALUE OVER A PERIOD. THUS FOR AC ALONE IT WILL READ 0.
 ii) MOVING IRON, HOT WIRE, ELECTRODYNAMIC & INDUCTION INSTRUMENTS READ RMS VALUE.

Now, I RMS value that is your reading of ammeter your this will be the whatever you will get this portion is the reading of ammeter A 1. This is your DC; this will read A 1 will read your 6 ampere. Now, come to RSM value.

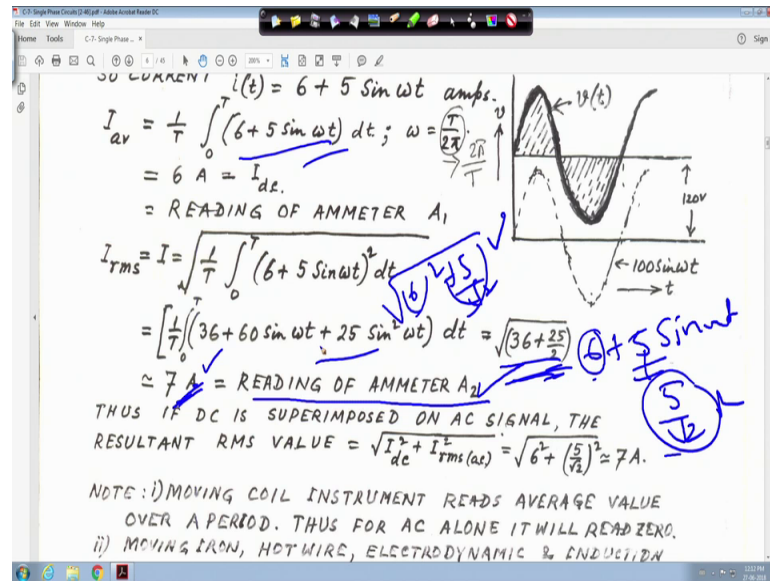
So, in this case I is equal to square root of 1 upon T 0 to T 6 plus 5 sin omega t square because I is equal to 6 plus 5 sin omega t square dt. Now, you just break it and just you integrate it. If you integrate it, you will find that your and you will use this relationship. After integrating, you have to use this omega is equal to 2 pi by T; that means, I am telling you somewhere it will come in integration you substitute omega is equal omega t is equal to 2 pi right. Then you substitute right. After the substitution, you will find integration of sin omega t 0 to T you will find this term will become 0 and this one also you make it your what you call that sin square omega t.

So, it will be 1 minus cos 2 omega t by 2 and then you integrate right. So, you will find that your this term only 2 from this integration, whatever will come it will be 25 by 2. So, integration of this term will vanish it will 0.

And, 36 will be there your T T will be cancel and under root 36 plus 2 pi by 2. So, I am not doing it here, but please do it. It is very interesting. You please do it, but you will use omega is equal to 2 pi by T. So, that is omega into capital T 2 pi.

Similarly, here also, in this integration also you will put this one omega t is equal to 2 pi omega capital T is equal to 2 pi right. So, because when you will integrate this that is your what you call sin omega t. So, it will be minus cos omega t upon omega right and there I put T is equal to capital T and at that time you have to use this thing and you simplify. So, we will get root over 36 plus 25 by 2.

(Refer Slide Time: 33:39)



So; that means, whatever it will come it will coming around say your 7 ampere. So, that will be the reading of ammeter a 2 that is the moving iron ammeter. It will measure this called RMS value. So, that means, moving iron ammeter that is ammeter A 2 will measure the RMS value and this is your RMS value. So, ultimately, what we got? Ultimately, what we got? Ultimately, the current value was 6 plus 5 sin omega t right that is your i t.

So, this is the fixed value constant value and this is the peak value, the peak value; that means, it is the peak value and we have seen that your peak factor your root 2; that means, the RMS value of the current will be actually 5 by root 2 ampere RMS value right.

So, ultimately that ultimately the effective value will be the 6 it is 6 square plus your 5 by root 2 square. So, whatever you get know 6 square 36 pi by root 2 square is equally 25 by 2 right. So, if you have this kind of thing directly you can write this RMS value will be under root of 6 square plus whatever value is come in this case pi by root to

square. So, that is your what you call this is what RMS value you are getting from this integration. So, please do this integration of your own. I told you everything right. So, please do this. Now, let me clear it. So, this is your ammeter your, what you call reading of the ammeter A2. It is a 7 ampere approximately.

(Refer Slide Time: 35:25)

The image shows a handwritten derivation for the RMS value of a signal $i = 6 + 5 \sin \omega t$. The derivation is as follows:

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T (6 + 5 \sin \omega t)^2 dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T (36 + 60 \sin \omega t + 25 \sin^2 \omega t) dt} = \sqrt{36 + \frac{25}{2}}$$

$$\approx 7 \text{ A} = \text{READING OF AMMETER } A_2$$

THUS IF DC IS SUPERIMPOSED ON AC SIGNAL, THE RESULTANT RMS VALUE = $\sqrt{I_{dc}^2 + I_{rms(ac)}^2} = \sqrt{6^2 + \left(\frac{5}{\sqrt{2}}\right)^2} \approx 7 \text{ A}$.

NOTE: i) MOVING COIL INSTRUMENT READS AVERAGE VALUE OVER A PERIOD. THUS FOR AC ALONE IT WILL READ ZERO.
 ii) MOVING IRON, HOT WIRE, ELECTRODYNAMIC & INDUCTION TYPE INSTRUMENTS READ RMS VALUE OF THE SIGNAL.

IMPORTANT TO REMEMBER: ✓
 WHEN AN A.C. SUPPLY OF 220 V IS REFERRED, IT IS MEANT ITS RMS VALUE. THE MAXIMUM VALUE IS $220\sqrt{2} \approx 310 \text{ V}$.
 IN CASE OF DC 220V IT IS 220V ONLY, THUS SHOCK LEVEL OF AC FOR SAME SPECIFIED VOLTAGE IS MORE THAN THAT OF DC VOLTAGE.

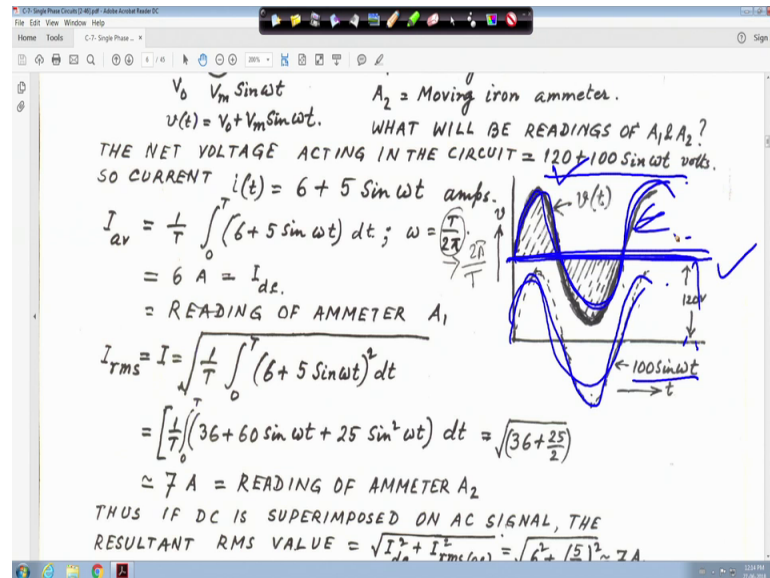
So, next is your something is written that you are moving iron where your something you should keep it in your mind that moving coil instrument. It is the average value moving iron hot wire electro dynamic and induction you are what you call type instruments read the RMS value of the signal right.

One thing is important to remember. When an AC supply of 220 volt is referred, it is meant the RMS value right. The maximum value is 220 into root 2. So, it is said 310 volts right. So, in case of DC 220 volts, it is 220 volt only. If it is DC 220 means, DC 220. If it is AC 220, means it is RMS value. You have to multiply by root 2, 310 volt.

That means that the shock level of AC for the same specified voltage is more than that of the DC voltage. That means, in AC voltage you will get the peak value shock and DC voltage you will get 220. I mean, in this DC voltage, the shock will be 220 volt only for AC supply 220 volt RMS means your shock level will be 310 volts. So, shock in AC will be more than the DC right. So, this is something I have written for you right.

So, that means, and this is that your what you call that is your just; that means, this is the wave form this is your $v(t)$ $v(t)$ is equal to your what you call your v your v , your $V_m \sin \omega t$ this is the plot of your $v(t)$ is shown right.

(Refer Slide Time: 36:53)



And this is your what you call that your 100 sin omega t right, 100 sin omega t and this volt this is your 120. This is 120 volts you DC this 120-volt DC is made it right. And this is your V_m is 100 sin omega t. If you if you add this 2, then this will be your what you call this $v(t)$ right; that means, this one your this plot this is 120 plus 100 sin omega t. This plot is this one and individually if you make it, this is 100 sin omega t and this is the DC means is the constant line this is 120 volt is mentioned. So, this is the resultant of this.

Thank you very much. We will be back again.