Fundamentals of Electrical Engineering

Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Lecture - 30
First order circuits (Contd.)

So, previously we have just saw we have just seen an typical example and one look this is a this is a course that most of the things we have to understand the fundamentals right. And one or two things I leave it up to you when we will solve some numerical why we have done so, you will try to answer that that they are. At the time if you cannot will response to your forum right. So, as many as I mean as much as possible we have tried to incorporate prior to the problems and understand and we have to see that whether everything in understanding is clear or not.

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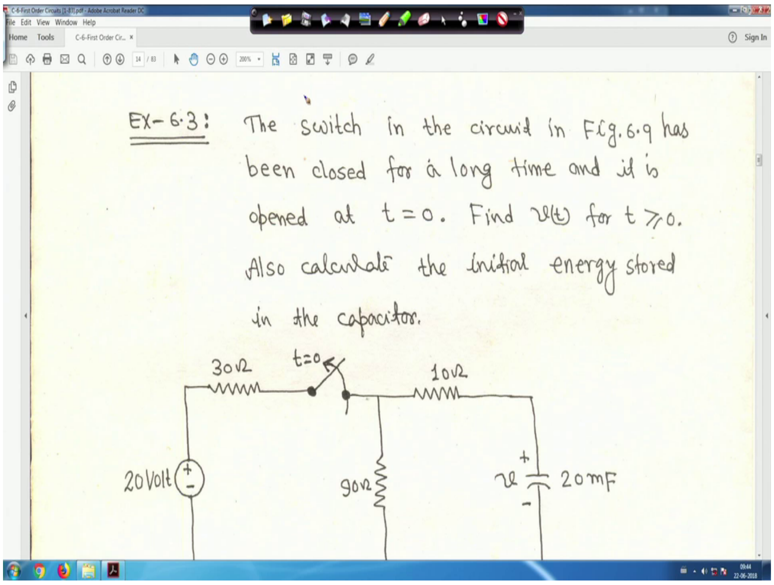

So, for example you take this example that your that switch in the your switch in the circuit, in shown free as being closed for a long time long time means, that it was a no it was closed it was right and that means, it has reached to steady state that long time switch was closed right and finally what happened if it is your you have to find out the initial condition for that right. And it will open at t is equal to 0 you have to find out v t for t greater than or equal to 0. Also you calculate the initial energy stored in the capacitor, so in this circuit that switch this switch was closed for long time.

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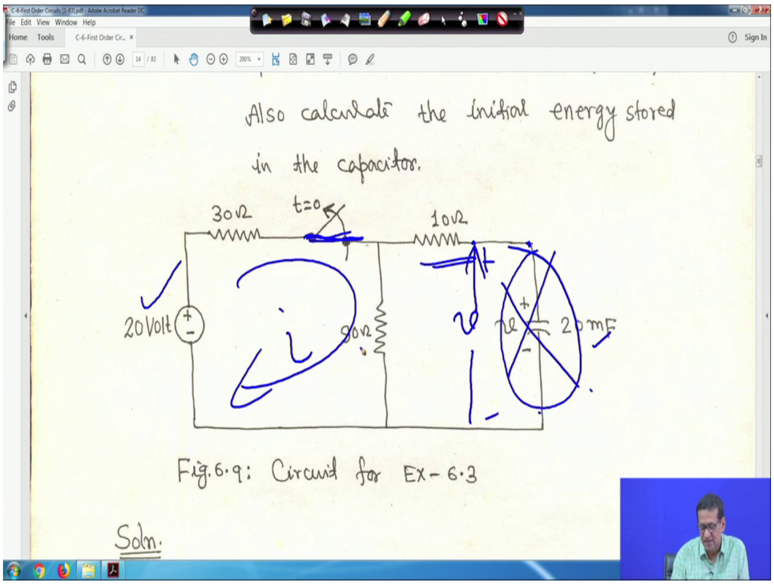

If the switch is closed for long time means, this switch was closed for long time right and that t your what you call t is equal to 0, the switch is just opened otherwise it was close. If switch was closed for long time you know that for the DC for the DC supply, the capacitor will be a like an open circuit right. So, it will act as a it will act as open circuit this two terminal will act as open circuit. So, in that case what will happen as this is open circuit means it is not there and that means, nothing is flowing in the 10 ohm resistance, and finally a current will flow through this only right.

So, if you try to find out that what is the; that if capacitor acts as open circuit, that means this is the voltage v say across the capacitor this 20 millifarad it is open circuit. So, in that case what will happen that you find this 30 ohm and this is your 80 it is just hold on let me see this is 90 ohm right. So this is your, I this is 90 ohm.

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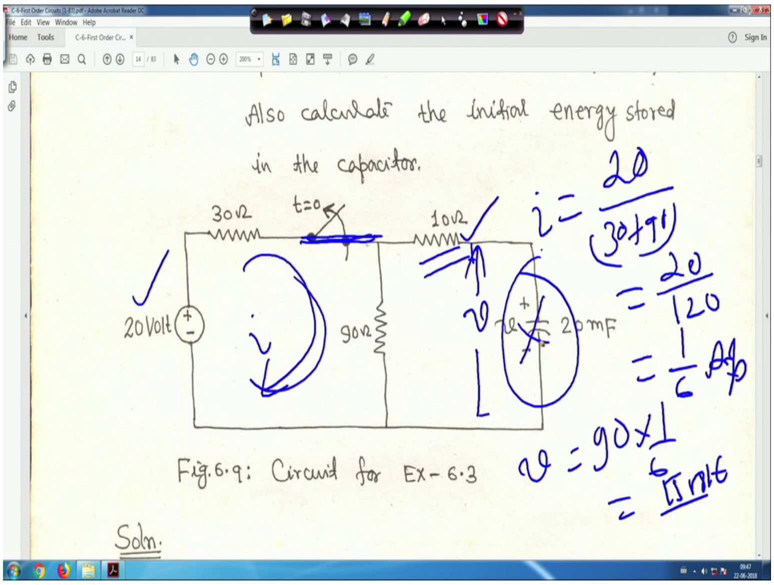

So, what is the current then i; i will be is equal to 20 divided by your 30 plus 90 right. So, it is 20 volt, so 20 divided by your 120 that is 1 upon 6 ampere that is the, that is the current your i then what is the voltage then? As this is your open circuit. So, it is not there and no current is flowing through this, so this is the voltage that opens circuit what you call that, voltage across the capacitor. When this was closed right this was because t is equal to switch was open and t is equal to 0 before that it was closed, so this current i is flowing right.

So, in that case what will happen the voltage across this will be 90 because, this no current is flowing here it is open circuit and 90 into your 1 by 6 that is your 15 volt right. So that means, that that means that your what you call when switch was closed for long time right, at that time voltage here what you call across this capacitor is just 15 volt. So, everything is given there, therefore if you look in to the circuit when switch is closed this is the equivalent circuit whatever I have done for you right.

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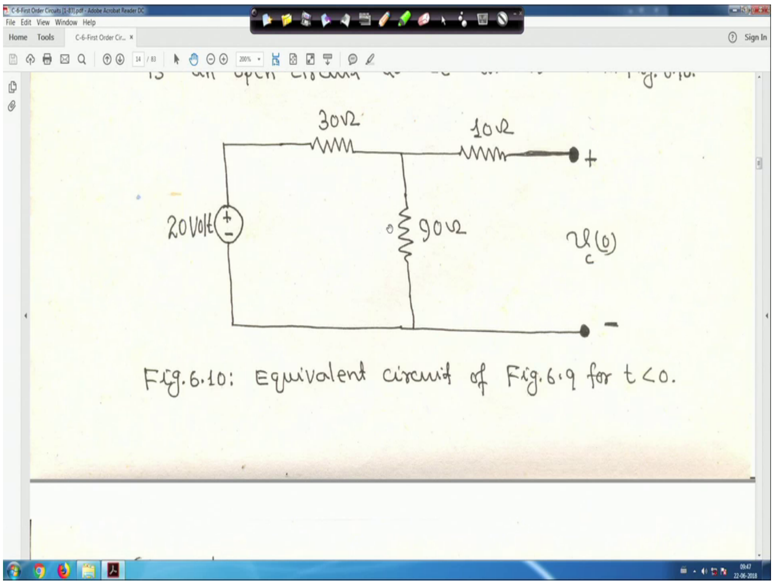

So, this is and this is initial condition that is that is why writing this is 0. So, current through this I told you 20 upon this is open this no current is flowing through 10 ohm resistance, because it is open circuit. So, i is current flowing through this I showed you that this 1 upon 6 ampere then v c 0 will be 90 into 1 upon 6. So, that is why it is 15 volt that is the initial voltage it is 15 volt.

Now, when t greater than 0 the switch is open and the equivalent rc circuit. Now, when t is equal to I mean at when switch is open then this part is gone when t greater than 0 so and switch t is equal to 0 that switch is opened; that means, this part is gone right. So, only this part is there so let me clear it.

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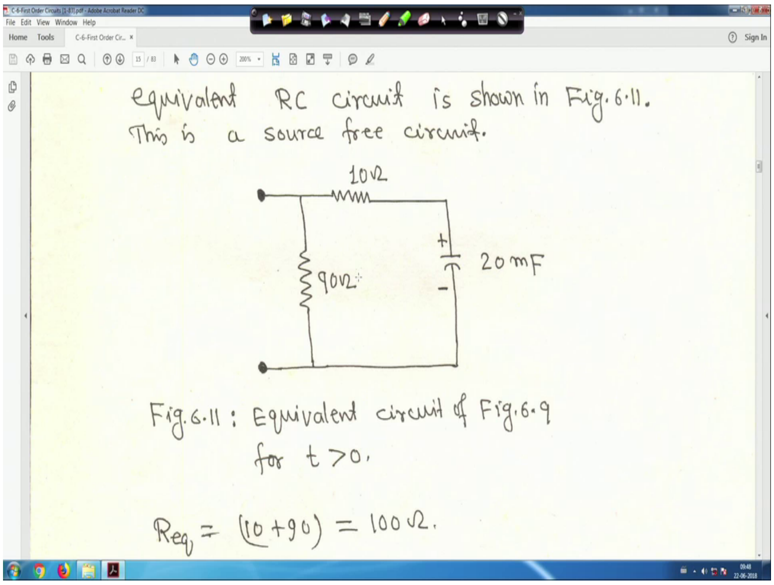

So, in this case your this is the equivalent circuit right, this is the x 10 ohm and 90 ohm are in series that it is 20 millifarad and initial capacitor was charged at v c 0 is equal to 15 volts that you have seen right. So, if then 40 then your R eq equivalent will be 10 plus 90 so 100 ohm.

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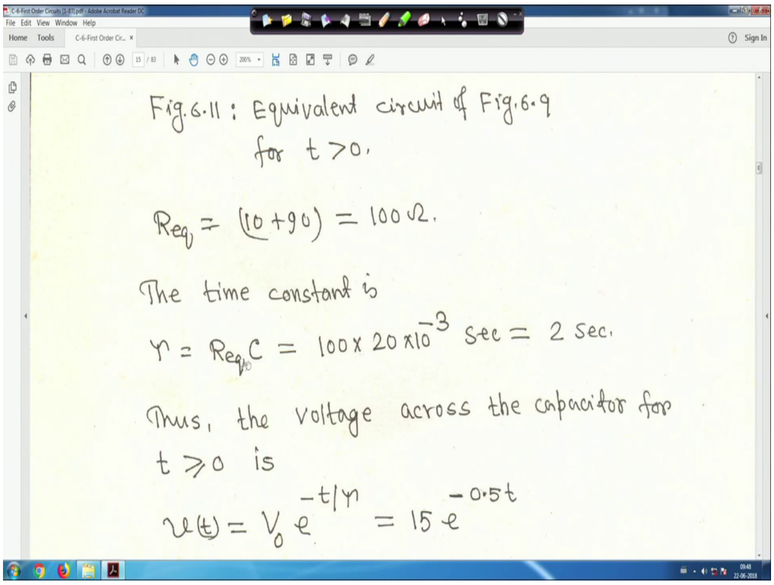

And time constant is C into R eq that is R eq into C, so R eq is equal to 100 ohm and C is equal to 20 millifarad, so 20 into 10 to the power minus 3 farad so it is second. So, 2 second tau is equal to 2 second. Thus the voltage across the capacitor for t greater than or equal to 0 we know this know v t is equal to V 0 e to the power minus t by tau. So, V 0 is equal to V C 0 is equal to 15 volt that we have got it that just hold on.

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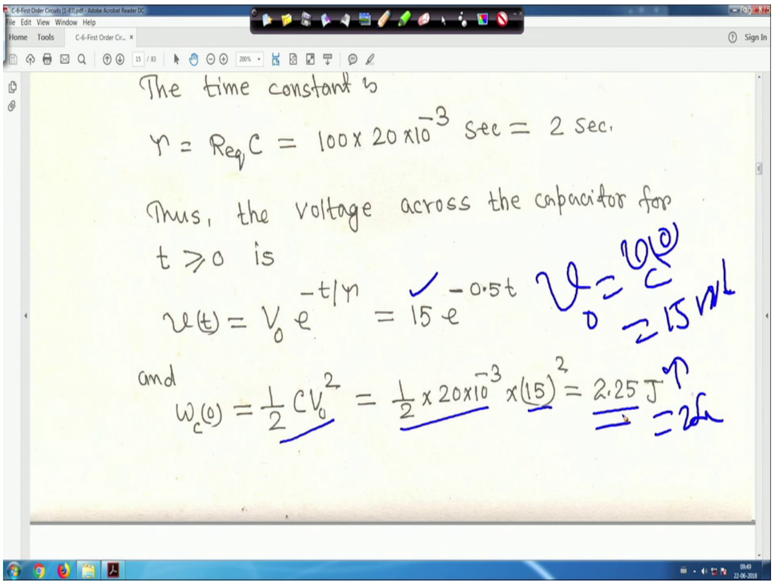

So, v 0 is equal to v c 0 is equal to 15 volt that this is the 15 and your tau is equal to tau is equal to 2 second. So, it is t by minus t by tau, so e to the power minus 0.5 t right and omega c 0 that is initially initial energy stored in the capacitor half C V 0 square. So, half C is 20 into 10 to the power minus 3 farad and V 0 square, so 15 square it is 2.25 volt right. So, next is that source free RL circuit, we saw till now we saw source free Rc circuit now will see is a source free RL circuit.

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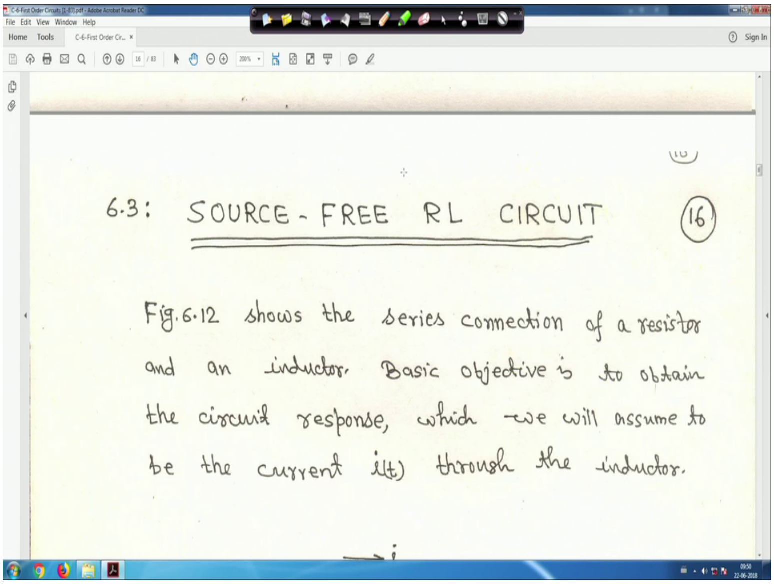

So, philosophy will remain same just as it is inductor, so just we have to see how it happens. So, this figure 12 shows the series connection of a resistor and inductor. Basic objective is to obtain the circuit response which will be natural response, because this is a source free circuit which will assume to be the current i t through the inductor.

So, this is the simple circuit this is inductor and this is resistor and this is the current i right and (Refer Time: 07:05) it is taken current your here it is plus minus here also it is plus minus, I mean I mean current is current is moving like this is current i right. So, that passive your what you call this your v R whatever we have taken that passive sign convention right

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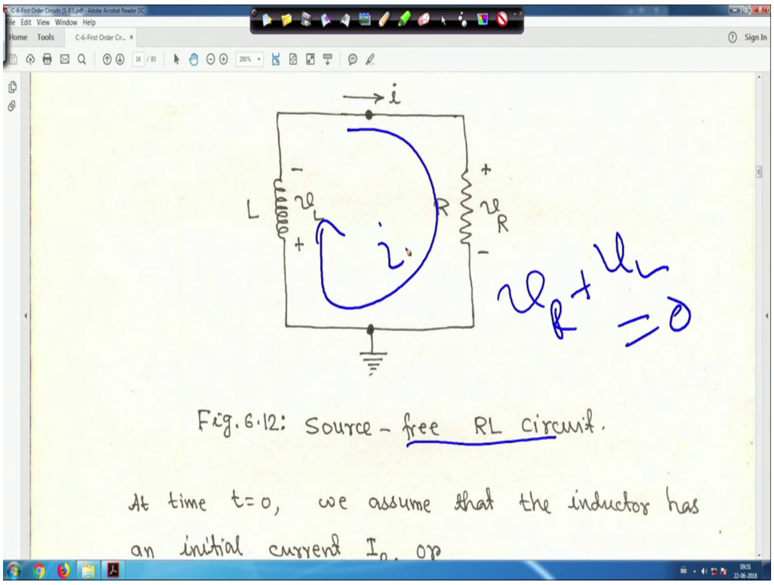

So, this is a source free RL circuit, now if you if you apply in this loop the KVL it will be v R plus v L is equal to 0 you apply KVL. So, that is why now you will know everything all these things have being discussed before. So, v R plus v L is equal to 0 right. So, let me clear it so you assume that at time t is equal to 0 we assume that the inductor as an initial current I 0 right. So, capital I suffix is 0 right or we can write small i that is 0 is equal to that is at t is equal to 0; this is the current i all the time perhaps we are not writing it, but this current it is actually it right.

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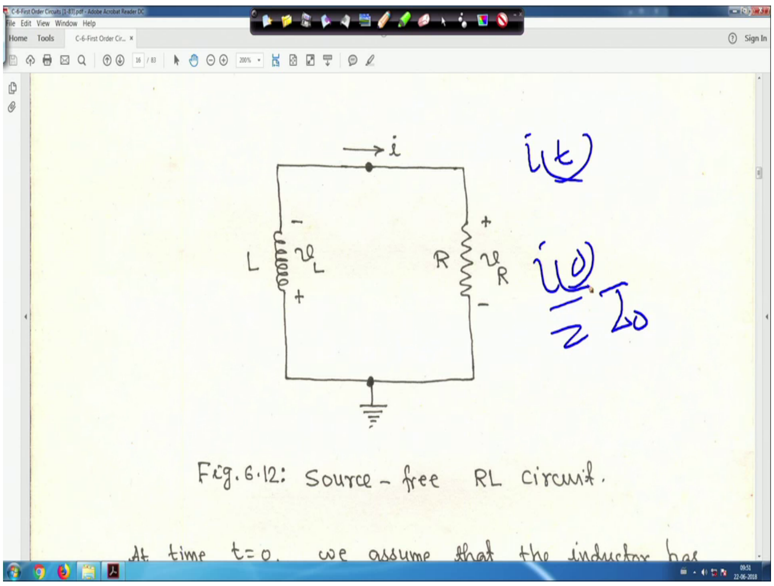

So, initial current we assume I 0 is equal to that capital I suffix 0, so I0 capital I suffix 0 right. So, this is the initial condition we assume that inductor has an initial current like this, so let me clear it.

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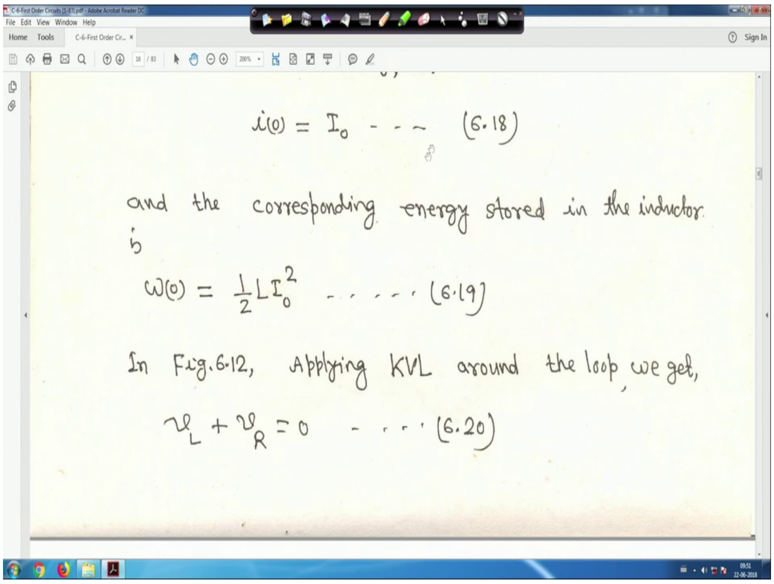

So, and the corresponding energy stored in the inductor that is initial W 0 is equal to half L capital I 0 square so this is equation. So, equation numbers are given understandable to you, so if you apply K VL along the loop we get I told you v L plus v R is equal to 0, in that circuit here v L plus v R is equal to 0.

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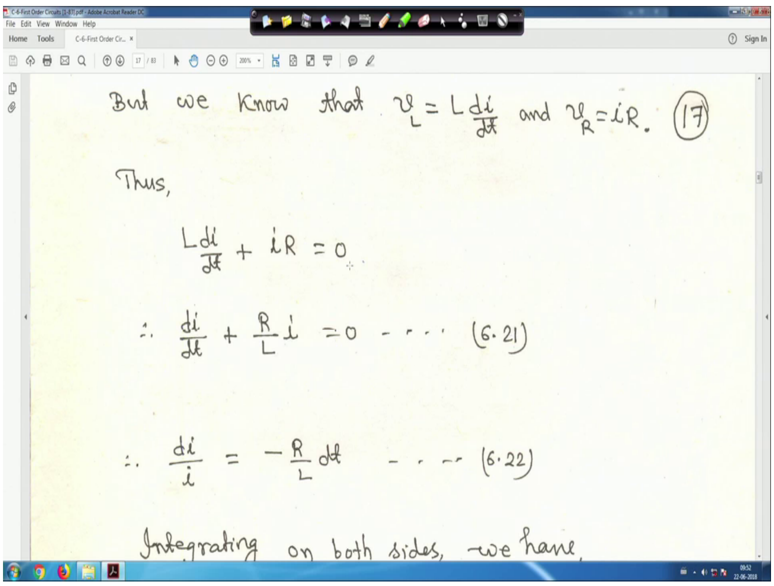

So, but we know that v L is equal to L into di by dt and v R is equal to I r. So, if you look into the circuit here it will be v L is equal to L into di by dt and here v R is equal to I R right. So that means, L into di by dt plus i R is equal to 0; that means, di by dt plus R upon L i is equal to 0 so or di upon i is equal to minus R upon L dt right, so integrate on both side.

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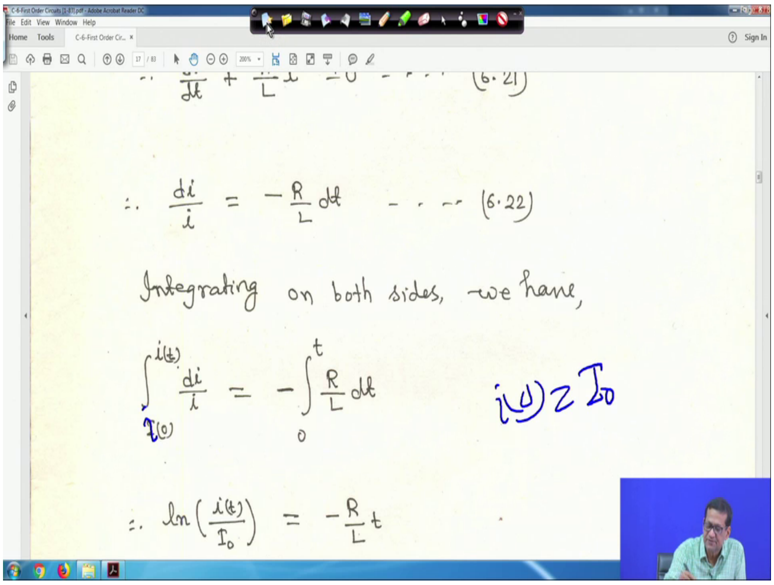

So, i actually varying from initial current I 0 that is I 0 to at some time t i t, so di upon i is equal to minus it is 0 at t is equal to 0 to t is equal to t right R upon L dt. So, this is at at your at t is equal to 0 that initial current was I 0 and here your I your what you call here it is your this one, just there should not be any confusion this one this limit I have written capital I 0 better it should be small i 0 right and small i 0 and I told you small i 0 is equal to capital I suffix 0 right. So, it is small i 0 that is better so it is and you this side right hand side is minus 0 to t R upon L dt now you integrate right let me clear it.

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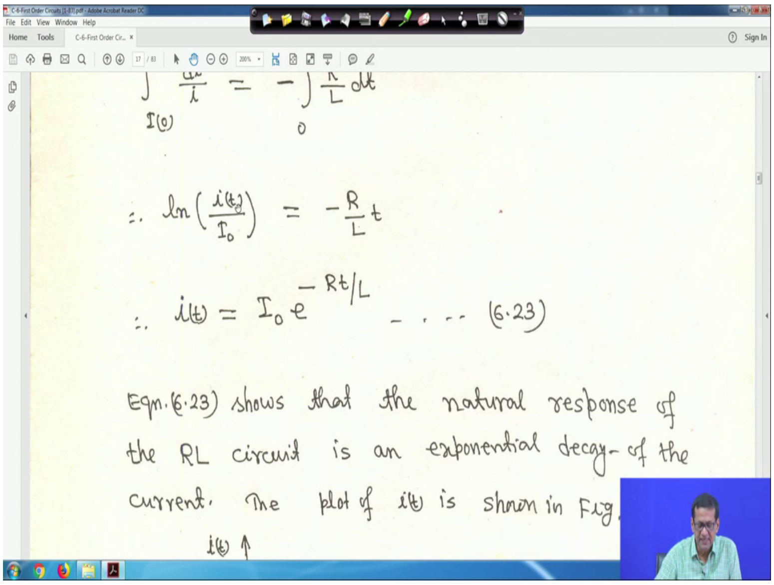

So, now if you integrate it, it will be natural log i t upon small i 0, actually this is small i 0 this is small i 0 and small i 0 is equal to I 0. So, it will be i t upon I 0 is equal to minus R upon L t. That means, i t is equal to I 0 into e to the power minus R t upon L, this is equation 6.23 this is chapter 6. So, this thing now that 23 was natural response of the RL circuit is an is an exponential decay of the current right.

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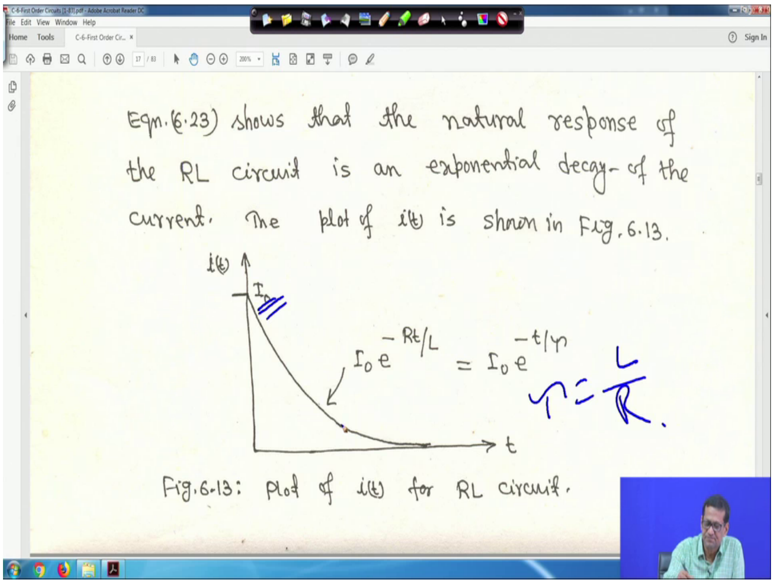

So, this is the here what you call the plot of your i t when t is equal to 0 when t is equal to 0, so i t is equal to this initial value that is I 0 and it is decay it is decaying right and your it is written is equal to I 0 is equal to t minus tau and tau is equal to L by R right, that is your time constant of the circuit right. So, for RL circuit time constant is L by R whereas, for capacitor it is your R C right.

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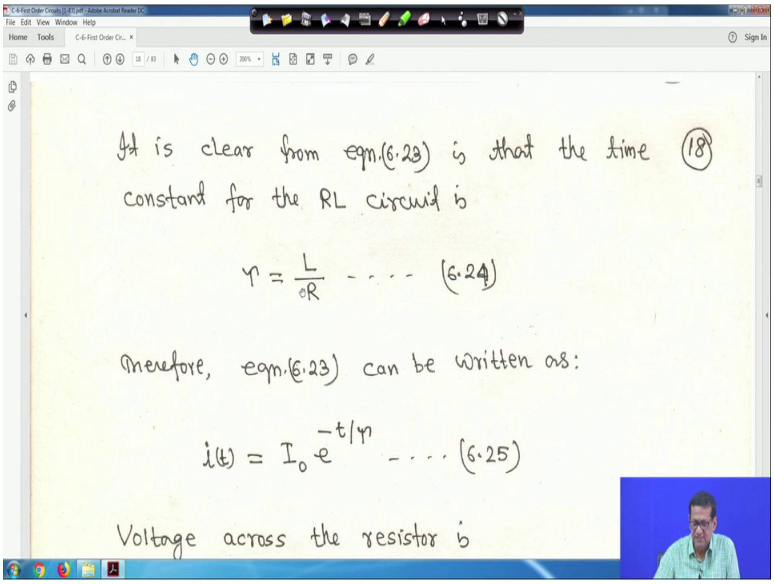

So, this is the plot so this is this is I explain tau is equal to therefore, i t is equal to I 0 e to the power minus t by tau. So, voltage across the resistor is v R t is equal to i into R. So, this is your, I and this is R so substitute I here it will be I 0 into R e to the power minus t upon tau this is equation 25.

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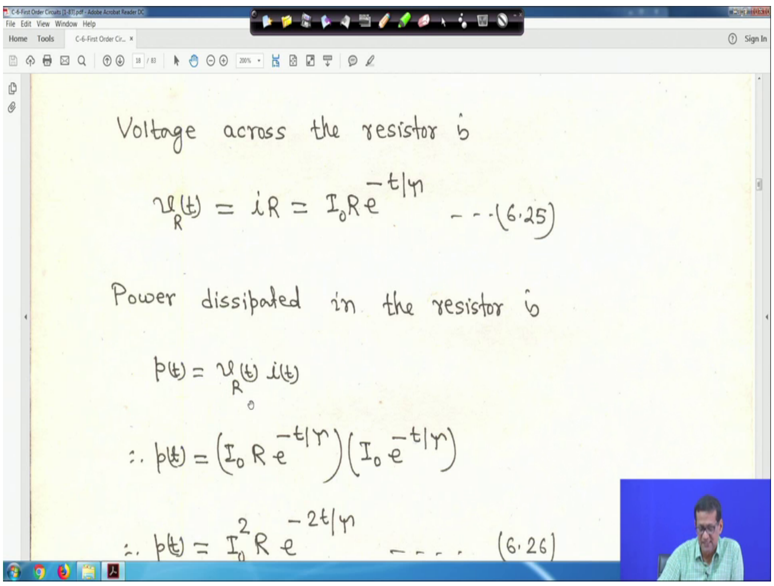

Now, power dissipated in the resistor is p t is equal to at any time t, p t is equal to v R t into i t. So, this is your V at t that is I 0 R e to the power minus t by tau, so I 0 R e to the power minus t by tau when i t is equal to I 0 e to the power minus t by tau; therefore, p t is equal to I 0 square R e to the power minus 2 t upon tau this is equation 26.

Now, energy absorbed by the resistor is right though energy at any time t W R t is equal to 0 you just integrate 0 to t p into dt. So, 0 to t and this is the expression for p that is equation 26, you put it here I just I 0 square R into e to the power minus 2 t upon tau dt you integrate and simplify if you do so.

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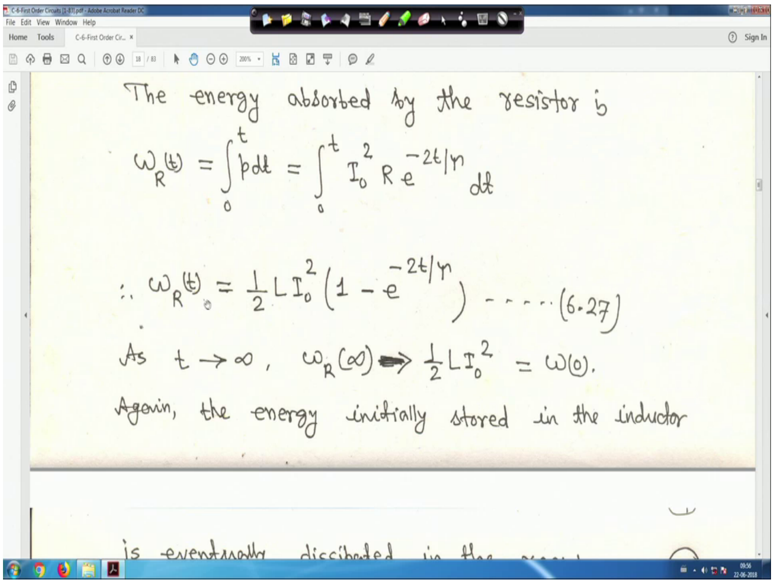

You will get half omega R t is equal to half L I 0 square into 1 minus e to the power minus 2 t upon tau this is equation 27. So, at t tends to infinity that is your omega R infinity is equal to half L I 0 square is equal to omega 0 omega initially also showed. Initially as stored half L I 0 square again the energy initially stored in the inductor is eventually dissipated in the resistor right. So, this is hope these things are understandable to you quite simple quite simple right.

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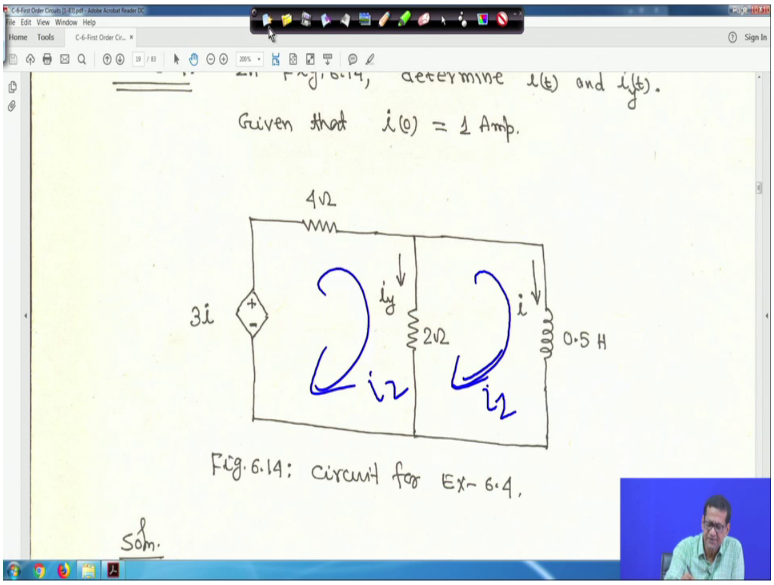

So, now next let us take 1 example, in this figure you have to determine i t and i y t given that I 0 is equal to 1 ampere. So, this is the circuit 1 here 1 d or what you call 1 dependent voltage source is there right 3 i plus minus terminal polarity has being marked. So, given that initial condition this is i and this is i y you have to find out i t and i y t given that I 0 is equal to 1 ampere. So, what you can do is you just if I if I make it like this before this thing, so this is this is one loop this is another loop you can take right and if this current is i 1 this current is i 2 right. So, you can make it that k we have to apply KVL and accordingly we have to solve it.

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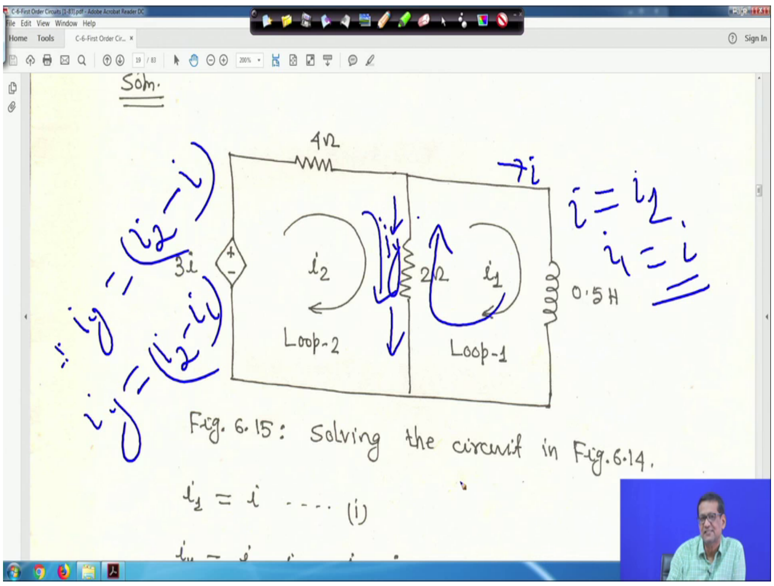

So, let us see so in this case this is that your call this is what I have taken right and the original circuit in the in the previous circuit it was given that this is my i and this is actually i y this was given right. So, basically the way we have taken the loop it is i is equal to actually i 1 or i 1 is equal to i. And i y current flowing through 2 ohm resistance direction is this way, so i y is equal to your in this direction we are taking, so is equal to i 2 minus i 1 right, because this i 2 this i 2 is going like this and this i 1 is going like this. So, resultant will be i 2 minus i 1 and that means my i y is equal to basically i 2 minus i 1 is equal to i, so it is i right.

So, that is all these things written, but just for the purpose of understanding I am writing here. So, next that means if you look into this you apply KVL here first in you apply that is whatever given the i 1 is equal to i and i 1 is equal to i 2 minus i 1 is equal to i 2 minus i, so here you apply KVL in this loop right. So, if you do so then you will get 0.5 it is 0.5 Henry given, so L l into din 1 by dt.

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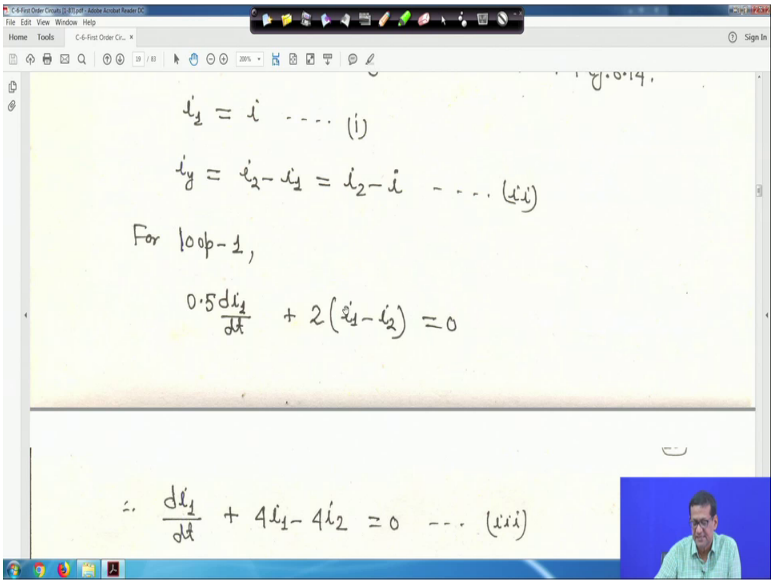

So, it is 0.5 di 1 upon dt then we are moving we are moving here clockwise. So, it will be 2 into plus 2 into i 1 minus i 2 you are moving like this because i 1 is going upward and i 2 moving downwards. So, resultant upward is i 1 minus i 2, so plus 2 into i 1 minus i 2 is equal to 0 or you just multiply both side by 2, then you will get di 1 by dt plus 4 i 1 minus 4 i 2 is equal to 0 this is the equation 3 given.

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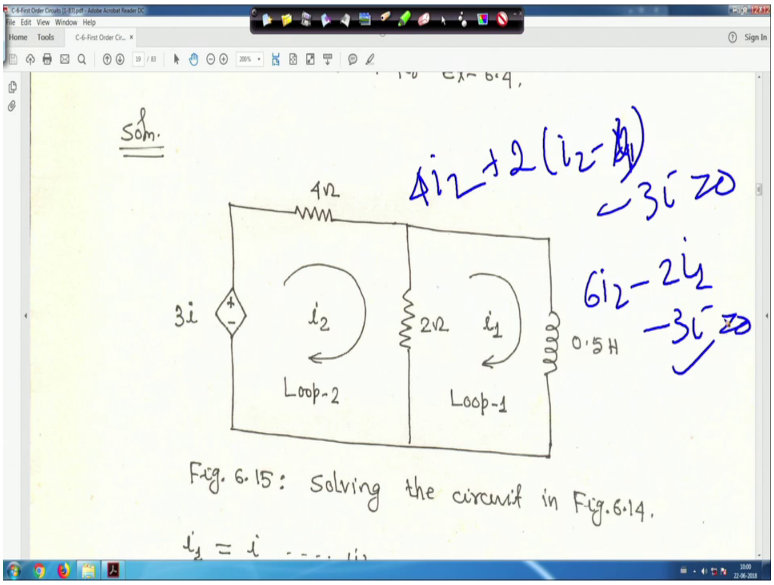

Now, similarly for loop 2 if you take loop 2, then your if you make it loop 2 it will be 4 i 2 4 i 2 right plus 2 i 2 i 2 minus i 1 2 i 2 minus i 1 right minus 3 i is equal to 0, because it is dependent voltage source is 3 i. So, in this case what will happen it will basically 6 i 2 minus 2 i 1 minus 3 i is equal to 0 this 1 I have written here right, for loop 2 this is 6 i 2 minus 2 i 1 minus 3 i is equal to 0 this is equation 4.

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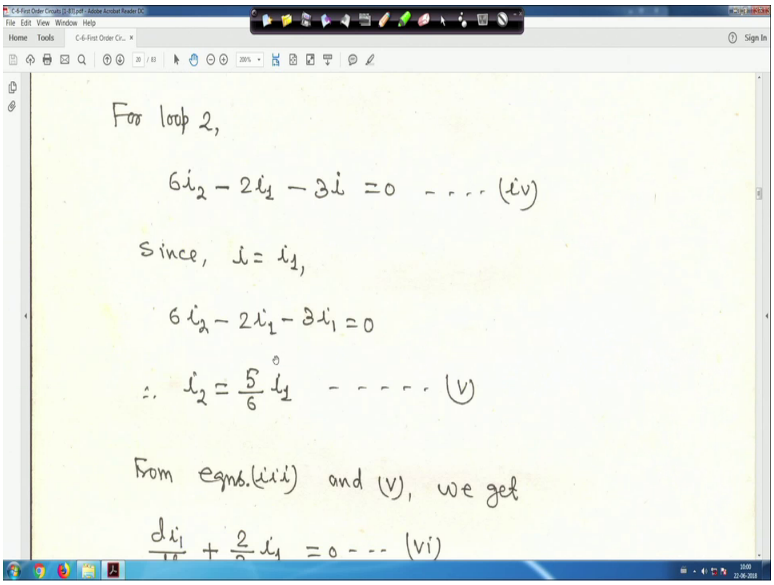

Now, since I is equal to i 1 right therefore, you put here 6 i 2 minus 2 i 1 minus 3 i 1 is equal to 0; that means, i 2 is equal to 5 upon 6 i 1; now from equation 3 and equation 5 right. In equation 3 actually you put this expression i 2 is equal to 5 upon 6 i 1 then you will get di 1 upon dt plus 2 by 3 i 1 is equal to 0, so this is equation 6 a. So, now what we know that i 1 is equal to i therefore, di upon dt is equal to minus 2 upon 3 i right or di upon I is equal to minus two third dt.

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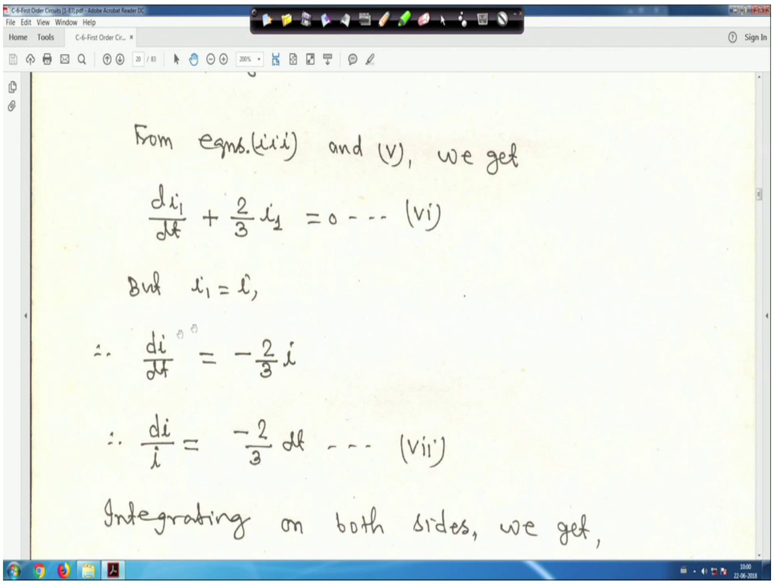

Now, you integrate for on both side at t is equal to 0 current was i 0 and at t is equal to time t current was i t di upon i is equal to minus 2 by 3 integration 0 to t dt right or i t is simply apply integration you will get i t is equal to i 0 e to the power minus 2 t upon 3 that is for t greater than 0, this we have seen this kind of integration earlier, so directly we are writing.

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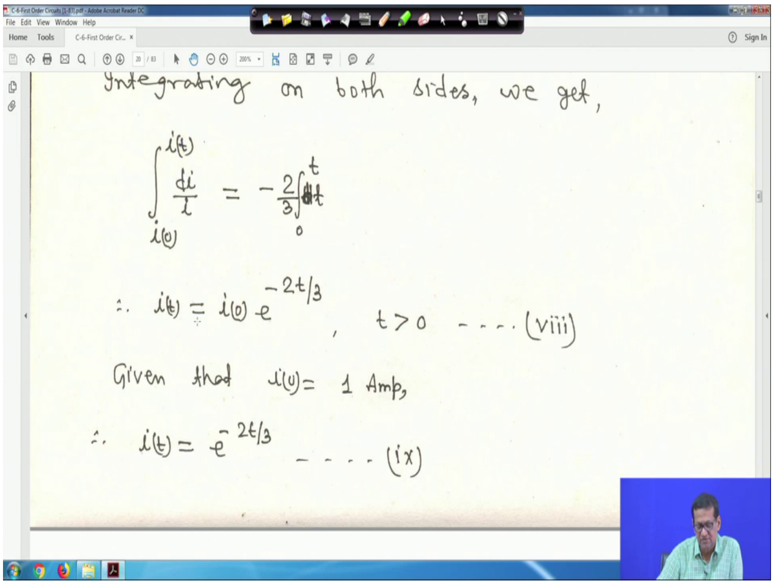

Now, given that it initially it was given that i 0 is equal to 1 ampere therefore, i t is equal to e to the power minus 2 t upon 3 because i 0 initial value is 1 ampere given, so put here i 0 is equal to 1. So, it will be e to the power minus 2 t upon 3 now voltage across the inductor is v is equal to L into di by dt L is 0.5 Henry right and you take the derivative of the current i t di t upon dt.

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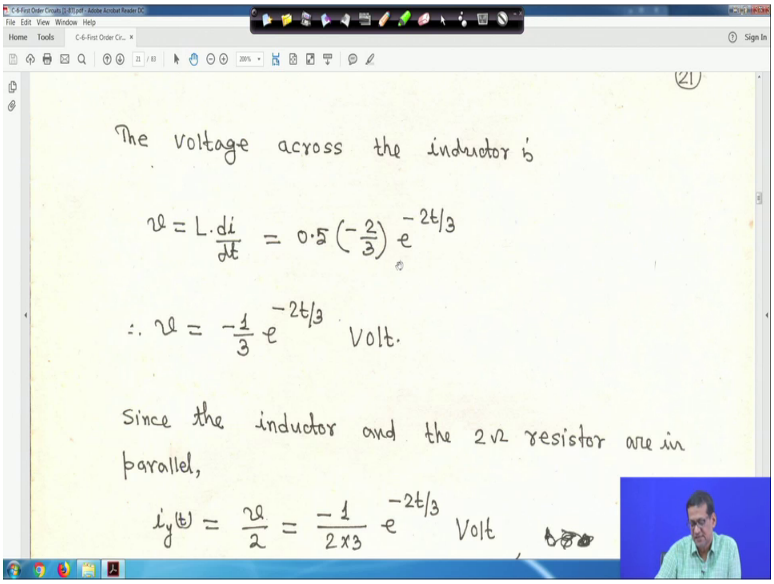

If you do so it will be 0.5 into minus 2 upon 3 into e to the power minus 2 t upon 3 therefore V is equal to minus 1 third e to the power minus 2 t upon 3 volt. Since the inductor and the 2 ohm resistor are in parallel right. So, I mean V we have got, but look at the circuit come to the circuit come to the circuit.

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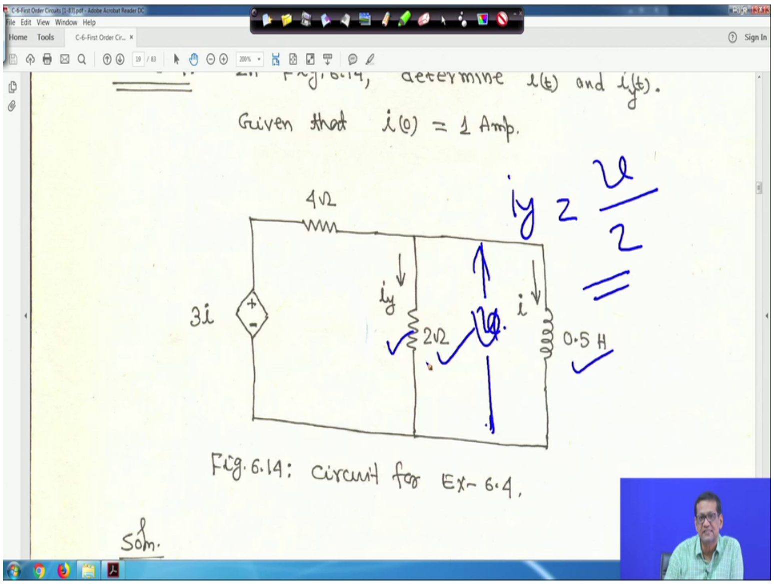

So, this voltage across the inductor we have got say this is voltage say v this is voltage v right. So, this 2 are in parallel 2 ohm resistance and 0.5 inductor both are in parallel right and this voltage is say v so that means, R i y will be this same voltage across a 2 ohm resistor, so it is V by this 2 ohm resistance right. So, this is actually your current i y v upon 2 right because this 2 ohm resistance is here, so let me clear it. So, will go back to that expression again. So, here it is i y time t v by 2 that 2 is the 2 ohm resistance right. So, that is the current flowing to 2 ohm resistance that is i y t.

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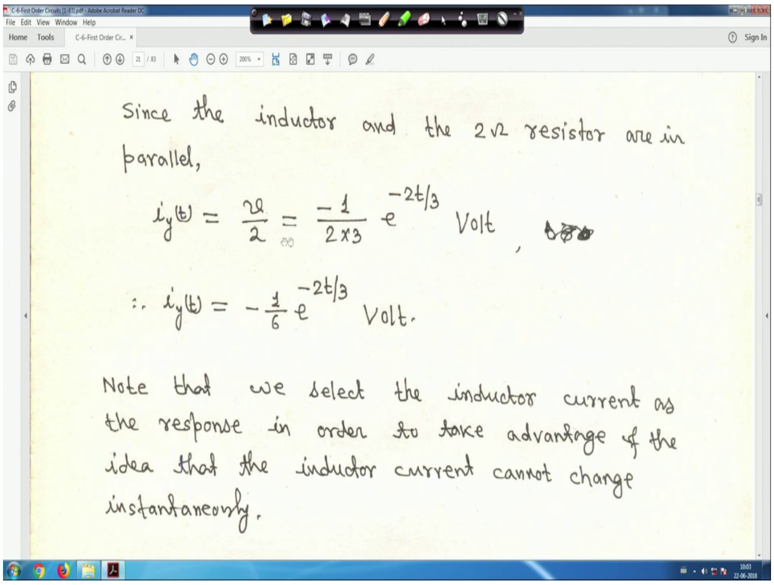

So, it is these expression you substitute v is equal to minus 1 upon 3, actually if you actually all the time I am not writing, but these expression actually it is all function of t all function of t right. So, all the time you are not writing but it is your, what you call it is un it is understandable it is understandable to you right. So, let me clear it so that means, in this case it will be minus 1 upon 6 e to the power minus 2 t upon 3 volt.

So, now note that we select the inductor current this is important will should keep it in your mind. Select the inductor current as the response in order to take advantage of the idea that the inductor current cannot change instantaneously right. Little bit more will see that right at the time of opening or closing switch will see that t is equal to 0 minus or t is equal to 0 plus. So, we will see later so but this is that inductor current your what you call as the response in order to take advantage of the idea that, the inductor current cannot change instantaneously right.

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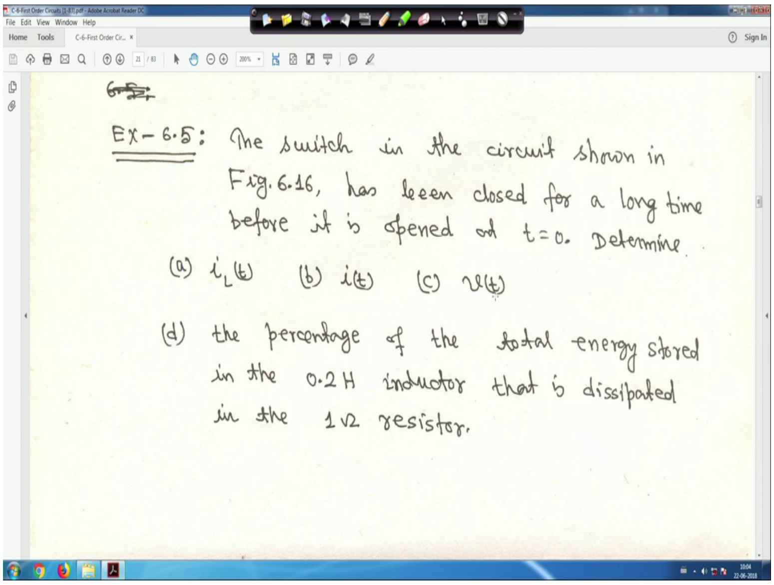

So this is 1 then another one another example is that your the switch in the circuit shown in figure 16, I will show you as been closed for a long time before it is open t is equal to 0. Determine i L t i t and v t that a b c that a is i L t b is it and c this 3 things you have to find out and fourth the d the percentage of the total energy stored in the 0.2 Henry inductor that is dissipated in the 1 ohm resistor right.

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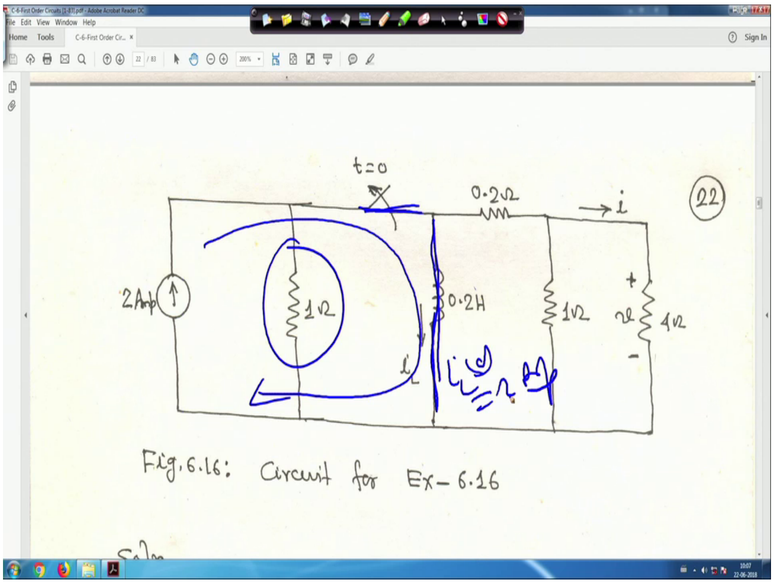

So, this circuit I will show you the switch in the circuit shown in has been closed for a long time before it is open at t is equal to 0 right. So, this is actually this is the circuit actually initially it was initially it was closed for a long time; that means, this circuit initially was closed for a long time and at t is equal to 0 it was open right. So, as the circuit initially was closed for a long time and at that time your how the inductor will behave right. So, for and for DC we have seen that capacitor acts as a open circuit and for the inductor it will act as a short circuit right.

So, if you if you when this switch was closed for long time, if you see the; what you call that equivalent circuit then you see how is it right; So, in this case what will happen that in this case what will happen here I have not give that equivalent circuit I have to make it for you.

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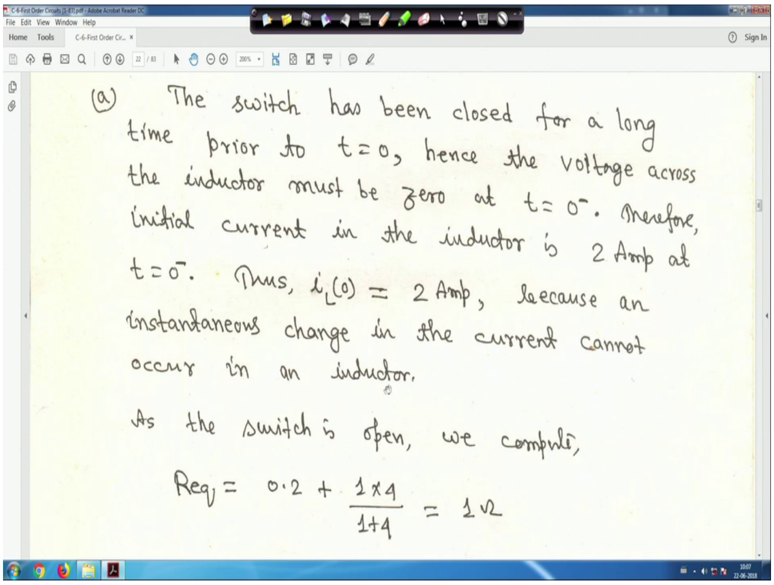

So, the switch has been closed for a long time and hence the voltage across the inductor must be 0 at t is equal to 0 minus, because it will act as a short circuit it will act as a short circuit right and therefore the initially current in the inductor is 2 ampere at t is equal to minus 0 that is i L 0 is equal. That means, this one actually this one it was it was closed for a long time, so inductor actually behaving as a short circuit.

So, what will happen in that case this 2 ampere current will flow through this inductor. So, i L 0 initially it will be 2 ampere right it will because, it is acting as short circuit because switch was closed for a long time after that it was open. So, in that case as it is short circuit this will be ineffective right, it is just take this short circuit path because particular it acts like a short circuit as an it an ideally inductor right. So, it is your i 0 your i L 0 is equal to 2 ampere the initial current.

So, of the inductor now I am clearing it, so now in this case right; so something is written for you read it same meaning. So, because an instantaneous change in the current cannot occur in an inductor, now as the switch is open we compute R eq. Now when switch is open now when this switch is open, your this is open this is open right, this switch is open means this is gone this is not there in the circuit, but we know the initial current the inductor i 0 is equal to 2 ampere.

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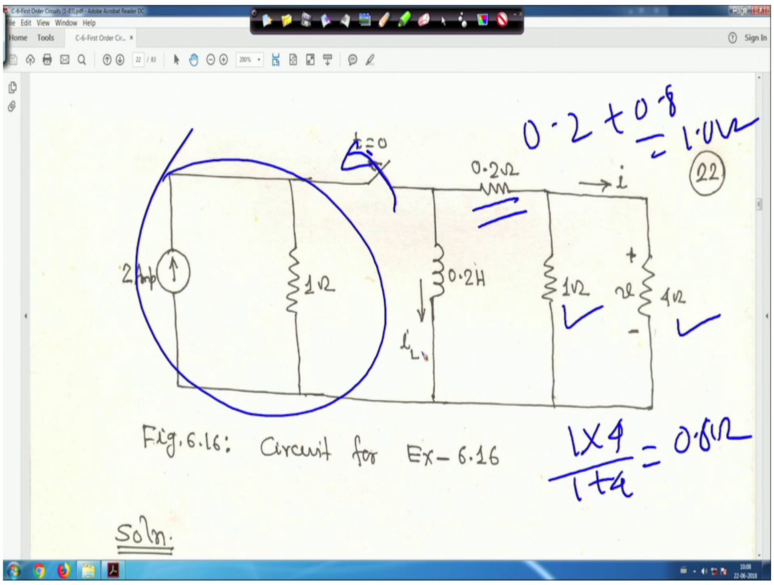

So, in that case what will happen if this is open this 1 ohm and 4 ohm they are in your what you call in parallel right; that means, their equivalent resistance will be 1 into 4 by 1 plus 4 is equal to your 0.8 ohm and with that this 2 ohm resistance is in series. Therefore, 0.2 plus 0.8 equivalent resistance is 1 ohm right and in that case time constant of the circuit will be tau is equal to L by R right.

So, it will be your it is L 2 Henry and it is R so it will be 0.2 second. Now another thing is this look at the your what you call the direction of the current, the current flowing through this 4 ohm resistance it is given it is taken as i right. So, first we have to find out that what is an this direction of the current here i L is downwards right. So, when you take the equivalent of this let me let me let me got through that let me go through that, therefore the tau is equal to I told you R eq is equal to 1 ohm and tau is equal to 0.2 second.

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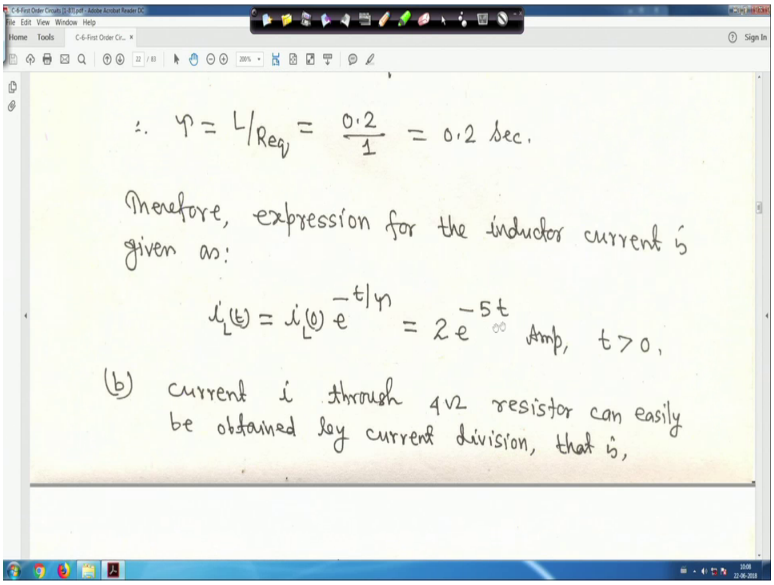

Therefore the expression for inductor current is we know that I L t is equal to i L 0 e to the power minus t upon tau this we know. So, i 0 is equal to i L 0 is equal to 2 ampere we have seen and tau is equal to your L by R. So, it will be your 0.2 t minus t tau is equal to 0.2 seconds minus t by 0.2, so it is it is e to the power minus 5 t ampere for t greater than 0.

Now, the current i through 4 ohm resistor can easily be obtained by current division that is i L t is equal to your i t is equal to minus i L t into 1 upon 1 plus 4, so if you come here if you if you come here right. So, whenever we you are what you call this the this 1 ohm and 1 ohm and 4 ohm they are in parallel, but this I current I is flowing through this 4 ohm resistance right.

So, if you if you come if you come like this that your what you call that from the current division. So, this circuit this part of the circuit is not there this part of the circuit is not there right and this is the your what you call the i L current and this is the current your what you call i is coming your through this way right.

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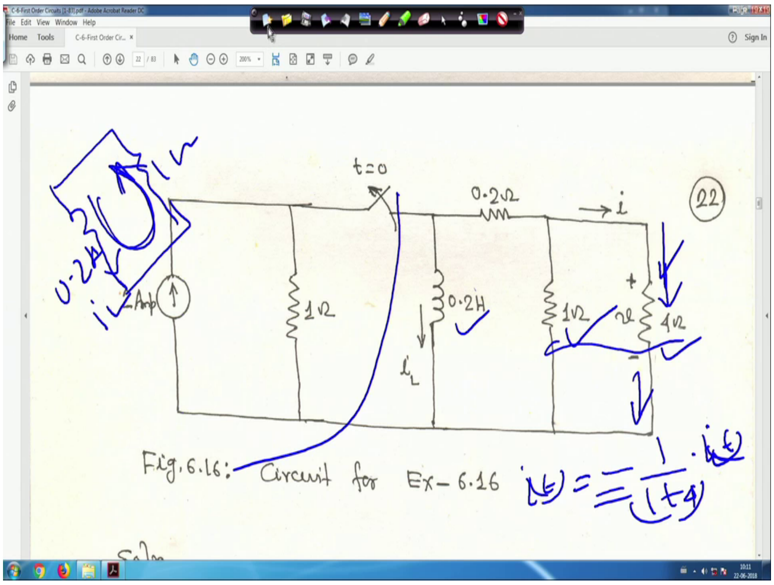

So, if you go for a your current division right then this i will be this i, if you put in terms of t does not matter i t is equal to it will be minus right is current this is 1 upon 1 plus 4 into your i L t right. So, this minus is taken because of the your what you call, if you just try to find out here what you call that your just your direction just for the way we have taken right.

So, so in the in that case actually this 2 things 2 your what you call, if your draw circuit like this that your 0.2 Henry right 0.2 Henry and this equivalent resistance was 1 ohm right and this is your i L this is your i L, that means current is flowing i L like this right so and this is this parallel equivalent is your 1 ohm that circuit I have not drawn, but just showing you and this is the 0.2 ohm Henry and I L we have taken like this.

So, this we have find out i L right, but as the current division is there. So, direction of this current that what you call is through i 4 ohm resistance it is given this way right. So, accordingly that your what you call this minus sign minus sign will be there, so anyway let me clear it.

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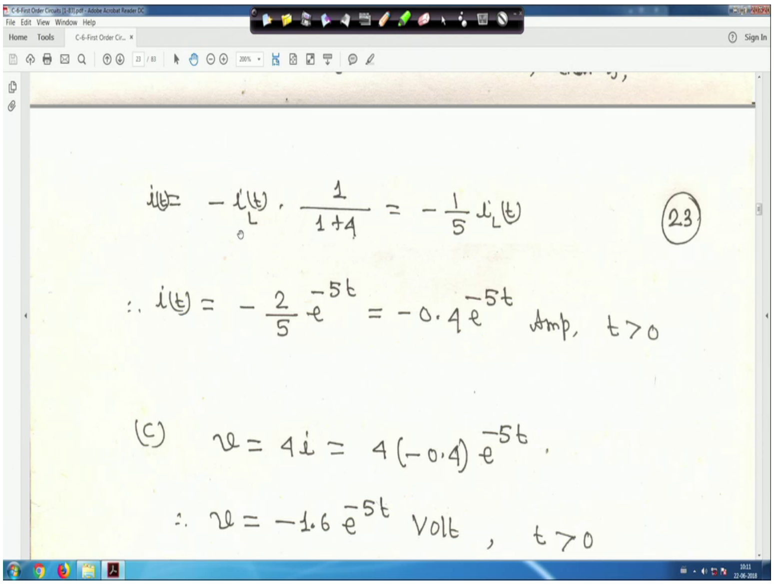

So, that is why your this i t is equal to minus i L into 1 upon 1 plus 4 that is minus 1 upon 5 i L t. So, that is i t is equal to minus 2 by 5 right e to the power minus this is the current division simply current flowing through this 4 and 1 ohm are in parallel. So, it will be current flowing through 4 ohm resistance will be your what you call that is 1 upon 1 plus 4 is equal to your that is 1 upon 5. So, minus 1 upon 5 i L t I told you that that your what you call that because i is minus right.

So, in that case i t is equal to basically minus 2 by 5 e to the power minus 5 by t because i L tis equal to 2 e to the power minus 5 t is equal to minus 0.4 e to the power minus 5 by t ampere for t greater than 0 right. So, now V is equal to your 4 i because 4 ohm resistance is there, so across 4 ohm resistance the voltage is V is equal to 4 i. So, 4 into minus 0.4 e to the power minus 5 t, so V is equal to minus 1.6 into minus 5 t volt for t greater than 0 right.

Now, next is the power dissipated in the 1 ohm resistor is that is p t is equal to v square by R. So, R is 1 ohm so that is 2.56 upon 1 e to the power minus your v square is your what you call it is b square. So, 1.6 square 2.56 and it is square so e to the power minus 10 t, so 2.56 e to the power minus 10 t watt that is t greater than for t greater than 0.

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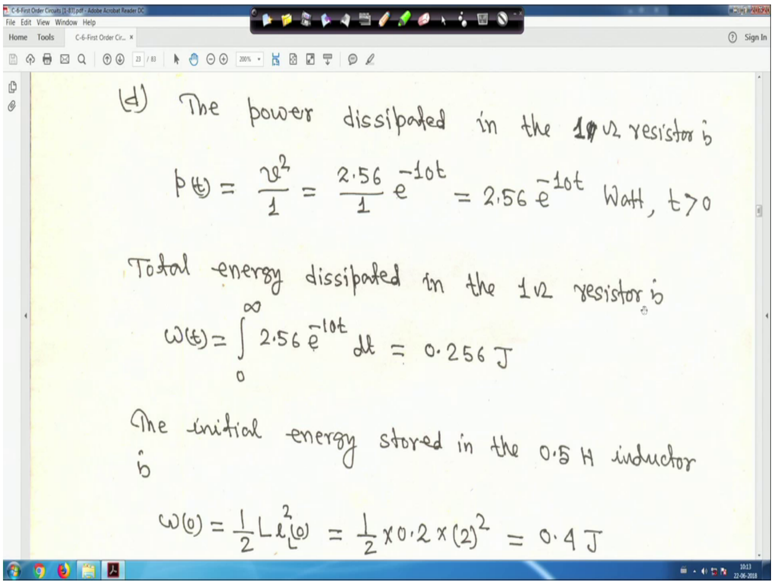

So, total energy dissipated in the 1 ohm resistor is that w t is equal to 0 to infinity 2.56 e to the power minus 10 t dt if you integrate you will get 0.256 joule right. So, the initial energy stored in the 0.5 ohm sorry 0.5 Henry inductor is half L i 0 L 0 square. So, half L is 0.2 Henry into L 0 initial value is 2 ampere, so 2 square that is 0.4 joule right therefore, the percent of energy dissipated in the 1 ohm resistor is that is 0.256 that is your energy dissipated in resistor 0.256 and initial energy stored was 0.4 joule, so 0.256 by 0.4 into 100 that is 64 percent right.

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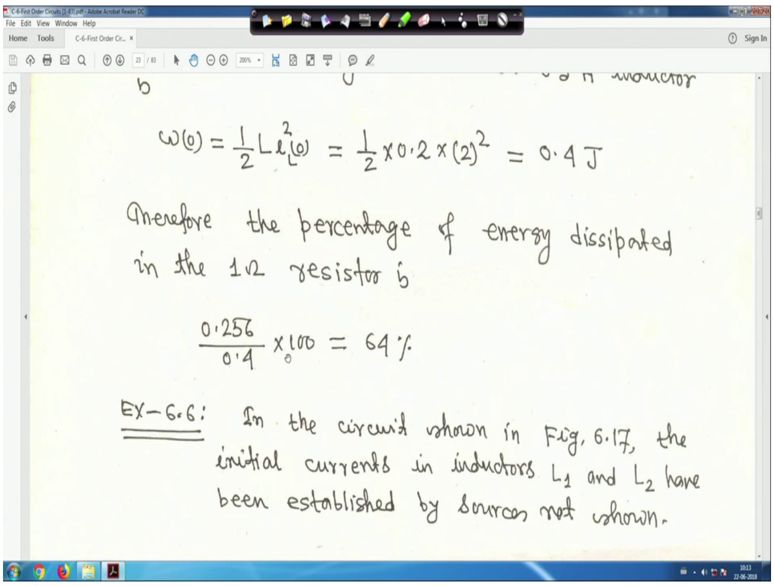

So, with this is a typical your example for your for circuit just looking at the circuit nothing to be hold it. As the switch initially was closed you find out what is the initial condition then switch is open and accordingly you solve right. And just looking at the plus sign or minus sign please look into the your what you call the direction of the current accordingly your otherwise the sign.

If you miss the minus sign or plus sign if something goes wrong, then your technique your numerical value numerical value may remain same, but the sign will create problem. So, little bit you should be careful right, so with your what you call. So, all these things I mean whatever the I mean what voltage, then energy stored in resistor 1 ohm, and percentage of the total energy stored everything is mentioned in this problem so.

Thank you very much we will be back again.