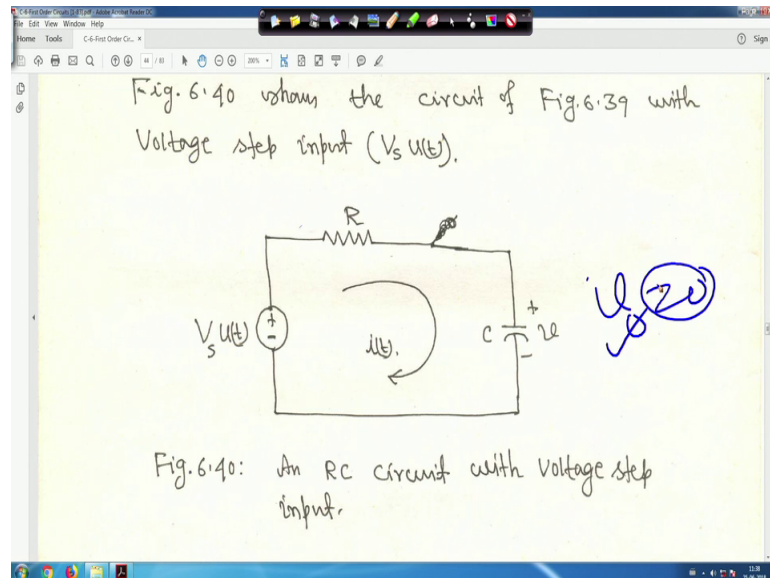


Fundamentals of Electrical Engineering
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Lecture - 33
First order circuits (Contd.)

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So, now this capacitor actually initially assumed, it was charged say some voltage say v_0 not necessarily that it will be charged all the time v_0 can be 0 also. But, we assume that this capacitor initially was charge at v_0 . Just to get generalized expression right. So, what you can do this right.

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Let us assume an initial voltage V_0 on the capacitor. But this is not necessary for the step response. Since the voltage of a capacitor cannot change instantaneously,

$$v(0^-) = v(0^+) = V_0 \quad \dots (6.41)$$

where

$v(0^-)$ = voltage across capacitor just before switching

$v(0^+)$ = voltage across capacitor just after switching.

So, let us assume when initial voltage V_0 on the capacitor, but this is not necessary for the step response right. Since, the voltage of a capacitor cannot change instantaneously that is $v(0^-)$ is equal to $v(0^+)$. What I want to mean for this circuit right for this circuit, that when switch was not close, it was open initially, suppose capacitor was this capacitor was charged at voltage $v(0^-)$ right.

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a constant dc voltage source.

Diagram showing a circuit with a DC voltage source V_s , a resistor R , and a capacitor C . A switch is shown closing at $t=0$. The voltage across the capacitor is labeled v_c .

$V_0 = v_c^+$

Fig. 6.39: An RC circuit.

Fig. 6.40 shows the circuit of Fig. 6.39 with voltage step input ($V_s u(t)$).

Suppose, this voltage if I write say it is say your $v(0^-)$ just I making it say $v(0^-)$. So as soon as the switch is close that your what you call capacitor voltage cannot change

instantaneously that means, it will be actually $v(0^-)$ plus. Just before switching and just after switching, it cannot change instantaneously; earlier also we have discussed this. So, $v(0^-)$ will be $v(0^+)$; this will just before switching, and just after switching right. So, let me clear this, because, sorry just hold on.

So, now let us come to the circuit. So, it is given $v(0^-)$ is equal to $v(0^+)$, this way I have written is equal to $v(0)$. Just your before switching and after switching that I can if voltage capacitor cannot change instantaneously, so this is equation 44, where $v(0^-)$ voltage across capacitor just before switching, and $v(0^+)$ voltage across capacitor just after switching right.

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$v(0^+) =$ Voltage across capacitor just after switching,

By applying KCL in Fig. 6.40, we have

$$\frac{v - V_s u(t)}{R} + C \frac{dv}{dt} = 0$$

$$\therefore \frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} u(t) \quad \dots (6.45) \quad i(t) = \frac{V_s u(t) - v}{R}$$

For $t > 0$, Eqn (6.45) becomes

Now, applying KVL, I told you it is v minus your what you call $V_s u(t)$ upon R plus $C \frac{dv}{dt}$, this already I have told you. And for t greater than 0, this $U(t)$ is 1. So, it will be basically v minus V_s upon R plus $C \frac{dv}{dt}$ are equal to 0. Actually, it was circuit was like this know I am drawing it here. This is your v ; your $U(t)$ right; this is $V_s u(t)$, this was your R right; and this side it was the capacitor, and plus minus this voltage was v right. So, some point if you current direction was taken like this, this $i(t)$ was taken like this right, $i(t)$ was taken like this, and $i(t)$ is equal to we wrote know, $i(t)$ is equal to that is your $i(t) = \frac{V_s u(t) - v}{R}$ that is your $V_s u(t) - v$ divided by R minus this V divided by R . This is in this direction clockwise.

Now, if you take at any point here say this point is a, now if I take one current is going like this just, you think like this. And one current i_c is going like this, although i_c is equal to minus thing. So, this point if you apply your what you call that your KCL, then this as direction is changed, so it will be v minus $V_s u(t)$. So, v minus $V_s u(t)$ by R divided by R plus your c this capacitor c into dv by dt is equal to 0 right. Now, for when this as soon as the switch is closed, for t greater than 0 that $u(t)$ is equal to 1 that is why this $u(t)$ you put here it is written. But, after that this $V_s u(t)$ $V_s u(t)$ it is written like this right, but $u(t)$ is equal to your 1 we will come to that.

(Refer Slide Time: 04:13)

Handwritten derivation on a digital whiteboard:

$$\frac{v - V_s u(t)}{R} + c \frac{dv}{dt} = 0$$

$$\therefore \frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} u(t) \quad \dots (6.45)$$

For $t > 0$, Eqn.(6.45) becomes

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} \quad \dots \text{(6.45 crossed out)}$$

So, this is actually this one if you simplify, it is dv by dt plus v upon Rc is equal to V_s upon Rc $u(t)$. Now, for t greater than 0 that means at that time $u(t)$ is equal to 1 for unit function we have seen. For t less than 0, $u(t)$ is 0; for t greater than 0, $u(t)$ is equal to 1 so that is why in the in the derivation now $u(t)$ is not there. Because, it is we are trying to obtain the response for t greater than 0, little bit understanding is required. For t greater than 0 your objective is to find out the response for t greater than 0.

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For $t > 0$, Eqn.(6.45) becomes

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} \quad \dots \text{--- (6.45)}$$
$$\therefore \frac{dv}{dt} = -\frac{(v - V_s)}{RC}$$
$$\therefore \frac{dv}{v - V_s} = -\frac{dt}{RC} \quad \dots \text{--- (6.46)}$$

Integrating both sides

So, this equation 45 becomes, so u is 1. So, it is dv by dt plus v upon RC is equal to V_s upon RC or dv by dt upon simplification, v your what you call minus v minus V_s upon RC or $d v$ upon v minus V_s is equal to minus dt upon RC , this is equation 46.

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$$\therefore \frac{dv}{v - V_s} = -\frac{dt}{RC} \quad \dots \text{--- (6.46)}$$

Integrating both sides

$$\int_{V_0}^{v(t)} \frac{dv}{(v - V_s)} = -\frac{1}{RC} \int_0^t dt$$
$$\therefore \ln(v - V_s) \Big|_{V_0}^{v(t)} = -\frac{t}{RC} \Big|_0^t$$

Now, you integrate both side. So, initially capacitor voltage across initial voltage across the capacitor was v_0 ; and at time t , it is v_t . So, you are making v_0 to v_t dv upon v minus V_s is equal to minus 1 upon RC that integration 0 to t dt right. So, if you

integrate this with natural log $\ln v$ minus V_s limit your integration limit is v_0 to v_t is equal to minus t upon RC here also 0 to t .

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$$\int_{V_0}^{v(t)} \ln(v - V_s) = \frac{-t}{RC}$$

$$\ln\left(\frac{v - V_s}{V_0 - V_s}\right) = \frac{-t}{RC}$$

So, this is equal to $\ln v$ minus V_s upon V_0 minus V_s is equal to minus t upon RC that means v_0 minus, so it is natural log means log of base e right. So, it is v minus V_s upon V_0 minus V_s is equal to e to the power minus t upon τ ; τ is equal to RC the time constant of RC circuit, this is RC . This is equation 47 right.

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$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}, \quad \tau = RC \quad (46)$$

$$\therefore v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \quad t > 0 \quad (47)$$

Thus

So, or $v(t)$ I mean upon simplification just cross multiply and simplify, you will get $v(t)$ is equal to actually what we what I am doing is this the up to this it is written your, what you call all the time not putting your, what you call $v(t)$, $v(t)$, $v(t)$ right. But, now this v is replaced by, it is understandable to you, it is function of time it is understandable to you. So, that it is $v(t)$ is equal to $V_s + (V_0 - V_s)e^{-t/\tau}$ for $t > 0$ right.

(Refer Slide Time: 06:37)

The image shows a handwritten derivation on a digital whiteboard. At the top, the equation $\therefore v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$ is written for $t > 0$, labeled as (6.48). Below this, the word "Thus" is written. A large curly brace defines the piecewise function for $v(t)$: $v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$, labeled as (6.49). At the bottom, a note says "Eqn (6.49) gives the total response of the".

Therefore, you can write because $v(t)$ is equal to $v(0)$, initially capacitor was charged at voltage $v(0)$ that is for t less than 0 that means complete response of voltage, you can write $v(t)$ is equal to V_0 for t less than 0 right. And, $v(t)$ is equal to this expression $V_s + (V_0 - V_s)e^{-t/\tau}$ for $t > 0$. This is we call the complete response much more we will see very soon right.

So, this is your what you call this way you can write, so for that this is called the complete response, that is in general minus infinity to your what you call that $t > 0$ means plus infinity. But, you know that say for earlier we have seen that up to 5 tau time constant, basically that responses or the I could say that dynamic circuit responses, it reaches to the steady state right for t is equal to say 5 tau. Because, it becoming this your what you call it is becoming your what you call less than 1 percent of the peak value right.

(Refer Slide Time: 07:38)

Eqn.(6.49) gives the total response of the RC circuit to a sudden application of dc voltage source, assuming the capacitor is initially charged.

$$v(t) = \begin{cases} V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases} \quad \dots (6.49)$$

So, 49 gives the total response of the R C circuit to a sudden application of dc voltage source, that is R C circuit we apply assuming the capacitor is initially charged right. Now, if it is assuming that initially uncharged, that means V_0 is equal to 0.

(Refer Slide Time: 07:51)

RC circuit to a sudden application of dc voltage source, assuming the capacitor is initially charged.

Let $V_0 = 0$ (assuming that the capacitor is initially uncharged), then Eqn.(6.49) reduces to

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases} \quad \dots (6.50)$$

So, if V_0 is equal to 0 in this expression, if V_0 is equal to we put it, in that case $v(t)$ will be is equal to 0, for t less than 0. And here also if you put V_s is equal to your what you call V_0 is equal to 0, it will V_s into 1 minus in bracket 1 minus e to the power minus t by τ .

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is initially charged.

Let $V_0 = 0$ (assuming that the capacitor is initially uncharged), then Eqn(6.49) reduces to

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases} \quad \dots (6.50)$$

Alternatively, Eqn(6.50) can be written as:

So, here it is 0 for t less than 0, and it is $V_s(1 - e^{-t/\tau})$ for t greater than 0, this is equation 50 right. So, that means alternately this equation you can write this, this equation other way that it is $v(t) = V_s(1 - e^{-t/\tau})u(t)$. Because, $u(t)$ written for $u(t)$ you know for t less than 0, $u(t)$ is equal to 1.

(Refer Slide Time: 08:27)

$$v(t) = V_s(1 - e^{-t/\tau})u(t) \quad \dots (6.51) \quad (47)$$

Eqn(6.51) gives the complete step response of the RC circuit when the capacitor is initially uncharged.

Current through the capacitor is obtained from Eqn(6.50), i.e.

$$i(t) = C \frac{dv}{dt} = \frac{C}{\tau} V_s e^{-t/\tau}, \quad t > 0$$

So, if you when t sorry $u(t)$ is equal to 0, for t less than 0. So, when t less than 0, $u(t)$ is equal to 0, then $v(t)$ will be 0, so it is 0. And for t greater than 0, $u(t)$ is 1; so when $u(t)$ is

equal to 1, it will be $V_s (1 - e^{-t/\tau})$. So, combining these two, we can write $v(t)$ is equal to $V_s (1 - e^{-t/\tau})$ for $t > 0$, this is equation 51. Hope this is understandable to you right, not very simple thing. So, this figure this equation 51 is that gives the complete response of the RC circuit, when the capacitor is initially uncharged right. So, current through the capacitor is obtained from equation 50, that is nothing but $i(t) = C \frac{dv}{dt}$. So, you take the derivative of your what you call your for $t > 0$, you take the derivative of this equation that $\frac{dv}{dt}$ for this $1 - e^{-t/\tau}$ for $t > 0$.

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Current through the capacitor is obtained from Eqn. (6.50), i.e.,

$$i(t) = C \frac{dv}{dt} = \frac{C}{R} V_s e^{-t/\tau}, \quad t > 0$$

$$\therefore i(t) = \frac{V_s}{R} e^{-t/\tau} u(t) \quad \text{--- (6.52)}$$

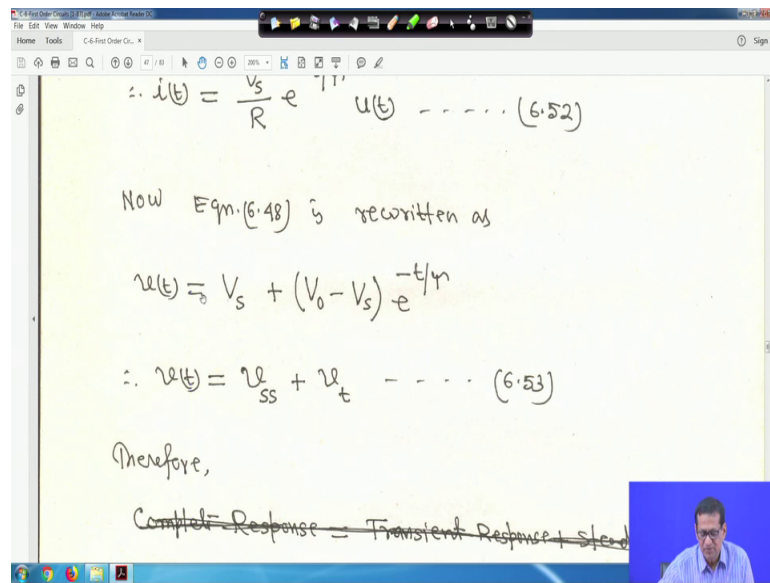
Now Eqn. (6.48) is rewritten as

$\tau = RC$
 $\frac{C}{R} = \frac{1}{RC} = \frac{1}{\tau}$

So, when you do so, $i(t)$ is equal to $C \frac{dv}{dt}$ that is here you have to write $t > 0$ right, that is C by τ then $V_s e^{-t/\tau}$, you take the derivative of this equation right. So, it will be your basically and your this is C by τ $V_s e^{-t/\tau}$. And τ is equal to RC right, so here τ is equal to say CR .

So, if you put τ is equal to CR , so C by τ is equal to C by CR ; C, C cancel, it will be 1 upon R that is why V_s upon $R e^{-t/\tau}$ right. So, and if you and in generally if you want, then $i(t)$ is equal to V_s upon $R e^{-t/\tau}$ again right. Here, here it is $u(t)$ same philosophy same philosophy for $t < 0$ $u(t)$ is equal to 0 , and for because circuit was open, so i was 0 . And for $t > 0$, $u(t)$ is equal to 1 , so that is why again it is $u(t)$ is multiplied right, because at that time $t < 0$ the switch was open right.

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The screenshot shows a digital whiteboard with the following content:

$$\therefore i(t) = \frac{V_s}{R} e^{-t/\tau} u(t) \dots\dots (6.52)$$

Now Eqn. (6.48) is rewritten as

$$v(t) = V_s + (V_0 - V_s) e^{-t/\tau}$$
$$\therefore v(t) = v_{ss} + v_t \dots\dots (6.53)$$

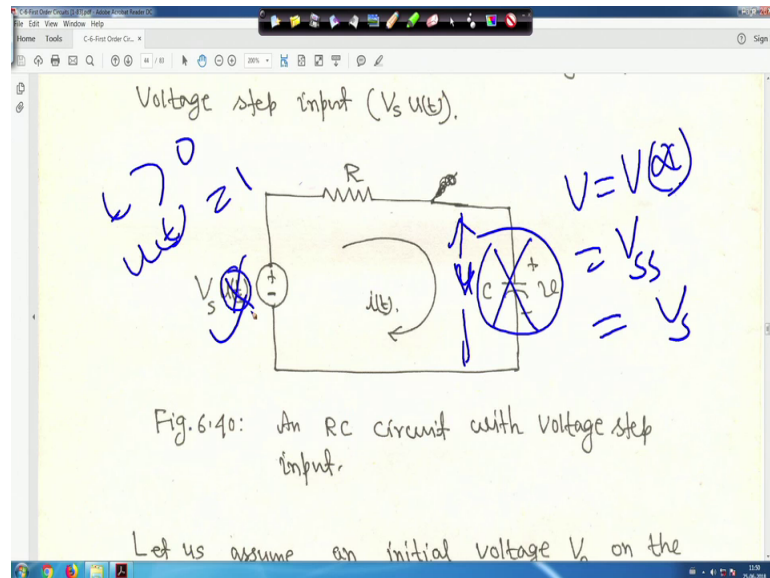
Therefore,

~~Complete Response = Transient Response + Steady State Response~~

So, now equation 48 is rewritten as, we have again this equation 40 if this one $v(t)$ is equal to $V_s + (V_0 - V_s)e^{-t/\tau}$, that means $v(t)$ is equal to can be written as $v_{ss} + v_t$ that is your steady state response plus transient response. So, therefore, complete response the $v(t)$ is the complete response is equal to steady state response plus transient response $v_{ss} + v_t$ right. This is your what you call v_{ss} is the steady state response and this is transient response.

Because, when t tends to infinity, if you look into that, when t tends to infinity, the circuit reaches to steady state. If you come to the circuit from the circuit only, because this we have to use again and again for solving your numericals. And we have to if you come to this circuit right, so our capacitor when this switch is closed for long time for do dc, because this switch is closed for long time, the capacitor act as your what you call open circuit right.

(Refer Slide Time: 11:56)



So, this is not required at that time no need to do that, because $u(t)$ at that time $u(t)$ is equal to 1 for t greater than 0. And if it is and that means, this is only V_s this is not required suppose V_s . And capacitor is open, then what is the voltage across this what is the voltage across capacitor is open. So, basically v is equal to your what you call in that steady state voltage in v infinity, sometimes, we call V_{ss} steady state right is equal to actually V_s this source voltage right. So, this is from your intuition, you can find out, because, to dc when switch is closed for long time, the capacitor act as a open circuit. So, this is actually just I am making it like this, and v will be is equal to V_s . And at that time circuit representation is like this, so $U(t)$ is equal to 1 right, so anyway.

(Refer Slide Time: 13:11)

Now Eqn. (6.48) is rewritten as

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$

$t \rightarrow \infty$
 $V(t) = V(\infty)$
 $= V_s = V_{ss}$

$$\therefore v(t) = \underline{v_{ss}} + \underline{v_t} \quad \dots (6.53)$$

Therefore,

~~Complete Response = Transient Response + Steady-state Response~~

~~where~~

Complete Response = Steady-state Response + Transient Response

So, again we will come back to this. So, this complete response is here also here also if t tends to infinity, here also if you make in this expression t tends to infinity, the second term will vanish. And it will be your I mean, if you if you move like this, just hold on. If you if t tends to infinity that means this term will vanish; that means, at that time V t will be V infinity that is your steady state value that is your V_{ss} will become your V_s this value right. So, that is your steady state value that is this is that means this is your steady state value V_s and this part is a transient part and this part is a steady state part. Let me clear it right.

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~~where~~

Complete Response = Steady-state Response + Transient Response

$$= v_{ss} + v_t \quad \dots (6.54)$$

Where

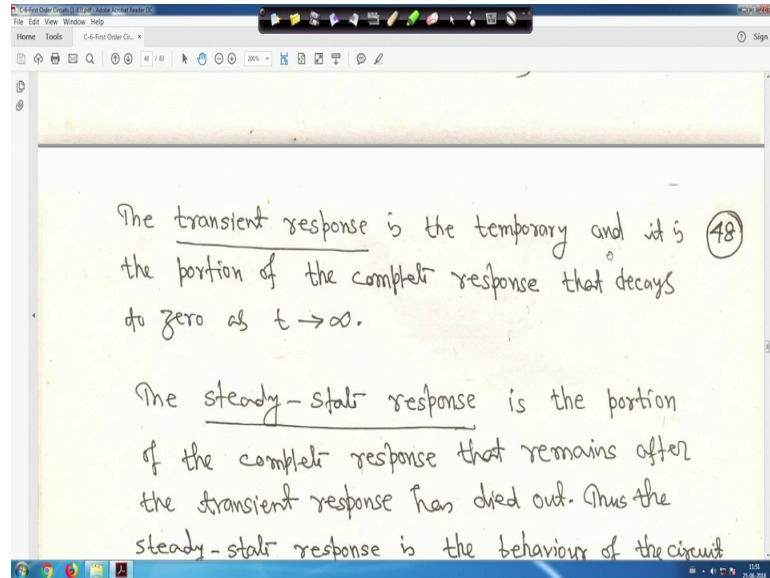
$$v_{ss} = V_s \quad \dots (6.55)$$

$$v_t = (V_0 - V_s)e^{-t/\tau} \quad \dots (6.56)$$

The transient response is the...

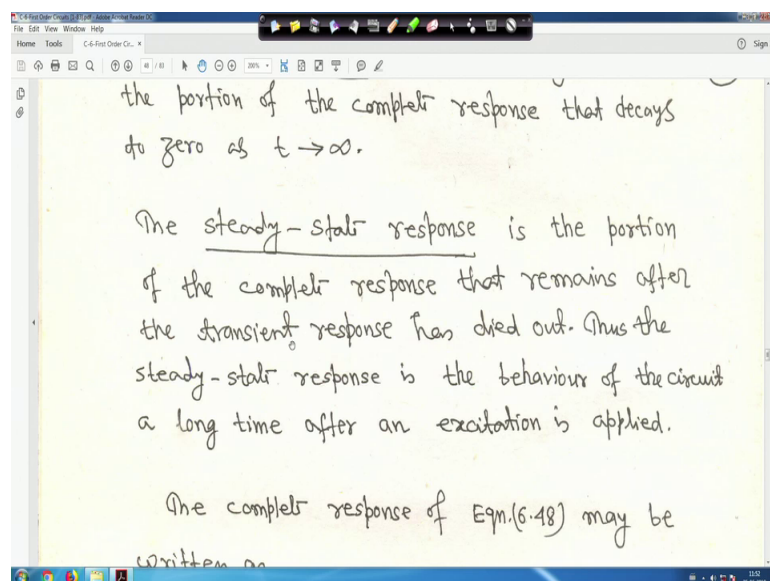
So, that is why V_{ss} is equal to V_s . And this is transient part, this equation 55; this is equation 56 right.

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So, transient response is the temporary and it is the portion of the complete response that is your that your, what you call that decays to 0 as t tends to infinity. Because, when t tends to infinity your I told you, when t tends to infinity, this part term will vanish right, so that means transient will vanish only left out portion will be that is steady state.

(Refer Slide Time: 14:16)



So, that require therefore and it is the portion of the complete response that decays to 0 at t tends to infinity. The steady state response is the portion of the complete response that remains after the transient response has died out, I mean transient is over right. Thus, the steady state steady state response is the behavior of the circuit a long time after an excitation is applied, that means you that your voltage source suppose it is switched on, and your steady state will achieved after long time right. So, the complete response of equation 48 may be written as, this can be written as something like this. if you come to this one, if you come to this response this one, I mean this one look this one, it is this is the thing you are obtaining at infinity and this is your initial value.

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Now Eqn. (6.48) is rewritten as $t \rightarrow \infty$

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$

$V(\infty) = V_s$
 $V(0) = V(0)$

$$\therefore v(t) = V_{ss} + v_t \quad \dots (6.53)$$

Therefore, $v(t) = V(\infty) + (V(0) - V(\infty))e^{-t/\tau}$

~~Complete Response = Transient Response + Steady-state Response~~

And this is again your at this V_s actually your, at t tends to infinity at t tends to infinity, your V infinity is equal to V_s right, that is steady state value. And initial value V_0 was is equal to I can make it V_0 right that means, this equation v_t , I can write like this V_s is equal to steady state value, that is V infinity plus I can make V_0 minus V infinity right, then your then your this one e to the power minus t by τ . That means, v_t will be V infinity plus v_0 minus v infinity e to the power minus t by τ . That means if you know, the initial value, if you know the final value, the steady state values, and if you know the time constant, easily you can compute v_t right, this you have to keep it in your mind for solving numerical. And for the for the exam also, you have to keep it in your mind right.

(Refer Slide Time: 16:10)

steady-state response is the behaviour of the circuit a long time after an excitation is applied.

The complete response of Eqn.(6.48) may be written as

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

where $v(\infty) + [v(0) - v(\infty)]e^{-(t-t_0)/\tau}$ (6.57)

$v(0)$ = initial voltage at $t = 0^+$
 $v(\infty)$ = final or steady-state value.

So, that means this one we can write the v at t is equal to v infinity, this is the final value v infinity plus initial value that is v 0 minus final value v infinity into e to the power minus t by τ , this is equation 57 right. So, v 0 is the initial voltage at say t is equal to 0 plus just after switching and because capacitor to capacitor the voltage cannot change instantaneously. So, v 0 minus is equal to v 0 plus. And v infinity final or steady state value right, so this equation v t can be written as v infinity plus v 0 minus v infinity bracket close e to the power minus t by τ right. Therefore, your note that if the switch changes position at t time t is equal to t_0 , so whatever we are considering that that is at t is equal to 0.

(Refer Slide Time: 16:53)

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \dots (6.57)$$

where

$v(0)$ = initial voltage at $t = 0^+$

$v(\infty)$ = final or steady-state value.

Note that if the switch changes position at time $t = t_0$ instead of $t = 0$, there is a time delay in the response so that Eqn(6.57) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau}$$

Now, suppose if the switch changes position at time t is equal to t_0 , t_0 is a naught is equal to 0 for example, so instead of t is equal to 0. There is a time delay in the response of that equation 57 right. So, there will be a time, in this case what will happen, in this case what will happen, before going to this thing in this case this will remain as say v infinity plus now this one, this was your there when t is equal to 0.

Now, switch this on at t is equal to t_0 , therefore initial values will be v , it is $v t_0$ minus v infinity will be there as it is, it is there as it is, then e to the power minus then t minus t_0 divided by τ . So, if this was actually a t is equal to 0, switch was just closed at t is equal to 0, 0 plus say right. So, but now if we make it t is equal to t_0 , then it will be instead of v_0 , it will be $v t_0$ right. But v in steady state will remain same v infinity plus your $v t_0$ minus v infinity then e to the power minus instead of t , it will be t minus t_0 divided by τ right. So, let me clear it.

(Refer Slide Time: 18:17)

delay in the response so that Eqn(6.57) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)] e^{-(t-t_0)/\tau} \quad \dots (6.58)$$

where
 $v(t_0) =$ initial voltage at $t = t_0^+$

Ex-6.11: In Fig.6.41, the switch has been in (A)

So, this is the expression $v(t)$ is equal to $v(\infty)$, what I told plus $v(t_0) - v(\infty)$ then e^{-t} to the power minus $t - t_0$ by τ this is equation 58. Now, where $v(t_0)$ initial voltage at t is equal to t_0^+ . If t_0 is equal to 0, then it will be $v(0)$ right. Here, also it will be $1 - e^{-t/\tau}$, if t_0 is equal to 0. So, this is that your what you call the RC circuit, that your what you call that your initially you have studied the source free, now you are studying that your circuit is involved with sources right.

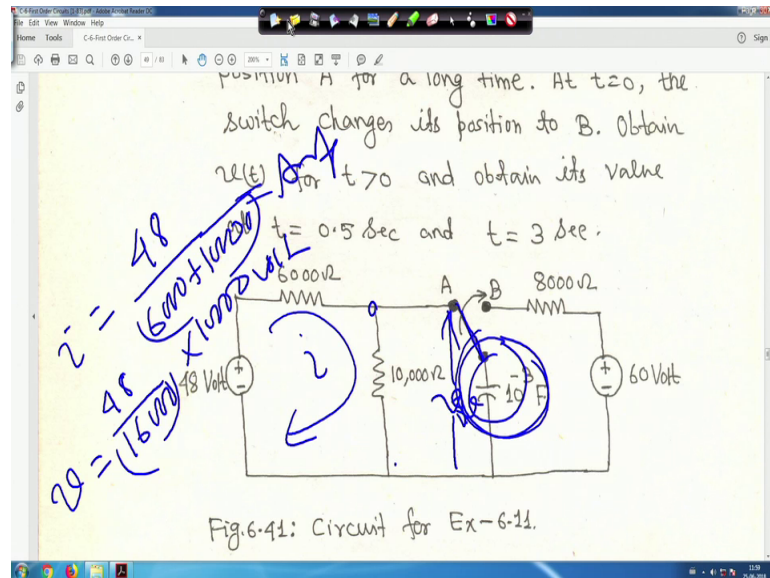
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Ex-6.11: In Fig.6.41, the switch has been in (A) position A for a long time. At $t=0$, the switch changes its position to B. Obtain $v(t)$ for $t > 0$ and obtain its value at $t = 0.5$ sec and $t = 3$ sec.

6000Ω A B 8000Ω

So, now look the example, so in figure this is 41, the switch has been in a position a for long time this that means, if you look into the circuit that, this is this was in this position for long time at position A, this is A right.

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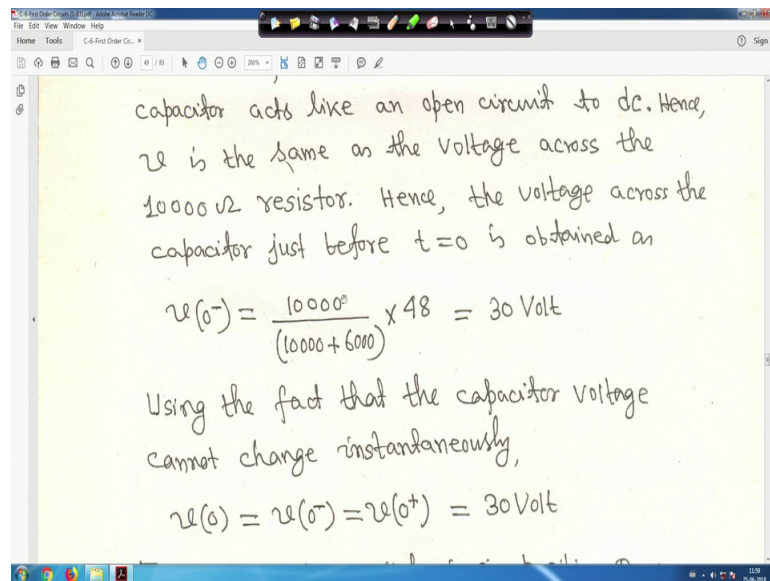
Now, at time t is equal to 0, the switch changes its position to B. Now, at t is equal to 0, the switch is your changing its position from A to B right, obtain $v(t)$ for t greater than 0, and obtain its value at t is equal to 0.5 second; and t is equal to 3 second. Now, this switch was at position for long time at A right, that means that means circuit at the time for that case circuit was a steady state, because, switch was at this position. And for long time, that means this capacitor is open circuited, and then voltage your this voltage $v(t)$ actually you have to find out across with your this thing this is the voltage B across the capacitor.

And capacitor value is 10 to the power minus 3 farad, so at that time this was open right. But, that means you have to find out the initial values of the your, what you call voltage across the capacitor initial, because switch is moved now. After that at t is equal to 0 switch will move from A to B. So, in that case, what will happen that first as it this is open circuit that means this voltage this voltage was $v(0)$ right. And say this is $v(0)$ and this is $v(0)$ right.

So, in that case, it is open so first you find out what is the current here, so current through this is i . And i equal to your 48 right divided by 6000 plus your 10000, this is

your writing on it all ampere right. And then what is the voltage across the capacitor, because capacitor is open circuit, this is open circuit. Therefore, your v is equal to you your this is the current 48 6000 plus 10000, this is 16000 into this is this point you find out the voltage across capacitor into your 10000 this much of volt. So, this is your v right, because switch was at this position, so capacitor was open circuited for long time. So, now let me clear it.

(Refer Slide Time: 21:20)



capacitor acts like an open circuit to dc. Hence, v is the same as the voltage across the 10000 Ω resistor. Hence, the voltage across the capacitor just before $t=0$ is obtained as

$$v(0^-) = \frac{10000}{(10000+6000)} \times 48 = 30 \text{ Volt}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 30 \text{ Volt}$$

So, whatever, whatever we have made it that this is whatever I have told that 10000 by 6000 plus 10000 48, it will be 30 volt right. That means, just before switching $v(0^-)$, the voltage that initial voltage across the capacitor was 30 volt, now switch is closed right.

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$$v(0^-) = \frac{10000}{(10000+6000)} \times 48 = 30 \text{ Volt}$$
 Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 30 \text{ Volt}$$
 For $t > 0$, the switch is in position B. Hence,

$$\tau = (8000 \times 10^{-3}) = 8 \text{ sec.}$$

Therefore, say your that means, your using the fact that the capacitor voltage cannot change instantaneously, so which switch is close $v(0)$ is equal to $v(0^-)$ is equal to $v(0^+)$ is equal to 30 volt right. So, now switch has moved from this position to that position right. So, now switch has moved here this part is not there, switch has moved from this position, so this part is not there.

(Refer Slide Time: 22:05)

at $t = 0.5 \text{ sec}$ and $t = 3 \text{ sec}$.
 The circuit diagram shows a 48V DC source on the left, a 6000Ω resistor in series with the source, and a switch that can connect to position A or B. Position A connects the circuit to a 10,000Ω resistor and a 10F capacitor in parallel. Position B connects the circuit to an 8000Ω resistor and a 10F capacitor in parallel. A 60V DC source is also shown on the right side of the circuit.
 Fig.6-41: Circuit for Ex-6-11.

$$\tau = 10^3 \times 8000 = 8 \text{ sec}$$

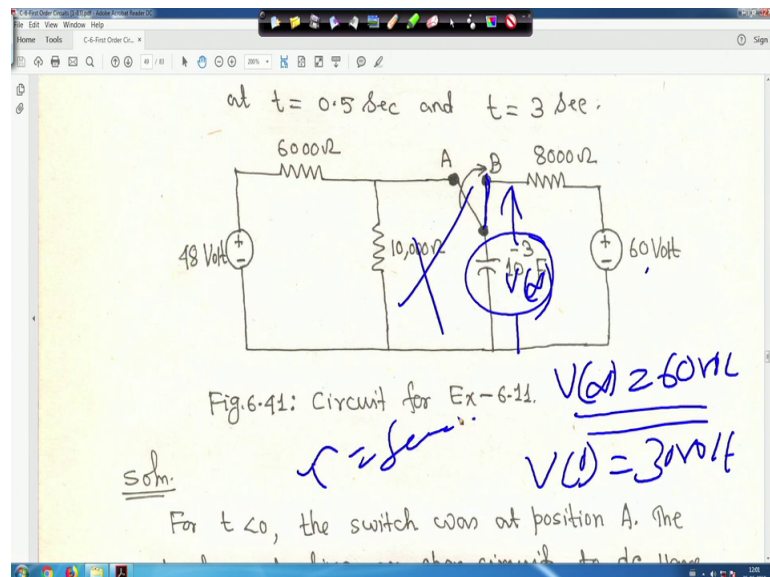
 soln.
 For $t < 0$, the switch was at position A. The

So, capacitor is your 10 to the power minus 3 farad and 8000 ohm. So, first you have to find out the time constant of the circuit, it will be then 10 to the power minus 3 into your

8000 right, so it is C R. So, it will become your 8 second right, this will be tau, because switch has moved from this to that so just let me clear it. So, in this case tau is equal to I told you 8 second. Now, since the capacitor acts like an open circuit to dc, at the steady state V_{∞} is equal to 60 volt.

Now, come to this circuit, now when switch position is your, what you call switch position is like this, this site is not there. So, switch is close at B moved from B. And for long time, that means capacitor will act as your, what you call an open circuit; at that time what is the value, that is the V_{∞} right.

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So, as it is open circuit, so V_{∞} is the final value will be just 60 volt, because this 60 volt is applied across, this and switch was closed at this position for long time that means this is open right. So, V_{∞} will be 60 volt and initial voltage V_0 your 30 volt we got right and tau, we got it is 8 second right. So, we will apply that formula. So, let me clear it.

(Refer Slide Time: 23:37)

Since the capacitor acts like an open circuit (50) to dc at steady state, $v(\infty) = 60$ Volt, thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$
$$\therefore v(t) = 60 + [30 - 60] e^{-t/8} = (60 - 30e^{-t/8}) \text{ Volt.}$$

At $t = 0.5$ sec

$$v(t=0.5) = 60 - 30e^{-0.5/8} = 31.817 \text{ Volt}$$

So, here it is from here it is v infinity 60 volt we explained. Now, v t is equal to you know this formula v t is equal to v infinity v t is equal to v infinity plus v 0 minus v infinity e to the power minus t by tau. So, v infinity is 60 volt; v 0 we calculated 30 volt 60 volt e to the power minus t by 8, so it is 60 minus 30 into e to the power minus t by 8 volt right. Now, at t is equal to 0.5 second, you put here at t is equal to 0.5 second, so v at t is equal to 0.5 is equal to 60 minus 30 into e to the power minus 0.5 by 8, so it will become 31.817 volt right.

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At $t = 0.5$ sec

$$v(t=0.5) = 60 - 30e^{-0.5/8} = 31.817 \text{ Volt}$$

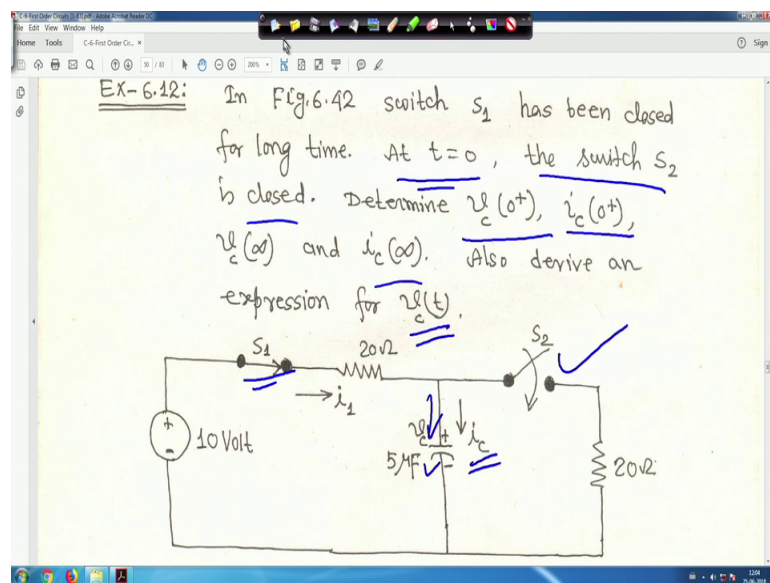
At $t = 3$ sec,

$$v(t=3) = 60 - 30e^{-3/8} = 39.38 \text{ Volt.}$$

Ex-6.12: In Fig.6.42 switch S_1 has been closed for long time. At $t=0$, the switch S_2 is closed. Determine $v_c(0^+)$, $i_c(0^+)$, $v_c(\infty)$ and $i_c(\infty)$. Also...

And another one is that at t is equal to 3 second. So, at t is equal to 3 second, you put here t is equal to 60 minus 30 into e to the power minus 3 by 8, so that is 39.38 volt these are the answers right. So, hope you have understood, hope you are understanding this. Just little bit you know, so many varieties of problems we are solving, just open any good book and just take few problems and just try yourself. Because, whatever various kinds of problems are there perhaps trying to cover all types of your problem, such that whatever book you will open, you will find problems are almost similar to that whatever has been done here right.

(Refer Slide Time: 25:10)



So, next one is that in figure 42, this is figure 42. You have to actually here know understanding is more required. Just blindly we cannot do this we have to understand the philosophy or physics while this switching circuit right. So, in figure 42, switch S_1 has been closed for long time this is your switch, this switch actually this switch S_1 was closed for long time right. And then your at t is equal to 0 that the switch S_2 is closed. The another switch is here S_2 that at t is equal to 0 switch S_2 is closed. You have to find out $v_c(0^+)$, $i_c(0^+)$ that is your $v_c(0^+)$ plus means this one, this one (Refer Time: 25:53) that is that your voltage across the capacitor $v_c(0^+)$.

So, $v_c(0^+)$ is nothing but the $v_c(0^-)$, because at the time of switching the voltage cannot be change instantaneously across the capacitor right. And you have to find out $v_c(\infty)$, that means this one, what will be the final value of this one. And $i_c(\infty)$ what

is the this i c infinity, what is the steady state value also derive expression for v c t, that is the voltage across the capacitor. All these things we have to do it right. And this is 20 ohm; and this switch was closed for long time. So, let me clear it.

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Fig. 6.42: Circuit for EX-6.12

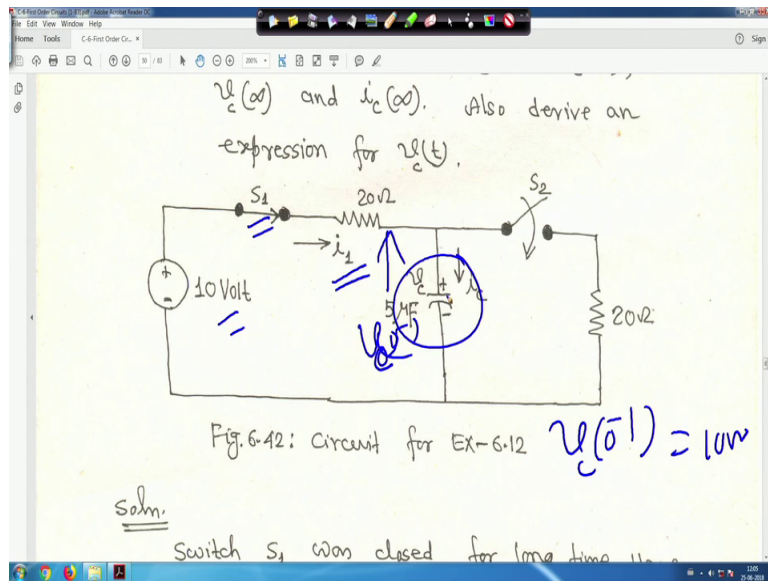
Soln.
Switch S_1 was closed for long time. Hence, before S_2 is closed, the capacitor is fully charged.

Thus $V_c(0^+) = V_c(0^-) = 10 \text{ Volt.}$ (51)

Fig. 6.43 shows the circuit when switch S_2 is closed.

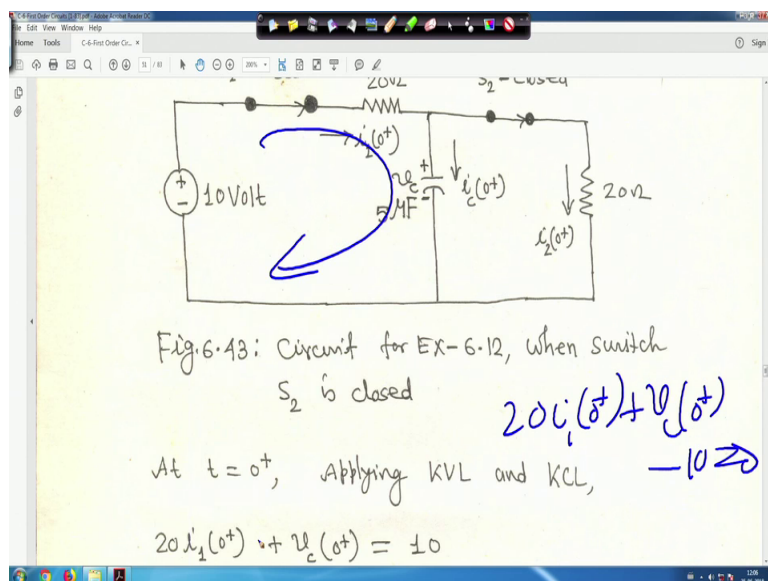
So, this is this switch S 1 was closed for long time. Now, look this switch was closed for long time, hence before S 2 is closed the capacitor is fully charged. That means, if capacitor is fully charged and this was is open, that means just let me make your thing clear. That means, this is this was open and this switch was closed for long time, that means capacitor is acting as an your open circuit.

(Refer Slide Time: 27:00)



That means, this is your say $v_c(0^-)$, your what you call that $v_c(0^-)$ minus right. So, as this is open, this is also open, so no current flowing through this. So, basically your $v_c(0^-)$ minus will be is equal to 10 volt, because, this is the voltage across the capacitor. So, let me clear it, so this is your $v_c(0^+)$ is equal to $v_c(0^-)$ is equal to 10 volt. Now, now top figure this set figure is that switch S_2 is closed right. Now, switch S_1 was closed; now at $t = 0$, switch S_2 is closed, now this switch is also closed. So, at the time of just your at the time of $t = 0^+$ you apply KVL and KCL right.

(Refer Slide Time: 27:43)



So, if you at the time of when switch is closed, just at the time of your this switch at the time of your what you call, just switch is closed right at that time you apply KCL and

KVL. So, here if you apply your KVL at your in this mesh right, so how it will look like, it will be $20 i_1 + v_c - 10 = 0$, just let me this thing. So, here we are writing $20 i_1 + v_c - 10 = 0$, that means $20 i_1 + v_c = 10$. I mean as soon as the switch is closed, little bit understanding is required, you apply here this your, what you call that your KVL. So, it will be $20 i_1 + v_c = 10$ plus that is just at the time of switching, just after switching plus this voltage at that time across the capacitor is $v_c - 10 = 0$ right. So, we know v_c , so let me clear it.

(Refer Slide Time: 28:53)

Fig. 6.43: Circuit for Ex-6.12, when switch S_2 is closed

At $t = 0^+$, Applying KVL and KCL,

$$20i_1(0^+) + v_c(0^+) = 10$$

$$\therefore 20i_1(0^+) + 10 = 10$$

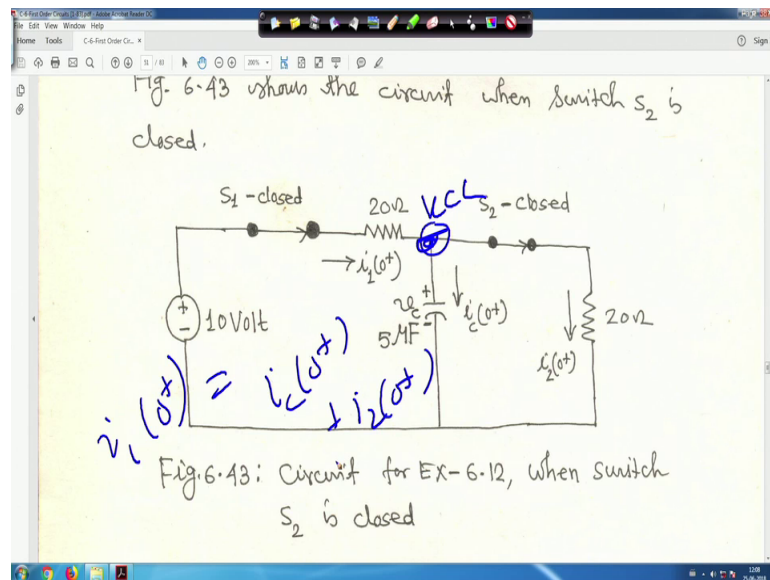
$$\therefore i_1(0^+) = 0 \text{ Amp}$$

$$i_2(0^+) = \frac{v_c(0^+)}{20} = \frac{10}{20} = 0.5 \text{ Amp}$$

Also,

So, in this case v_c is equal to your this thing 10 volt. So, i_1 is equal to 0 ampere right. Here, your v_c is equal to $v_c - 10 = 0$ volt. Because, two capacitors at the time of switching again and again I am telling voltage cannot your, what you call across the capacitor voltage cannot change instantaneously. So, in that case i_1 is equal to 0 ampere. Now, come to i_2 . Now, i_2 plus your this is your i_2 plus and at that time voltage across the capacitor that is v_c , so it is 20 ohm is connected across this, so i_2 will be v_c by 20. So, i_2 is equal to your v_c by 20. So, v_c is 10 volt, so 10 by $20 = 0.5$ ampere right.

(Refer Slide Time: 29:50)



Also, if you apply here at this point, if you apply at this point your KCL, you apply at this node right, then you will get your $i_1(0^+) = i_c(0^+) + i_2(0^+)$ right, so that means I am writing here $i_1(0^+) = i_c(0^+) + i_2(0^+)$ right, you apply KCL at this point.

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$$i_2(0^+) = \frac{v_c(0^+)}{20} = \frac{0}{20} = 0.5 \text{ amp}$$

Also,

$$i_1(0^+) = i_c(0^+) + i_2(0^+)$$

$$\therefore i_c(0^+) + 0.5 = 0$$

$$\therefore i_c(0^+) = -0.5 \text{ Amp.}$$

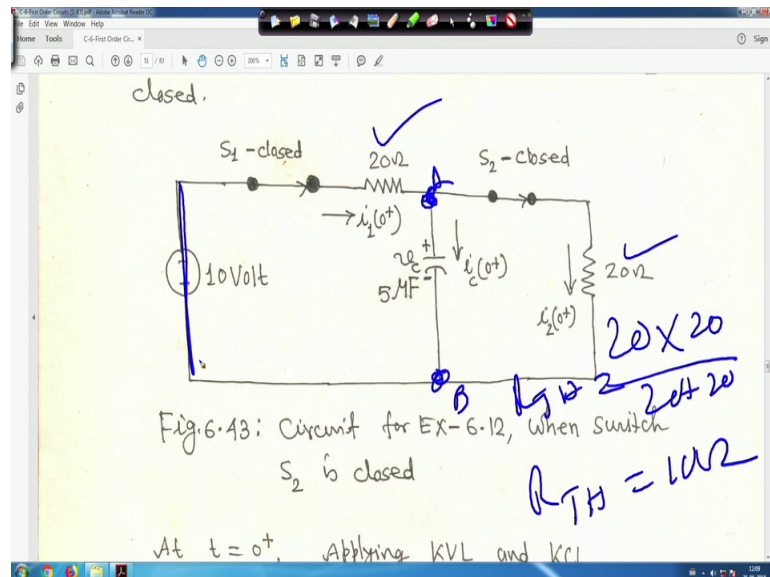
$$\tau = CR_{Th} = 5 \times 10^{-6} \times \frac{20 \times 20}{(20+20)} = 5 \times 10^{-5} \text{ sec.}$$

For determining τ , voltage source was short circuited and Thevenin resistance seen by capacitor was obtained

So, let me go to this. So, here $i_1(0^+) = i_c(0^+) + i_2(0^+)$. So, your what you call $i_1(0^+)$ was 0. So, $i_c(0^+) + i_2(0^+) = 0.5 = 0$. So, $i_c(0^+) = -0.5$ ampere. And τ is equal to C into $R_{Thevenin}$. So, it is 5×10^{-6} into 10 to the

power minus 6, c value is given and R Thevenin means, this 1 right across the I mean you have to find out R Thevenin. So, this is a this is the terminal, this is the terminal for the capacitor right. When you find the R Thevenin, this voltage source is sorted, we have seen this right.

(Refer Slide Time: 31:00)



And this 20 ohm and this 20 ohm, both are in parallel right. So, it will be 20 because at this point say, this is point A, this is point B, we are trying to find out R Thevenin, so it will be 20 into 20 by 20 plus 20 this is your R Thevenin you have to find out. Therefore, R Thevenin is equal to 10 ohm right. So, because this we have seen before that your what you call how to find out your what you call R Thevenin right.

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The whiteboard shows the following handwritten text and equations:

$$i_c(0^+) + 0.5 = 0$$
$$\therefore i_c(0^+) = -0.5 \text{ Amp.}$$
$$\tau = CR_{Th} = 5 \times 10^{-6} \times \frac{20 \times 20}{20+20} = 5 \times 10^{-5} \text{ sec.}$$

For determining τ , voltage source was short circuited and Thevenin resistance seen by capacitor was obtained

At $t = \infty$, capacitor acts as open circuit, (52)
thus,
$$i_c(\infty) = 0$$

So, this so we got this one your, what you call this R Thevenin is equal to directly, we are writing 20 into 20 upon 20 plus 20. So, basically it is your, what you call that is 10 ohm, so multiplied by 5 into 10 to the power minus 6, so it will be 5 into 10 to the power minus 5 second right.

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The whiteboard shows the following handwritten text and equations:

and Thevenin resistance seen by capacitor was obtained

At $t = \infty$, capacitor acts as open circuit, (52)
thus,
$$i_c(\infty) = 0$$
$$v_c(\infty) = 10 \times \frac{20}{40} = 5 \text{ Volt.}$$

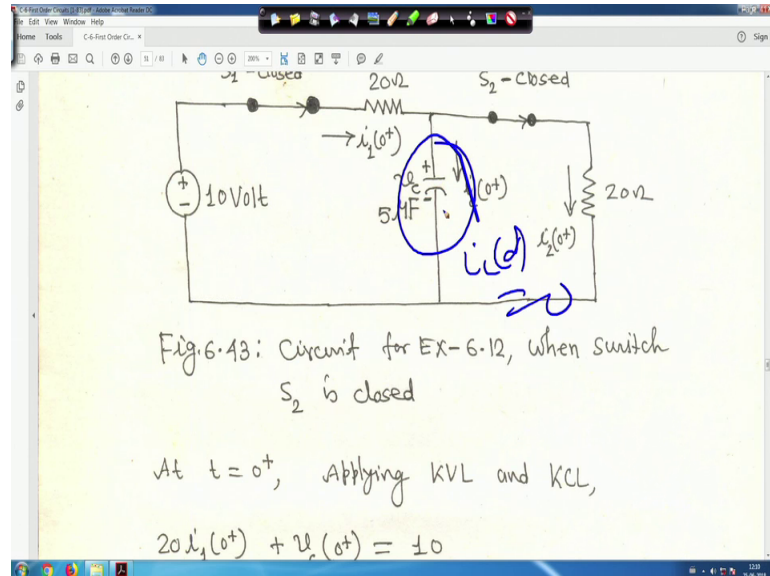
We know

$$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)] e^{-t/\tau}$$

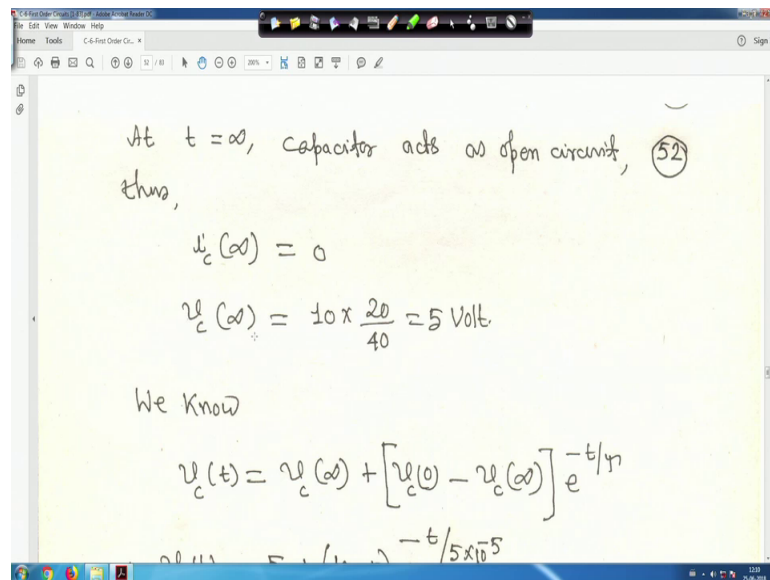
And at t is, at t tends to infinity capacitor acts as an open circuit. So, for this thing switch is closed; and for it was it remain closed for long time, so capacitor will act as open

circuit. So, i_c infinity is equal to 0, because capacitor will be an open circuit, when switch is closed for long time.

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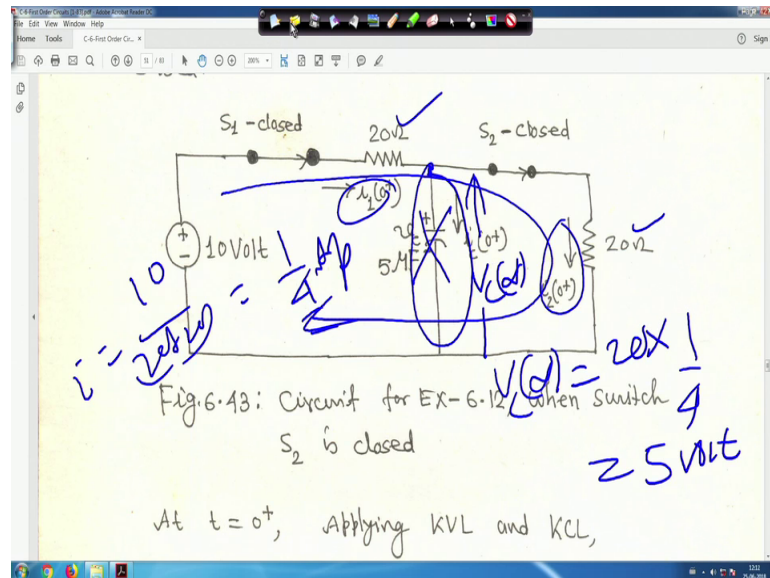


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Therefore, right, i_c infinity 0 and v_c infinity will be 10 into 20 by 40 is equal to 5 volt. So, if you come here what will be your v_c infinity right. So, if you look, if you look into that the capacitor acts an open circuit acts as an open circuit right. So, question is that this is open, this is your this is open right, so this is open circuit, so find out what is the current flowing through this what is the current flowing through this.

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So, this is open circuit, so what is the current flowing through this the i it is 10 by 20 ohm plus 20 ohm. So, 20 plus 20 so 40 that is 1 by 4 ampere this is your what you call the current flowing and this voltage your and this 1 by 4 ampere. So, at that time nothing is there it is a steady state, it is a steady state, it is at infinity. That means and then open circuit voltage, I mean voltage across the capacitor that v_c infinity right, that is v_c infinity this is the this is your voltage v_c infinity this is the voltage right is equal to your this 20 into this 1 by 4 ampere, that is equal to 5 volt right, because, capacitor is open circuit at that time. So, current you have to flowing like this and this capacitor is connected this 20 ohm resistance is connected across the capacitor right. So, you can find out easily v_c infinity 20 into 1 by 4 , so 5 volt right. So, let me clear it.

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The image shows a software window with a yellow background containing handwritten mathematical work. At the top, it says "We know". Below that is the general formula for a capacitor voltage response: $v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)] e^{-t/\tau}$. This is followed by a specific substitution: $\therefore v_c(t) = 5 + (10 - 5) e^{-t/5 \times 10^{-5}}$. The final result is: $\therefore v_c(t) = (5 + 5 e^{-2 \times 10^4 t}) \text{ Volt, } t > 0$. At the bottom, there is an example problem: "EX-6.13: In Fig. 6.44, determine $v_c(t)$ after the switch is closed. Given that $v_c(0^+) = 2 \text{ Volt}$."

So, that means $v_c(0)$, $v_c(\infty)$ all are known to us right so that is your 5 volt. This is $v_c(\infty)$ directly I have written here I hope you understood this right. Therefore, $v_c(t)$ is equal to $v_c(\infty)$ the same formula, that $v_c(\infty)$ plus $v_c(0)$ minus $v_c(\infty)$ e to the power minus t by tau generalized formula, you substitute $v_c(\infty)$ 5 volt, $v_c(0)$ 10, $v_c(\infty)$ 5, and e to the power minus t, then by 5 into 10 to the power minus 5 that is tau time constant for t greater than 0. So, that means $v_c(t)$ is equal to 5 plus 5 e to the power minus 2 into 10 to the power 4 t your, that means that is for t greater than 0. Hope you have understood this, little bit understanding is required.

Thank you very much, we will be back again.