Fundamentals of Electrical Engineering
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Lecture – 35
First order circuits (Contd.)

So, come back to another your problem. Then, we will see that your what you call that R L circuit and this right now.

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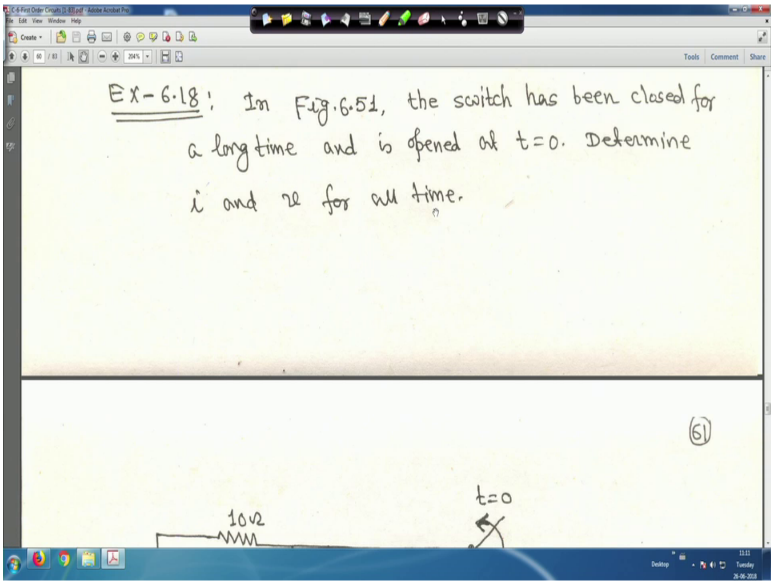

So, in figure I will come to that. In figure 51, the switch has been closed right for a long time and it open at t is equal to 0. You have to determine i and v for all time right. So, it is given that switch has been closed for a long time and that means, this switch was closed for a long time and your I mean it was closed for a; this switch was closed for a long time.

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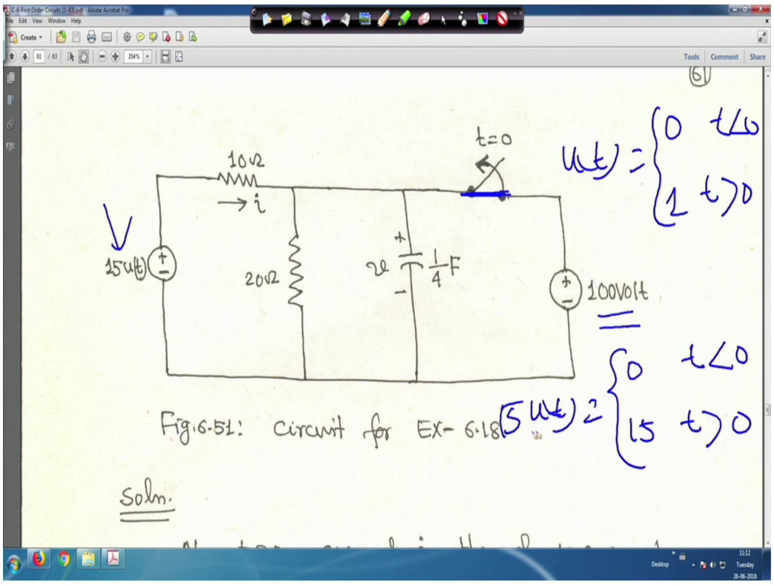

And at t is equal to 0, this switch is opened right.

Now question is, this one your this 100 volt source is there, but one step input is also there here that is 15 u t volt, right. So, in general your when you write 15 of u t that is equal to your this one is equal to 0, when t less than 0 and is equal to 15, when t greater than 0 right. So, in that case that means, in general your u t actually is equal to 0 for t less than 0 and it is 1 for t greater than 0, right.

So, that is why this 15 u t your for t is equal to 0 for t less than 0, that is your what you call the past history; that means, let me clear it. So that means this switch was closed for long time.

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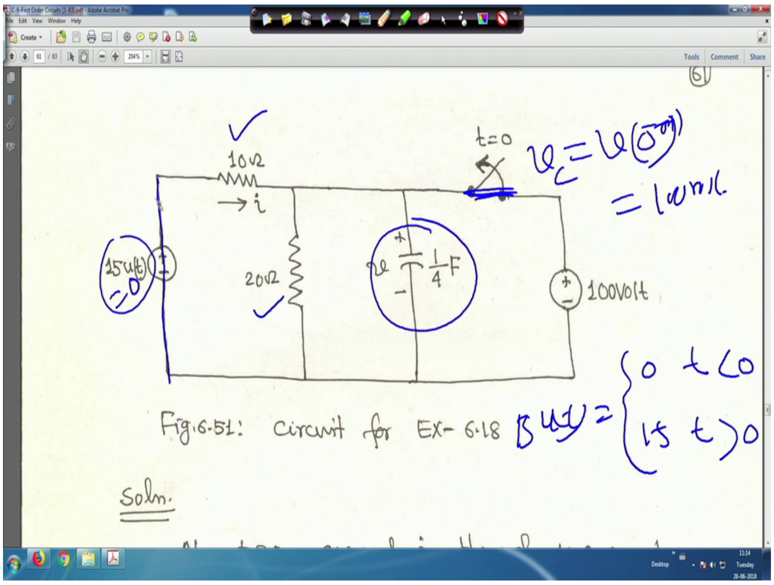

So, in this case, what will happen that is your for t less than 0. So, in that case what will happen, that 15 u t right. So, is equal to I told you it is 0, for t less than 0 and it is 15, for t greater than 0.

That means, in that case, what will happen? This is your what you call this, this was closed for a long time and that means, this the 15 u t voltage source, actually this is becoming 0 that means, it will be like this, right. So, just join it because this thing is actually is equal to 0 because switch was closed for long time, right. So, as switch was closed for a long time that means, to this and this 100 volt DC source is there, this is close; that means, this capacitor will act as an open circuit right.

So, from the I mean if it acts as a open circuit; that means, the switch was closed for a long time; that means, your v c that is v, v c is equal to say v, right is equal to 0 minus sorry 0 minus is equal to this 100 volt right. So, this is your what you call, this is your v c 0 minus. And at the same time, you can easily and as this 10 ohm and 20 ohm are in your what you call are in parallel right because this voltage is 0. So, it is short I mean just for the purpose of explanation. So, and your what you call, easily you can compute I because this will act this will act as a open circuit. So, equivalent circuit is shown. I have drawn it here, right. So, in this case everything is written there, whatever I said everything is explained here, right.

It is 15, so in this case what will happen the circuit will be like this.

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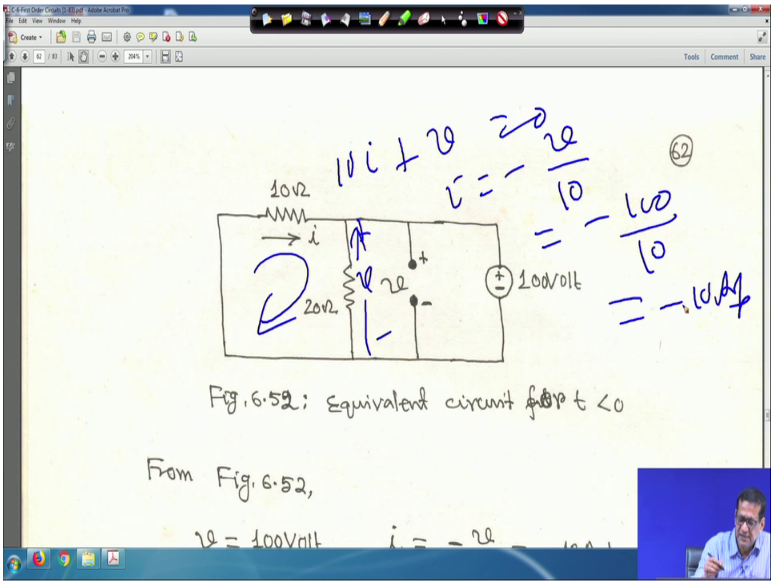

So, this is the capacitor was there it will act as an open circuit and this is i and this 10 ohm and 20 ohm are in parallel and this is 100 volt, right. So, v 0 plus your v is equal to 100 volt and i will be is equal to minus v by 10, that is minus 10 Ampere. Actually, you have if you I mean, just 1 minute.

If you try to find out this, I the direction is taken in this way direction is taken in this way. So, if we say that I know just for your this thing, this voltage and this plus minus voltage means this is the voltage v, right. So, this is plus, this is minus.

So, it is basically what will happen, 10 i, if we move like this, 10 i plus v is equal to 0. So, i is equal to minus v by 10 that is equal to minus 100 by 10 that is equal to minus 10 ampere, right. So, let me clear it. So, in this case, that is why it is given i is equal to minus v by 10, it is minus 10 ampere and the capacitor at capacitor voltage I mean if you look into the circuit when switch is open at t is equal to 0, the capacitor volt that means, as soon as it is open, it this voltage source should not be there because it is disconnected and that is at t is equal to 0 it is open. So, for t greater than 0, this 15 volt step input will be here.

And at the end, capacitor voltage cannot change instantaneously. So, in that case what will happen v 0 minus is equal to v 0 plus, that will be your 100 volt.

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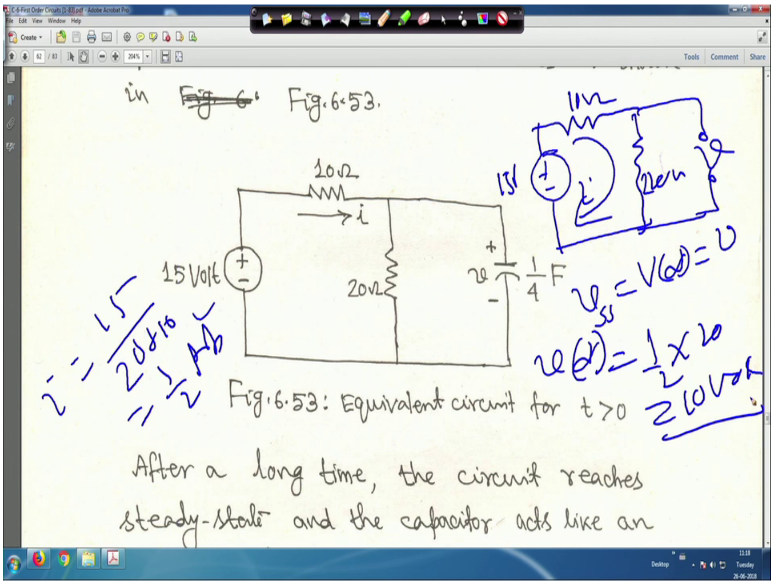

So, now in this case, 15 volt source is there and this is 10 i and this is your v 1 1 upon 4 Farad. Now, after a long time, I mean if you look into that the since the capacitor voltage cannot change instantaneously, so v 0 is equal to v 0 minus that is 100 volt. For t greater than 0, the switch is open and the 10 volt source is disconnected from the circuit.

That I told you, then this 15 u t volt source is now operative in the circuit it is like this. Now, at steady state, if you if you look in to the steady state that means, what will be your v infinity. Suppose at steady state this capacitor will act as open circuit; that means, circuit will be something like this, right. This is your 15 volts, this is 10 ohm this is your 20 ohm and this capacitor is acting as open circuit at steady state to the DC. So, in that case what will happen, this V s v ss is equal to actually v infinity is equal to is v, this is your v at steady state

Now, as you can easily at this is open circuit. So, this is your 10 ohm and this is your 20 ohm. So, you can easily find out what is the current i. So, this i is given it is open circuit. So, i is actually is equal to 15 divided by 20 plus 10, that is 30. So, is equal to half ampere that is your i, right. Then, what is v infinity, then v infinity will be that is a steady state value, this is open circuit this I is flowing this i is half and it is 20. So, that is equal to your 10 volt, right. So, v infinity is the 10 volt, right. So, this circuit is drawn for you. So, in this case, your this is v infinity is equal to 10 volts.

Now, when we will find out the your R Thevenin. So, R Thevenin means your for getting Thevenin I showed you the circuit, this is for looking from this point you have to find out.

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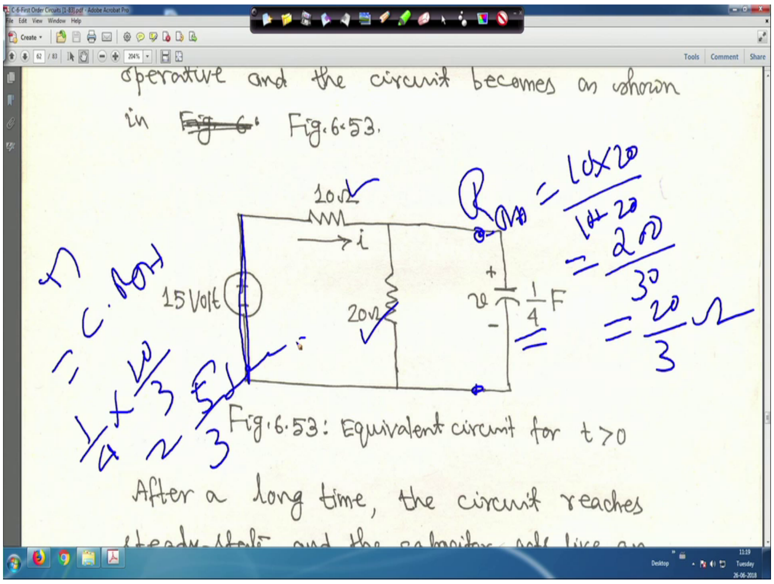

At that time, you have to short this one. So, 10 ohm and 20 ohm are in parallel, right because we have to short this. So, in this case your R Thevenin will be 10 into 20 by 10 plus 20, right. So, it will be 200 divided by 30 is equal to 20 by 3 ohm, right. That is R Thevenin therefore; time constant tau will be is equal to c into R Thevenin. So, c is given your one upon 4 Farad. So, it is 1 upon 4 into 20 by 3.

So, that is equal to 5 by 3 second. This is the tau right. So, let me clear it all this calculations are there, but I am making it such that. So, in this case, your this is tau that is 5 by 3 second and R Thevenin, you got 20 by 3.

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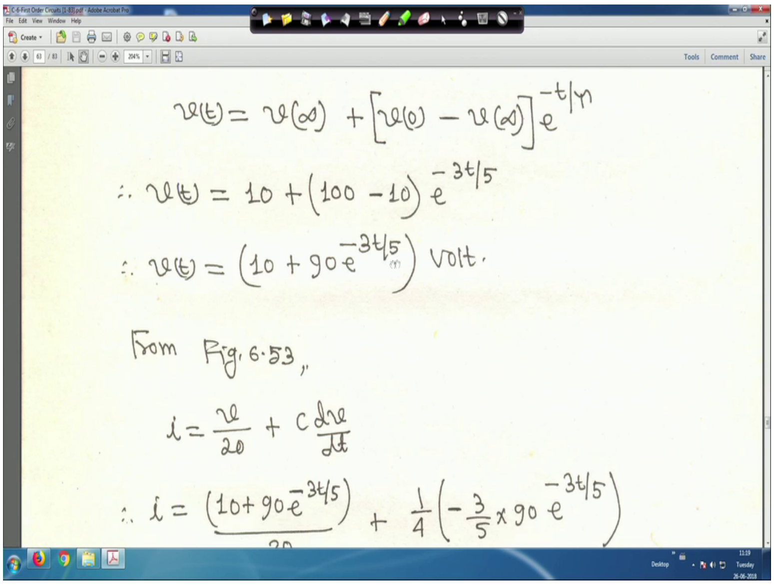

Now, we know this formula that v t is equal to v infinity plus v 0 minus v infinity e to the power minus t upon tau. So, v infinity we got 10, v 0 is equal to 100 and v infinity 10. So, 100 minus 10 into tau is equal to your 5 by 3. So, it will be e to the power minus 3 t by 5. So, v t will be 10 plus 10 into e to the power minus 3 t by 5 volt, right.

Now, from figure 53, i is equal to, now if you come to figure your 53 that i is equal to, so, one thing is there that this voltage.

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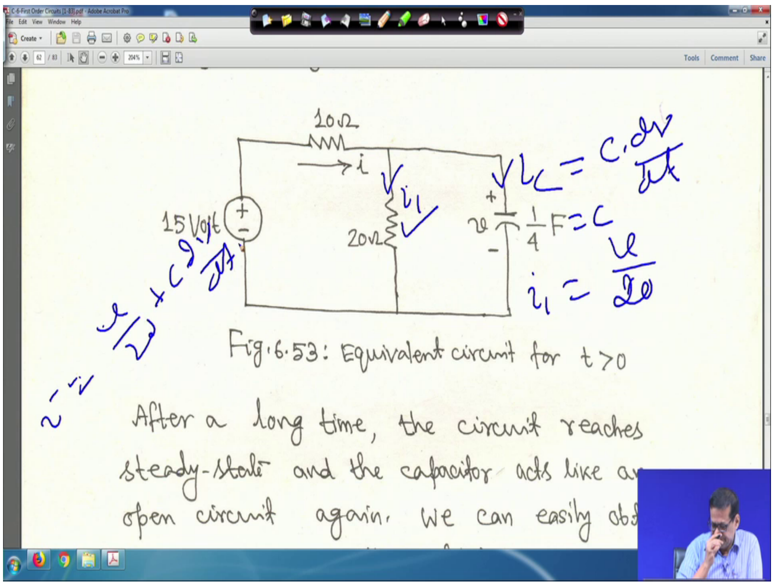

This voltage v means no other element is there. So, same voltage we will impress across your 20 ohm resistance. So, this current say it is i 1 and through this capacitor say it is i your i c, then i 1 will be say v by 20, this is i 1 and i c will be this c, this is your c, the capacitor c into d v by d t, right.

That means, i is equal to your v by 20 plus c into d v by d t, right. So, let me clear it. So, if you from that only we are writing this equation that i is equal to v by 20 plus c into d v by d t. So that means your what you call that you put that expression of v, right. So, every time I am not writing that your v t and v t.

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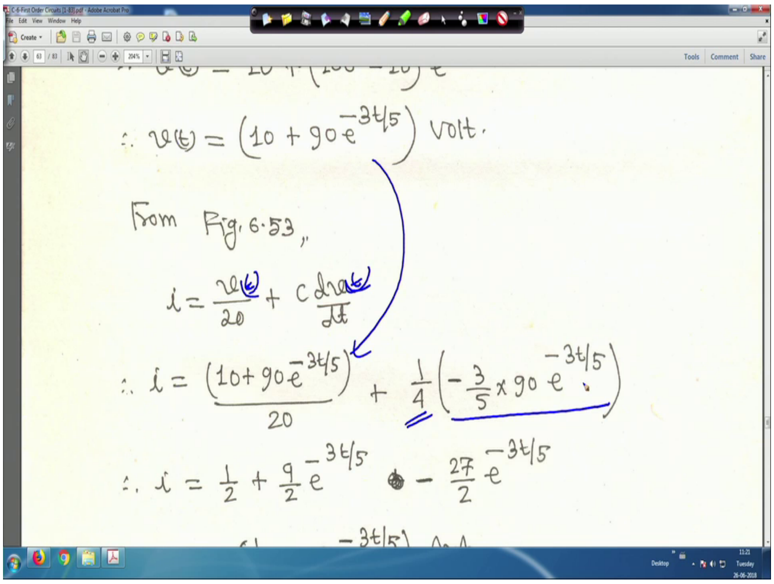

If you want you can put it this is v t, but it is understandable, it is understandable right. So, put the expression of v t here, this expression you put it here right and take the derivative of this one. So, this is the second term that c is equal to 1 upon 4 Farad into d v by d t right.

So, let me clear this. So, in this case your i is equal to after your making simplification half plus 9 by 2 e to the power minus 3 t by 5 minus 27 upon 2 e to the power minus 3 t by 5; that means, i is equal to half minus 9 e to the power minus 3 t by 5 ampere right. Note that this v plus 10 i is equal to 15 is satisfied.

If you go to this circuit right, your go to this circuit, I mean if you your suppose I want to apply KVL right, I want to apply KVL. So, this voltage v means suppose if I make it like this, this voltage v means it is 10 i plus v minus 15 is equal to 0; that means 10 i plus v is equal to 15; you check it, it will be satisfied right.

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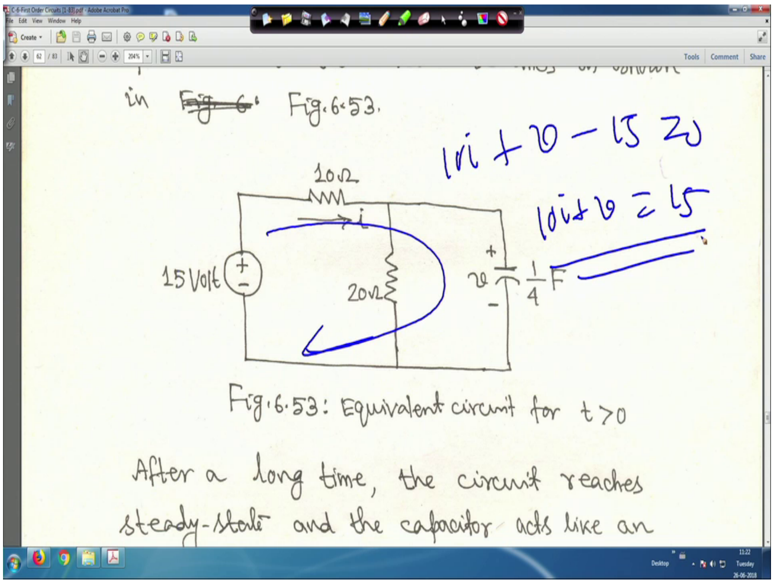

So, let me clear it, sorry. So, that is why it is written this is actually satisfied. Therefore, for all t v is equal to 100 volt that is your v 0 minus is equal to 100 volt for t less than 0 and 10 plus 90 e to the power minus 3 t by 5 volt for t greater than 0. And similarly for i, for t less than 0 it is minus 10 Ampere, you have computed and for t greater than 0 it is half minus 9 e to the power minus 3 t by 5 ampere that is t greater than 0, right. So, another way, now this is I hope you have understood this. Things are not at all difficult one, right.

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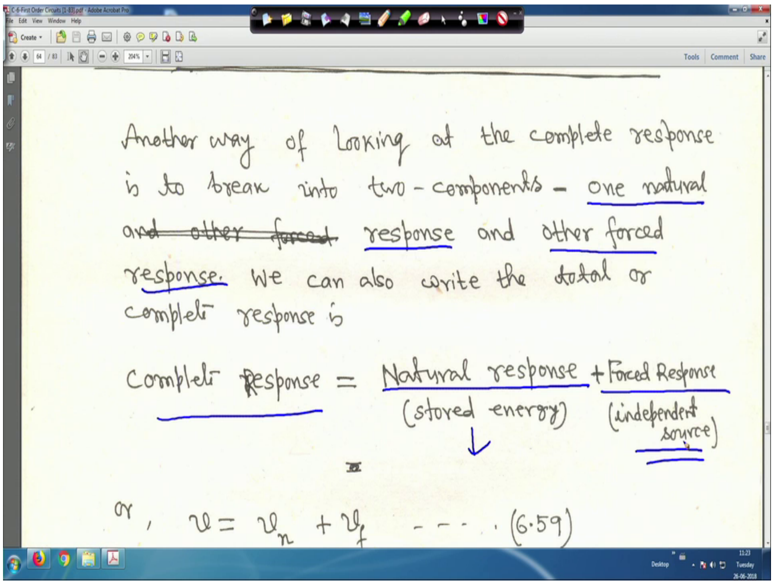

So, another way of looking at the complete response is to break into two components. One is natural response and other forced response, I mean the response has two things. You can break it. One is natural response and other is the forced response right. So, we can also write the total complete response. So, complete response is equal to natural response that is stored energy, that is your source the circuit that is the natural response we got plus forced, that is independent source may be voltage source or current source independent source, right. So, let that means, let me clear it.

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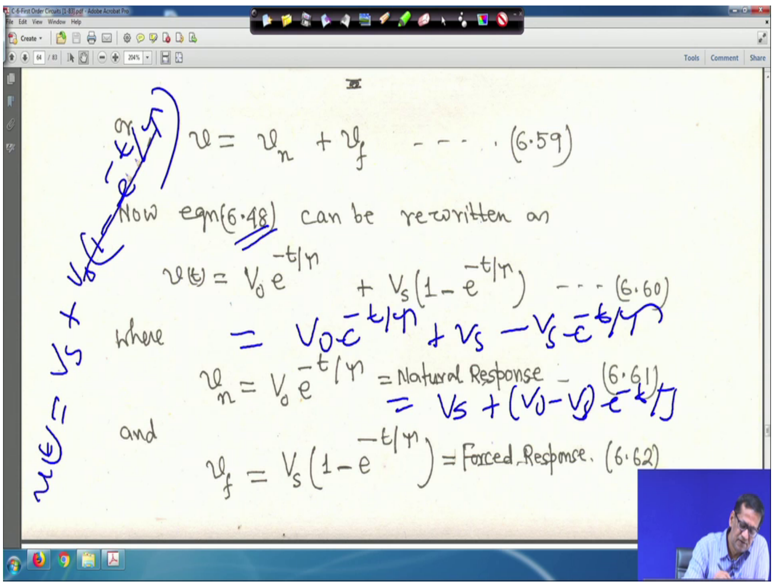

That means v is equal to natural response v n plus forced response v f this is the equation 59 right.

Therefore, equation from equation 48, it can be rewritten as I am writing here equation 48, from equation 48, we are writing this actually equation 40 somewhere I am writing say here v t, v t is equal to this equation was equation 40 V s plus your v s, then your V 0, you take V 0 common right, then one your 1 minus e to the power minus t upon tau right. I mean just, just this equation you can write that V 0 it is 1 your what you call it is v 0, then into your just one minute e to V 0 e to the power minus t by tau then plus v s, then V 0 will be your just 1 minute let me make it here.

This is V 0 e to the power minus t by tau plus V s minus V s e to the power minus t by tau, right. So, that is actually is equal to your V s plus V 0 minus V s e to the power minus t by tau. So, it is V 0 minus your sorry, let me clear it. V s

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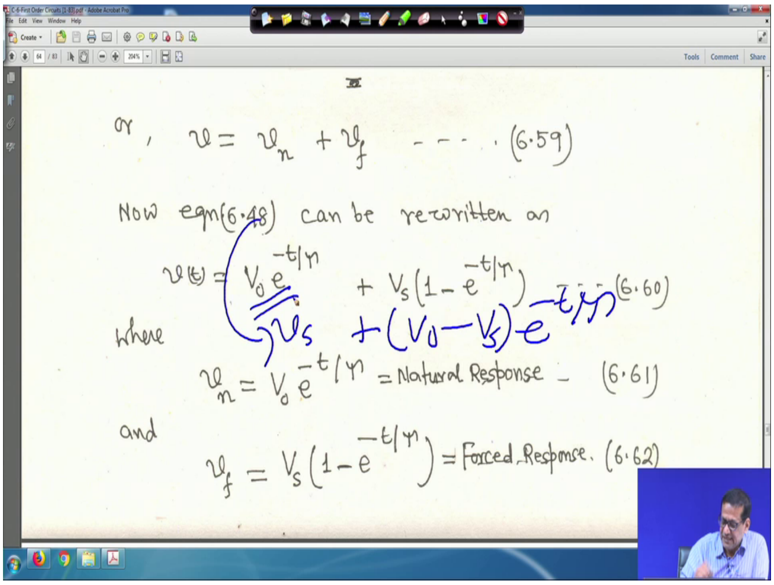

So, it is actually your V s right plus V 0 minus V s e to the power minus t by tau, that is your equation 48 right. So, this is actually v infinity plus V 0 minus v infinity e to the power minus t by tau. So, if you that V 0 e to the power minus t by tau, you write it here and they are V s minus V s e to the power minus t by tau you take V s common. So, it will be 1 minus e to the power e to the power minus t by tau, right.

So that means, that v n the let me clear it; that means, the v n that is your first term if V 0 e to the power minus t by tau that is natural response for source the circuit, we have seen that right. For source the circuit, we have seen that and this is actually we call forced response that is some that is the due to that external force, that is your nothing but that your voltage source, right. So, this is actually v n and this is your v s. So, v n is equal to V 0 e to the power minus t by tau, the natural response and V s is equal to V s 1 minus e to the power minus t by tau; that is your forced response, right.

So, let me clear it. So, this way this equation your equation 48 can be written in this way. So, this is equation 59, 60, 61 and 62.

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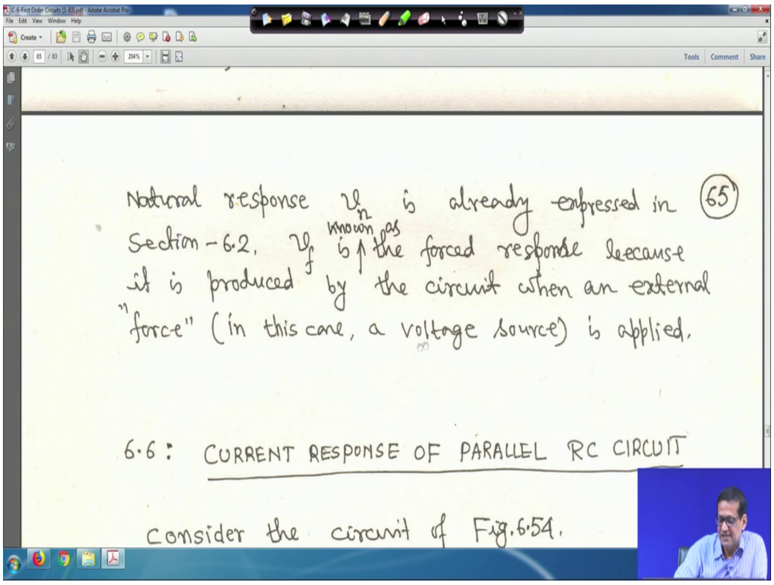

Therefore natural response v n is already expressed in section 6.2 that we have done it. So, v f is known as the forced response because it is produced by the circuit when the external force is applied that is in this case, it is a voltage source, right now, that current response of parallel RC circuit.

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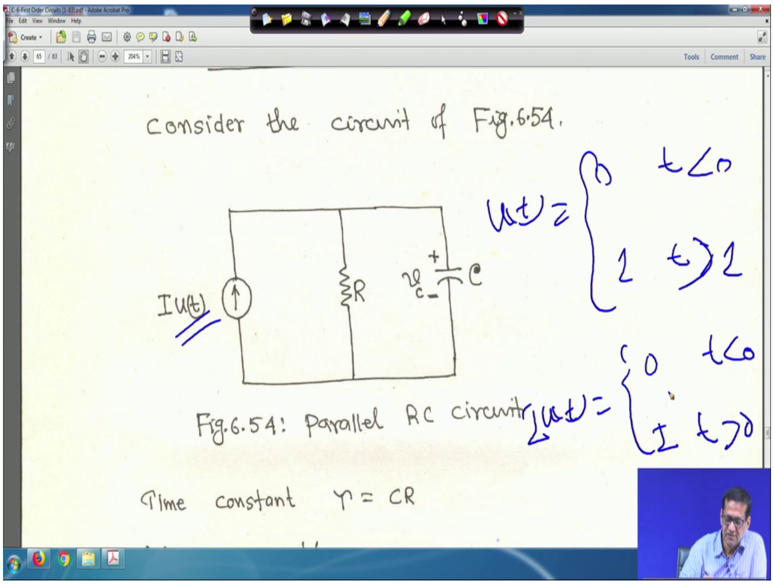

So now, consider that your the circuit that is your parallel RC circuit, but a current source is there. So, in this case also, it is i u t is given and you know that u t is equal to it is 0, for t less than 0, it is 1 for t greater than 1, right. Therefore my i u t is equal to 0 for t less than 0 and it is i for t greater than 0. So, step input right and this is a parallel R C circuit. So, let me clear it.

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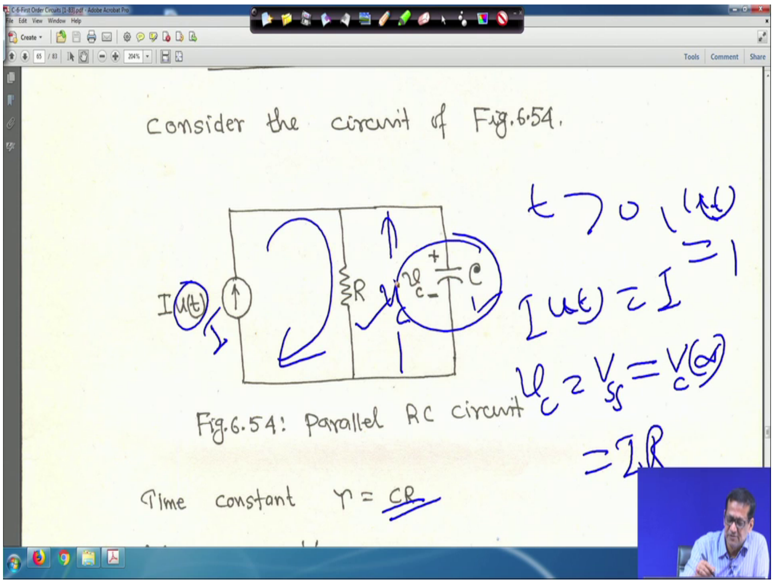

So, if is suppose when your what you call that when for t greater than 0, right, so I u t will be is equal to I because u t is equal to 1, right. So, in this case if u t is not there for t greater than 0 say, then it represent by for your understanding it is I, right. And if that is your what you call, if the if the circuit has it is steady state, then this will act as a open circuit, this will act as a open circuit; that means, the voltage across this capacitor v c that is we call it what you call that is your steady state response or your that is v c infinity, right. So, in that case v c will be v steady state is equal to same thing v c infinity.

And this I current will be flowing like this, so, it will be is equal to I into R, right. I hope this is understandable to you. So, in this based on that and tau is equal to your time constant, that is C into R, right. So, using this natural response and forced response, so we will try to obtain the your what you call expression of the your voltage or current whatever you want. So, let me clear it. So, in this case your, so initially that is why I told you that v c your and initially we will assume that initial voltage of the capacitor was v c 0 is equal to v 0 that we are assuming it, it may be 0 also, but we are assuming that v c 0 is equal to v 0 right and at steady state I told you, v c infinity is equal to I R.

So, we know this from equation 648, that v c 2 sorry v c t is equal to v c infinity plus v c 0 minus v c infinity e to the power minus t by tau, right. Therefore, v c t is equal to I R, v c infinity we have got I R plus v 0 because initial value of capacitor we assume it is charged at v 0. So, v 0 v c infinity is I R. So, v 0 minus I R e to the power minus t by tau or we can right v 0 e to the power minus t by tau, I mean this v 0 into multiplied by e to the power minus t by tau, then plus I R minus I R e to the power minus t by tau. So, take I R common, 1 minus e to the power minus t by tau. This is equation 63.

So, v n is natural response that is v 0 e to the power minus t by tau like a source pre circuit and v f is equal to forced response that is I R into 1 minus e to the power minus t by tau, right. So, after this, so this is simple parallel R C circuit we take current source. So, I step your step input right. So now, next is the step response of an R L circuit. So, this is the circuit, this next we will go for step response of an R L circuit.

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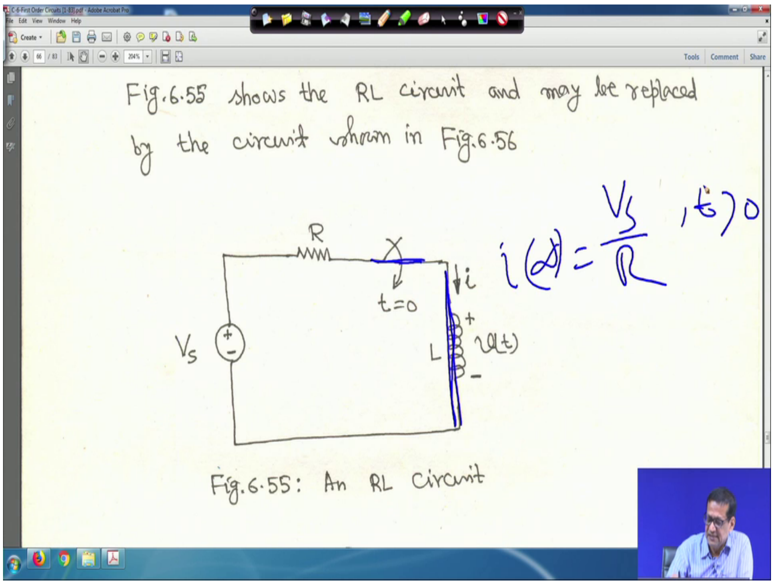

So, in this case that figure 55 shows R L circuit and may be replaced by the circuit shown in figure 56. So, this is your R L circuit. Suppose that t is equal to 0, the switch is closed right.

So, this is a your what you call a simple the way we considered the R C circuit. Here also, it is simple RL circuit, this is a L and voltage across L that is inductor is v t right and polarity is marked L, I sorry plus minus and this current i is flowing. Now, your in general when the switch before moving further, when the switch is closed it will be like this and at steady state, this inductor acts as a short circuit, right.

A steady state inductor acts as a short circuit and if this is the current i that means, our steady state I infinity will be V s upon R, right. That is of course, t greater than 0 after switching switch is closed, right. Inductor acts as your what you call that your short circuit, capacitor acts as open circuit to steady state of DC. So, let me clear it.

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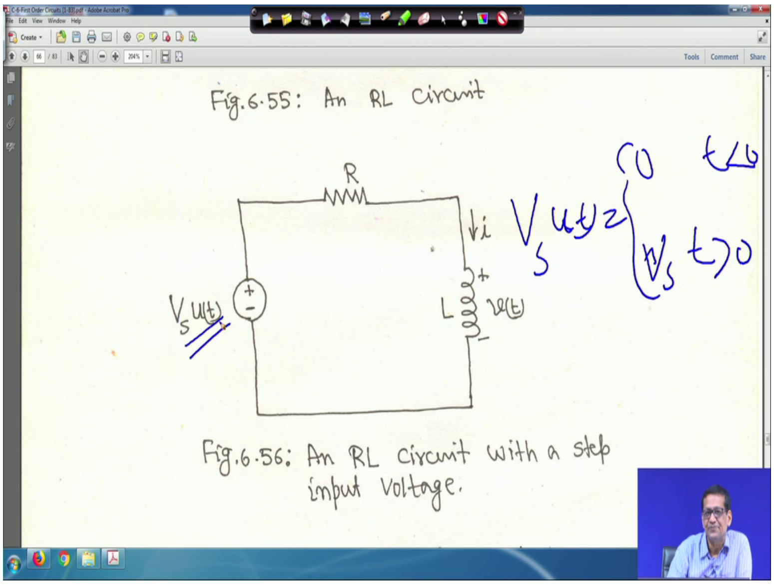

So, this circuit it can be redrawn like this, that this will be v step input V s u t and r and this is L same as before v t at same thing it is same thing that V s u t right is equal your 0 for t less than 0 and it is it is equal to V s for t greater than 0, right. So, that is why this is written as V s u t. So, let me clear it. So, our objective is to find the inductor current i, this is the inductor current i, right. As the circuit response, we will use the technique that equation 59 to 62, that is this equation to 59 to 62; that means, this your what you call this equation this equations that from 59, 60, 61, 62 you using this v is equal to v n plus v f.

Similarly, we apply for i is equal to i n plus i f, same concept. So, in this case in this case, what we will do that we write i is equal to i n plus i f, i n is the natural response of the current and i f is the forced response of the current. So, natural response is always a decaying your what you call exponential, right. So, that we have seen that it is also we can write i n is equal to say a into e to the power minus t upon tau, right and it is L R your simply L R, R L circuit.

So, tau is equal to L by R, the time constant say this is equation 67 and a is a constant which you have to determine and we know that natural response is always a what you call decaying exponential.

So, directly I write i n is equal to a into e to the power minus t by tau right. So, A can be determined later, we will see that. Now, forced response is the value of the current after a long time after the switch figure is closed, right. So, we always know that after 5 time constant, that is 5 tau, the natural response essentially dies out. So, at that time t greater than equal to 5 tau, so, inductor becomes a short circuit and the voltage across it is 0, right.

So that means, I told you initially for this circuit when switch is your what you call for this circuit, when switch is closed and placed it for a long time, so inductor acts as a short circuit. So, naturally your what you call you can that your i infinity is equal to V s upon R right. So, this is your i f that is the forced response, this is nothing but i infinity is equal to what you call V s upon R right.

So, if it is, so then equation 67 and 68, it can be written as i is equal to, it is natural response a is equal a into e to the minus t by tau and this is your forced response V s upon R, that is the steady state one, that is the i i f is equal to nothing but i infinity, right.

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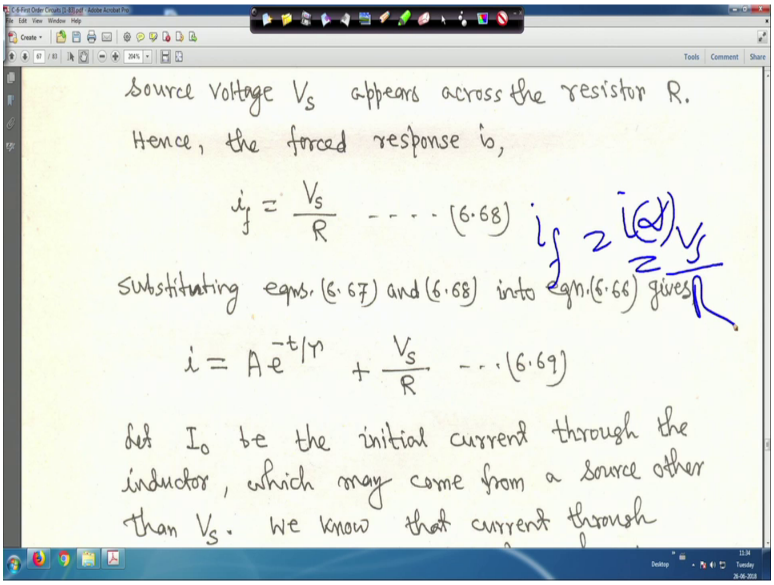

So, i f the forced response is nothing but I steady state that is i infinity that is V s upon R, right. So, that and we assume let me clear it we assume that i 0 be the i 0 be the initial current to the inductor, right.

We will assume that which may come from a source other than V s. So, we know that current through inductor cannot change instantaneously; that means, at the time of switch, at the time switch is closed i 0 plus is equal to i 0 minus is equal to i 0 because current through inductor cannot change instantaneously. Therefore, at t is equal to 0, i is equal to i 0. So, in equation 69 becomes that mean here your what you call here, at t is equal to 0, you put i is equal to capital I suffix 0, right.

So, if you do so, that I 0 is equal to then we will get capital I 0, we will get a plus V s upon r from which you will get a is equal to I 0 minus V s upon R. This A value in put is in equation 69, then you will get i t is equal to i 0 minus V s upon R e to the power minus t up t by tau plus V s upon R. This is equation 72, therefore, it is equal to this term writing first V s upon R plus i 0 minus V s upon R e to the power minus t upon tau, right. So, this equation is nothing but look into that this is i infinity V s upon R is i infinity here, right.

And this i 0 is the initial value that is your i 0, again it is i 0 minus I infinity e to the power minus t upon tau same as before right; for i 0 is equal to capital I suffix 0 and I told you i infinity is equal to V s upon R, right. So, note that if the switching takes place at time t is equal to t 0, if it is at t switching takes place at t is equal to t 0, then it will be then it will be that it will be i t 0.

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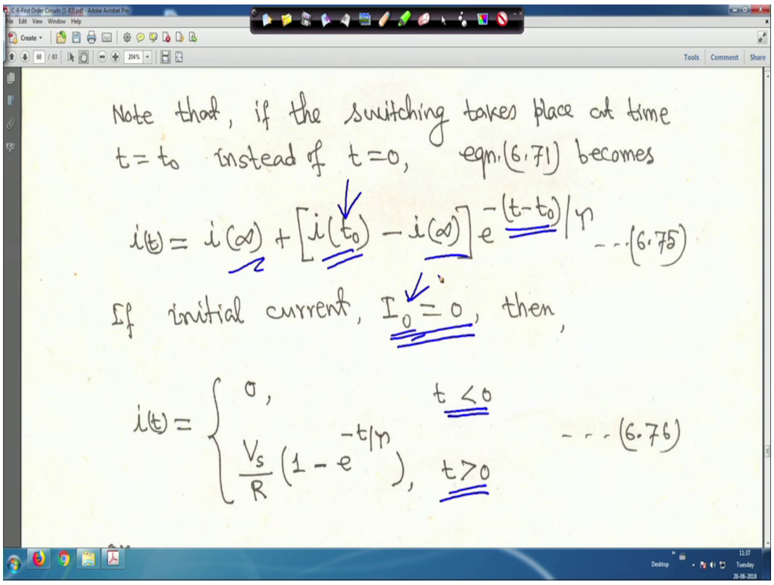

Because at t is equal to t 0, you have to get the initial value that is at t 0 that is it 0. So, i infinity is there, i infinity is there and here instead of e to the power t by tau here, it will be e to the power minus t minus t 0 by tau right. So, if that current I 0 is 0 suppose if initial current is 0, then we can write i t is equal to 0 for t less than 0.

And i t is equal to because I 0 is equal to your what you call, I 0 is equal to 0, then for and then it is V s upon R 1 minus e to the power minus t by tau if t greater than 0 and if we assume the initial current I 0 is equal to 0, this is equation 76, right. So, or we can write simply this thing because it is t less than 0, t greater than 0, you can make it like this i t is equal to V s upon R, 1 minus e to the power minus t upon tau into u t.

So, u t is 0 for t less than 0. So, it will become 0 as if for t greater than 0, u t is 1 it will become this thing. So, this 2 this 2 your what you call current thing can combine on that can be written in one term that is V s upon R, 1 minus e to the power minus t upon tau into u t. This is equation 77, right. So, equation 77 actually give the step response of the R L circuit with I 0 is equal to 0.

The voltage across the inductor can be obtained from equation 6 your 77, right. So, that is v t is equal to l into d i t by d i upon d t. So, you take the derivative of this one, you take the derivative of this one d i d t right. If you do so, you will get V s upon R into L by tau into e to the power minus t upon tau.

But L upon R is tau. So, tau, tau will be cancelled. So, it will be v t is equal to V s, e to the power minus t upon tau, but here u t is attached right; that means, for t less than 0, u t is 0. So, it will be 0 right and for t greater than 0, u t is 1, it will be v t will be is equal to V s e to the power minus t by tau, right. So that means, if you see this, so the step response of i t and this thing.

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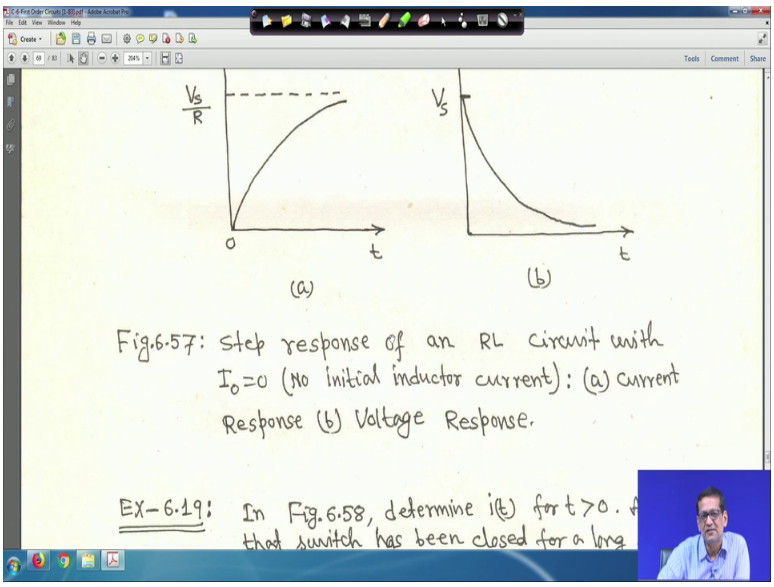

So, when switch is closed for a long time for R L circuit, right.

So, current is the your i infinity is the V s by R. So, current is going to the steady state value V s upon R. Generally, after 5 tau tau is the time constant it will achieve nearly V s upon R. And at for t when t is greater than equal to 5 tau, the V s is exponentially decaying. So, it will be your v t is decaying. So, it is gradually going to 0. So, at t greater than equal to 5 tau, it will become almost 0, right.

So, this is the current plot and this is the voltage plot, for your what you call that is your R L circuit is the step input right. So, this is the step response of an R L circuit with I 0 is equal to 0, no initial inductor current, right; a current response and second is the voltage response so.

Thank you very much. We will be back again.