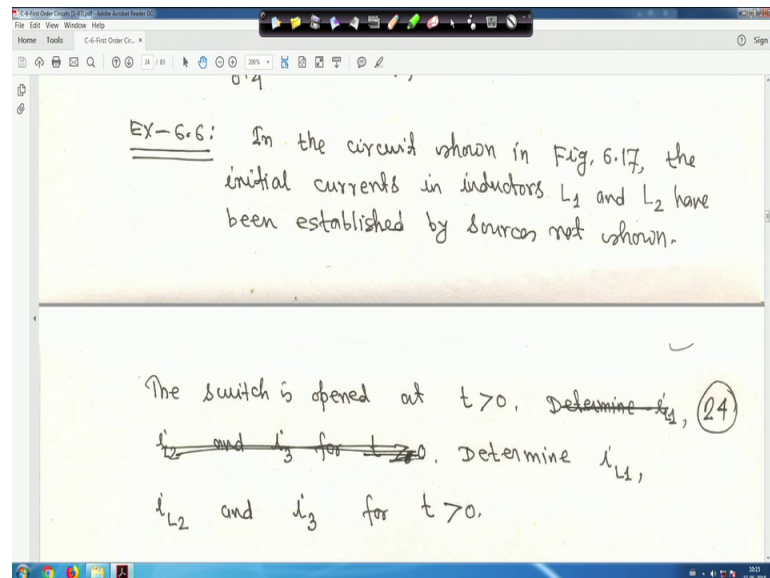


Fundamentals of Electrical Engineering
Prof. Debapriya Das
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Lecture – 31
First order circuits (Contd.)

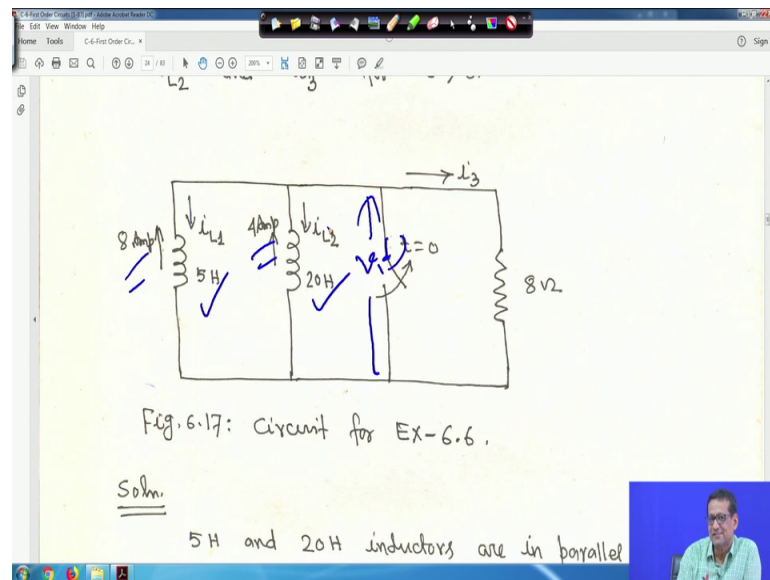
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So, next is your in the circuit shown in figure 17, the initial currents in inductor L_1 and L_2 have been established by the sources not shown. You assume that there are some sources, circuit I will show you later. So, there are some sources and in initial your what you call currents in the inductors, 2 inductors are there in parallel and it has been established right. So, the switch is opened at t greater than 0 and determine i_{L_1} , i_{L_2} and i_{L_3} , for t greater than 0.

The once for capacitor case RC circuit with 2 coil problem, where 2 capacitors were in series, but in this case a similar type of problem is taken when 2 inductors are in parallel. Here also, only one case, I will not tell you the reason, this is you to find out that you to try to think that why it has been taken right. And finally, if you have any difficulty you have the forum, there will there we are giving your all answers right. So but this I will leave up to you 1 or 2 things I will leave up to you. So, this is very interesting problem and just look at the circuit, right.

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All the direction of the current i_{L1} , i_{L2} and this is another i_3 is given right and your initial current through the inductor, it is given; 8 ampere and 4 ampere. This is also marked in that circuit, this is 8 ampere and this is 4 ampere. And this inductor is 5 Henry this is 20 Henry. And in problem it is said that in the circuit shown in this the initial currents L_1 and L_2 have been established by the sources right, not shown no need actually we assume that it was established. The switch is open at t greater than 0 right and you have to determine i_{L1} , i_{L2} and i_{L3} for t greater than 0.

It is the $i_{L1}(t)$, $i_{L2}(t)$ and $i_3(t)$ for t greater than 0 and switch is opened at t greater than 0; that means, at this time switch has opened right. Now look at this, so 5 if you look into that this 5 Henry and 20 Henry are in parallel. So, it is like a resistor you have to find out equivalent, the way we have found out equivalent resistance there also you have to find out equivalent inductance.

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Fig. 6.17: Circuit for Ex-6.6.

Soln.

5 H and 20 H inductors are in parallel. Hence,

$$L_{eq} = \frac{5 \times 20}{5 + 20} = 4 \text{ H.}$$
$$i_{L_1}(0) = 8 \text{ Amp; } i_{L_2}(0) = 4 \text{ Amp.}$$
$$\therefore i_{L_{eq}}(0) = i_{L_1}(0) + i_{L_2}(0) = 8 + 4 = 12 \text{ Amp.}$$

So, 5-ohm Henry and 20 Henry inductors are in parallel, therefore, L equivalent is equal to 4 Henry right. So, this is very simple. Now initial current that 8 ampere and your 4 ampere is given for inductor 1 and inductor 2 that is $i_{L_1}(0)$ and $i_{L_2}(0)$.

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$$i_{L_1}(0) = 8 \text{ Amp; } i_{L_2}(0) = 4 \text{ Amp.}$$
$$\therefore i_{L_{eq}}(0) = i_{L_1}(0) + i_{L_2}(0) = 8 + 4 = 12 \text{ Amp.}$$

Equivalent circuit is shown in Fig. 6.18.

So, here it is $i_{L_1}(0)$ is equal to 8 ampere and $i_{L_2}(0)$ is equal to 4 ampere. Therefore, $i_{L_{eq}}$ equivalent inductor current will be $i_{L_1}(0)$ plus $i_{L_2}(0)$ that is 12 ampere right. So, this is actually units that is $i_{L_1}(0) + i_{L_2}(0)$ is equal to $i_{L_1}(0) + i_{L_2}(0)$; that is 8 plus 4 is equal to 12 ampere, both we are adding. So and equivalent your what you call inductance is 4 Henry,

the parallel equivalent is 4 Henry and this is the voltage $v(t)$ we have taken some point we have taken $v(t)$ and this is the current i_3 . If you look into that this is open as soon as you this is actually open.

As soon as you open it, these 2 parallel equivalent we have taken and this is the current i_3 . So, and this is 8-ohm resistance. So, this is the equivalent circuit right.

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$\tau = L_{eq}/R = 4/8 = 0.5 \text{ sec.}$
 Therefore,
 $i_3(t) = 12 e^{-2t} \text{ Amp, } t \geq 0^+$
 $v(t) = 8 i_3(t) = 96 e^{-2t} \text{ Volt.}$
 at $t=0,$
 $v(0) = 96 \text{ Volt.}$

$i(t) = I_0 e^{-t/\tau}$
 $I_0 = 12 \text{ A}$

Now, in this case tau is equal to L_{eq} upon R . So, your L_{eq} is equal to 4 Henry and R is equal to 8 ohm. So, 4 by 8; so 0.5 second, so tau is equal to 0.5 second. Therefore, you know that i_3 generally we know $i(t)$ is equal to i_0 to the power minus t upon tau. So, here also $i_3(t)$ is equal to initial current to initial this current that i_0 is equal to 12 ampere right.

And your tau is equal to we got that is your somewhere I have mentioned that 0.5 second. So, basically we know this we know this one; this is for i_3 , but in general we got know $i(t)$ is equal to your i_0 , say, capital I_0 or small i_0 , so this thing your e to the power minus t upon tau right. So, in this case you are what you call that tau is equal to your 0.5 second that is why it is coming $2t$ and this I_0 is equal to that combined inductors are that this thing your initial current is equal to 12 ampere.

That also we have made it 8 plus 4 is equal to 12. So, that is why it is $12 e$ to the power minus $2t$ ampere right it is given $t > 0$ plus right. So, that therefore, voltage

across the 8 ohm let me clear it. Therefore, the voltage across the 8-ohm resistance is $v(t)$ is equal to $i(t) \cdot 8$ because this $i(t)$ current is flowing through 8-ohm resistance that is given in the circuit. So, $8 \cdot i(t)$ if you multiply, it will be substitute $i(t)$ here substitute $i(t)$ here if you multiply it will be $96 e^{-2t}$ to the power minus $2t$ volt.

So, at t is equal to 0 you put here at t is equal to 0, then $v(0)$ will be 96 volt right, so that means, this one if you go to this circuit then initial voltage; that means, at t is equal to 0 this voltage will be 90 or 96 volt right.

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at $t=0$,
 $v(0) = 96$ Volt.

$v = L \frac{di}{dt}$
 $di = \frac{1}{L} v dt$

$\therefore i_{L1}(t) = \frac{1}{L_1} \int_0^t 96 e^{-2t} dt - i_{L1}(0)$

$\therefore i_{L1}(t) = \frac{1}{5} \int_0^t 96 e^{-2t} dt - 8$

$\therefore i_{L1}(t) = (1.6 - 9.6 e^{-2t})$ Amp, $t \geq 0$

So, next one is that next is that when you are writing, that you are we know that your what you call that $i_{L1}(t)$ right that is your $v(t)$ is equal to in general $v(t)$ is equal to L into $d i$ upon $d t$.

Same for inductor 1 and inductor 2 the parallel inductors 2 are there right. So, when you make it, I mean we know that in general v is equal to L into $d i$ by $d t$ that we know right; that means, $d i$ is equal to your what you call your $d i$ is equal to 1 upon L right, then v into $d t$. If you integrate this like this, so, 1 upon L in general it is $v dt$. So this is my voltage v and this for inductor 1 and this is L_1 , this is everything is ok, but after this minus $i_{L1}(0)$ is there because initial current was there 4 ampere. So, why we have taken here the minus sign right? So minus or plus, this is a question to you right, L same as capacitor I put this question. So, hope when you will go through this video lecture right.

So, this 1 or 2 things I am leaving up to you just to see whether you can answer correctly or not you just put the answer in the forum. At that time, we will see it otherwise, I will answer immediately do not worry do not worry, but just leaving up to you just have some thinking; everything, we are trying to tell everything from my side I am trying my best to tell everything, but this is 1 or 2 I am leaving it to you that just think right.

Similarly, your what you call and if you integrate and if you put this value this 8 ampere right, i_{L1} 108 ampere, so it is coming actually your $1.6 - 9.6 e^{-2t}$ ampere, this is for t greater than or equal to 0 right. Next similarly, the same question was there for when 2 capacitors were in series right.

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$$\therefore i_{L1}(t) = \frac{1}{5} \int_0^t 96 e^{-2t} dt - 8$$

$$\therefore i_{L1}(t) = (1.6 - 9.6 e^{-2t}) \text{ Amp, } t \geq 0$$

Similarly,

$$i_{L2}(t) = \frac{1}{20} \int_0^t 96 e^{-2t} dt - i_{L2}(0)$$

$$\therefore i_{L2}(t) = -(1.6 + 2.4 e^{-2t}) \text{ Amp, } t \geq 0$$

Similarly, i_{L2} also here it was 5 Henry L_1 was 5 Henry, here it is 20 Henry and it is 0 to t $96 e^{-2t} dt$; so, minus $i_{L2}(0)$ right, so here also minus sign is there.

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Similarly,

$$i_{L2}(t) = \frac{1}{20} \int_0^t 96e^{-2t} dt - i_{L2}(0)$$
$$\therefore i_{L2}(t) = -(1.6 + 2.4e^{-2t}) \text{ Amp, } t \geq 0.$$

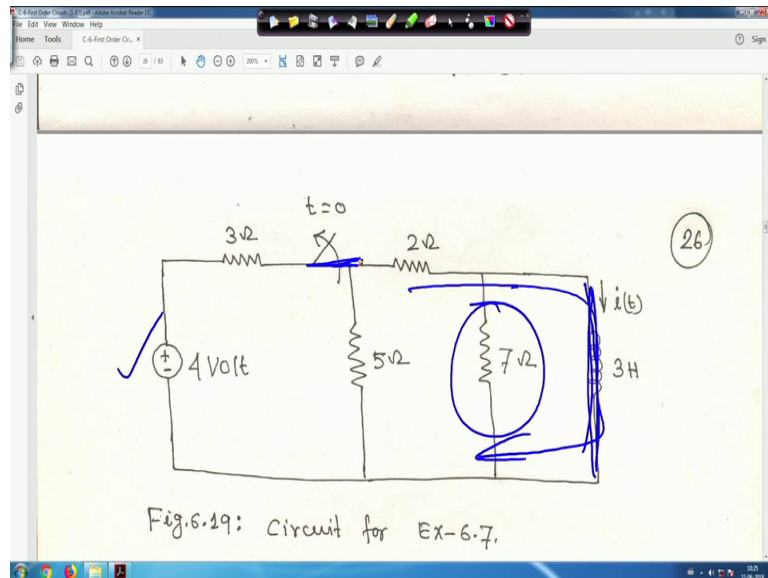
Ex-6.7: The switch in the circuit of Fig.6.19 has been closed for a long time. At $t=0$, the switch is opened. Determine $i(t)$ for $t > 0$.

So, this is a question to you that why it is? So if you put all if you do the integration and simplify right and you put $i_{L2}(0)$ initial condition that $i_{L2}(0)$ I think it was 4 ampere, you put it here 4 ampere and you will get $i_{L2}(t)$ is equal to this simplification minus in bracket 1.6 plus 2.4 e^{-2t} bracket close ampere, this is for i_{L2} .

Problem is very simple, but only thing that i_{L1} and i_{L2} why minus sign, right? This is a question to you. So, after this is a typical problem everything is and this voltage v_t means this whenever we are taking that your what you call at t is equal to 0. So, this voltage v_t ; that means, this is the voltage we have taken, when this switch is opened right. So, this is my v_t and this same voltage is impressed across this one and across this one. That is why that i_{L1} and i_{L2} we have obtained and initial current is given right.

So, only thing is that minus i_L , why minus $i_{L1}(0)$ and why minus $i_{L2}(0)$ in this 2 expression have been taken? This is a question for you right. So, next one is that another example in this case, the switch in the circuit. Example 7 the switch in the circuit I will show you has been closed for a long time right and at t is equal to 0. The switch is opened determine $i(t)$ for t greater than 0. You have to find out t the switch was closed and has been closed for a long time.

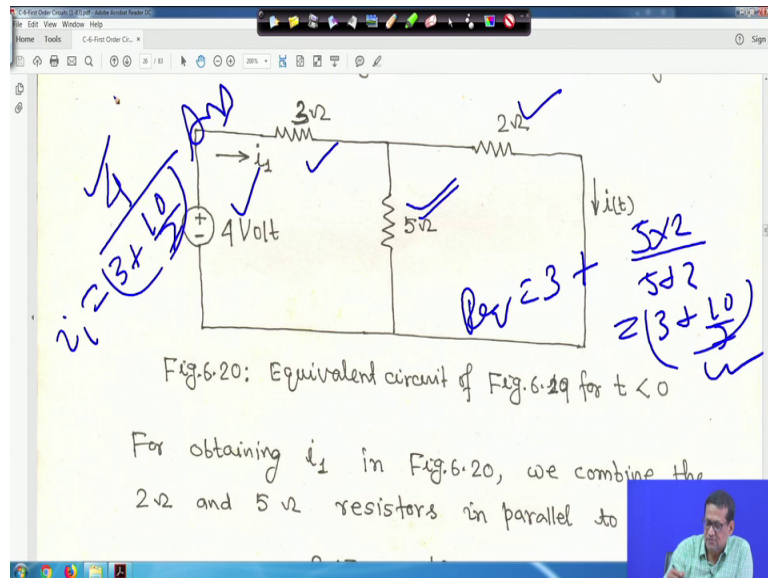
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If this switch is closed for a long time; that means, this switch was closed right the switch was closed and in that case what will happen? If the switch is closed for a long time then this inductor will behave as to DC because it is a DC source it will be as short circuit.

In that case, what will happen? This resistance as though current will flow through this resistance it will be completely isolated because, there just I am telling the current flow will take like this current flow will take like this. Because, this is the switch was closed for a long time. So, for DC when switch is closed for long time inductor will act a short circuit whereas capacitor act as open circuit this things you have to keep it in your mind.

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So, if it is so just looks just see the how is the circuit. I think here I might have made it for you. So, this if it is so this is the equivalent circuit I told you this thing will not be there. This 7Ω will not be there because this inductor is short circuited. So, that no current will flow; so circuit will be like this. So, this 7Ω should not be there. So, that is why it is 4V and some current i_1 is flowing here right, i_1 is flowing and this 5Ω and 2Ω are in parallel in this current; it is $i(t)$ right. Now if you just say if you look into this then your that for obtaining the current i_1 , it is simple DC circuit, it is simple.

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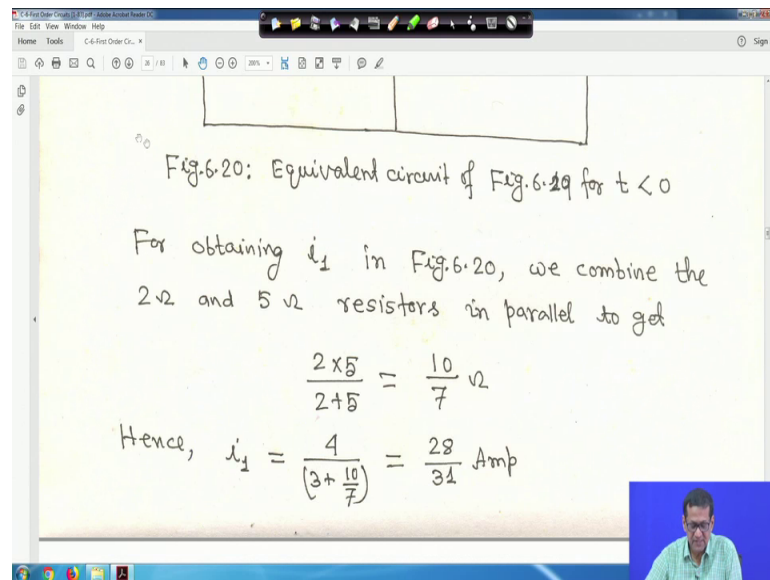


Fig.6.20: Equivalent circuit of Fig.6.19 for $t < 0$

For obtaining i_1 in Fig.6.20, we combine the 2 Ω and 5 Ω resistors in parallel to get

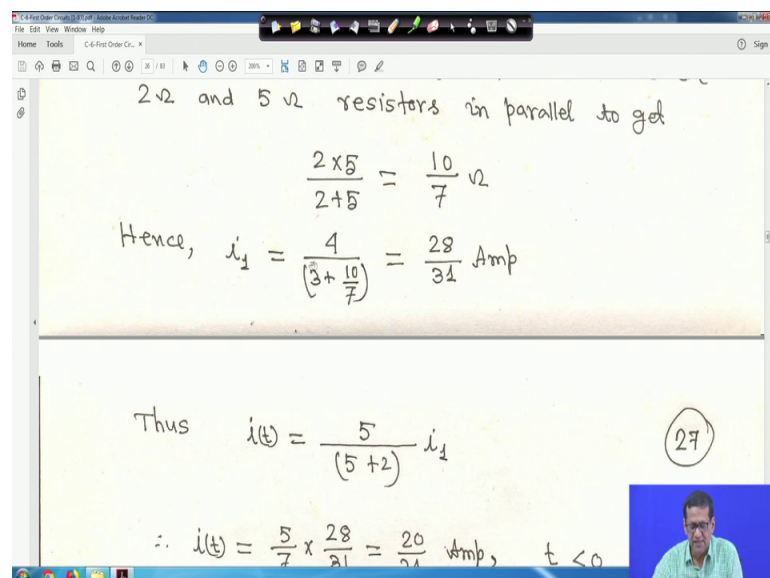
$$\frac{2 \times 5}{2 + 5} = \frac{10}{7} \Omega$$

Hence, $i_1 = \frac{4}{\left(3 + \frac{10}{7}\right)} = \frac{28}{31} \text{ Amp}$

For obtaining the current i_1 this 5 ohm and 2 ohm, they are in parallel right with that this 3 ohm in series.

So, R equivalent will be 3 plus your 5 into 2 by 5 plus 2 that is your 3 plus 10 by 7 ohm right. This is R equivalent and therefore, therefore, i_1 therefore, this i_1 will be your, I mean if you take this equivalent one then I am not drawing the second circuit is understandable to you; therefore, it will be 4 divided by 3 plus 10 by 7 ampere because this source is 4 volt this source is 4 volt right. So, let me clear it so.

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2 Ω and 5 Ω resistors in parallel to get

$$\frac{2 \times 5}{2 + 5} = \frac{10}{7} \Omega$$

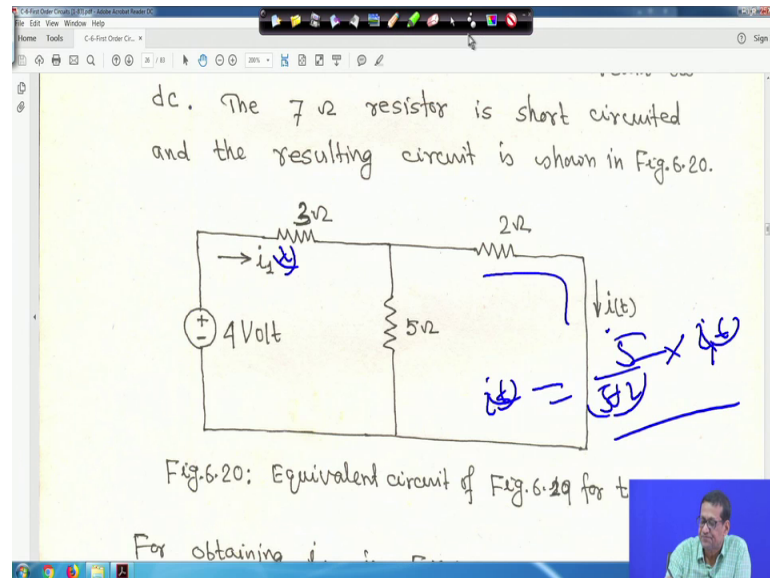
Hence, $i_1 = \frac{4}{\left(3 + \frac{10}{7}\right)} = \frac{28}{31} \text{ Amp}$

Thus $i(t) = \frac{5}{(5+2)} i_1$

$\therefore i(t) = \frac{5}{7} \times \frac{28}{31} = \frac{20}{31} \text{ amp, } t < 0$

So, if this is your what you call that i_1 is equal to $4 \text{ by } 3 \text{ plus } 10 \text{ by } 7$ 28 upon 31 ampere. Now, this current division i_t will be $5 \text{ by } 5 \text{ plus } 2$ into i_1 right now, this i_1 you know this i_1 you know.

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So, your i_t will be this i_t will be in terms of t say here also if you want it is a function of t say. So, it will be the current division what is the current flowing through this 2-ohm resistance that is your i_t is equal to $5 \text{ by } 5 \text{ plus } 2$ into your i_1 right.

Is a current division so $5 \text{ by } 7$ into i_1 ? So, let me clear it sorry right.

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Thus $i(t) = \frac{5}{(5+2)} i_1$ (27)

$\therefore i(t) = \frac{5}{7} \times \frac{28}{21} = \frac{20}{31} \text{ amp, } t < 0.$

Since the current through an inductor cannot change instantaneously,

$i(0) = i(0^-) = \frac{20}{31} \text{ amp.}$

So, in this case this is 5 by 5 plus 2 into i_1 . So, that is your $i(t)$ your what you call $i(t)$ will be your this is actually just 1 minute one correction I will make it. So, better you do not put it I simply write your I simply write it $i(t)$. So, 20 upon 31 ampere for $t < 0$ right it is initial condition. So, by mistake I have made it $i(t)$. So, t better not right simply you put I right.

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$i(t) = \frac{5}{7} \times \frac{28}{21} = \frac{20}{31} \text{ amp, } t < 0.$

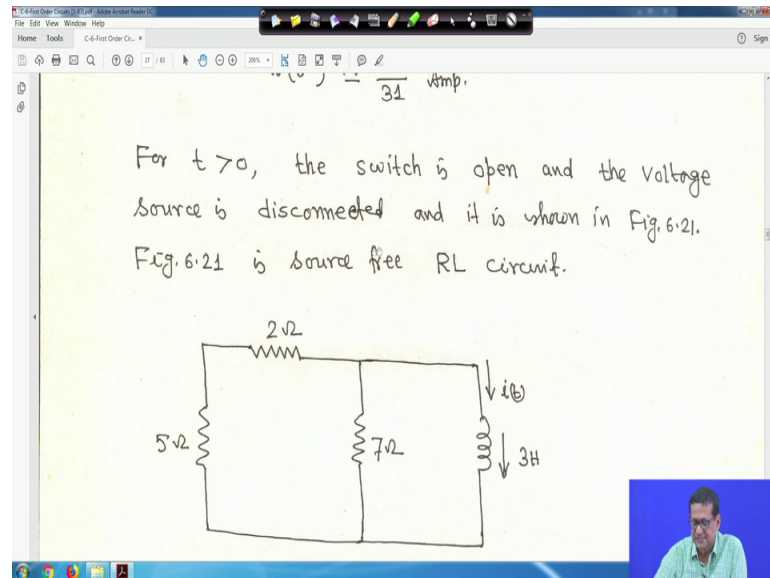
Since the current through an inductor cannot change instantaneously,

$i(0) = i(0^-) = \frac{20}{31} \text{ amp.}$

For $t > 0$, the switch is open and the voltage source is disconnected and it is shown in Fig. 6.21. Fig. 6.21 is source free RL circuit.

So, let me clear it so since the current through an inductor cannot change instantaneously; that means, $i(0)$ is equal to $i(0^-)$ which is equal to 20 upon 31 ampere. This is the initial thing right for t greater than 0 the switch is open right.

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And the voltage source is disconnected and it is shown in figure this thing at t greater than 0 the switch is open. So, come to the circuit first right the circuit first at t greater than 0 the switch is open the switch is open means this part is not there this is part is not there right. So, if this part is not there let us see the how the circuit is right.

So, in this case in this case, that your this $i(t)$ is there 3 Henry 3 Henry inductor is there and this 2 ohm and 5 ohm are will be will be in series right so; that means, that means this 2 plus 5.

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Fig. 6.21 is source free RL circuit.

Fig. 6.21: Equivalent circuit of Fig. 6.20 for $t > 0$.

From Fig. 6.21;

$$\frac{7 \times 7}{7 + 7} = \frac{7}{2} \Omega$$

This is your 7 ohm and this 7 ohm 7 ohm are in parallel. So, equivalent will be 7 into 7, right by your 7 plus 7. That is actually the 7 by 2 ohm right actually 7 by 2 ohm. So, this equivalent is 7 by 2 ohm then your tau will be L upon R whatever time constant will come your L is 3 Henry.

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Fig. 6.21: Equivalent circuit of Fig. 6.20 for $t > 0$.

From Fig. 6.21;

$$R_{eq} = \frac{(5+2) \times 7}{(5+2) + 7} = \frac{49}{14} \Omega$$

The time-constant is

$$\tau = \frac{L}{R_{eq}} = \frac{3}{(49/14)} \text{ sec} = \frac{6}{7} \text{ sec.}$$

So, in this case, your R eq is equal to 49 by 14 that is 7 by 2 ohm right I didn't written for that, but it is 7 by 2 ohm so time constant is L upon Req.

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The screenshot shows a digital whiteboard with the following content:

$$i(t) = i(0) e^{-t/\tau} = \frac{20}{31} e^{-7t/6} \text{ Amp}$$
$$\therefore i(t) = 0.645 e^{-7t/6} \text{ Amp, } t > 0.$$

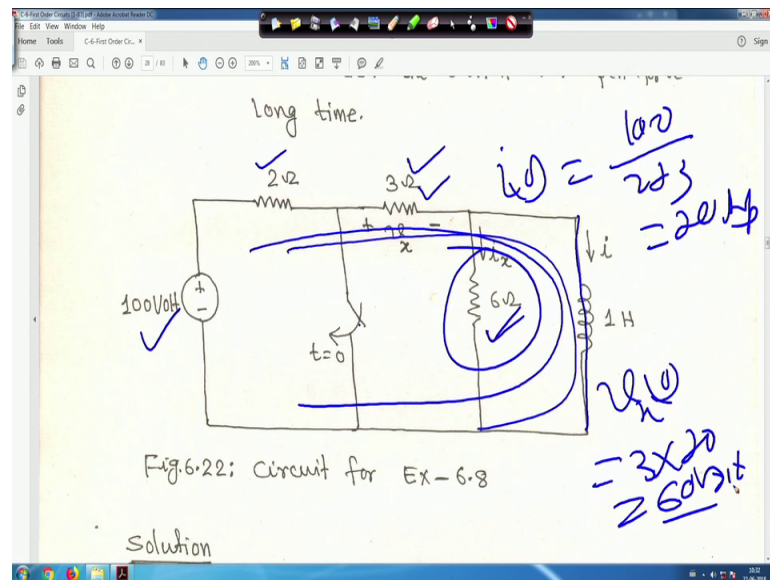
Ex-6.8: In the circuit shown in Fig. 6.22, determine i_x , v_x and i for all time. Assume that the switch was open for long time.

A small video inset in the bottom right corner shows a man speaking.

So, 3 upon 49 by 14; so 6 by 7 second so thus $i(t)$ is equal to $i(0) e^{-t/\tau}$ this we know. So, that is 20 upon 31 and τ you got 6 by 7 second you substitute you will get 20 upon 31 $e^{-7t/6}$ ampere.

So, $i(t)$ is equal to; that means, $i(t)$ is equal to $0.645 e^{-7t/6}$ ampere that is for $t > 0$. So, this is a simple problem only you have to little bit your what you call some basic to that dc. If the switch is closed for long time, that inductor will behave as a short circuit and for the capacitor it will behave as a for DC that open circuit. This thing you have to keep it in your mind right. So, this is I hope I hope you have understood this simple problem.

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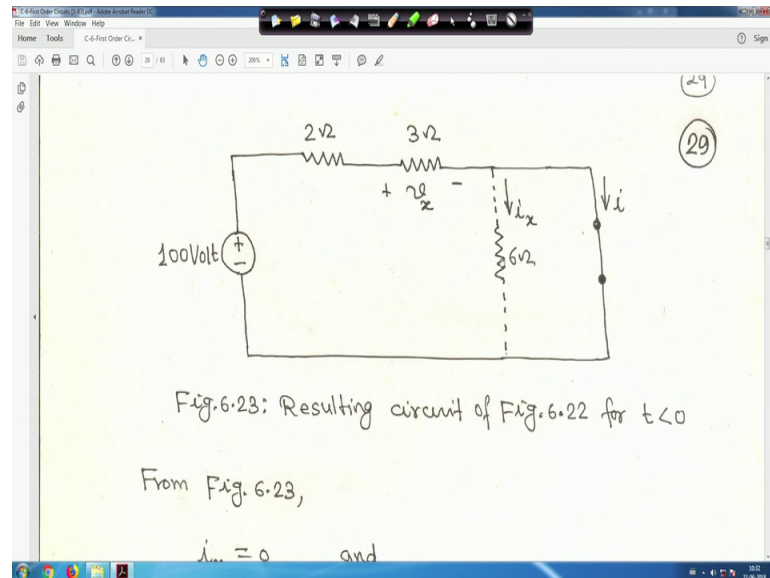
So, next one is in the circuit shown in figure 22, you have to determine i_x and v_x and i that is your you have to determine your i_x this is your i_x then your v_x that is the voltage across this 3-ohm resistance and i . Can you this is the i for all time right that is i which current flowing to one in the inductor assume that the switch was open for a long time here in this case it is given opposite that switch was opened for a long time right. So, if switch was opened for a long time, first you have to see that initial condition right so let me clear it.

So, when the switch was open for a long time. So, this will be the circuit because if switch is open for a long time if the switch is open for a long time; that means, that it is a DC volt 100 10 volt right and inductor behaves as a short circuit inductor behaves as a short circuit so, it is short. So, in that case what will happen that if it is open for a long time, the current will flow like this and there will be no current in the 6-ohm resistance because it because it behaves like a short circuit. So, there will be no current here. So, it will take this path it will go like this. So, no current will flowing so initial current to i_x 0 is 0 right and initial through the inductor initial current is it is 2 ohm and 3 ohm will be this is open right.

It has open for a long time and only at t greater than 0 it was closed. So, it was open for a long time so initial current that i_0 will be 100 divided by 2 plus 3 that is 20 ampere that is the initial current that is flowing to the inductor right and that i_x 0 at it will be 0. So,

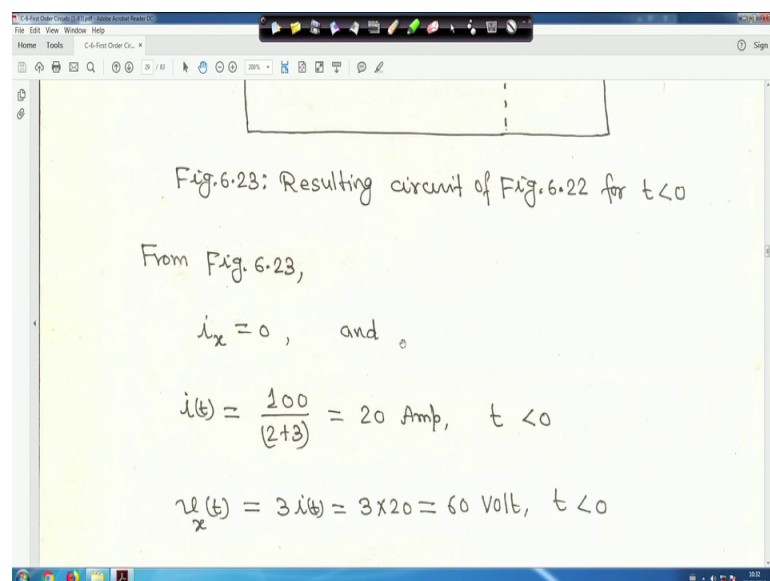
so let me clear it and if you try to find out the initial voltage $v_x(0)$ that will be your 3 into 20. So, it will be 60 volt right, that mean if you want to find out what is the initial voltage $v_x(0)$ it will be 3 into 20 that is your 60 volt right.

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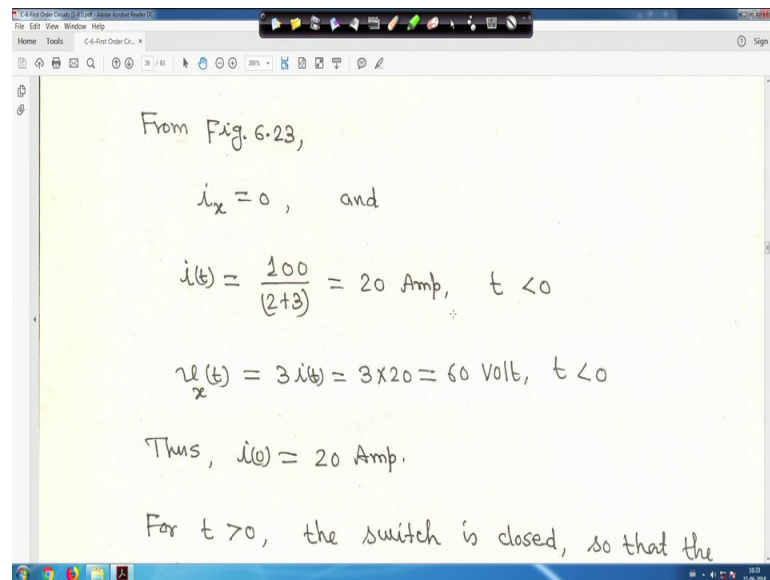
So this is the circuit I have drawn. So, 6 ohm is totally isolated right it is it is no current is flowing here while switch was open for long time and this is 2 ohm 3 ohm are in series I made it for you.

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So, $i_x(0)$ is equal to 0 I told you as i_x is equal to 0 initial and it is.

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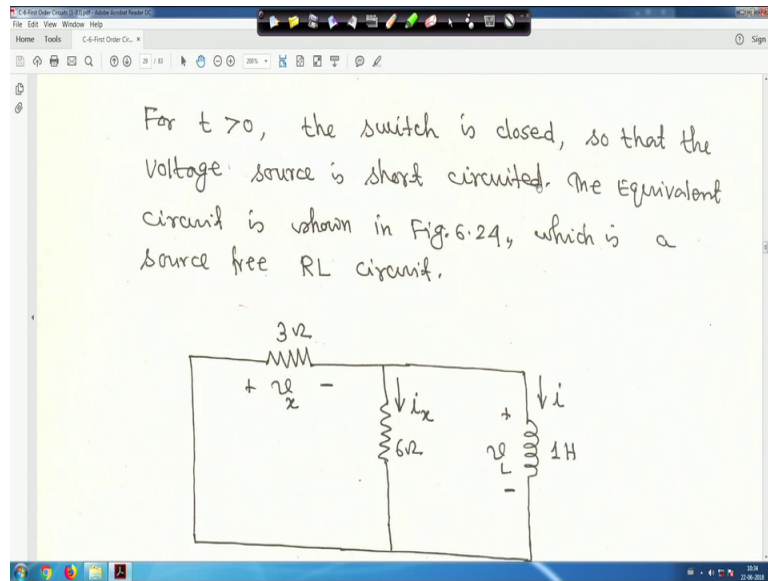
The image shows a digital whiteboard with handwritten mathematical work. The text is as follows:

From Fig. 6-23,
 $i_x = 0$, and
 $i(t) = \frac{100}{(2+3)} = 20 \text{ Amp, } t < 0$
 $v_x(t) = 3i(t) = 3 \times 20 = 60 \text{ Volt, } t < 0$
Thus, $i(0) = 20 \text{ Amp.}$
For $t > 0$, the switch is closed, so that the

Actually your, what you call this is actually a switch was a closed for this thing. So, I suggest a better instead of writing t you just write I initial condition the way I wrote $i(0)$ with do not write t , this is while making it I made some your what you call the writing error. So, it just make $i(0)$ is equal to 20 ampere and I made this one $v(0)$ right that is your 60 volt.

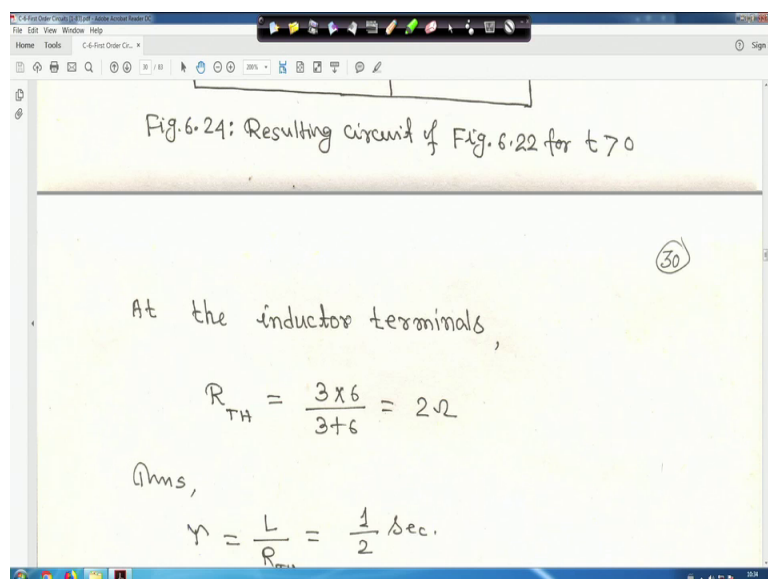
This is because this is for t less than 0 of course, one thing is there as I have mentioned t less than 0 or t less than 0. So, $i(t)$ here also you can make it for to all t less than 0 it is also $i(t)$ also is valid, but you just make it for $i(0)$ and $v_x(0)$ both are valid, but still I have made it, because as we have as we have mention it here if you have mention I overlook this one if you have mention in this one then $i(t)$ and $v_x(t)$ is if it is not mentioned then you put initial value that $i(0)$ and $v(0)$ right. So, and that is why $i(0)$ is equal to I write it 20 ampere that is the initial current right. So, let me clear it.

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So now, for t greater than 0 the switch is closed right switch is closed. So, that the voltage source is short circuited; that means, as soon as you if you look into the circuit when if you look into the circuit at t is equal to 0 this switch is closed; that means, this voltage is this voltage source is short circuited because this is short right. So, what will be the equivalent your what you call equivalent circuit in that case. So, in that case what will happen? That this is your equivalent circuit voltage source is short circuited. So, this 3 ohm and 6-ohm resistor are in parallel right and there equivalent will be 6 into 3 by 6 plus 3. So, 2 ohm right so and this is one Henry inductor.

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So, that means if you are writing parallel resistance, but the same time at the inductor terminal if you make it like this R the venin that is nothing but the 2 resistors are in parallel 3 into 6 by 3 plus 6 is equal to 2 ohm right. And tau is equal to we are writing L upon R the venin that is L is 1 Henry.

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the inductor terminals,

$$R_{TH} = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

Ans,

$$\tau = \frac{L}{R_{TH}} = \frac{1}{2} \text{ sec.}$$

Hence,

$$i(t) = i(0) e^{-t/\tau} = 20 e^{-2t} \text{ Amp, } t > 0.$$

Applying KVL, we have;

Here L is given 1 Henry and your this one given that 2 your R the venin is 2. So, half second tau is equal to half second.

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$$\tau = \frac{L}{R_{TH}} = \frac{1}{2} \text{ sec.}$$

Hence,

$$i(t) = i(0) e^{-t/\tau} = 20 e^{-2t} \text{ Amp, } t > 0.$$

Applying KVL, we have;

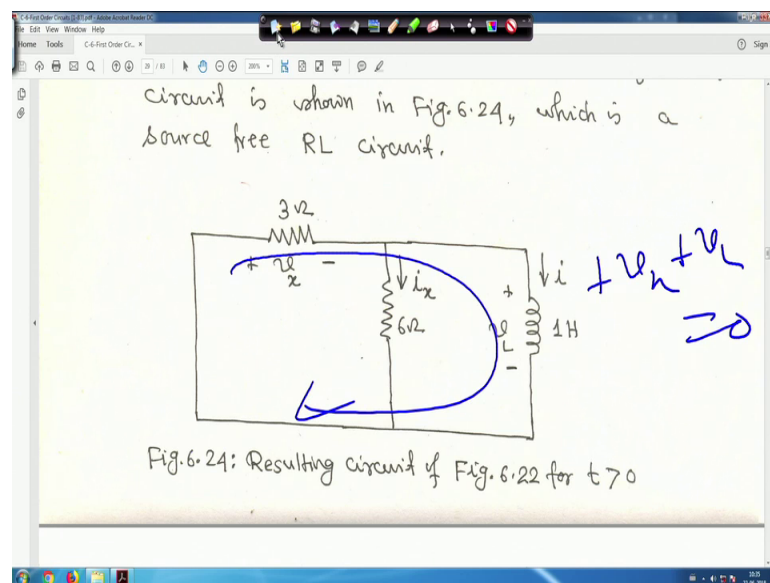
$$v_R(t) + v_L(t) = 0$$

$$\therefore v_L(t) = -v_R(t) = -L \cdot \frac{di}{dt} = -(1)(20)(-2) e^{-2t}$$

Now, we know that $i(t)$ is equal to $i(0)$ the source p circuit we know I_2 is equal to $i(0)$ e to the power minus t upon τ . So, $i(0)$ is equal to 20 and τ your τ is equal to half so e to the power minus 2 t ampere for t greater than 0 right.

Applying KVL we have $v_x(t) + v_L(t)$ is equal to 0 right now if you apply KVL in the circuit if you apply KVL here if you apply KVL here.

(Refer Slide Time: 20:51)



So, if you look this that this is v plus v_x plus v_L is equal to 0 right. So, apply KVL in the circuit. Let me clear it right.

(Refer Slide Time: 21:08)

Handwritten mathematical derivation showing the application of KVL to the circuit. It starts with the current expression $i(t) = 20e^{-2t}$ Amp, $t > 0$. Then it applies KVL: $v_x(t) + v_L(t) = 0$. This leads to $v_x(t) = -v_L(t) = -L \cdot \frac{di}{dt} = -1 \cdot (20) \cdot (-2) \cdot e^{-2t} = 40e^{-2t}$ Volt, $t > 0$. Finally, it shows $i(t) = \frac{v_L(t)}{L} = 40e^{-2t}$.

So, $v_x(t)$ is equal to minus $v_L(t)$ is equal to minus $L \frac{di}{dt}$. So, it is L is 1 Henry minus 1 and $\frac{di}{dt}$ is you take the derivative of this 1; so, 20 into minus 2 into e to the power minus 2t. So, $v_x(t)$ will be $40 e^{-2t}$ volt that is for t greater than 0 right.

(Refer Slide Time: 21:31)

Handwritten derivation showing the calculation of voltage $v_x(t)$ and current $i_x(t)$ over time $t > 0$.

$$\therefore v_x(t) = -v_L(t) = -L \cdot \frac{di}{dt} = -(1)(20)(-2)e^{-2t}$$

$$\therefore v_x(t) = 40e^{-2t} \text{ Volt, } t > 0$$

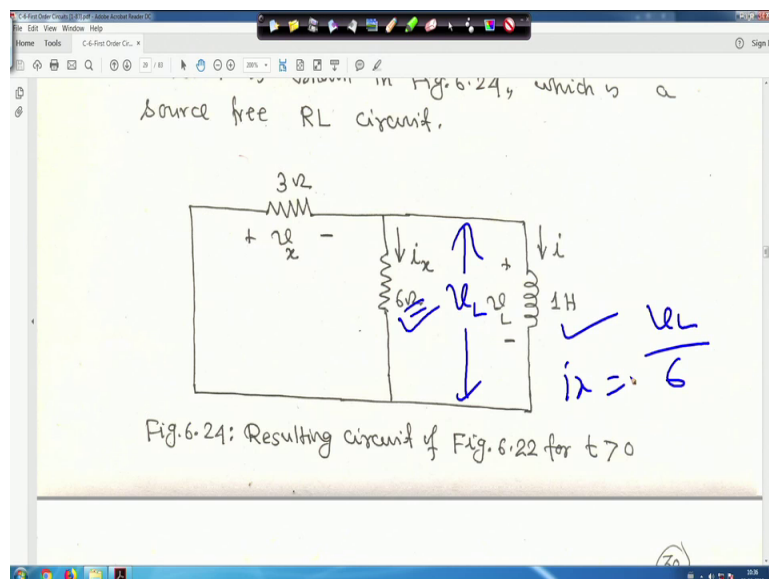
and

$$i_x(t) = \frac{v_x(t)}{6} = -\frac{40e^{-2t}}{6}$$

$$\therefore i_x(t) = -\frac{20}{3}e^{-2t}, \text{ Amp, } t > 0$$

So, and $i_x(t)$ is equal to $v_L(t)$ upon 6 because this whatever voltage you got $v_L(t)$ right this 6 ohm and this 6 ohm and 1 Henry are in parallel.

(Refer Slide Time: 21:45)



So, whatever you get across this is v_L right then i_x will be your in general v_L not writing t . Later, we will see that understand this divided by 6-ohm resistance is the current right.

(Refer Slide Time: 22:03)

Handwritten mathematical derivation on a digital whiteboard:

$$\frac{d}{dt} \dots (20)(-2)e^{-2t}$$

$$\therefore v_x(t) = 40e^{-2t} \text{ Volt, } t > 0$$

and

$$i_x(t) = \frac{v_L(t)}{6} = -\frac{40e^{-2t}}{6}$$

$$\therefore i_x(t) = -\frac{20}{3}e^{-2t}, \text{ Amp, } t > 0$$

Thus for all time,

$$i_x(t) = 0 \text{ Amp, } t < 0$$

So in this case, we are writing $i_x t$ is equal to $v_L t$ upon 6. So, v_L your what you call $v_L t$ is given that you are we have got $v_L t$ is equal to your minus $v_L t$ is equal to minus $L \frac{di}{dt}$ this much right.

(Refer Slide Time: 22:19)

Handwritten mathematical derivation on a digital whiteboard:

Applying KVL, we have;

$$v_x(t) + v_L(t) = 0$$

$$\therefore v_x(t) = -v_L(t) = -L \cdot \frac{di}{dt} = -(20)(-2)e^{-2t}$$

$$\therefore v_x(t) = 40e^{-2t} \text{ Volt, } t > 0$$

and

$$i_x(t) = \frac{v_L(t)}{6} = -\frac{40e^{-2t}}{6}$$

Additional handwritten notes in blue ink:

$$v_L(t) = -\frac{v_x(t)}{2t} = -\frac{40e^{-2t}}{2t}$$

So, $v \times t$ is equal to this much and $v L t$ we are writing minus why because here this $v \times t$ is equal to minus $v L t$ and $v \times t$ will get this much; that means, our $v L t$ is equal to your minus $v \times t$. So, that is actually minus $40 e^{-2t}$ that is why this minus sign is here.

Right so, minus $40 e^{-2t}$ upon 6 so $i \times t$ is equal to then minus 20 upon $3 e^{-2t}$ ampere this is it is set for all time.

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The whiteboard content is as follows:

$$\therefore i_x(t) = -\frac{20}{3} e^{-2t}, \text{ Amp, } t > 0$$

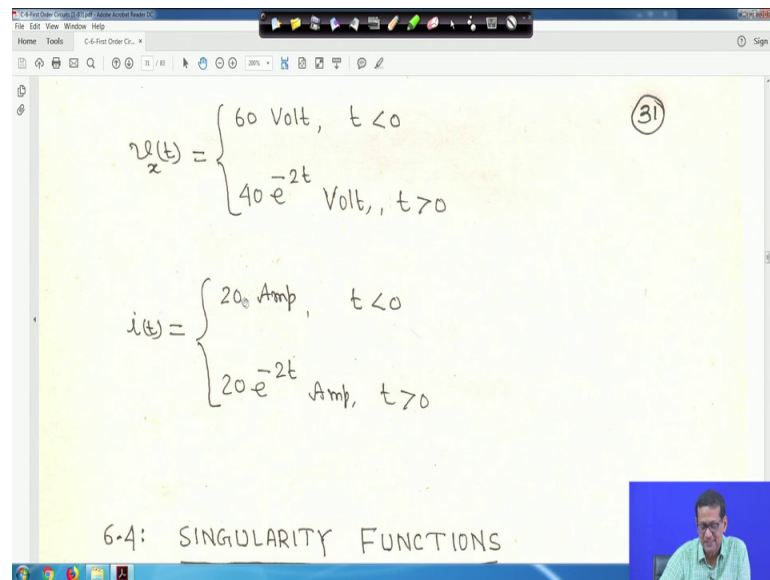
Thus for all time,

$$i_x(t) = \begin{cases} 0 \text{ Amp, } t < 0 \\ -\frac{20}{3} e^{-2t} \text{ Amp, } t > 0 \end{cases}$$

(60 Volt, $t < 0$) (31)

That means, if you mention this, $i \times t$ is equal to 0 ampere for all t less than 0 and $i \times t$ is equal to minus 20 upon $3 e^{-2t}$ ampere for t greater than 0. This way you should write when you say all t means it is actually in general it is actually minus infinity to plus infinity right. So, that is why for 0 ampere it is like this t less than 0 and it is $i \times t$ minus 20 upon 3 this we got.

(Refer Slide Time: 23:21)


$$v_x(t) = \begin{cases} 60 \text{ Volt}, & t < 0 \\ 40 e^{-2t} \text{ Volt}, & t > 0 \end{cases} \quad (31)$$
$$i(t) = \begin{cases} 20 \text{ Amp}, & t < 0 \\ 20 e^{-2t} \text{ Amp}, & t > 0 \end{cases}$$

6-4: SINGULARITY FUNCTIONS

Similarly, for voltage we got that 60 volt that t less than equal to 0 right that is and 40 e to the power minus 2 t while you mention this one at that time $v \times t$ is equal to 60 volt. Right, if you do not if you do not try to mention then we will assume that it is initial value 0, but for anything less than t less than 0 it is 60 volt right. Similarly, it is also initially 20 ampere you got that is for all t less than 0 and because switch was closed for I think in this case it was open for a long time and $20 e$ to the power minus 2 t ampere 40 greater than 0.

If says all time then in this way you will write the answer right this way you write the answer in general, it is minus infinity to plus infinity whatever break up is there for time you write right.

(Refer Slide Time: 24:07)

6.4: SINGULARITY FUNCTIONS

Singularity functions are functions that either are discontinuous or have discontinuous derivatives. A basic understanding of singularity functions will help us to make sense of the response, especially the step response of RC or RL circuits. Singularity functions are also called switching functions.

So, next is I hope up to source I know this is up to this is source free circuit whatever we have studied. So, for right so next is that a singularity functions because, we have to take a now you are your what you call that where source is there in the circuit. So, before that little bit we were learn about we will learn about singularity function.

So, singularity function are function that either are discontinuous or have discontinuous derivatives right. So, a basic understanding of the singularity function will help us to make sense of the response specially the step response of RC or RL circuit right. So, singularity function are also called the switching function right.

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Singularity functions will help us to make sense of the response, especially the step response of RC or RL circuits. Singularity functions are also called switching functions and very useful in circuit analysis.

In circuit analysis, the three most widely used singularity functions are the unit step, the unit impulse and the unit ramp functions, Basic unders

So, sometimes it is called all I have underlined the switching function and very useful for circuit analysis right.

(Refer Slide Time: 25:07)

and very useful in circuit analysis.

In circuit analysis, the three most widely used singularity functions are the unit step, the unit impulse and the unit ramp functions, Basic understanding of these three functions help to make sense of the first-order circuits following a

So, in circuit analysis the 3 most widely used singularity function are the unit step function the units impulse and the unit ramp. These 3 functions one is unit step the unit impulse and the unit ramp all are underlined right. So, basic understanding of these 3 functions helps to make sense of the first order circuit right.

(Refer Slide Time: 25:28)

6-4.1: UNIT STEP FUNCTION

The unit step function $u(t)$ is 0 for $t < 0$ and 1 for $t > 0$.

In mathematical forms,

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \quad \dots (6.28)$$

So, following a your what you call sudden application of an independent DC voltage or current source first we will understand this one by one. Then we will go to that your what you call that your at that time source will be there in the circuit at that time, you will see the transient response and you will find things are very easy right.

So, unit step function first right basically unit step function this kind of singularity function step function, ramp function all set of things you will find mostly used in the circuit thing as well as in the control system right. So, for unit step function that $u(t)$ is 0 for t your what you call for t less than 0 right and for and 1 for t greater than 0. So, $u(t)$ is equal to 0 for t less than 0 and $u(t)$ is equal to 1 for t greater than 0 right. So, this we will define the unit function your what you call the unit step function.

(Refer Slide Time: 26:31)

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \quad \dots (6.28)$$

The unit step function is undefined at $t=0$, where it changes suddenly from 0 to 1. It is a dimensionless quantity. Fig.6.25 shows the unit step function.

$u(t)$ ↑

So, so it is, but one thing is there it is undefined at t is equal to 0, but when it is it is undefined at t is equal to 0 it is less than 0 t less than 0 it is 0 t greater than 0 is 1, but is undefined at t is equal to 0 right where it changes suddenly from 0 to 1. And second thing is it is dimensionless quantity this thing we have to keep it in our mind it is dimensionless quantity. So, figure 25 shows the unit step function I will show you the figure. So, this is the unit function def time your def for all time how you define unit time function.

(Refer Slide Time: 27:08)

dimensionless quantity. Fig.6.25 shows the unit step function.

$u(t)$ ↑

↑

→ t

Fig.6.25: The unit step function.

Instead of $t=0$, if the sudden change occurs at $t=t_0$ ($t_0 > 0$), the unit step function

So, this is your unit time function, but remember at t is equal to 0 it is undefined right. So, in for this case $u t$ is defined this is unity and this is like this we have drawn because for t less than 0 it is your 0 so, but at t is equal to 0 your what you call it is undefined. So, instead of t is equal to 0 if the sudden change occurs at t is equal to t_0 that is t_0 greater than 0 right. So, what will happen? Suppose, instead of t is equal to t_0 , if the sudden changes occur at t is equal to t_0 where t_0 greater than 0.

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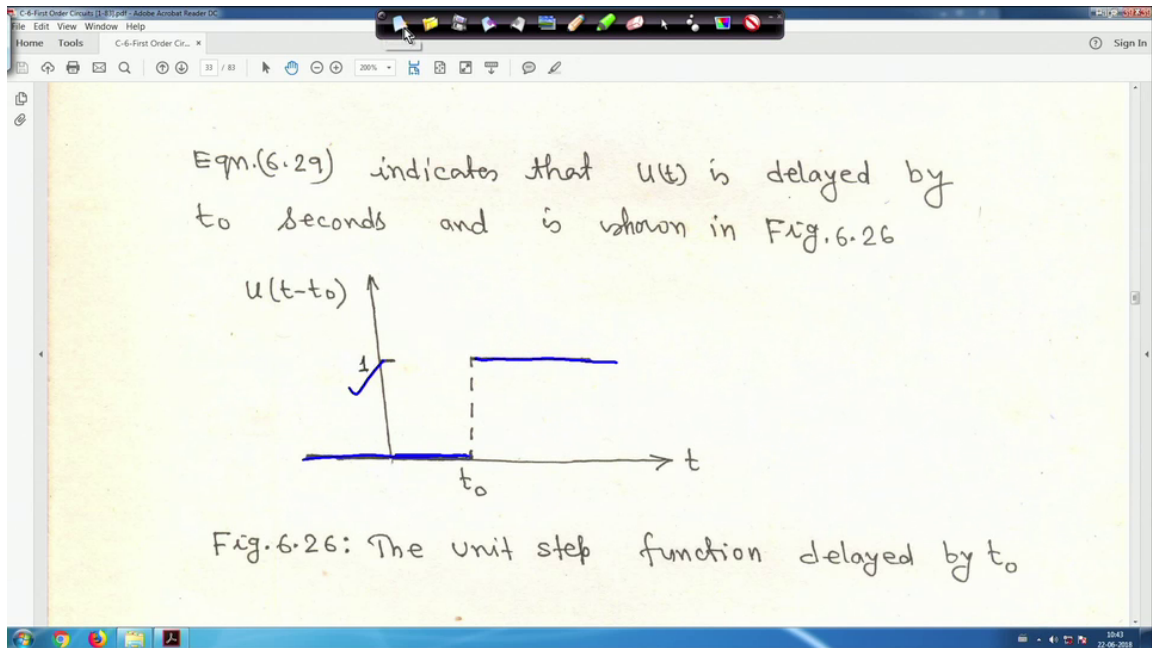
at $t = t_0$ ($t_0 > 0$), the unit step function can be expressed as

$$u(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases} \quad \dots (6.29) \quad (33)$$

Eqn.(6.29) indicates that $u(t)$ is delayed by t_0 seconds and is shown in Fig.6.26

The unit step function can be expressed as that we can write if t_0 greater than 0 at a t is equal to t_0 then $u t$ minus t_0 is equal to 0 for t less than t_0 and it is 1, for t greater than t_0 and at t is equal to t_0 it is undefined right.

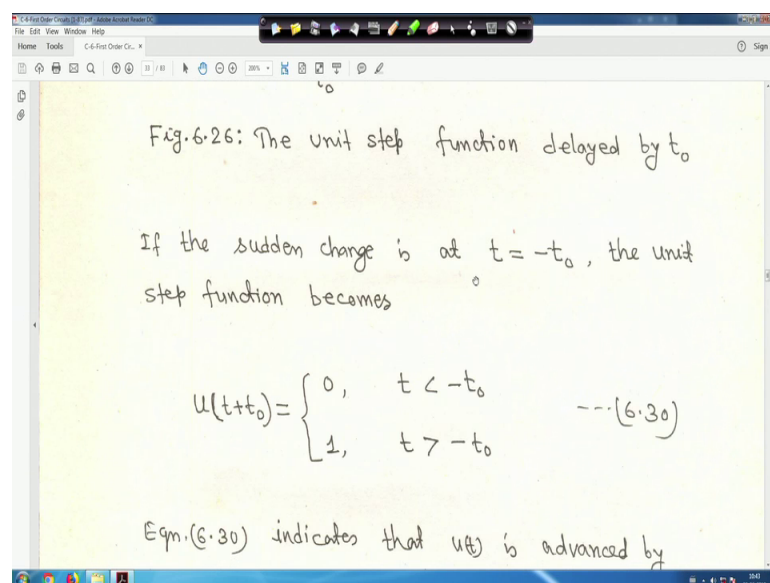
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So, 29 indicates that u that your u t is delayed by t_0 second and is shown in figure. So, it is delayed right so, it is delayed it is it is made it has been made like this it this thing is your less than t_0 and this is your greater than t_0 and right and this is your 1. So, this is u t minus t_0 that it delayed by t_0 .

Now, if it is advanced by t_0 how it will look like.

(Refer Slide Time: 28:36)



So, next is if the sudden change at t is equal to minus t_0 , the unit step function become in that case it is $u(t + t_0)$ is equal to 0 for t less than minus t_0 and is equal to 1 for t greater than minus t_0 .

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If the sudden change is at $t = -t_0$, the unit step function becomes

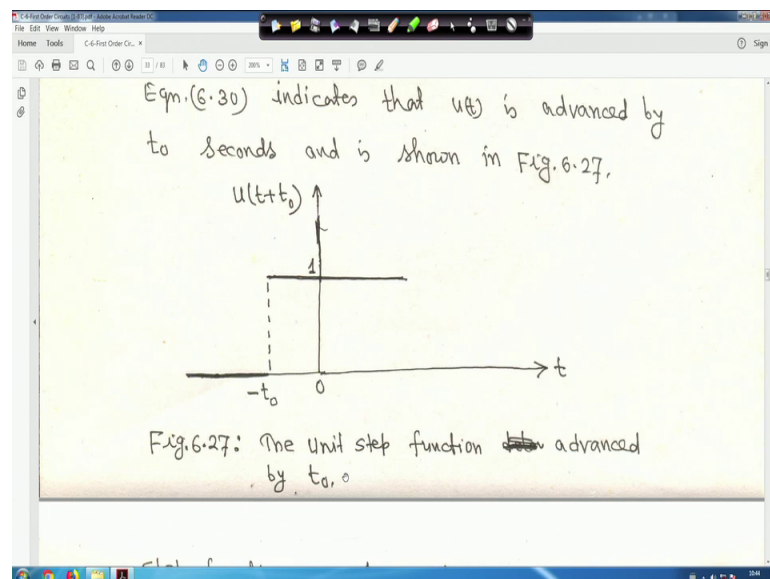
$$u(t+t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases} \quad \text{---(6.30)}$$

Eqn.(6.30) indicates that $u(t)$ is advanced by t_0 seconds and is shown in Fig.6.27.

$u(t+t_0)$ ↑

So, this is equation 30 say; so this equation 30, it indicates that $u(t)$ is advanced by t_0 second and is shown in figure 27.

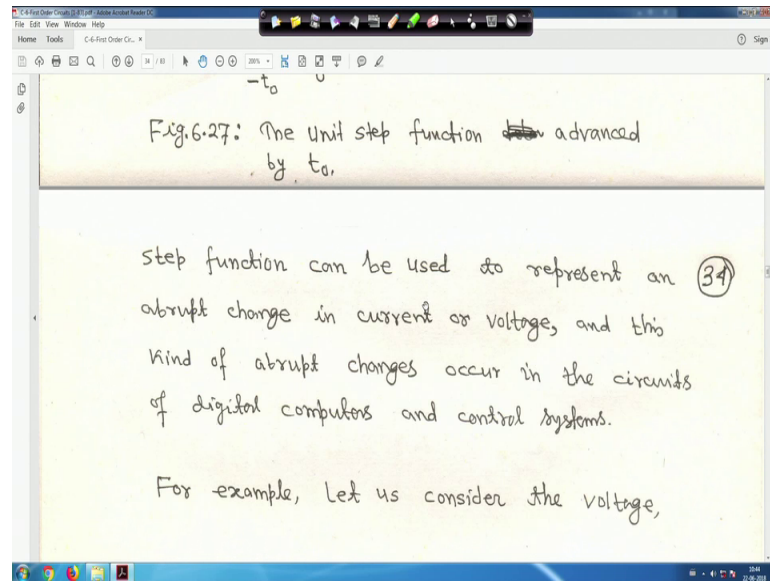
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So, it is advance. So, this is minus t_0 right. And this is 1 and this is the thick line again I am not marking by blue color this is a thick line for less than minus t_0 this less than

minus t_0 is 0. So, this is thick line and this is your greater than t_0 right and it is it is $u(t - t_0)$ plus t_0 time right. So, unit is your unit step function advanced by t_0 . So, this 3 that $u(t - t_0)$; that means, $u(t - t_0)$ this is $u(t)$ hope you have understood this $u(t - t_0)$ this is your $u(t - t_0)$ this is the plot and this is your $u(t + t_0)$ right if it is advanced by t_0 . So, this is the plot how you will show the unit step function right.

(Refer Slide Time: 29:44)



So, step function can be used to represent an output abrupt sorry abrupt change in current or voltage and this kind of abrupt changes occur in the circuit of digital computers and control systems right.

(Refer Slide Time: 29:59)

abrupt change in current or voltage, and this kind of abrupt changes occur in the circuits of digital computers and control systems.

For example, let us consider the voltage,

$$v(t) = \begin{cases} 0, & t < t_0 \\ v_0, & t > t_0 \end{cases} \quad \dots (6.31)$$

can be expressed in terms of the unit step

For example, let us consider the voltage let us consider the voltage $v(t)$ is equal to 0 for t less than t_0 and $v(t)$ is equal to v_0 for t greater than t_0 this is equation 31. Suppose, this voltage we are writing like this if it is 0 t less than t_0 and v_0 t greater than t_0 you are writing like this.

(Refer Slide Time: 30:18)

can be expressed in terms of the unit step function as:

$$v(t) = v_0 u(t-t_0) \quad \dots (6.32)$$

At $t_0 = 0,$

Handwritten notes in blue ink:
 $t < t_0$
 $u(t-t_0) = 0$

It can be expressed in terms of unit step function as $v(t)$ is equal to $v_0 u(t - t_0)$. We solve this step function your this thing $u(t - t_0)$ is equal to for t less than t_0 it is 0 and for t greater than t_0 it is 1 and multiplied by $u(t - t_0)$. So, this

voltage this voltage right if it is given like this we write $u(t - t_0)$ we know for $t < t_0$ it will be 0. So, $v(t)$ will be 0 and for $t > t_0$ you are your $u(t - t_0)$ will be 1.

So, in that case $v(t)$ will be v_0 right this we have seen from that, this we have seen from that right. When $t < t_0$ your $u(t - t_0)$ is equal to 0. So, in that case $v(t)$ will be 0, right.

(Refer Slide Time: 31:18)

$$v(t) = \begin{cases} 0, & t < t_0 \\ v_0, & t > t_0 \end{cases} \quad \dots (6.31)$$

can be expressed in terms of the unit step function as:

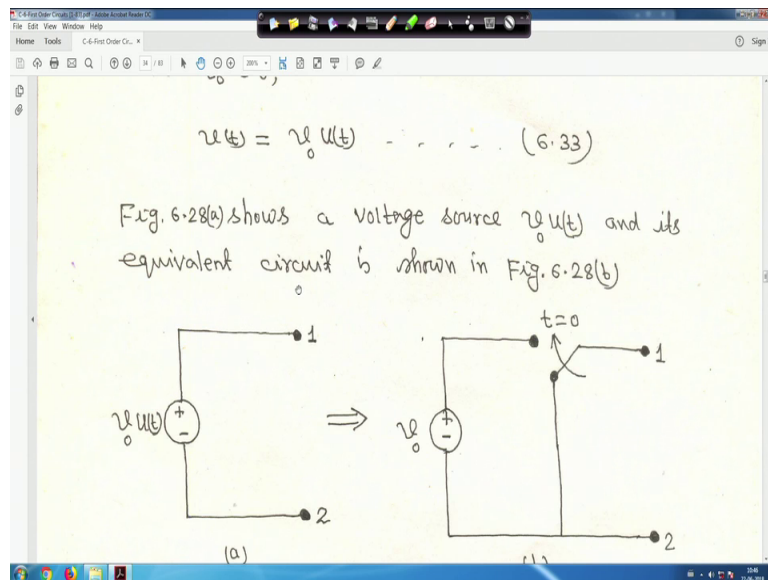
$$v(t) = v_0 u(t - t_0) \quad \dots (6.32)$$

At $t_0 = 0$, $= v_0$

$t > t_0 \quad u(t - t_0) = 1.$

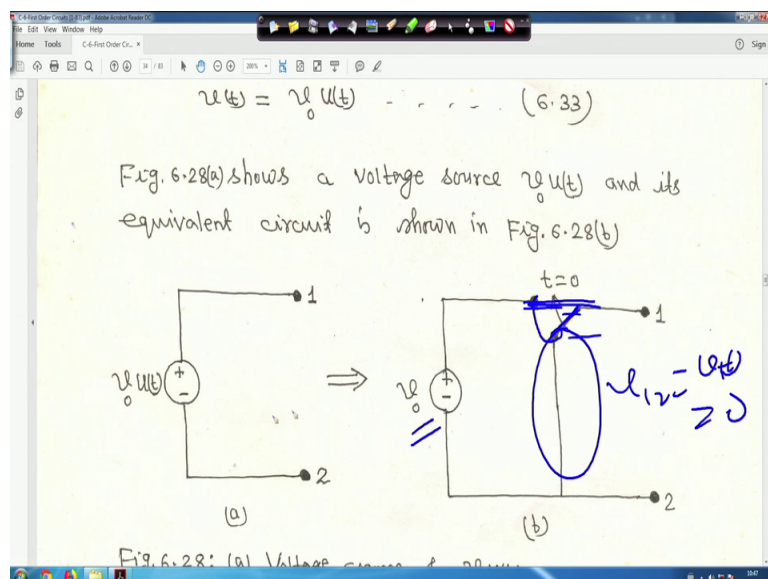
Now, let me clear it so similarly, when your $t > t_0$ $u(t - t_0)$ is equal to 1; that means, this one will be v_0 right. So, this expression this whole thing equation 31 can be written in this form, right. So, it is understandable right this equation can be represented by equation 32 right.

(Refer Slide Time: 31:43)



So, let me clear it and at t is if t is equal to 0, if $t < 0$ sorry if $t = 0$ is equal to 0 then this 32 can be written as $v(t) = v_0 u(t)$ right. So, with this just show you the circuit right. So, figure 28 a shows a voltage source $v_0 u(t)$ and it is equivalent circuit is shown.

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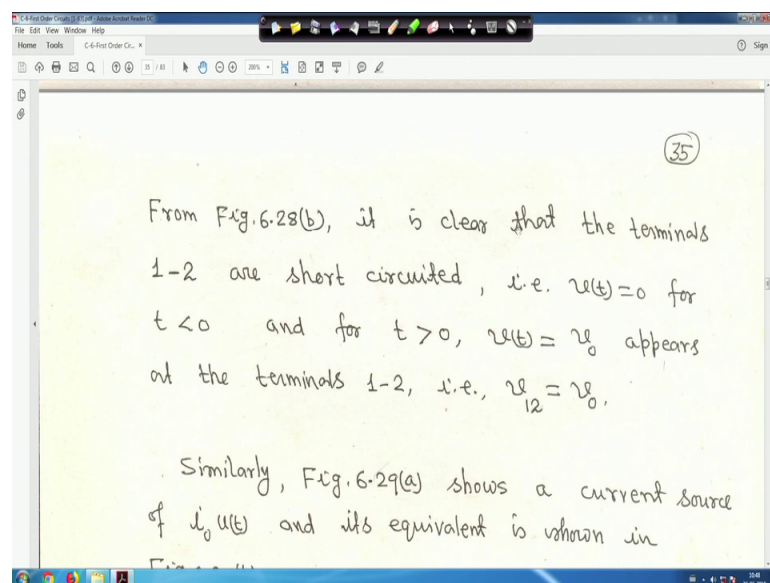


So, this is $v_0 u(t)$ is of voltage source is given right for $t < 0$ it is your what you call $u(t) = 0$ and for $t > 0$ it will be v_0 . So, this when the switch position is like this so 1 and 2 this is actually your what you call that it is 0 right because 1 and 2 is short circuit if 1 if it is if it is 1 and 2 make like this.

So, if the switch position sorry if the switch position is like this then it is short circuit right. So, in that case you can say that $v_{12} = 0$ for example, right. So, in that case this is what you call there will be no output voltage that v_{12} is 0 that v_{12} is equal to v_{12} is equal to v t is equal to 0 right, but when switch position had gone from this to this I mean when it is there when it is connected forget about this, when it is connected at that time your v_{12} is equal to v t is equal to v_0 right.

So, this if you draw v_0 at this circuit then your what you call let me clear it this is the equivalent circuit from here. If you pick v_0 and this is your how you will make in a switching logic right how you will make it as switching logic; so, just one just one second right. So, this way you can make it, so voltage source v_0 and it is equivalent circuit.

(Refer Slide Time: 33:22)



And whatever I said it is explanation given explanation is given when the your what you call it is clear that terminal 1 and 2 are short circuited that is $v_{12} = 0$, for t less than 0 and for t greater than 0, I told you $v_{12} = v_0$ appears at the terminal one and 2 that is $v_{12} = v_0$ right. So, today sorry today I have to this I will for this up to this.

Thank you very much we will be back again.