Fundamentals of Electrical Engineering
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Lecture - 32
First order circuits (Contd.)

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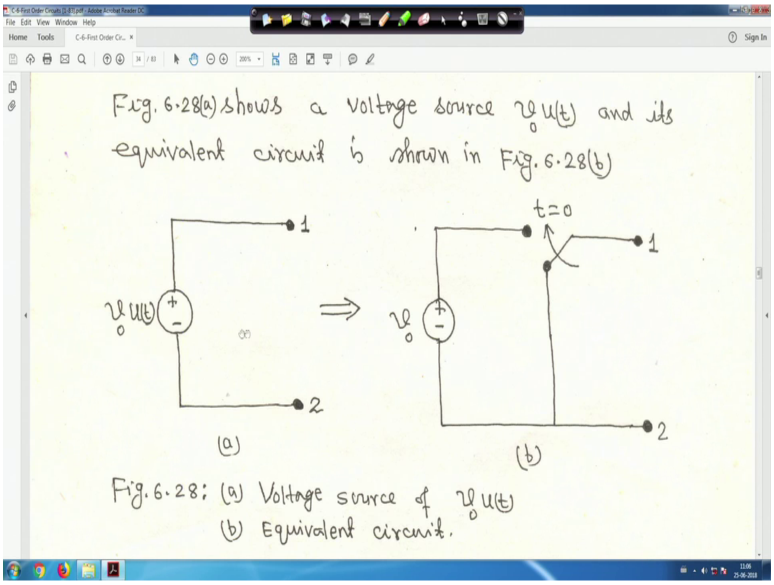

So we just we have seen that your v 0 u t that we have explained already how it can be represented this is v 0 into u t, it is right and step unit function we have given for t less than 0, that u t is 0 and it is if it is greater than 0 then t greater than 0, then u t is 1; so that means, when the switch in this position just we have explained 1 and 2 are short circuited; that means, your v 1 2 is equal to your output voltage whatever it is that is 0 right? And that is your and second thing is if switch moves from this position to that position at t is equal to 0, then at that time your what you call v 1 2 is equal to v 0 that is v t right is equal to v 0.

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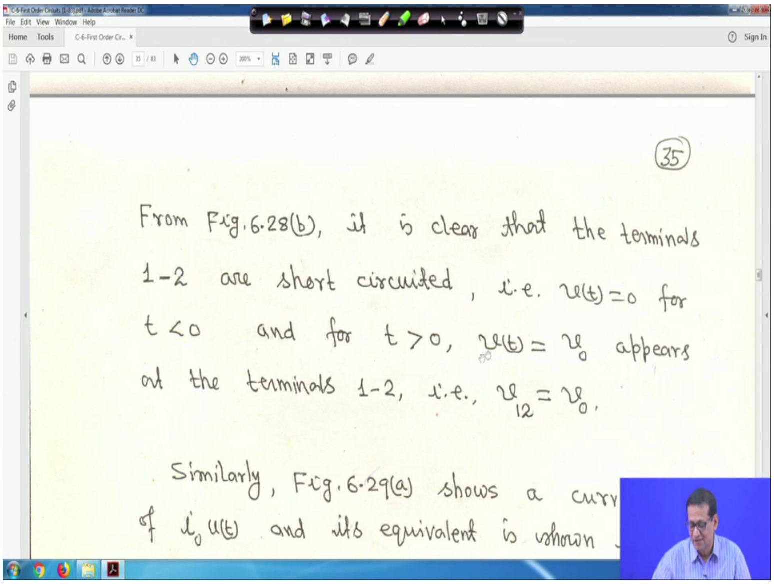

So, this is all have been explained here.

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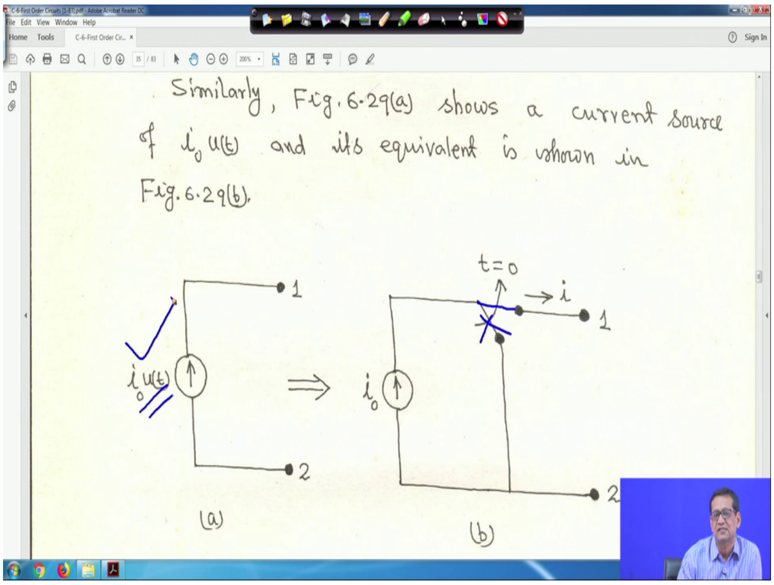

Similarly, for the current source if you take; I mean suppose if you take a current source that i 0 u t and its equivalent circuit is shown; this is i 0 into u t. So, we have explained the step unit function that for t less than 0, that is your u t is equal to 0 and t greater than 1 sorry t greater than 0, that u t is equal to 1. So, when switch in this position whatever shown in this figure at t is equal to your what you call it is switch is open at t equal to 0, but when switch is in this position that is t less than 0, in that case this is open; that means, your i t is equal to 0, right? And as soon as switch moves from this position to that position I mean, if the switch is moving from this to that; that means, if it is connected here and it is not there, at that time i t is equal to i 0 right.

So, this is your this is the meaning of i 0 into u t right; that means, for t less than 0 your what you call that your u t is equal to 0. And t less than t greater than your 0 that your what you call that u t is equal to 1 that unit step function. So, this is i mean, when you will solve the your what you call; that your DC at the transient problems at that time we will consider this kind of source, such that v into u t or i into u t we will later right.

So, this is your i meaning of i 0 u t is like this, right? So, that is the idea rather than then moving like this, if we represent circuit, if we represent the source voltage source or current source right, by we multiplied by unit your step function, how when we will solve the circuit at that time we will know how it behaves, right. So, this is your, I am clearing it. So, this is your representation of current source.

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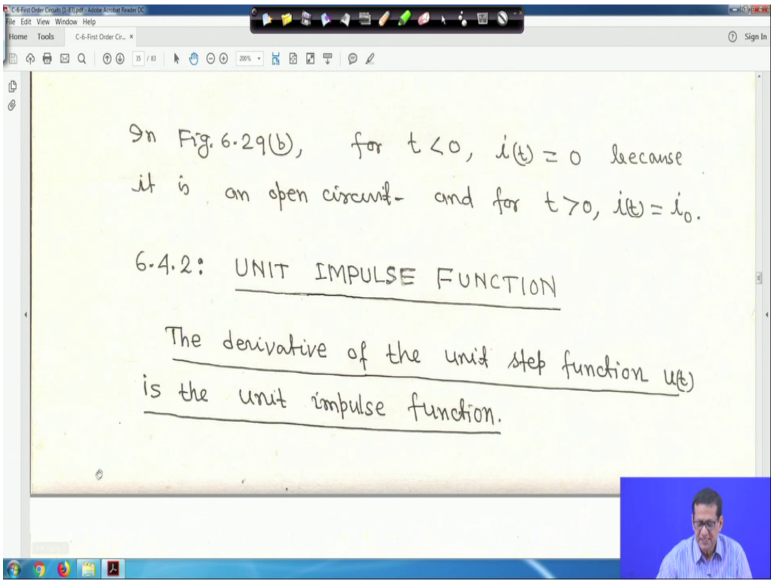

Now, next is that unit impulse function, right. Actually, physically unit impulse function is like ideal sources volt ideal sources or resistor that your unit in physically it is not realizable, but it is a very strong mathematical tool right, for circuit analysis. So, the derivative of the; it is underlined only the derivative of the unit step function u t is the unit impulse function.

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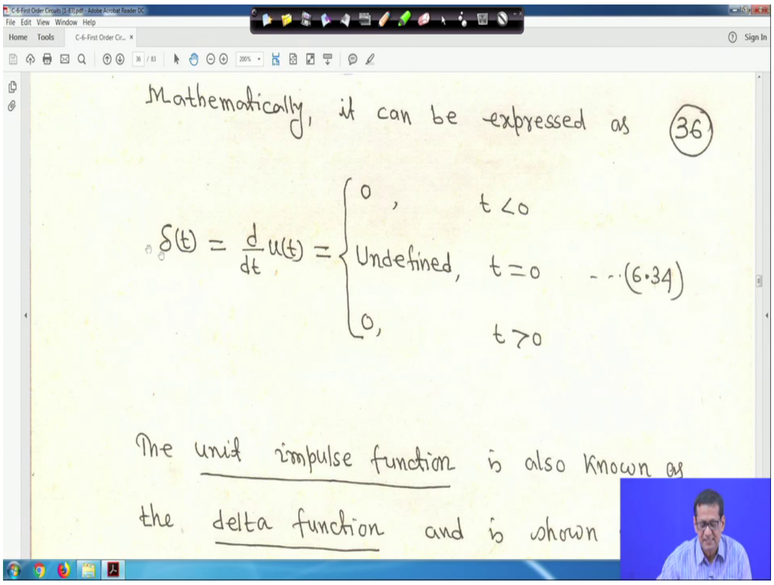

Now, mathematically it can be expressed as; delta t we represent delta t your what you call that unit impulse function right. So, it is unit impulse function. So, delta t is equal to d t of u t, it is it is equal to 0 for t less than 0. It is t is equal to 0, it is undefined right and t greater than 0 again it is 0, right. So, this is actually what you call? This is the definition of unit impulse function right.

So, it is also known sometimes we call as a delta function, in it in a book you will find this unit impulse function also is termed as delta function, right?

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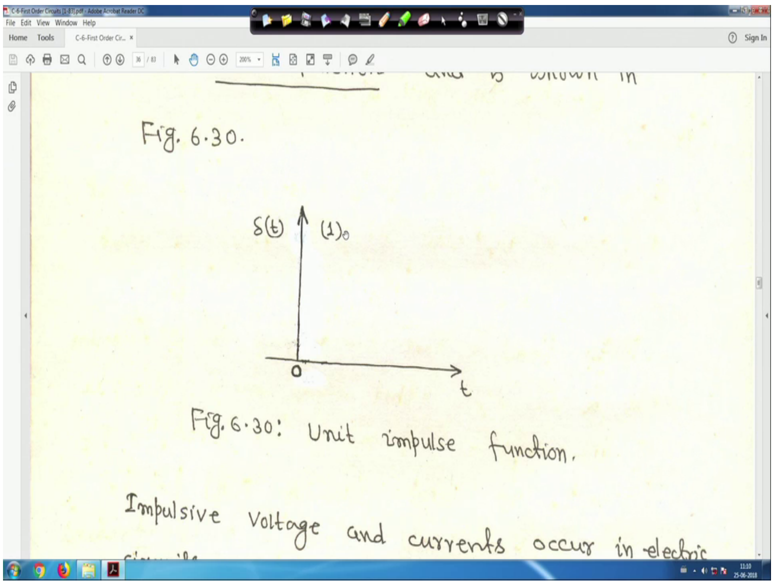

And it is shown in figure this one this is t and this is say it is 0, time 0 and this is delta t. This 1, if you will find it is marking 1 means it unit area is 1 right that is strength, we will see later much after that right.

It can so, but this is your what you call, unit impulse function, the representation right. And this 1 we will see later it is unit area it is called the strength of the your unit impulse function we will see just after few minutes.

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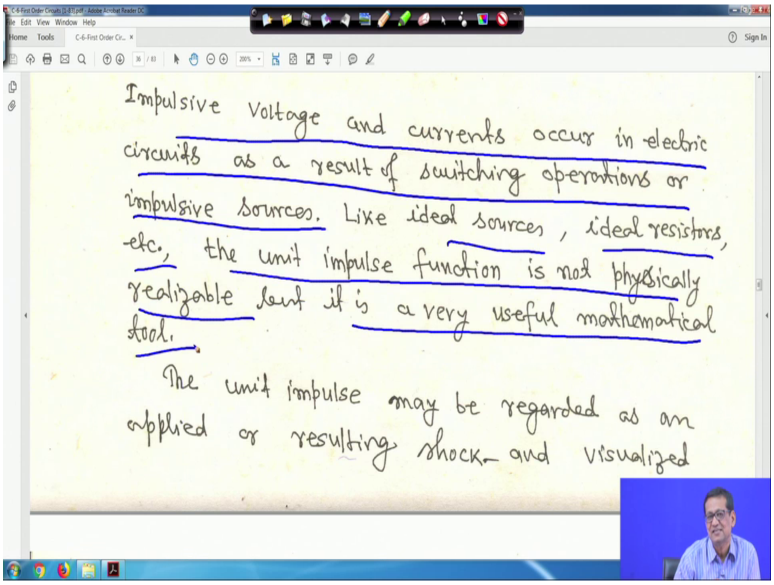

So, impulsive voltage and currents occur in electric circuit as a result. I mean, if you can you might have experienced also, that impulsive voltage and currents occur in electric circuits as a result of switching operation or impulsive sources right.

Like ideal sources ideal resistor etc right, the unit impulse function is not physically realizable right, but it is very useful for mathematical tool. So, that means, physically it is a your what you call; say if it is not your realizable, but it is a very useful mathematical tool right, for analysis of various circuit. So, though just let me clear it.

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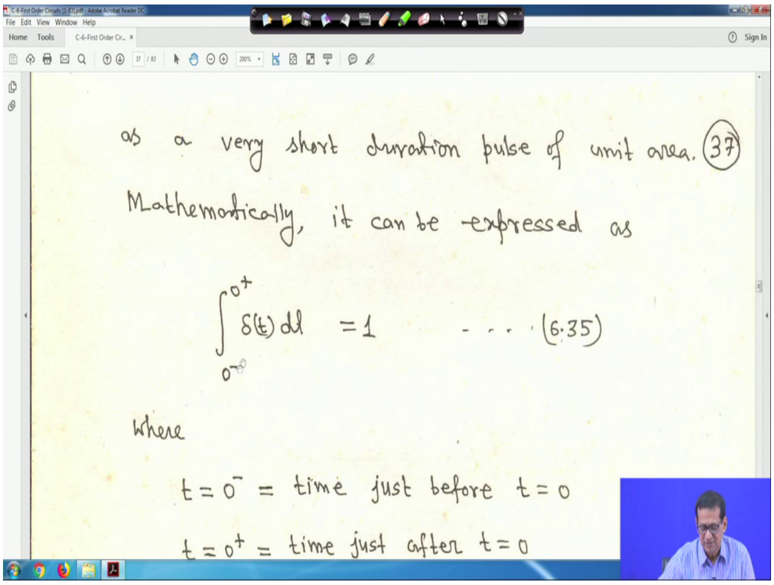

So, the unit impulse may be regarded as an applied or resulting shock and visualized as a very short duration pulse of unit area. Mathematically, actually we can express this that your integration just before switching that is 0 minus to 0 plus delta t d t is equal to 1.

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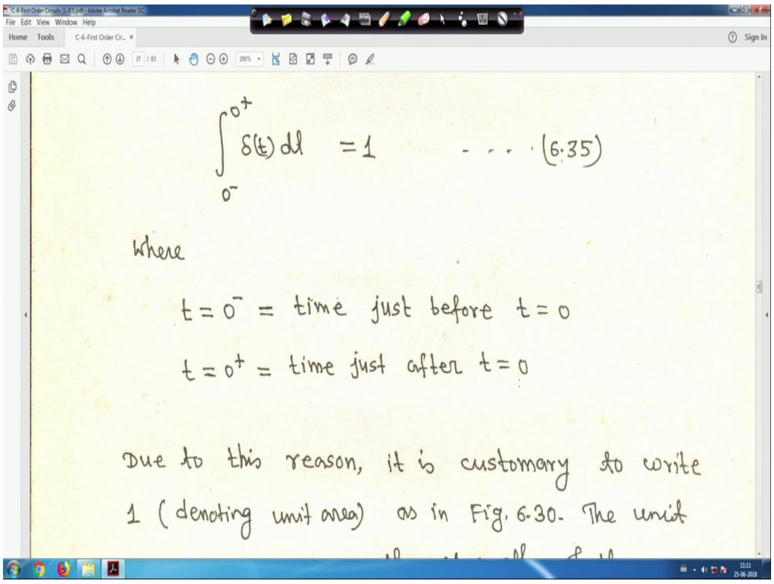

It is area is what you call? It can be mathematically it can it can be expressed as 0 minus to 0 plus delta t d t is equal to 1 right, this is unit impulse function but this area is 1, t is equal to 0 minus time just before t is equal to 0 and t is equal to 0 plus time just after t is equals to 0, right? So, you have to remember this that from 0 minus to 0 plus delta t integration of delta t dt is equal to 1; this is equation 35.

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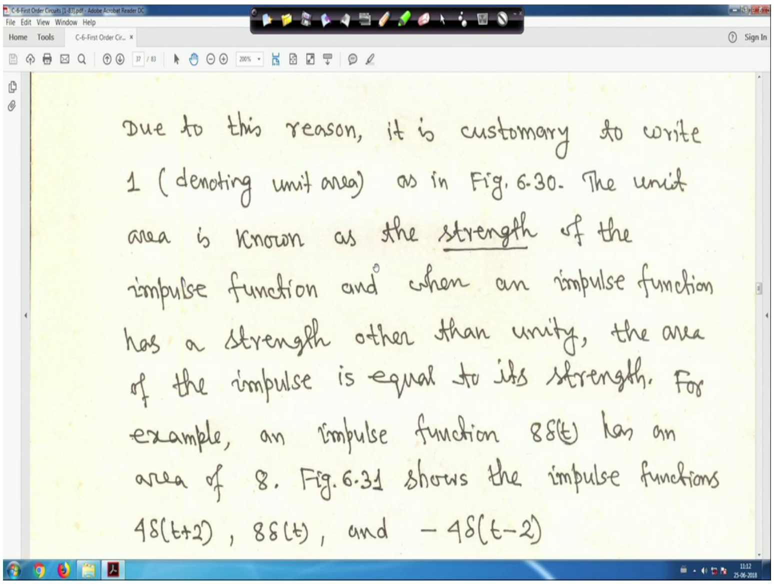

But to this reason, it is customary to write 1 denoting unit area, 1 that figure I told you here right, it is in this figure here I told you that it is 1 means it is denoting your unit area.

So, but see how is it. Due to this reason, it is customary to write 1, that is denoting unit area as in figure 30 I showed you. The unit area is known as the strength of the impulse function and when an impulse function has a strength other then unity, the area of the impulse is equals to it is strength right. So, actually area of the your what you call unit impulse function that is actually strength ok. I have underlined here for you underlined here right. So, for example, an impulse functions say here just here.

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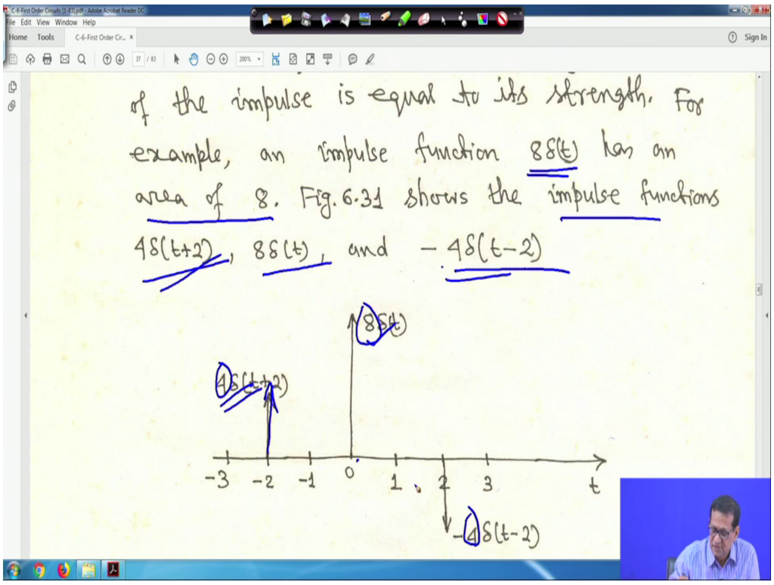

An impulse function like 8 delta t has an area of 8, right? Similarly, you are what you call an impulse function of 4 delta t plus 2 8 delta t and minus 4 are shown in here, right. So, this is 8 delta t, so it is delta t, so it is this is the time 0. So, it is you can write this 8 delta t.

Now, for 4 t delta plus for delta t plus 2 that is t is equal to minus 2, right. So, it will start from here. This is 4 delta t plus 2, it is just a just impulse response right? And similarly it is minus 4 delta t minus 2. So, t is equal to 2 and it is minus, so it is minus 4 delta t minus 2, this is the representation of the impulse function. In this case it is strength is 8, in this case it is strength is 4, in this case also it is strength is 4, right. So, let me clear it, so this is 3 impulse function I just showed you.

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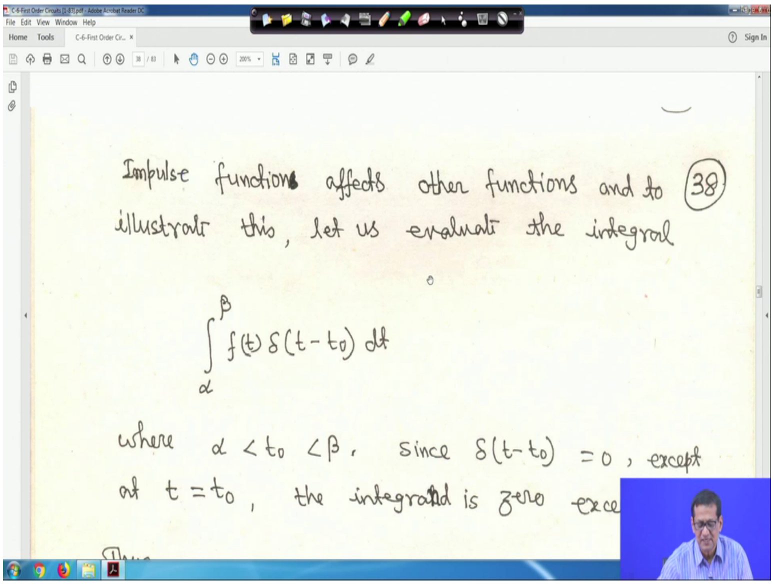

So, impulse function affects other function and to illustrate this, let us evaluate the integral. I mean it affects the that impulse function affects the other function also.

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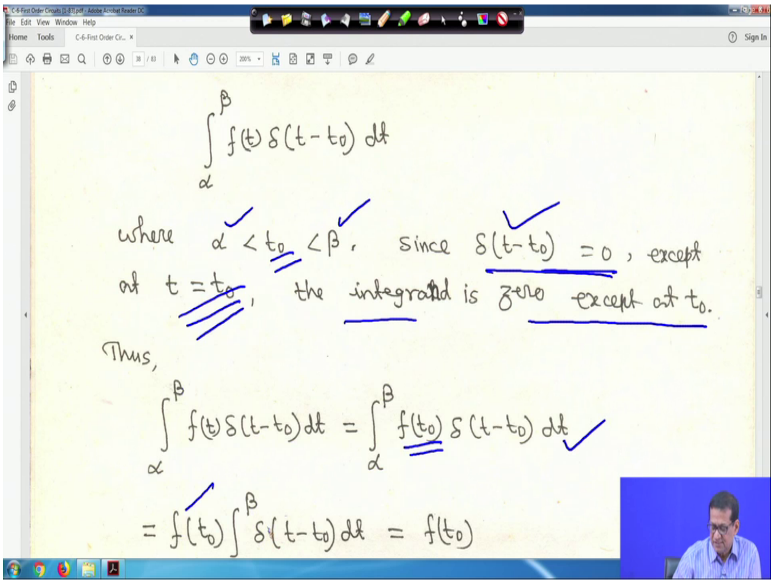

For example, you take a you take that integration alpha to beta ft delta t minus t 0 dt right. Now t 0 is lying in between alpha and beta.

Since, delta t minus t 0 is equal to 0, except at t is equal to t 0 because at t is equal to t 0 it is undefined you just saw, right? There it was delta t less than 0 greater than 0 whatever is given 0 0, but at t is equal to 0 it is undefined, but here we are taking delta t minus t 0.

So, it is 0, but except at t is equal to t 0. So, the integrate is 0 except at t is equal to t 0 because it is undefined. Therefore, alpha to beta f t delta t minus t 0 d t you can write, at t is equal to t 0 you can write because at t equal to t 0 right. Except this delta t minus t 0 is equal to 0 except at t 0 is equal to t 0. Therefore, a t is equal to t 0 alpha beta it is f t 0 delta t minus t 0 d t or is equal it is f t 0 will come out it is here right. Then integration alpha to beta your delta t minus t 0 d t, just let me go little bit up right.

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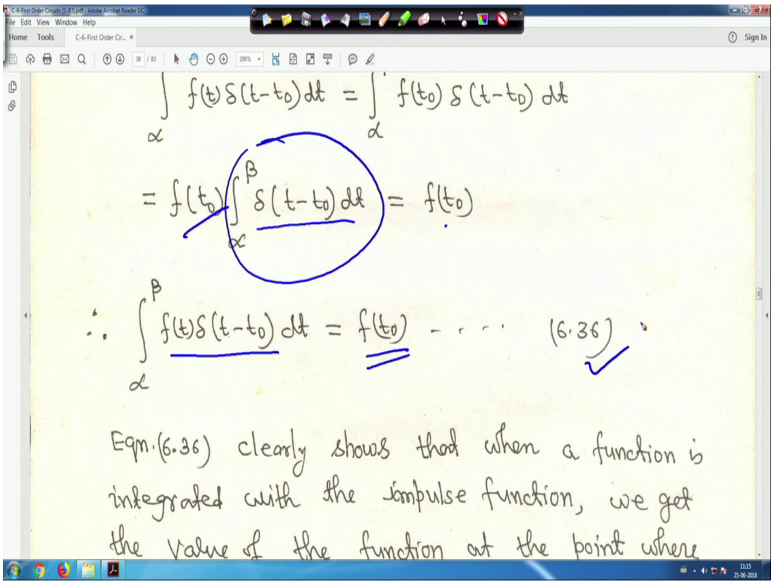

So, here now this is this is come out right. So, alpha to beta delta t minus t 0 d t is equal to f t 0 because this is actually you take that area is equal to your what you call; that unity 1 right. So, that is equal to f t 0, therefore, alpha to beta f t delta t minus t 0 d t is equal to f t 0. So, this is equation 36 right; that means let me move little bit up.

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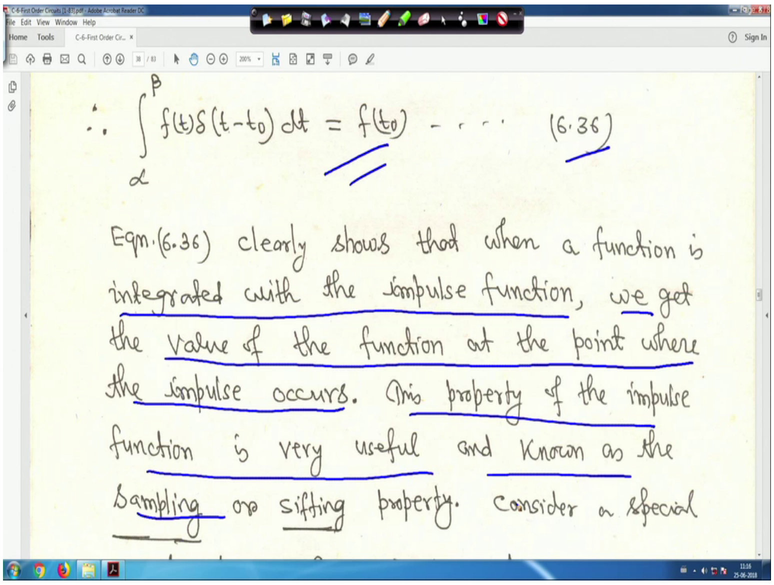

That means, that this equation 36 actually this is equation this is your equation this is your equation this is your equation 36. It is clearly shows that a function is integrated with the impulse function. We get the value of the function at the point where the impulse function impulse occurs right. This property of the impulse function is very useful and known as the sampling property or sifting property right; so, this is this is very important.

So, but in this course we will study little bit only right, more things are available in a course like your signals and networks, right? That is a different thing, but here just little bit little bit we will learn little bit we will learn. So, let me clear it.

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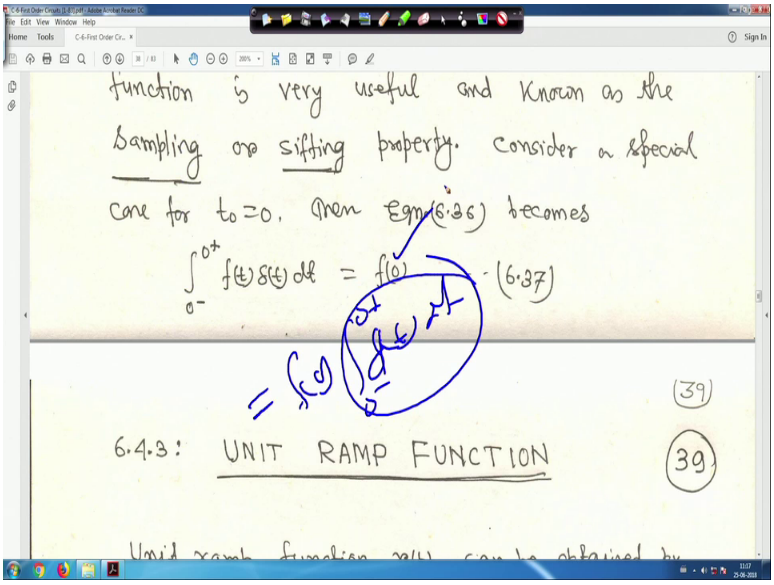

Now, therefore, consider a special case for t equal t 0 equal to 0. Then equation 36; that means, this equation, if t is equals your here it is if t 0 is equal to 0, here it is t 0 is equal to 0. If t 0 is equal to 0 then it will be say in between 0 minus to 0 plus f 0 delta t dt is equal to f 0.

So, here similarly, that means, if t is t 0 is equal to 0 then it will be 0 minus to 0 plus integration f t d t f t delta t d t and that is f 0 because this one this one just I am making it. This one is equal to your f 0 then integration 0 minus to 0 plus delta t d t right and this part is unity, so it is become f 0 right? So, let me clear it. So, this is all regarding impulse function. This is just to for the sake of completeness of this thing just I have taken this right.

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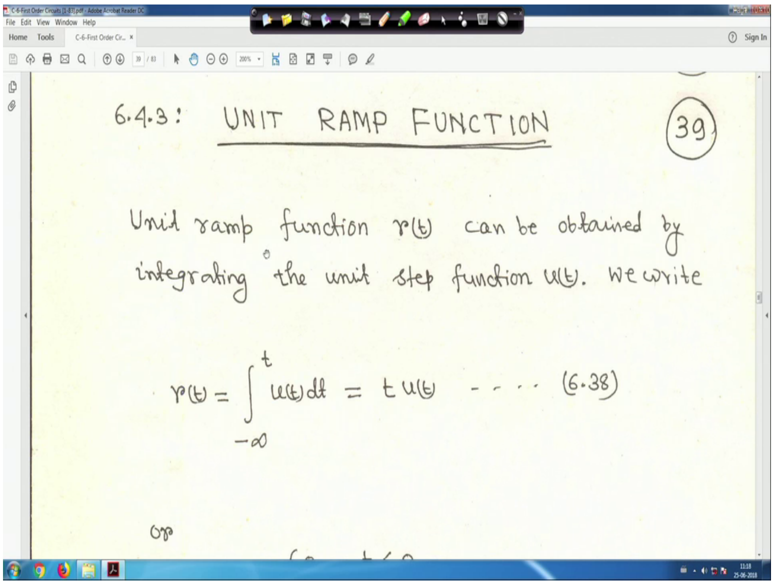

There are many other singularity function, but we will we will come up to ramp function some example. So, unit ramp function, in this case unit ramp function r t can be obtained by integrating the unit step function u t.

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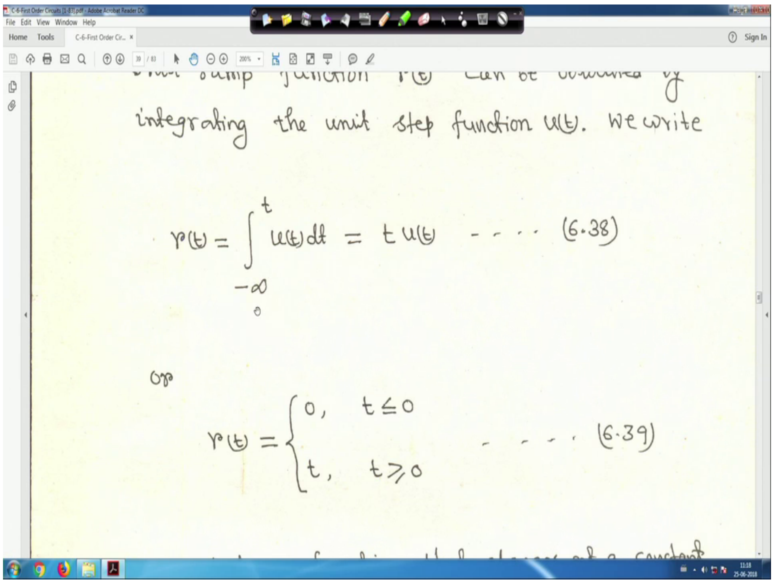

If you integrate unity step function u t, you can write r t is equal to your minus infinity to t u t d t is equal to t into u t right? So, r t is equal to actually 0, for t less than equal to 0 and r t is equal to t for t greater than equal to 0 right that is why; minus infinity to t u t d t is equal t into u t. This is equation 38, this equation can be written like this; that r t is equal to 0 or t less than equal to 0 and is equal to t for t greater than or equal to 0. This is equation 39, this is that your what you call; it is integration of the unit step function. That is your what you call unit your if you just want to obtain you need a ramp function.

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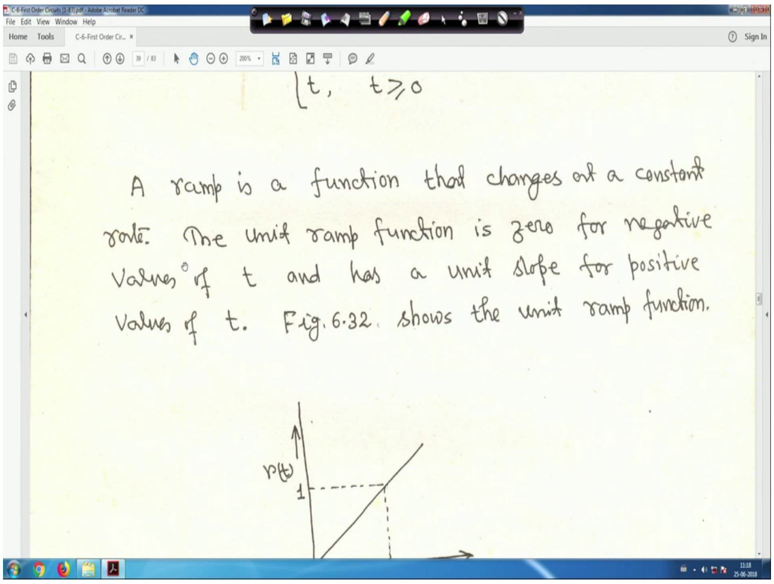

So, a ramp is a function that changes at a constant rate right. So, basically this is equation of straight line. So, the unit ramp function unit is 0, for negative values of t and has a unit slope for the positive values of t.

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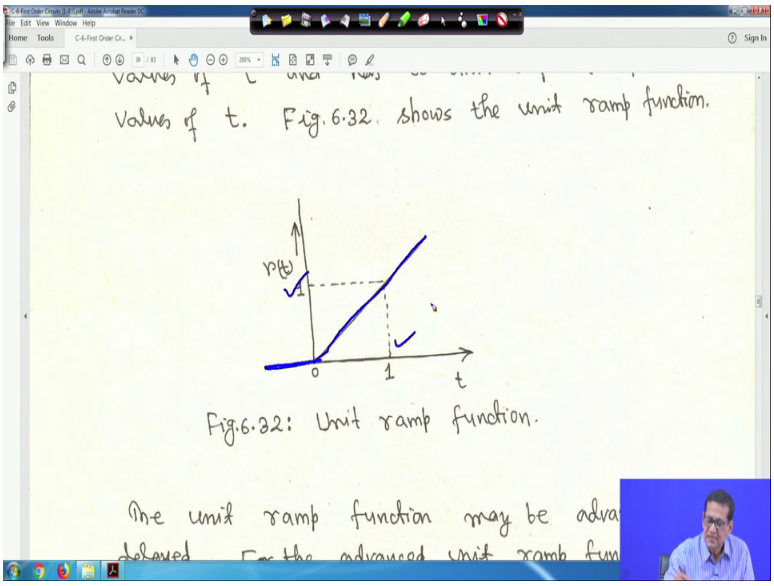

So, figure 32, shows the unit ramp function; that means, for a negative value of t, this is that which is plot like this, it is plotting like this. So, negative value of t it is 0 and for positive value of t it is shown right. And this is unit this is also 1 right. So, this is a unit step, your what you call the ramp function.

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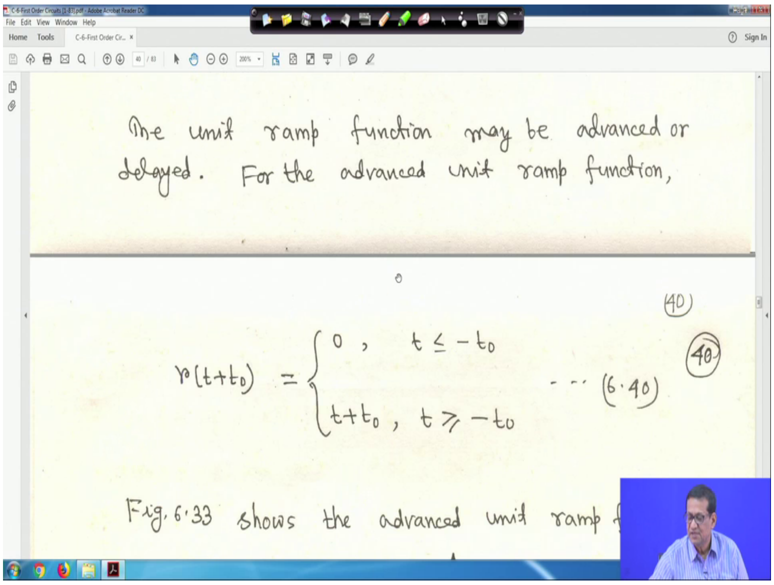

So, next, the unit ramp function may be advanced or delayed right? Previously also if so in t 0 right it may be advanced by time t 0 or delayed by time t 0. For the advanced unit ramp function right, this is for advanced r t plus t 0. That is if it is 0, if t less than equal to minus t 0 and it is t plus t 0 if t greater than equal to minus t 0, this is equation 40, right?

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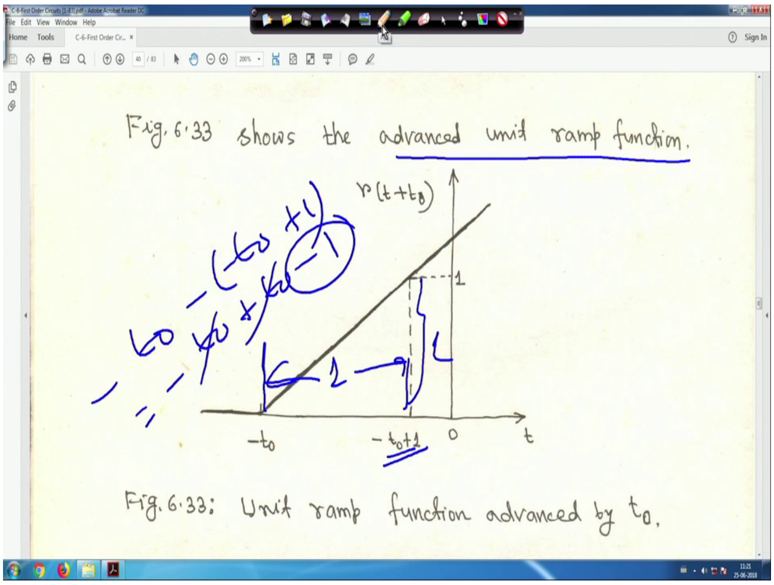

So, if you if you plot it right, so shows the advance unit ramp function. So, r t plus t 0 so it will here it is your t plus t 0 t is equals to minus t your t 0 and this is your minus t 0 plus 1. And this difference I mean only distance, only distance this difference actually this difference actually it is 1, in terms of just length this 1 and this is this is also 1, so the this height is also 1.

So, your what you call; slope is also 1. This is you call that advanced unit ramp function, this is advanced unit ramp function, right? And this is this point is minus t 0 plus 1. So, it is minus t 0, if you take the this side negative will come, but distance is 1 minus t 0 minus your t 0 plus 1 right. So, it is basically minus t 0 plus t 0 minus 1. So, this will be cancel minus 1 because it is on the negative axis negative side of the time axis, but distance is 1 right.

So, let me clear it, so this is your unit ramp function advanced by t 0.

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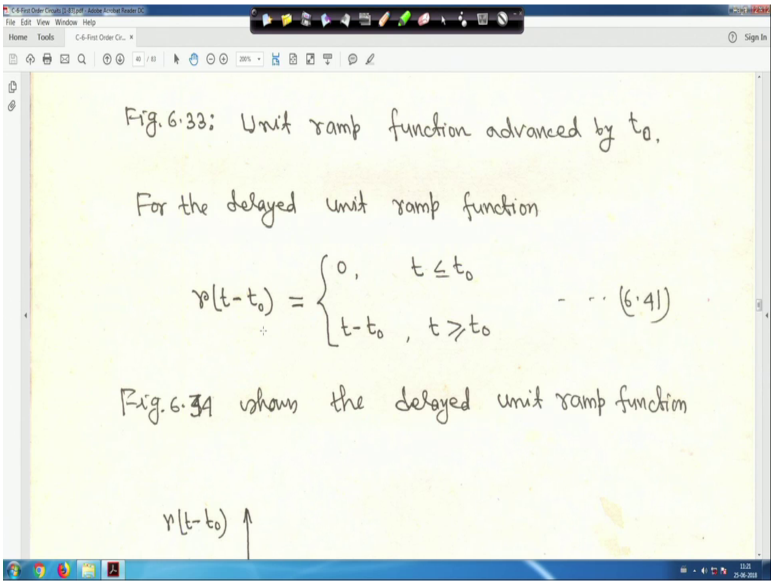

Now if it is delayed, if for that if that if it is delayed ramp function then it will be r t minus t 0, right? That mean it is it is equal to 0 for t less than equal to t 0 and it is t minus t 0 for t greater than t 0 right. So, this is the figure 34 shows the delayed unit ramp function, this is your delayed unit ramp function right.

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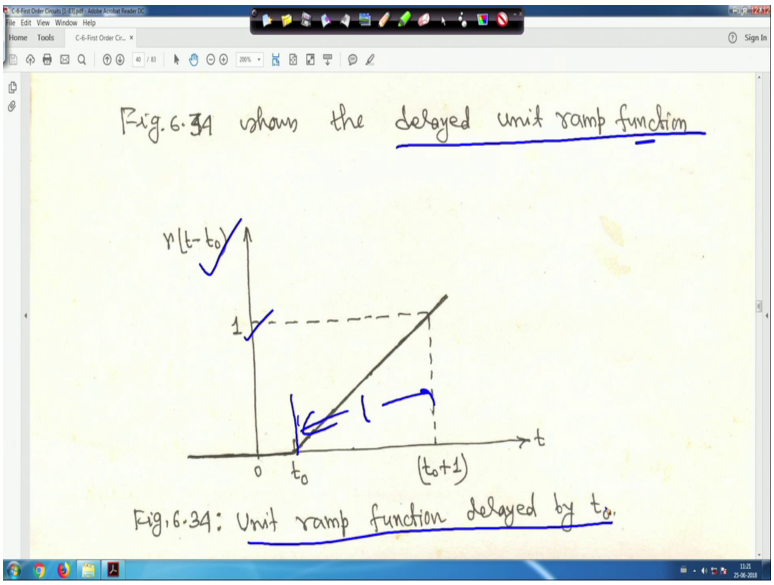

So, it is starting from t 0 and this distance again your this distance is again 1 right, t 0 plus 1 minus t 0 1 this1. So, unit ramp function and this r t minus t 0, this is the plot of the your what you call unit ramp function delayed by t 0; that means, when it is delayed it will be t equal to t 0 on the right on side, that is with the first coordinate right. And when you are that is or starting from here, but this is your this will be less than t 0, it is 0.

Similarly, when it is advanced it is coming it is starting from somewhere at t is equal to minus t 0 right. So, this is your unit ramp function delayed by t 0.

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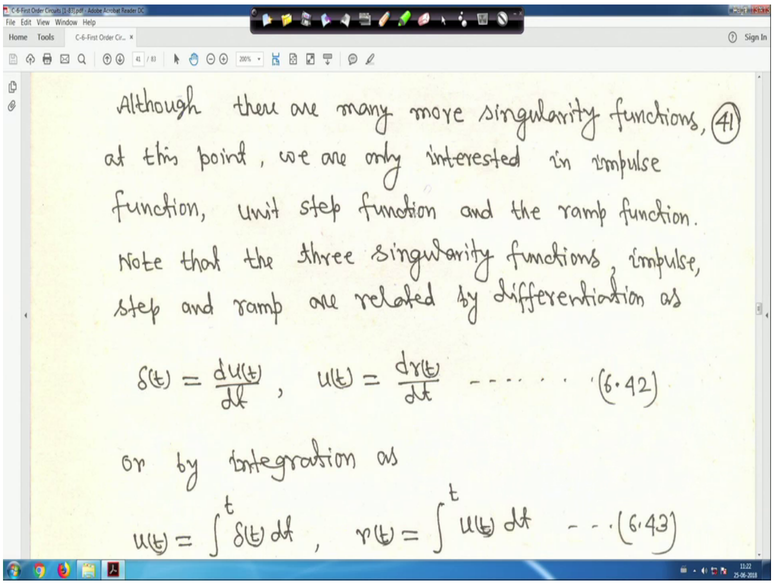

So, next is, so, although there are many more singularity functions at this point, but we in this course we will just for the sake of completeness, little bit just we have given the flavor of this 3 different functions right. We are only interested in impulse function, unit step function and the ramp right. Note that 3 singularity functions impulse step and ramp are related to differentiation as follows.

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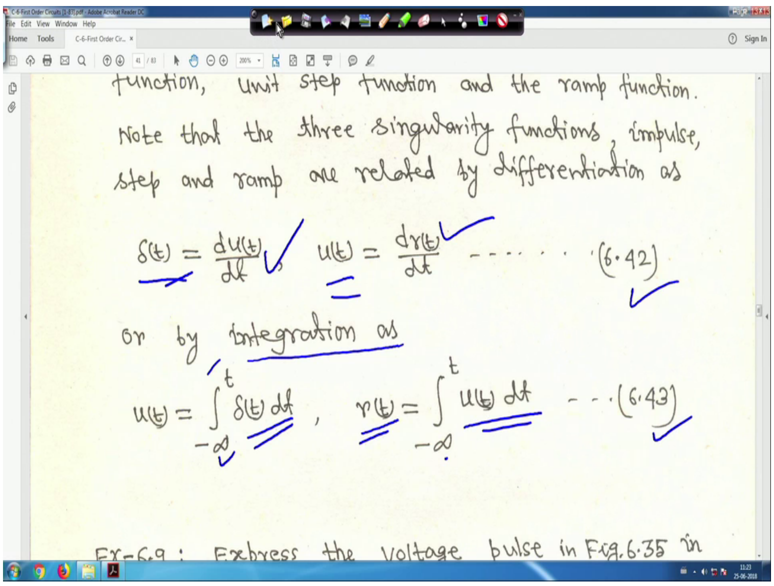

So, if you look into that; that your delta t is your differentiation of d u t by d t right. Similarly, u t also is differentiation of ramp function d r t by d t right or by integration or by integration u t will be minus infinity to t delta t d t right. And similarly for ramp function r t is equal to minus infinity to t u t d t right. If you this is delta t is equal to differentiation of u t and u t is equal to differentiation of r t, but other way if you integrate u t will be it is your minus infinity to t delta t d t and r t will be minus infinity to t u t d t. This is 42 and this is equation 43, right. So, next take a 1 or 2 simple example.

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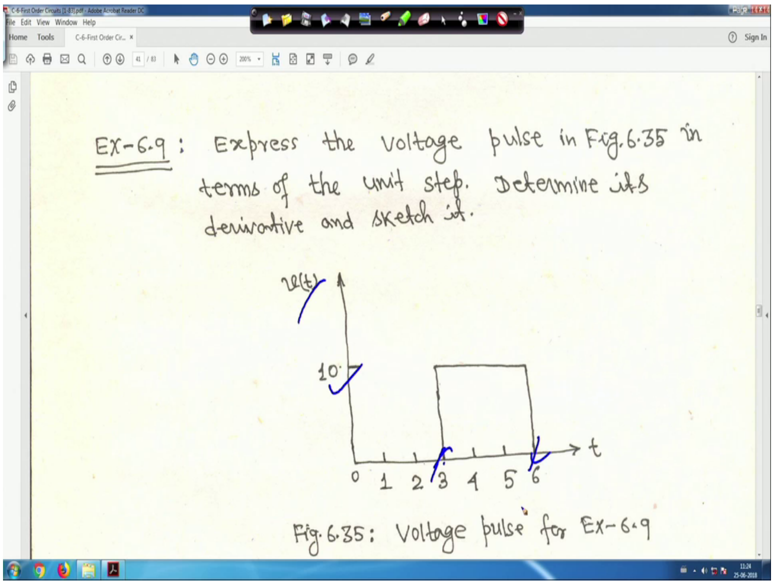

It is given that express the voltage pulse right in figure 35, this is figure 35, in terms of the unit step determines it is derivative and sketch it. So, it is looks like it is a gate function actually, it is a gate function. Actually at time t is equal to 3 second and actually idea is like this, this graph is like this. So, it is a voltage pulse so at time t is equal to 3 it is switched on and say time t is equal to 6, it is switched off right.

So, it is a gate function so at; that means, you have to see that this voltage this is v t and this voltage is 10 volt right. So, v t basically it is a function of your what you called unit step function. So, at t is equal to 3, second say it is switched off switched on and t is equal to 6 it is switched off. And it is a gate function, so how we will get it? Let me clear.

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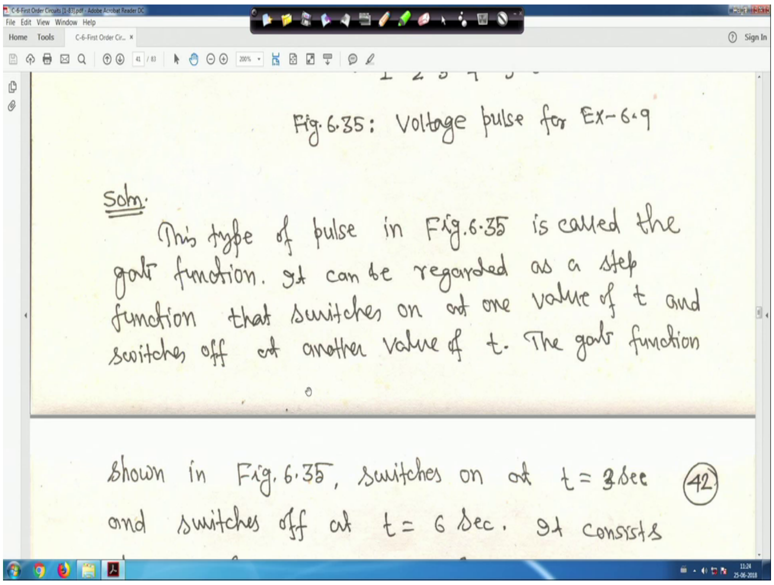

So all these things are written for you so right, so whatever I am telling.

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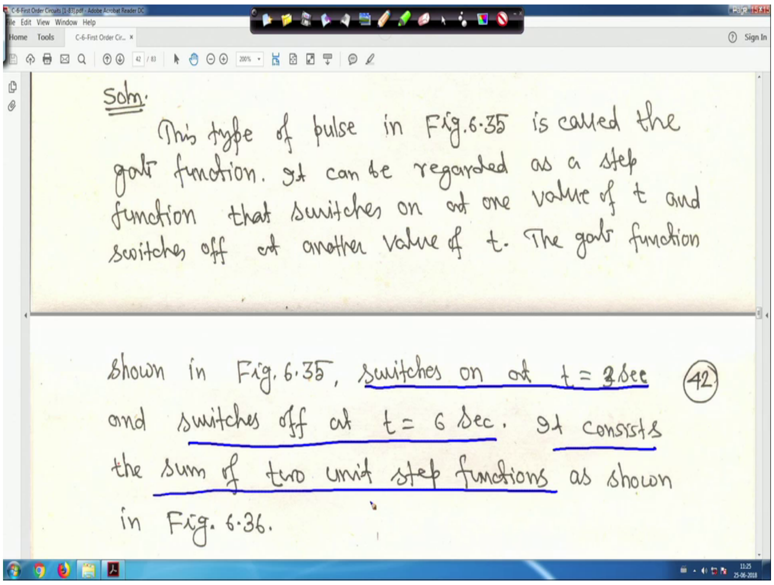

So, it is a gate function. So what I said? A t 3 second it is switches off and t 6 second at t is equal to your 3 second, it is switching on switches or switches on at t is equal to 3 second, h ere it is it is switches on at t equal to 3 second and switches off at t equal to 6 second; that means, it consists of sum of 2 unit step functions as shown in figure 36. So, let us see how is it?

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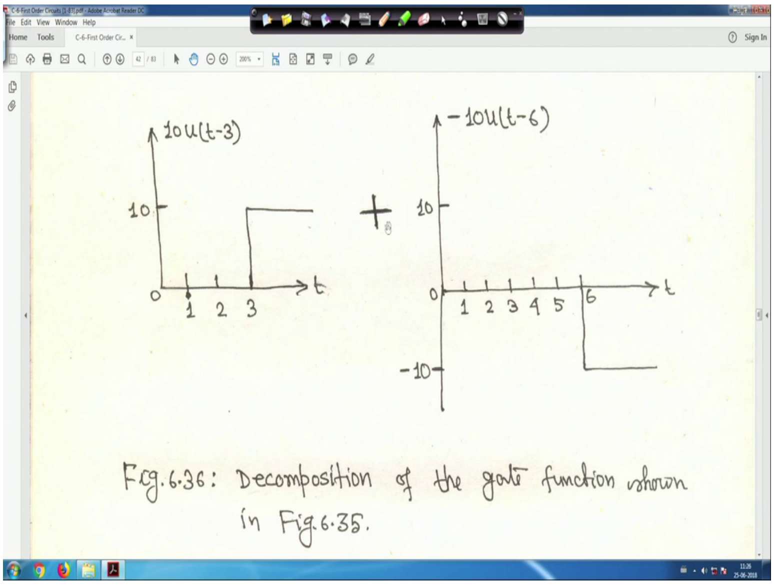

So, now when it is switching on it is a value has given 10 volt, v t right, it is a 10 volt. So, when it is switching on at t is equal to 3, so, this will be actually 10 u t minus 3, right. And when it is switch off switched off at t is equal to 6 second it will be minus 10 u t minus 6, right. So, at the time of switching on it will be 10 u t minus u 3 right, plus and at the time of switching off at t equal to 6 second it will be minus 10 u t minus 6.

Now, if you; that means, this function actually this function, this gate function actually is equal to sum of 2 unit steps a function that is this is one plus symbol is here, I made it plus right is equal is this one; that means, this one you can; that means, mathematically if you write like this; it will be 10 u t minus 3 minus 10 u t minus 6 because if this one plus this one. This function this function that is figure 35, this function is equal to this function, this these 2 unit step function, this is these 2 your what you call step as summation of these 2 step function.

So, in this case this is a decomposition of the gate function; this one and this one.

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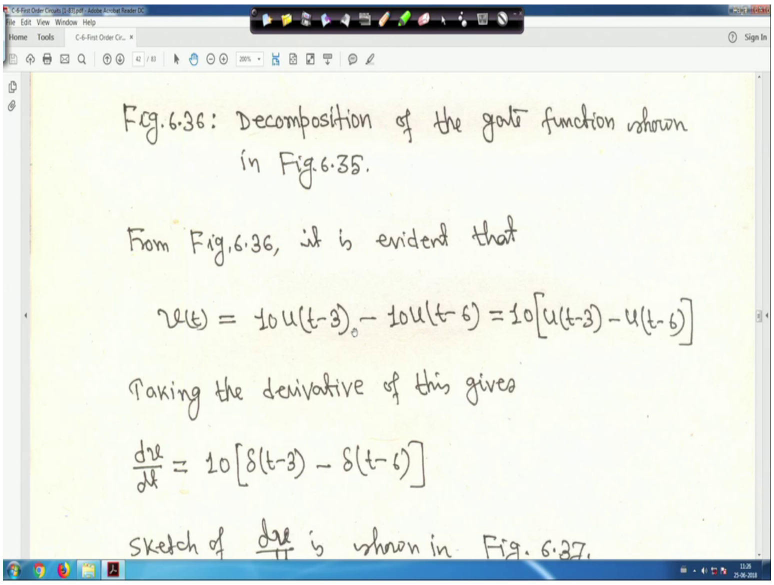

So, if you represent now, v t is equal it will be 10 u t minus 3 that is your; this one minus 10 u t minus 6; that means, it will be 10 u t minus 3 minus 10 u t minus 10, you take common u t minus u t minus u t minus 6 right. This is your representation of v t that gate function.

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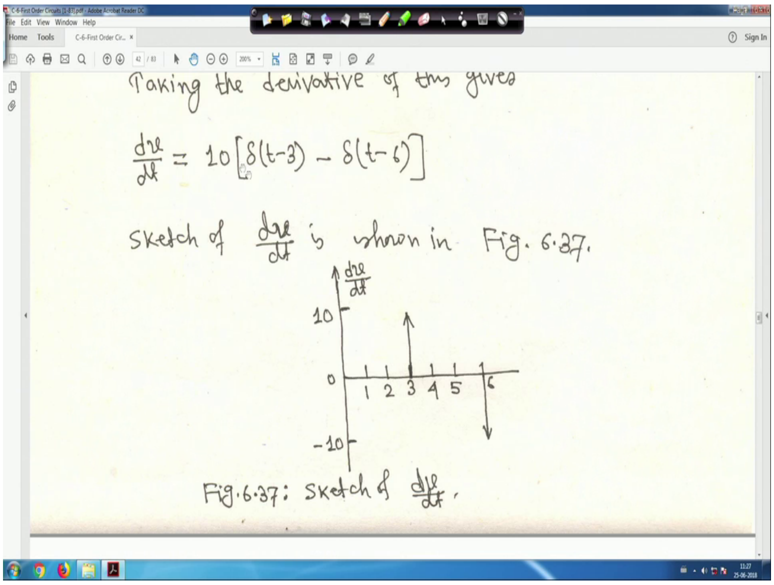

And if you take the derivative of this one d v by d t, it if you take derivative it will be your delta function that is your impulse function it will be 10 delta t minus 3 minus delta t minus 6. Here we have seen know this one that your what you call delta function is equal to your derivative of u right and a unit a step function is derivative of the ramp right.

So, if you take the derivative of this one, if you take the derivative of this one d v by d t is equal to 10, it will be delta t minus 3 if you take the derivative of unit function. It will be your delta function delta t minus 3 minus delta t minus 6, right. And now if you sketch del your d v by d t, so it is a delta function. So, at t is equal to 3 it is strength is 10. So, it will be 10 and at t is equal to 3 second it is plotted. Another thing is at t equal to 6 that is minus 10 it is t is equal to at t equal to 6 it is minus 10. So, this is plus 10 this is minus 10 it is plotted right.

So, because it is 10 into delta t minus strength is 10, right. So, this is how one can plot your what you call that your gate function and this is the things are very simple let me tell you things are very simple.

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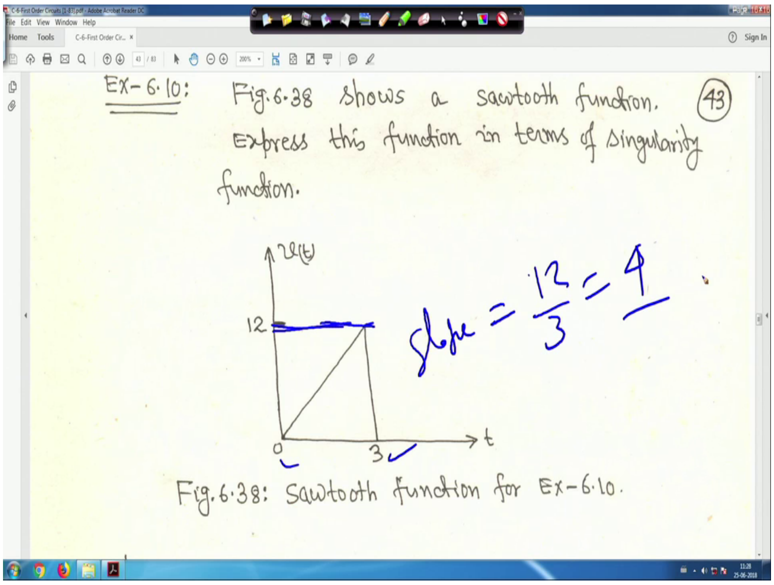

So, another one we will take thing this is like this figure it shows a sawtooth function, express this function in terms of singularity function, it is a sawtooth function is given right. So, it is basically a ramp function and your another thing is that is a gate function right. So, it is a slope it is this height, it is this height it is 12 right and this is from here to here it is 3. So, it is slope is equal to your 12 by 3 is equal to 4, the slope is equal to 4 for this straight line right.

Now, let me clear it. So how we will represent this one? How we will represent the v t right? Let us see.

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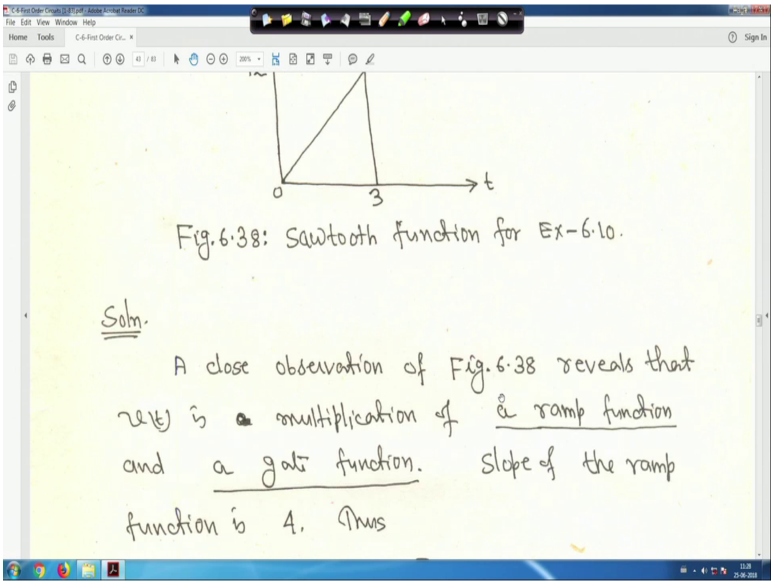

So, a close observation reveals that v t is multiplication of a ramp function, it is a ramp function. And another thing as a gate function and the gate function and this at t is equal to 3. Suppose your what you call, at t is equal to 3 it is your what you call that yours it is going on a ramp and at t is equal to 3 switches off right.

So, a close observation of figure 38 it reveals that v t is a multiplication of a ramp function and a gate function.

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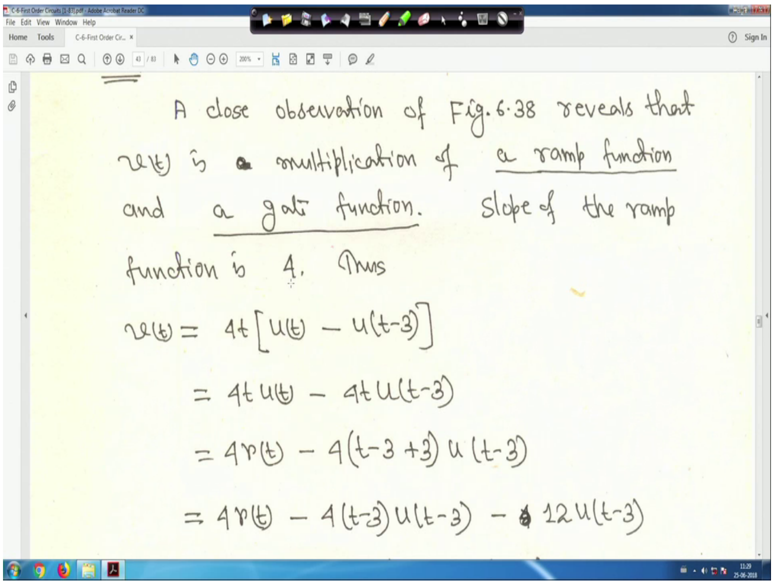

So, slope of the ramp function is 4, I told you it is slope of the ramp function is 4. Therefore, v t will be that they are what you call it will be 4 t then it will be u t minus u t minus 3 right. This way you can write because slope is 4, slope is 4 and it is a product of ramp function and a gate function right.

So, it will be slope is 4, so and is a ramp function is basically it is a straight line passing through the origin right. So that means, your this your this equation of this straight line will be your what you call; v t for this straight line only v t is equal to 4 t right. And then that means, we can represent 4 t u t and minus that your u t minus 3. Suppose at this point it is switches off, so 4 t u t minus 4 t u t minus 3, right.

So, in this case this, this one I mean it is understandable to you right. This slope is 4 slope is 4 equation of that straight line is 4 t and at t is equal to 3 second suppose something is switched off, then it will be your 4 t u t minus 4 t u t minus 3 right. So, that is your 4 t u t minus u t minus 3. So, multiply this. So it will be 4 t u t minus your 4 t u t minus 3.

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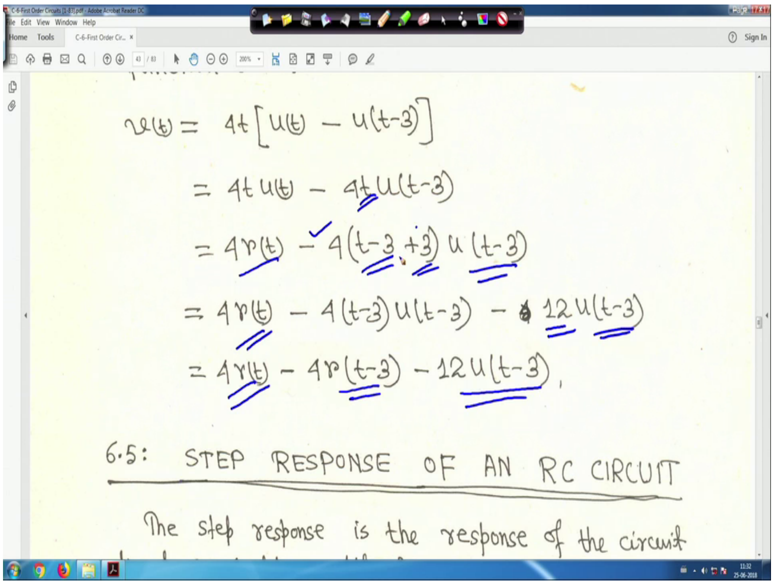

Now, this one this t, this t we can write it is t minus 3 t minus 3 plus 3, right. This equal to you can write this is 4 r t, 4 r t as it is. So, minus 4 t minus 3 into u t minus 3 and this is minus plus minus. So, minus 12 it is u t minus 3. So, it is 4 r t minus 4 t minus u t minus 3 minus 12 u t minus 3, that is equal to this is 4 r t ramp function minus 4 r t minus 3. That is your advanced or delayed we have already shown in the ramp function right, so, minus 4 r t minus 3 minus 12 u t minus 3. So, this is the answer, this is the answer right.

This is little bit little bit before moving to your step response of an RC circuit considering some sources. So, little bit you know particularly voltage source, when you will apply when you will solve the circuit little bit idea is required. So, that is why; so this is a simple thing and I told you how to break it. Little bit problem not much problem, we will study on this, but just to you know give you a some ideas; that is why we have made we have made it like this, some example right, other singularity function we will not consider in this course; this is just little bit of basic right. So, next is step response of an RC circuit.

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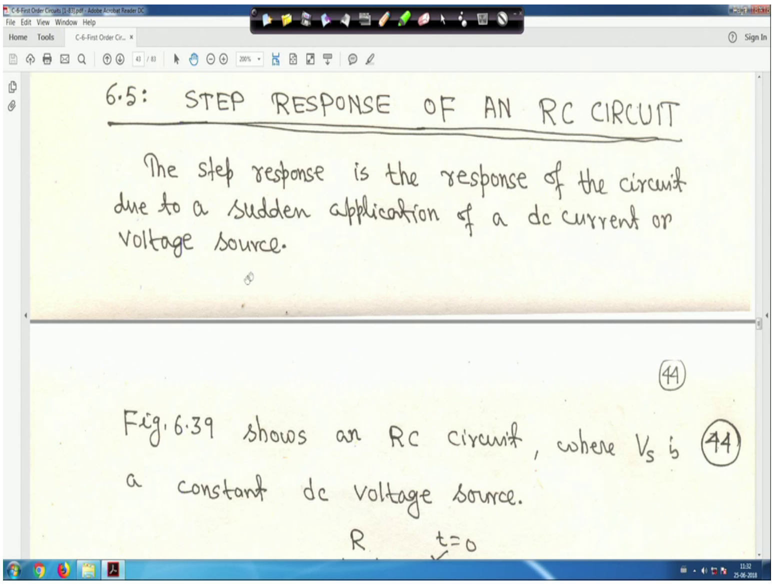

Now, the step response is the response of the circuit due to a sudden application of dc current or voltage source because our interest here is dc transient and that is first order circuit either RC circuit or RL circuit right. So, in this course we will not consider RLC circuit or LC circuit; that is second order circuit we will not consider. And so, the step response is the response of the circuit due to a sudden application of a dccurrent or voltage source.

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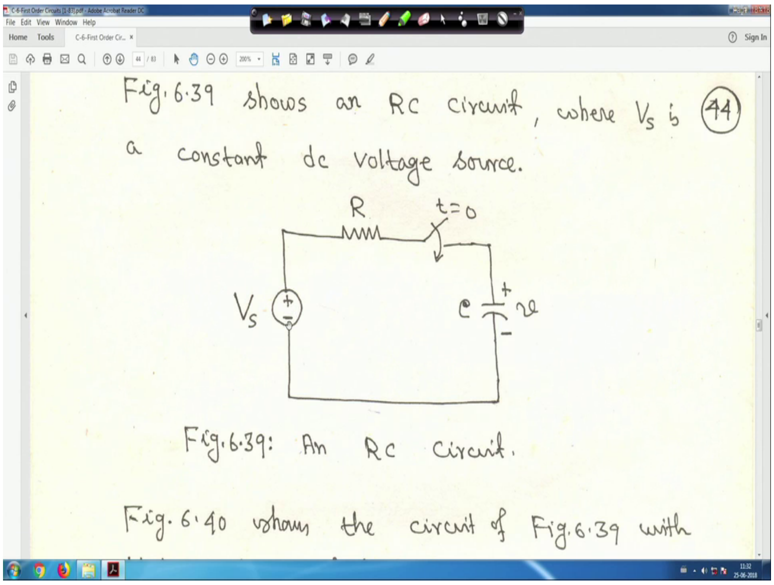

So, figure 39 this is your figure 39 right is shows RC circuit, where V s is a constant DC voltage source. This is actually your RC circuit, this is RC circuit and V s is a constant voltage source. And one is capacitor is there it may be initially charged maybe initially uncharged. So, at your for t less than 0 this switch was opened and at t at that at your what you call; at t greater than 0 the switch is closed right 0 minus of 0 plus just before switching it is 0 minus and just after switching it is 0 plus right. So, switch is closed right for t greater than 0.

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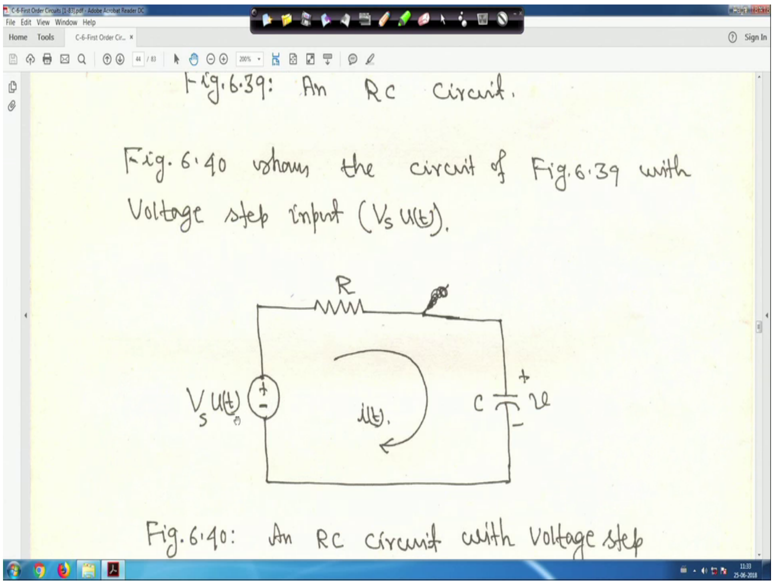

So, this circuit it this is the circuit and this can be represented as this is an RC circuit, this can be represented as that is V s u t that is; source voltage is this is your what you call this switch is closed. And it can be written as V s u t then R and C. Actually idea is simply like this for t less than 0 we know the unit step function for t less than 0 u t is equal to 0. And for t greater than 0 u t is equal to 1.

So, when it was t less than 0 this voltage source is completely disconnected, right? So, nothing is showing in the circuit, so in that case u t is equal to 0. So, here also if u t your u t is equal to your 0. And for your t greater than 0 right u t is equal to 1 when switch is closed that is why; this can be represented this circuit can be represented as like this V s into u t right. Earlier we have seen that the step function unit step function right.

So that means, that an RC circuit with voltage step input right.

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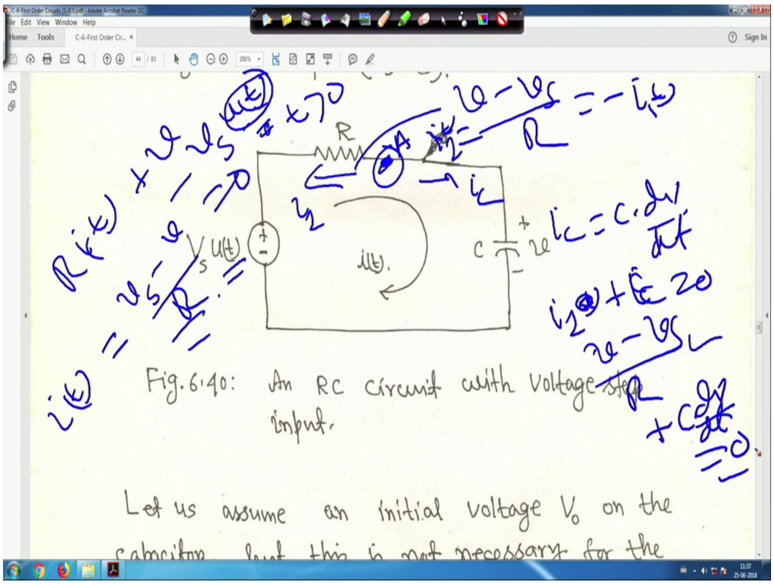

So, basically it is a voltage step input. I mean from here when it is open is ok, but as soon as you close the switch and if it is going as a step input. So, that is why it is a that your circuit of figure 39 with voltage step input V s u t. And this is the current i t this is the current i t right. So, before moving further, before moving further just one thing that current direction I have taken like this. Suppose if you take a if you take a your what you call that node like this from here, first let us see what is i t, you apply KVL. So, it will be R then i t moving like this clockwise right your plus your v right and minus V s right, V s u t you can write, but for t greater than 0 if greater than 0 then u t it is equal to 1 right. So that means, your it will be V R i t plus v minus equal to your is equal to 0 right; that means, my i t will be is equal to your this V s minus v divided by R. When for t greater than 0 so it is closed u t is equal to 1 right so; that means, i t is equal to this direction.

If you take, if you take current direction is like this; then this one will be v minus V s just it will be reversed it will V minus V s upon R this is the current. And this side, I mean this current if we take say i 1. And this side if we take this is your basically i 1 right is equal to minus of i t, t I am not putting here, if you want you can and this side you say i c right; that means, my i c is equal to c into d v by d t right; that means, at this point suppose this node is A, suppose you create like this and I apply KCL at this point.

So that means, my i 1 t rather i 1. I can make i 1 plus your i c is equal to 0; that means, my this is be V minus V s divided by R plus c into d v by d t is equal to 0 right. All the direction is taken like this, but if I if you apply KCLs at this point you take and this direction in this direction the current this. When we change this direction suppose this way it will V minus V S and for t greater than 0 u t is equal to 1 it will be V s, right? And that means, from this equation we have to try to solve this circuit.

Thank you very much, we will be back again.