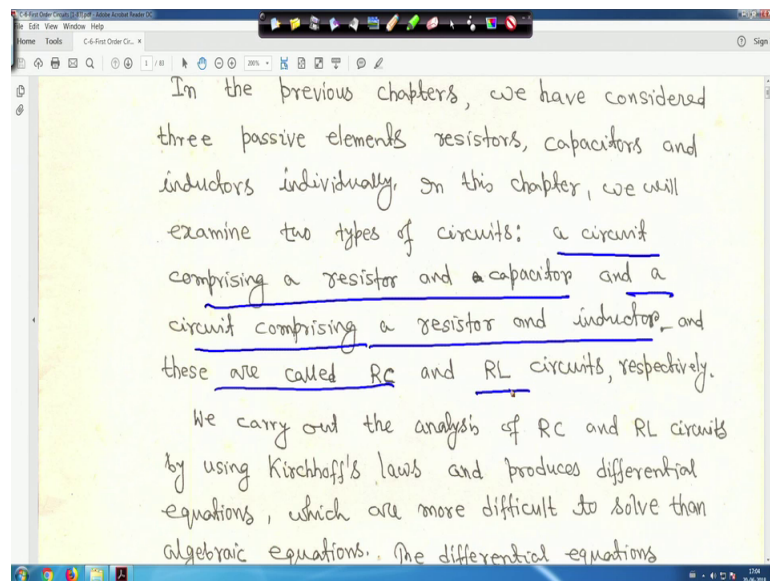


Fundamentals of Electrical Engineering
Prof. Debapriya Das
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Lecture - 29
First order circuits

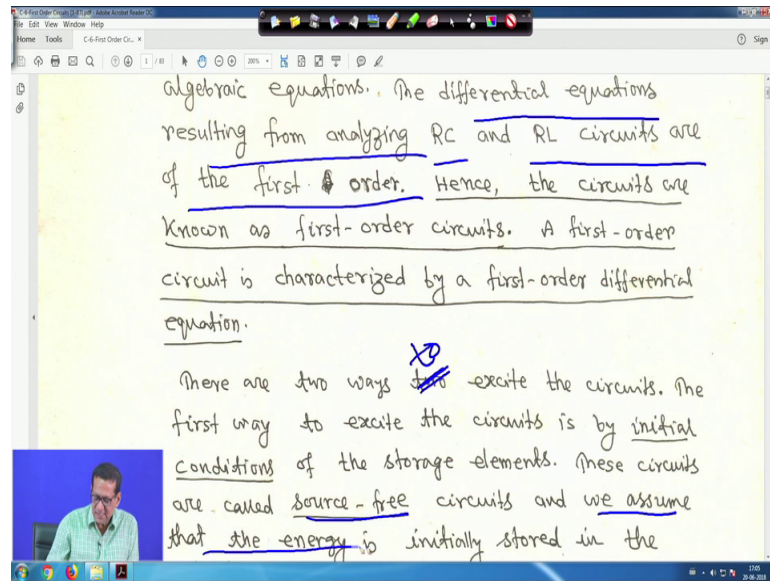
So next we will start the First order circuit, this is your chapter 6 right. So, and here we will see the dc transient particularly the, your R L circuit and R C circuit only the we will see that why it is called first order circuit also. So, when this topic is over we will go for ac circuit. So, we start first single phase then 3 phase right. So, first some introduction I have made it for you that in the previous chapters we have considered 3 passive elements resistors capacitors and inductors individually.

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So, in this chapter, we will examine your 2 types of circuit. A circuit you are comprising a resistor and a capacitor and a circuit comprising a resistor and your inductor. I mean we will see the 2 types of circuits that is a circuit comprising a resistor and capacitor and we will see a circuit comprising a resistor and inductor and these are called R C circuit and R L circuit with respectively right. We carry out the analysis of R C and R L circuit by using your what you call KCL your KCL and KVL and produces different equations, which are more difficult to solve then as every equations right. So, just let me this thing.

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The screenshot shows a digital whiteboard with handwritten text. The text is as follows:

Algebraic equations. The differential equations resulting from analyzing RC and RL circuits are of the first order. Hence, the circuits are known as first-order circuits. A first-order circuit is characterized by a first-order differential equation.

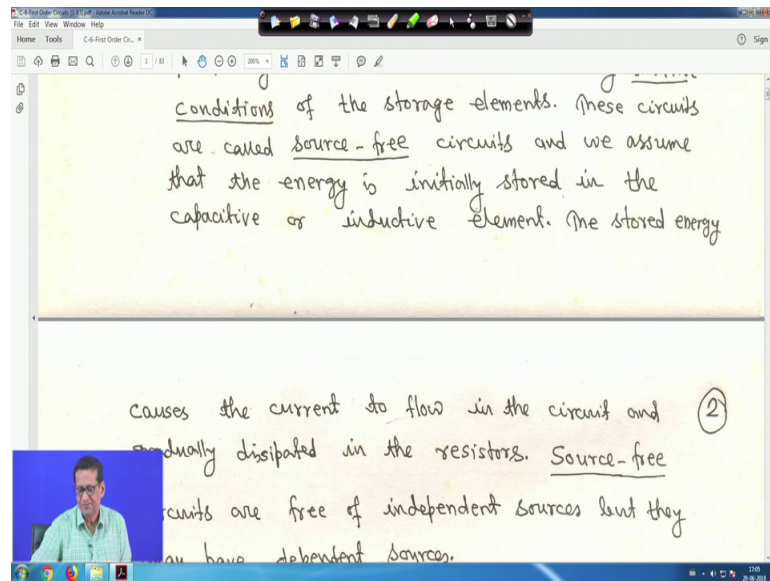
There are two ways ~~to~~ excite the circuits. The first way to excite the circuits is by initial conditions of the storage elements. These circuits are called source-free circuits and we assume that the energy is initially stored in the

The slide also features a small video inset in the bottom-left corner showing a man speaking.

So, the differential equations resulting from analyzing RC and RL circuit, that is your the differential equation resulting analyzing that RC and RL circuits are of the first order right hence it is underlined the circuits are known as first order circuits. A first order circuit is characterized by first order differential equation. So, there are two ways it will be 2 right, I am correcting it right.

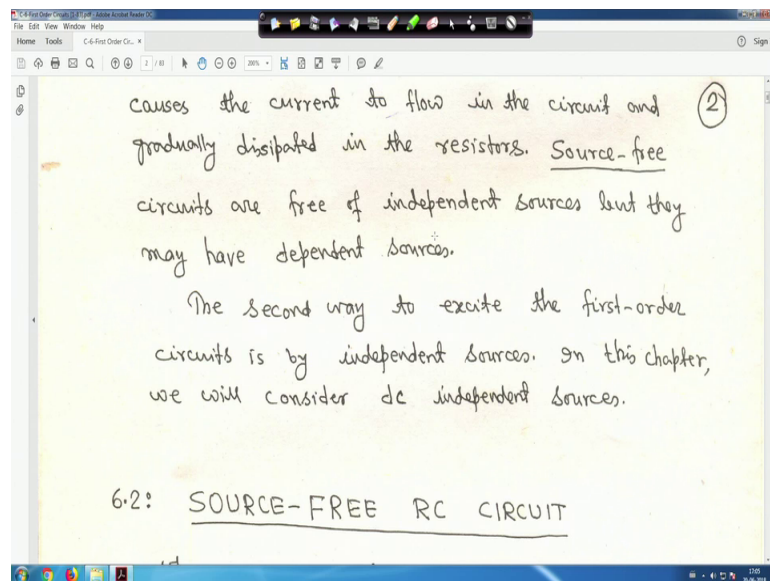
So, there are two ways to excite the circuit, the first way to excite the circuit is by initial condition of the storage element right that will be source free circuit right this circuit are called source free circuit; that means, circuit is excited by the initial your what you call that your conditions of the storage your storage elements and this these circuits are source free circuit right. So, actually in that case we assume that the energy is initially stored in that your what you call in that circuit element, just scroll one in that a stored in the capacitor or an inductive element right.

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So, this stored energy causes the current to flow in the circuit and gradually dissipated in the resistor.

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So, source free circuits are free of independent sources, but they may have dependent sources. So, it may have I mean it is it is there will be no independent sources, but they may have dependent sources. The second way second way to excite the first order circuit is by independent sources. So, in this chapter or in this topic we will consider dc independent sources right first will see the source free circuit.

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6.2: SOURCE-FREE RC CIRCUIT

When dc source of a RC circuit is suddenly disconnected, a source-free RC circuit occurs. The energy already stored in the capacitor is gradually dissipated in the resistors.

Fig. 6.1 shows a series combination of a resistor and an initially charged capacitor. main objective is to determine the circuit response.

So, whenever you see source free circuit here no then we will see when I will solve numerical 1 or 2 cases or 2 3 cases I will tell you the solution and this way, but here 1 and 2 cases I will not tell you that how 1 and 2 cases obtained I will give you your what you call that you think that how it has come, but that will come later. So, when dc source of a R C circuit is suddenly disconnected, a source free R C circuit occurs right. So, the energy already stored in the capacitor is gradually dissipated in the resistor.

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Fig. 6.1 shows a series combination of a resistor and an initially charged capacitor. main objective is to determine the circuit response.

Fig. 6.1: A source-free circuit

So; that means, figure 1 suppose it shows us series combination of a resistor and an initial energy your charge initial sorry initially charged capacitor. So, main objective is to examine the circuit response.

So, in this case this, this capacitor was initially charged and resistor is there; all though i C and i R it is a it is a simple circuit, but at this point if you apply KCL, I if you take like this that i C is going this way and i R is going this way both are leaving. So, i C plus i R equal to 0 right already the same circuit, but for some analysis we will do that and this is your this side is your voltage right this is the voltage, v is the voltage that was the capacitor or across the resistor it is same right.

So, it is a source free circuit there is no independent source in the circuit, assuming that this capacitor was initially charged and if you apply KVL at this point it will be i C plus i R is equal to 0 right so, this is a source free circuit.

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Assume that the voltage across capacitor is $v(t)$.
Since the capacitor is initially charged, we assume that, at time $t=0$, the initial
voltage is
 $v(0) = V_0$ (6.1)
Stored energy in the capacitor is

Now so, assume that the voltage across the capacitor is v_t since the capacitor is initially charged we assume that time t is equal to 0 the initial voltage V_0 is equal to V_0 right.

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Voltage is

$$w_c(0) = V_0 \quad \text{--- (6.1)}$$

Stored energy in the capacitor is

$$w_c(0) = \frac{1}{2} C V_0^2 \quad \text{--- (6.2)}$$

In Fig. 6.1, applying KCL at node a,

$$i_c + i_R = 0 \quad \text{--- (6.3)}$$

So, in this case we assuming t is equal to 0 that is V_0 is equal to your capital V suffix 0 is the initial voltage. Now, stored energy in the capacitor initially that $w_c(0)$ is equal to half $C V_0$ square right just hold on. So, just hold on. So, in this case the stored energy in the capacitor is this $w_c(0)$ is equal to half $C V_0$ square right. So, applying KCL at node I told you i_C plus i_R is equal to 0 right. So, let me clear it.

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In Fig. 6.1, applying KCL at node a,

$$i_c + i_R = 0 \quad \text{--- (6.3)}$$

By definition, we know $i_c = C \frac{dv}{dt}$ and $i_R = \frac{v}{R}$,

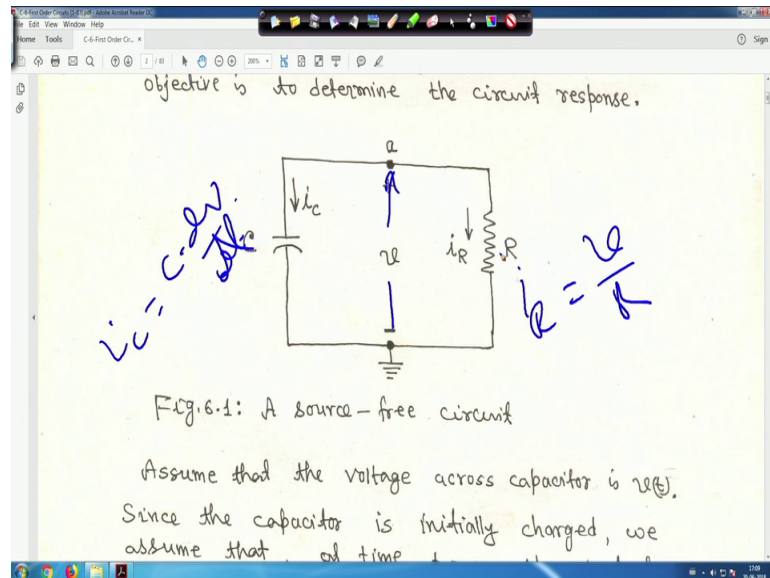
Thus,

$$\therefore C \frac{dv}{dt} + \frac{v}{R} = 0 \quad \text{--- (6.4)}$$
$$\therefore \frac{dv}{dt} + \frac{v}{RC} = 0 \quad \text{--- (6.4)}$$

This is a first-order differential equation,

So, by definition we know that i_C is equal to C into dv by dt , and at the same time i_R is equal to v by R .

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If you come to this circuit if you come to this circuit that your this voltage I told you, this voltage is your v . So, $i R$ is equal to v upon R right because same voltage that was the capacitor across a resistor.

So, v is equal to $i R$ so, $i R$ into $i R$ so, $i R$ is equal to v by R similarly here also your $i C$ is equal to C into dv by dt right. This v is across the same this capacitor as well as the resistor. So, let me clear it. So, in this case your this is C into dv by dt plus v by R is equal to 0 because $i R$ is equal to v by R is equal to 0 or dv by dt plus v by $R C$ is equal to 0.

(Refer Slide Time: 07:28)

Eqn (6.4) can be written as,

$$\frac{dv}{v} = -\frac{1}{RC} dt \quad \dots (6.5)$$

Integrating both sides, we get,

$$\ln(v) = -\frac{t}{RC} + \ln(A) \quad \dots (6.6)$$

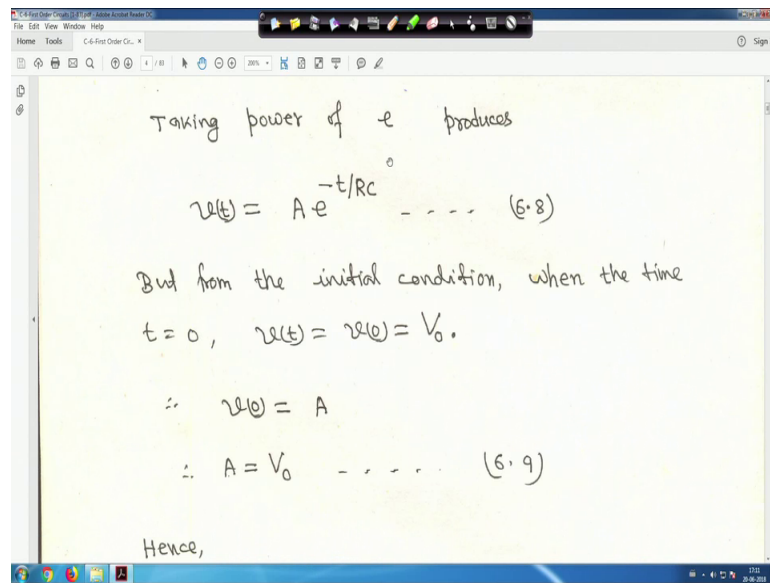
where A is the integration constant. Thus,

$$\ln\left(\frac{v}{A}\right) = -\frac{t}{RC} \quad \dots (6.7)$$

So, this is a first order differential equation, since only the first derivative of v is involved. So, equation 4 can be written as we can write dv by v is equal to minus 1 upon R C dt. So, this one you can write that you are dv by v is equal to minus 1 upon R C dt this because equation 5. Integrate both side this side you will get ln v natural log minus t upon R C plus some constant ln a we have taken instead of a or c 1 C 2 directly we have taken ln a for simply simplicity so, this is equation 6.

Now, A is the integration constant, thus ln if you I mean if you make this one ln v bring this one to this side, left side minus ln A right is equal to minus t upon RC. So, this side this side can be written as ln V upon A is equal to minus t upon R C that is what it is written here right so, this is equation 7. So, let me clear it.

(Refer Slide Time: 08:37)



The image shows a digital whiteboard with handwritten mathematical derivations. The text is as follows:

Taking power of e produces

$$v(t) = A e^{-t/Rc} \quad \text{--- (6.8)}$$

But from the initial condition, when the time $t = 0$, $v(t) = v(0) = V_0$.

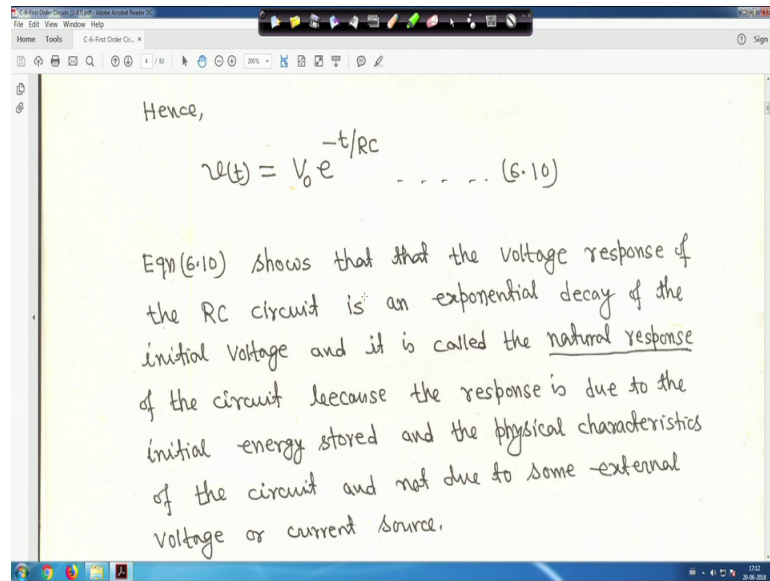
$$\therefore v(0) = A$$
$$\therefore A = V_0 \quad \text{--- (6.9)}$$

Hence,

So, taking power of e I mean now if you take power of e it is given. So, $v(t)$ is equal to A into your e to the power minus t upon $R C$, actually it was given here no here it was given here right. So, here you are here it is given \ln means it is log of base e . So, v upon A is equal to e to the power minus t upon $R C$ or v is equal to A into e to the power minus t upon $R C$ as e is a function of t that is why I writing dt right.

So, just so, that is why you are writing $v(t)$ is equal to $A e^{-t/Rc}$. Now, but from the initial condition when the time t is equal to 0 , $v(t)$ is equal to $v(0)$ is equal to capital V suffix 0 right. Therefore, at t is equal to 0 if you write, then it will become this part will become 1 and it will $V(0) = A$. So, that is A is equal to $V(0)$ right; because $v(0)$ is equal to earlier we are writing v initial condition $v(0)$ is equal to capital V suffix 0 . So, this is equation 9 I am not given equation number therefore, my $v(t)$ is equal to $V(0) e^{-t/Rc}$ this is equation the 10.

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Hence,

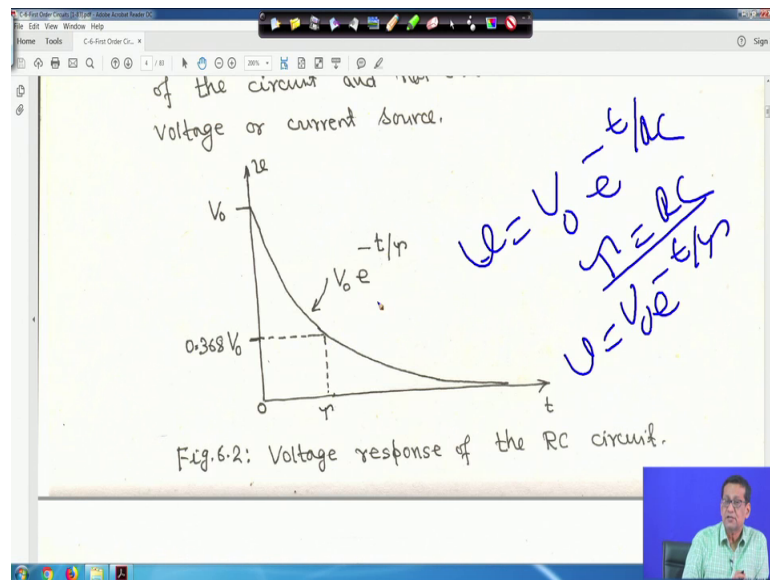
$$v(t) = V_0 e^{-t/RC} \quad \dots \dots (6.10)$$

Eqn (6.10) shows that that the voltage response of the RC circuit is an exponential decay of the initial voltage and it is called the natural response of the circuit because the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source.

Equation 10, shows that the voltage response of the R C circuit is an your what you call is an exponential decay of the initial voltage, and it is called the natural response of the circuit because it is given e to the power minus t upon RC. So, naturally it is the exponential decay of the initial response, and it is called the natural response of the circuit because this response is due to the initial energy stored and the physical characteristic of the circuit and not due to the external voltage or current source right.

That means this is called your natural response of the circuit, because the response is due to the initial energy stored and the physical characteristic of the circuit right and not due to some external voltage or current source, because there is a source free circuit. So, this is called the natural response right of the circuit. So, let me clear it.

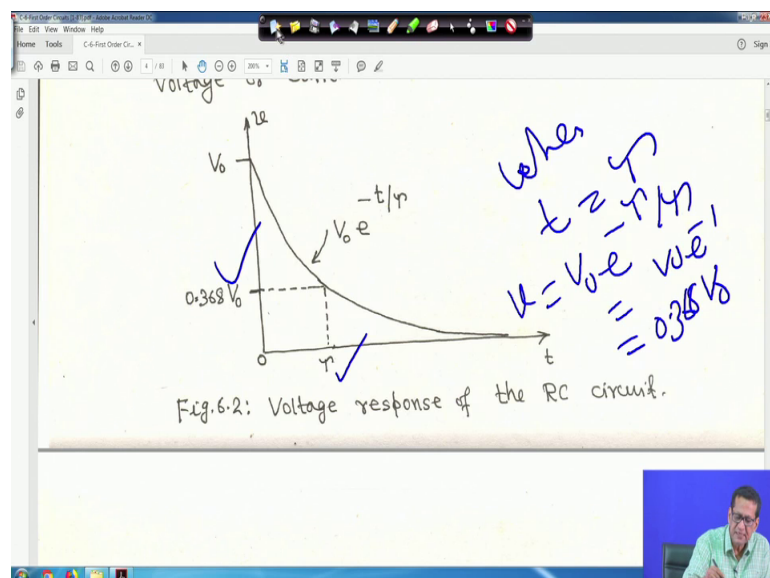
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So, now if we plot if we plot that V is equal to $V_0 e^{-t/\tau}$, that is your that is V is equal to your $V_0 e^{-t/\tau}$, if we take τ is equal to RC then V is equal to $V_0 e^{-t/RC}$, τ is equal to RC τ actually is called your time constant right will see that.

So, that is why this plot is $V_0 e^{-t/\tau}$ now let me clear it. So, voltage response this is the voltage response of the RC circuit now and when I will come to $t = \tau$, but I am telling.

(Refer Slide Time: 11:37)



That when t is equal to τ , when t is equal to τ then v will be is equal to $V_0 e^{-1}$ to the power minus τ by τ that is $V_0 e^{-1}$ to the power minus 1 that will become $0.368 V_0$ that is here it is written here. So, when t is equal to τ , then it is $0.368 V_0$ right. So, let me clear it now I will come to some explanation. So, figure this is your figure 2 this is your figure 2 figure 2 shows the natural response.

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⑤

Fig.6.2 shows the natural response. At $t=0$, we have the correct initial condition $v(0) = V_0$ and as time t increases, the voltage decreases toward zero. The rapidity with which the voltage decreases is expressed in terms of the time constant, denoted by τ , and expressed as:

$$\tau = RC \quad \dots \quad (6.11)$$

Therefore

At t is equal to 0 we have the correct initial condition V_0 your what you call your v is equal to V_0 at is equal to 0. So, it is v is equal to V_0 right and as time t increases the voltage decreases towards 0. The rapidity, which we with which the voltage decreases is expressed in terms of the time constant denoted by τ right and expressed as τ is equal to RC this is the time constant of that circuit. Therefore, equation 10 can be expressed as v t is equal to $V_0 e^{-t/\tau}$ I told you.

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The screenshot shows a presentation slide with handwritten text and equations. At the top, it states $\tau = RC$ (6.11). Below that, it says "Therefore, Eqn. (6.10) can be expressed as:" followed by the equation $v(t) = V_0 e^{-t/\tau}$ (6.12). Then, it specifies "At $t = \tau$," and derives $v(\tau) = V_0 e^{-1} = 0.368 V_0$ (6.13). The final sentence reads: "From eqn. (6.13), we can state that the time constant of a circuit is the time required". A small video inset of a presenter is visible in the bottom right corner of the slide.

So, at t is equal to τ V τ is equal to $V_0 e^{-1}$ that also I told you is equal to $0.368 V_0$; that means, this one $0.368 V_0$ right.

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The screenshot shows a presentation slide with handwritten text. The first sentence is: "the response to decay 36.8 percent of its initial value." Below this, it explains: "Table-6.1 shows the value of $v(t)/V_0$. From Table-6.1, it is seen that the voltage $v(t)$ is less than 1 percent of V_0 after $t = 5\tau$. Thus, it is customary to assume that the capacitor is fully discharged (or charged) after five time constants. In other words, it takes $t = 5\tau$ for the circuit to reach its steady state when no changes take place with time."

So, that means equation 13 we can state that the time constant of a circuit is the time required for the response to decay, 36.8 percent of its initial value. This is 0.368; that means, if you take in percentage that is 36.8 percent of initial value is V_0 right.

We can state that the time constant of the circuit is the time required for the response to decay 36.8 percent of its initial value. Now table I have make a table, table 1 shows the

value of $v(t)$ by V_0 from there your from table 1 it is seen that the voltage $v(t)$ is less than 1 percent of V_0 after t is equal to 5 tau. I mean this is $v(t)$ is equal to $V_0 e^{-t/\tau}$ the power minus t upon tau; that means, $v(t)$ by V_0 is equal to $e^{-t/\tau}$.

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TABLE-6.1: Values of $v(t)/V_0 = e^{-t/\tau}$

t	$v(t)/V_0$
τ	0.3678
2τ	0.1353
3τ	0.0498
4τ	0.01832
5τ	0.0067

When t is equal to tau says becoming 0.3678 when 2 tau 0.1353 and so on up to when 5 tau it is 0.0067 less than 1 percent right less than 1 percent. Therefore that when t is equal to 5 tau right, it is less than 1 percent thus it is your here what you call customary to assume that the capacitor is fully discharged right fully discharged or fully charged when it will be in other way right after 5 time constant.

In other wards it takes t is equal to 5 tau for the circuit to reach its steady state, when no changes takes your when no changes takes place with time; that means, for this graph if you come if you go up to tau, 2 tau up to 3 tau up to 5 tau. So, the time is equal to t is equal to 5 tau after that you will find almost no change right it is almost this is the steady state. So, that is your what you call this stable shows that this table shows right.

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The screenshot shows a digital whiteboard with a table and handwritten text. The table is as follows:

3 τ	0.0498
4 τ	0.01832
5 τ	0.0067

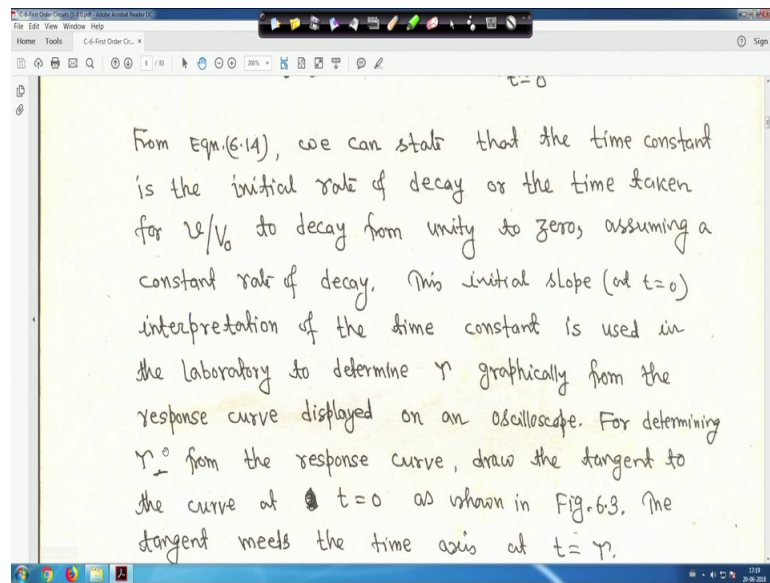
Below the table, the text reads: "The time constant may be viewed from another perspective. Evaluating the derivative of $v(t)$ in Eqn.(6.12) at $t=0$, we get

$$\left. \frac{d}{dt} \left(\frac{v}{V_0} \right) \right|_{t=0} = \left. -\frac{1}{\tau} e^{-t/\tau} \right|_{t=0} = -\frac{1}{\tau} \quad \text{---(6.14)}$$

So, the time constant may be viewed from another perspective. Evaluating the derivative of $v(t)$ in equation 12 at t is equal to 0. So, it in equation 12 if you come to the these equation 12 this is your equation 12, $v(t)$ is equal to $V_0 e^{-t/\tau}$ right take the derivate of this right I will come to that you take the you take v by V_0 then $e^{-t/\tau}$ to the power minus t by τ right. So, d/dt of v by V_0 you take at t is equal to 0, it becomes minus 1 by τ $e^{-t/\tau}$ and at t is equal to 0 right it becomes minus 1 upon τ right so, this is equation 14.

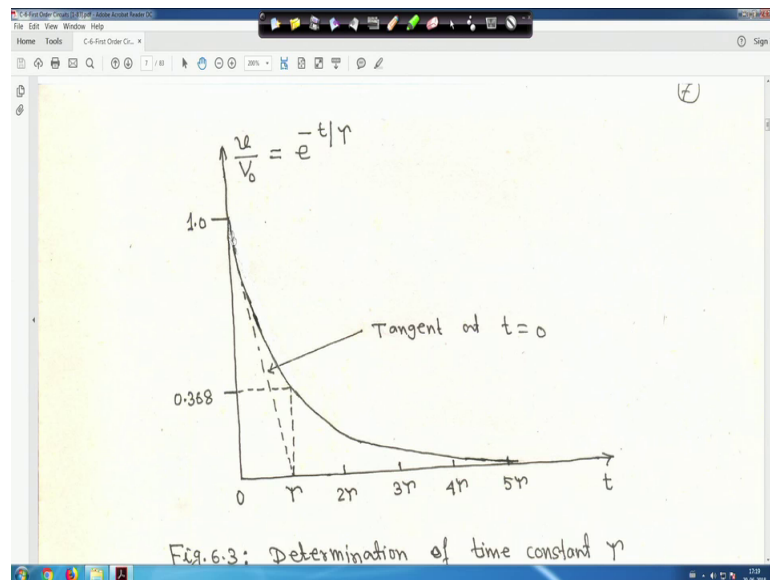
Now from equation 14 we can state that the time constant is the initial rate of decay or the time taken for v by V_0 to decay from unity to 0 assuming a constant rate of decay this initial slope at t is equal to 0, your what you call interpretation of the time constant is used in the laboratory to determine τ graphically from the response curve displayed on an oscilloscope right.

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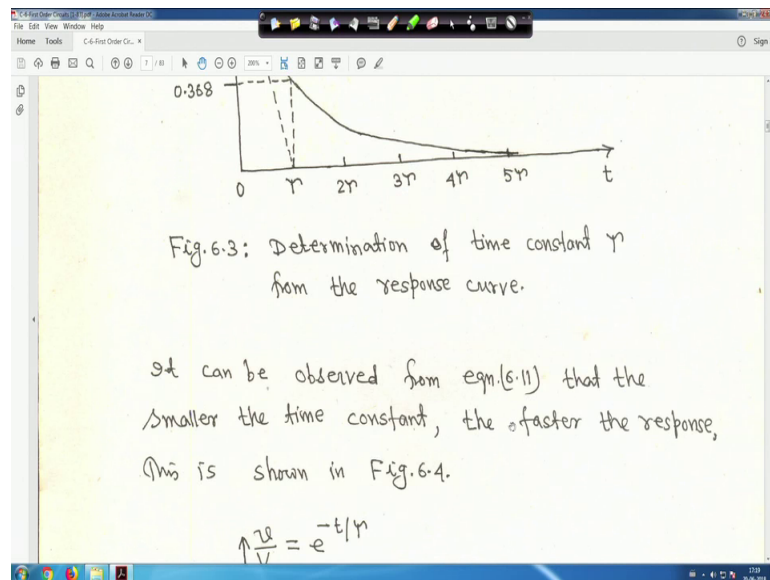
So, for determining tau from the response curve, draw the tangent to the curve at t is equal to 0; that tangent meets the time axis at t is equal to tau; that means, that t is equal to 0 at t is equal to 0 if you draw a tangent it will meet here at tau right here it will meet at tau right.

(Refer Slide Time: 16:34)



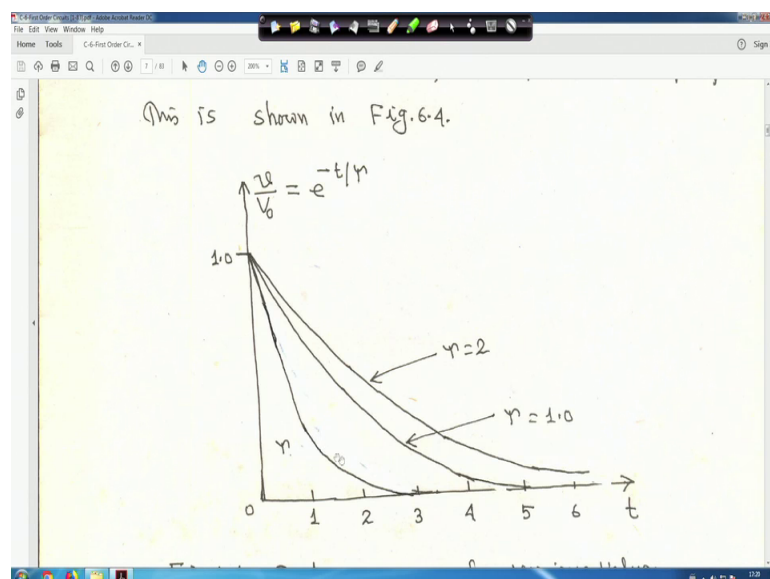
And this tangent is a at t is equal to your 0, somewhere from somewhere you draw the tangent and it will meet at tau right.

(Refer Slide Time: 16:50)



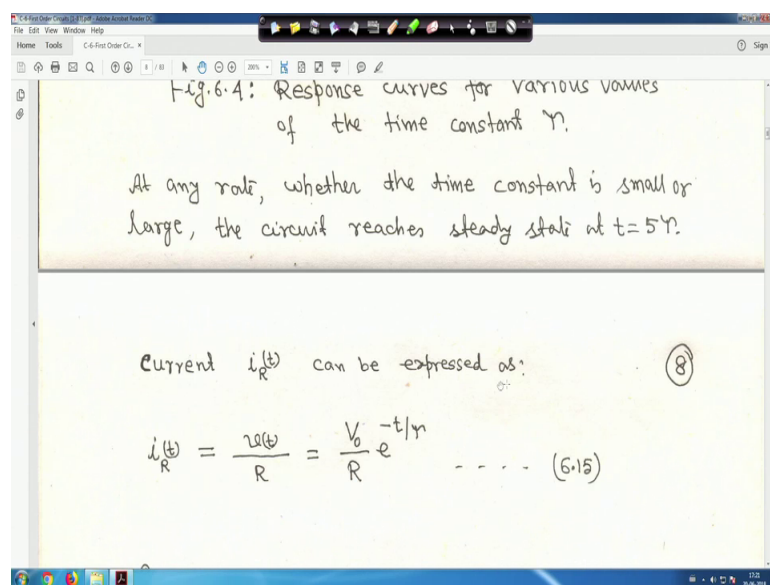
So, this is determination of time constant tau from the response curve, this way also you can find out your time constant at the laboratory, where whatever graph you get from that you can do it. And then 2 tau 3 tau up to 5 tau it is shown right. So, it can be observed from equation 11 if you come to equation 11 right hold on this is tau is equal to R C right. So, it can be observed from equation 11 that the smaller the time constant the faster the response this is shown in figure 4.

(Refer Slide Time: 17:27)



I mean if the when tau is equal to 2 response is slower, when tau is equal to 1 much faster than tau is equal to 2, if you further tau decreases response will become faster right. So, this side is taken v upon V 0 is equal to e to the power minus t upon tau the ratio is taken on the y axis. So, this is the response curve for various values of the time constant tau. At any rate whether the time constant is small or large, the circuit reaches steady state at t is equal to 5 tau because it is less than 1 percent that your what you call the ratio whatever you have taken right. So, circuit reaches steady state at t is equal to 5 tau right.

(Refer Slide Time: 18:12)



Therefore, current $i_R(t)$ can be expressed as $i_R(t) = v(t) \text{ upon } R$ we have seen in the circuit same thing. So, it will be $V_0 \text{ upon } R e^{-t/\tau}$ this is the current $i_R(t)$ right. The power dissipated in the resistor is $v(t) i_R(t)$. So, if you put $v(t) = V_0 e^{-t/\tau}$, and $i_R(t) = \frac{V_0}{R} e^{-t/\tau}$ multiply you will get $V_0^2 \text{ upon } R e^{-2t/\tau}$ this is equation 16 right.

(Refer Slide Time: 18:47)

The power dissipated in the resistor is

$$p(t) = v(t)i_R(t) = \frac{V_0^2}{R} e^{-2t/\tau} \quad \dots (6.16)$$

The energy absorbed by the resistor up to time t , is,

$$w_R(t) = \int_0^t p(t) dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau} dt$$
$$\therefore w_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), \quad \tau = RC \quad \dots (6.17)$$

So, the energy absorbed by the resistor up to time t that $w_R(t)$ is $\int_0^t p(t) dt$ right p is a function t , 0 to t this is v equation 16, V_0^2 upon R $e^{-2t/\tau}$ the power minus $2t$ upon τ dt right. If you entry at this you will get your τ is equal to RC that time constant half $C V_0^2$ into $1 - e^{-2t/\tau}$. Note that as t tends to infinity $w_R(\infty)$ is equal to half $C V_0^2$ that is $w_C(0)$ right.

So, initially initial energy stored in the capacitor is eventually dissipated in the resistor. So, if your t tends to infinity, then it is simply becoming half $C V_0^2$ because this term will not be there and is equal to initially stored $w_C(0)$ right. So, initial energy stored in the capacitor eventually dissipated in the resistor. So, we will take some example.

(Refer Slide Time: 19:47)

$$w_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-t/RC}), \quad \tau = RC \quad \dots (6.17)$$

Note that as $t \rightarrow \infty$, $w_R(\infty) = \frac{1}{2} C V_0^2 = w_C(0)$.

Initial energy stored in the capacitor is eventually dissipated in the resistor.

Ex-6.1: In Fig. 6.5, $v_C(0) = 10$ Volt, determine v_C , v_x and i_x for $t > 0$.

The circuit diagram shows a 5 ohm resistor in series with a 0.1F capacitor. The capacitor voltage is v_C . This combination is in series with another 5 ohm resistor. The current through this resistor is i_x . A 15 ohm resistor is connected in parallel across the output terminals, with voltage v_x across it.

So, this is hope this source free R C circuit will be understandable to you right actually nothing is there only very simple thing very simple thing right

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$$v_C(t) = 20 \text{ volt, determine } v_C, v_x \text{ and } i_x \text{ for } t > 0.$$

$$(5 + 15) = 20 \Omega$$

$$\frac{5 \times 20}{(5 + 20)} = 4 \Omega$$

Fig. 6.5: Circuit for Ex-6.1.

Soln.

So, here your a simple circuit of a this thing is taken, a series parallel your parallel circuit right. So, 5 ohm resistor is there, 0.1 microfarad capacitor sorry 0.1 farad is there, voltage across is v_C this 5 ohm resistor and this is 5 and 15 both are in series and taken separate given separately and current through is i_x . And voltage across 15 ohm resistor is v_x you have to you have to find out initial values of the your capacitor voltage that is

your 10 volt is given $v_c = 0$. You have to determine we have to determine v_c that is your this one then v_x and i_x for $t > 0$, these we have to find it out right. Now, first we obtain the equivalent or Thevenin resistance across the capacitor terminal.

(Refer Slide Time: 20:45)

Soln. (9)

First we obtain the equivalent or Thevenin resistance across the capacitor terminals.

$$\therefore R_{eq} = R_{TH} = \frac{5 \times (5 + 15)}{5 + (5 + 15)} = 4\Omega$$

Equivalent circuit is shown in Fig. 6.6

0.1F \quad \quad \quad $R_{eq} = 4\Omega$

So Thevenin resistor, Thevenin resistance so, we have studied right Thevenin equivalent. So, in this case if you try to find out your what you call that your that your across these this 2 terminal, what will be the Thevenin resistance though this 5 ohm and 15 ohm so, 20 ohm. So, it is here it both are in series. So, let us see it is equal to 20 ohm; that means, and 5 ohm these 2 are in parallel. So, it will be 5 into 20 divided by 5 plus 20. So, it will be 4 ohm right 100 by 25. So, 4 ohm.

So, let me clear it. So, if you if you find out R_{eq} is equal to $R_{Thevenin}$ it is 5 into 5 15 plus 5, and 5 plus 5 plus 15 this is same thing your what you call 4 ohm right I have little all these things have written like this right, but there I showed the calculation.

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Equivalent circuit is shown in Fig.6.6

Fig.6.6: Equivalent circuit for the circuit of Fig.6.5.

The time constant is

Therefore, equal equivalence circuit will be this capacitor it is a source free circuit. So, 0.1 farad, and this R this is Req is equal to 4 remember for this kind of thing we have to make the R Thevenin first equivalent right. Let us see how things happen and this voltage is this voltage is v; that means, across the 0.1 farad capacitor it is be across R also it is your Req also it is v right. So, let me clear it. So, the time constant is tau is equal to we have seen RC. So, it is R eq here into c. So, R eq is 4 into C 0.1 so, 0.4 second right.

(Refer Slide Time: 22:32)

The time constant is

$$\tau = R_{eq} C = 4 \times 0.1 = 0.4 \text{ sec.}$$

Thus

$$v = V_0 e^{-t/\tau}$$
$$V_0 = v(0) = v_c(0) = 10 \text{ Volt, } \tau = 0.4 \text{ sec}$$
$$\therefore v_c = v = 10 e^{-t/0.4} = 10 e^{-2.5t} \text{ Volt.}$$

From Fig. 6.5, we can use voltage division to get

And v is equal to you know $V_0 e^{-t/\tau}$ to the power minus t by τ , but V_0 is given 10 volt. So, V_0 is equal to V_0 is equal to $v_c(0)$ is equal to 10 volt it is given and τ is equal to 0.4 second therefore, v_c is equal to v is equal to $10 e^{-t/0.4}$ to the power minus t upon 0.4. So, that is $10 e^{-2.5t}$ volt, this is the simple thing from the previously developed your formulas we are just putting the data.

(Refer Slide Time: 23:02)

Thus

$$v = V_0 e^{-t/\tau}$$

$$V_0 = v_c(0) = v_c(0) = 10 \text{ Volt}, \quad \tau = 0.4 \text{ sec}$$

$$\therefore v_c = v = 10 e^{-t/0.4} = 10 e^{-2.5t} \text{ Volt.}$$

From Fig. 6.5, we can use voltage division to get v_x

$$\therefore v_x = \frac{15}{(15+5)} v = 0.75 (10 e^{-2.5t}) = 7.5 e^{-2.5t}$$

and $i_x = \frac{v_x}{15} = \frac{7.5 e^{-2.5t}}{15} = 0.5 e^{-2.5t} \text{ Amp.}$

So, from figure 6.5 we use the voltage division to get v_x right. Now if you come to this if you come to this figure so, first suppose if this voltage is v .

(Refer Slide Time: 23:15)

Ex-6.1: In Fig. 6.5, $v_c(0) = 10 \text{ Volt}$, determine v_c , v_x and i_x for $t > 0$.

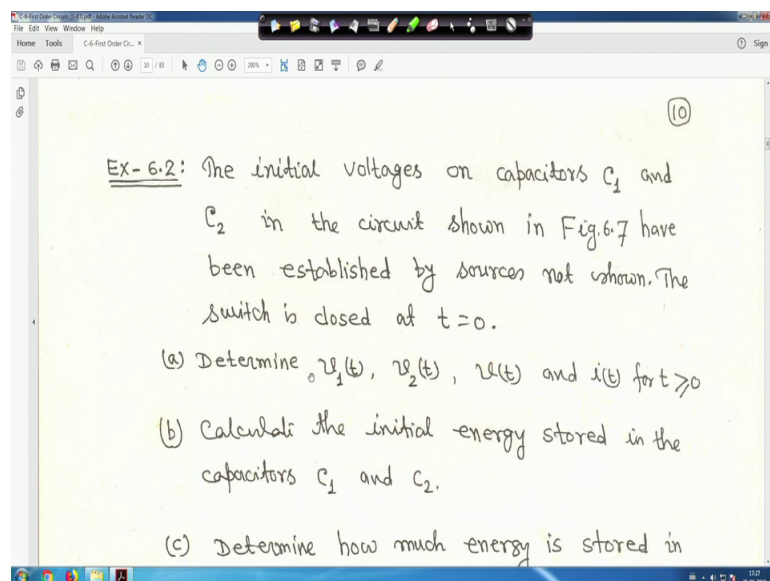
Fig. 6.4: Circuit for Ex-6.1.

$v_x = \frac{15}{(15+5)} v$

Suppose if this voltage is v right then voltage division this is $v \times$. So, voltage division because 5 and 15 ohm are in series so, it will be 15 by 15 plus 5 into this v that will be your $v \times$, this voltage across this is v . So, it will be 15 divided by 15 plus 5 because it is 15 here it is 5 into your this voltage, this voltage v that is voltage division right. So, let me clear it. So, this is what we have done here this is $v \times$ is equal to 15 upon 15 plus. So, it will be v , v we have got $10 e$ to the power minus 2.5 t , v is equal to $v c$ v is equal to $v c$. So, it is 0.75 into $10 e$ to the power minus 2.5 t .

So, it is $7.5 e$ to the power minus 2.5 t volt right and $i \times$ is equal to $v \times$ upon 15, because across this across 15 ohm resistance the voltage is $v \times$. So, naturally you can take $i \times$ is equal to $v \times$ by 15. So, $v \times$ we have got. So, $i \times$ is equal to $v \times$ upon 15 so, it will be $7.5 e$ to the power minus 2.5 t upon 15 is equal to $0.5 e$ to the power minus 2.5 t ampere this is your $i \times$ right this answer all answers you have got. Now, next example I hope you are understanding this I hope you are understanding this.

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So, the initial voltage on the capacitor C_1 and C_2 in the circuit, I will show you shown in figure 7 have been established by sources not shown sources not shown, but initial conditions have been established this is the language of the problem the switch is closed at t is equal to 0. Therefore you determine v_1 to v_2 to sorry $v_1 t$ $v_2 t$ then $v t$ and $i t$ for t greater than 0 right. Sorry next you calculate the initial energy stored in the capacitors C_1 and C_2 right.

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(c) Determine how much energy is stored in the capacitors as $t \rightarrow \infty$

(d) Show that the total energy delivered to the $250\text{ k}\Omega$ resistor is the difference between the results obtained in (b) and (c).

The diagram shows a circuit with a 4V DC source on the left, a switch at the top, a $5\mu\text{F}$ capacitor C_1 in the middle, and a $250\text{ k}\Omega$ resistor on the right. The voltage across the capacitor is labeled $v_1(t)$ and the voltage across the resistor is labeled $v(t)$. A time marker $t=0$ is shown at the switch.

Next is determine how much energy stored in the capacitor and t tends to infinity and so that the stored energy delivered to that 250 kilo resistor is the difference between the result obtained in b and c. So, these are the 4 thing you have to make it right.

(Refer Slide Time: 25:47)

between the results obtained in (b) and (c).

The diagram shows a circuit with a 4V DC source on the left, a switch at the top, a $5\mu\text{F}$ capacitor C_1 in the middle, a 24V DC source at the bottom, a $20\mu\text{F}$ capacitor C_2 in the middle, and a $250\text{ k}\Omega$ resistor on the right. The voltage across C_1 is $v_1(t)$, across C_2 is $v_2(t)$, and across the resistor is $v(t)$. A time marker $t=0$ is shown at the switch. Blue checkmarks are present next to the components.

Fig.6.7: Circuit for EX-6.2.

Now, here one thing I will not tell you I will just this thing, but first this is the circuit is given and look at the polarity right. So, here we have taken we want this is the capacitor C_1 it is 5 microfarad this is capacitor C_2 20 microfarad, and this is plus minus some where I have marked plus minus. So, this is v_1 t v_2 t right.

So, voltage means across C 1 and voltage means across C 2 and this is 250 kilo ohm resistor and across the voltage is v_t and current flowing through is i_t and switch was such capacitor was charged, switch was open now switch is closed at t is equal to 0 right. So, question is look at the polarity here polarity is given here it is minus, but here it is minus and here it is plus this is the this is there tricks in the problem.

So, I will simply write it and one only 1 or 2 place I will tell you look at this when you solve this problem, you will see that the polarity of this whatever it is given, it is minus this is plus and again here it is plus here it is minus. So, you can easily you can easily your what you call, you can easily make it that how what is the initial conditions right. So, let me clear it. So, this all these things are given, I mean all these your what you call this is given and you have to find out right.

(Refer Slide Time: 27:13)

Fig. 6.7: Circuit for EX-6.2.

soln.

(i) From Fig. 6.7, C_1 and C_2 are in series, hence,

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left(\frac{1}{5} + \frac{1}{20} \right)^{-1} \text{ MF} = 4 \text{ MF}$$

Given that,

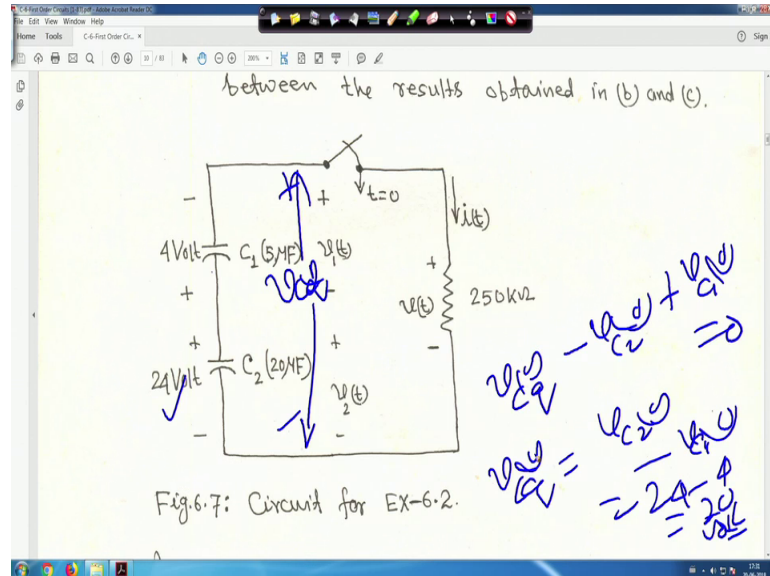
$$V_{C_1}(0) = 4 \text{ Volt} ; V_{C_2}(0) = 24 \text{ Volt},$$

(ii)

So, now question is and initial voltage that is given 4 volt and here it is 24 volt right, but look at the polarity and accordingly we have to choose the sign of that what you call that initial voltage. So, and so, these polarity is marked. So, figure this actually this 2 capacitors are in series 5 microfarad and 20 micro C 1 and C 2 so, its equivalent to the way you do the parallel resistor. So, therefore, here it is C_{eq} is equal to 1 upon C 1 plus 1 upon C 2 to the power minus 1 reciprocal right.

So, $\frac{1}{5} + \frac{1}{20}$ its reciprocal it becomes 4 micro farad that is c equivalent and given that $v_{c1}(0)$ just I am writing that 4 volt and $v_{c2}(0) = 24$ volt right. So, this is the initial condition therefore, $v_{c_{eq}}$ will be $v_{c2}(0) - v_{c1}(0)$ that is 20 volt.

(Refer Slide Time: 28:11)



Because if you try to take your what you call a your this thing suppose if I want to take this eq equivalent right that is $v_{c_{eq}}(0)$, suppose if I want right and if this is my this is my plus and this is my minus. So, it will be $v_{c2}(0)$ right then minus your $v_{c1}(0)$ then plus $v_{c1}(0)$ is equal to 0 right.

So, I am just looking at that polarity; that means, $v_{c_{eq}}$ is equal to $v_{c2}(0) - v_{c1}(0)$. So, it is given 24 volt that $v_{c2}(0) = 24$ and that is given 4. So, it is 20 volt right. So, this is that your what you call $v_{c_{eq}}(0)$ suppose if you look like this right.

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$v_{c_2}(0) = 4 \text{ Volt} ; v_{c_2}(0) = 24 \text{ Volt},$
 $\therefore v_{C_{eq}}(0) = v_{c_2}(0) - v_{c_1}(0) = 24 - 4 = 20 \text{ Volt}$
 $\therefore v(0) = V_0 = v_{C_{eq}}(0) = 20 \text{ Volt}.$
 Equivalent circuit for the circuit of Fig. 6.7 is shown in Fig. 6.8.

So, that is why that is why that your $v_{c_2}(0)$ is equal to 20 volt and V_0 is equal to capital V_0 for $v_{c_2}(0)$ is equal to 20 volt because this is the equivalent circuit source free R C circuit whatever we have done and this is 4 micro farad and this is 250 kilo ohm.

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shown in Fig. 6.8.

$C_{eq} = 4 \mu F$
 $250 \text{ k}\Omega$
 $v(t)$
 $i_c(t) \rightarrow$
 $i_c = -i_k$

Fig. 6.8: Equivalent circuit for the circuit of Fig. 6.7.

Same we are drawing this is $v(t)$ this is the equivalent circuit of that one right. Now the time constant is τ is equal to RC_{eq} 250 into 10 to the power 3 it is kilo ohm. So, converted into ohm into 4 into 10 to the power minus 6, 4 micro farad so, that is 1 second

thus the expression for $v(t)$ is we know $V(t)$ is equal to $V_0 e^{-t/\tau}$ upon τ .

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The time constant is

$$\tau = RC_{eq} = 250 \times 10^3 \times 4 \times 10^{-6} = 1 \text{ Sec.}$$

Thus the expression for $v(t)$ is

$$v(t) = V_0 e^{-t/\tau} = 20 e^{-t} \text{ Volt}$$

and

$$i(t) = \frac{v(t)}{R} = \frac{20 e^{-t}}{250 \times 1000} = 80 e^{-t} \mu\text{Amp}$$

So, it is $20 e^{-t}$ to the power minus t volt. So, $i(t)$ is equal to from this circuit simply $v(t)$ upon R the earlier we have seen. So, it is $20 e^{-t}$ to the power minus t by 250×1000 so, $80 e^{-t}$ to the power minus t micro ampere right.

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By definition, we can calculate the expressions for $v_1(t)$ and $v_2(t)$:

$$v_1(t) = -\frac{1}{C_1} \int_0^t 80 \times 10^{-6} e^{-t} dt - 4$$

$$\therefore v_1(t) = -\frac{10^6}{5} \int_0^t 80 \times 10^{-6} e^{-t} dt - 4$$

$$\therefore v_1(t) = (16 e^{-t} - 20) \text{ Volt}$$

Handwritten note: $i_C + i_R = 20$

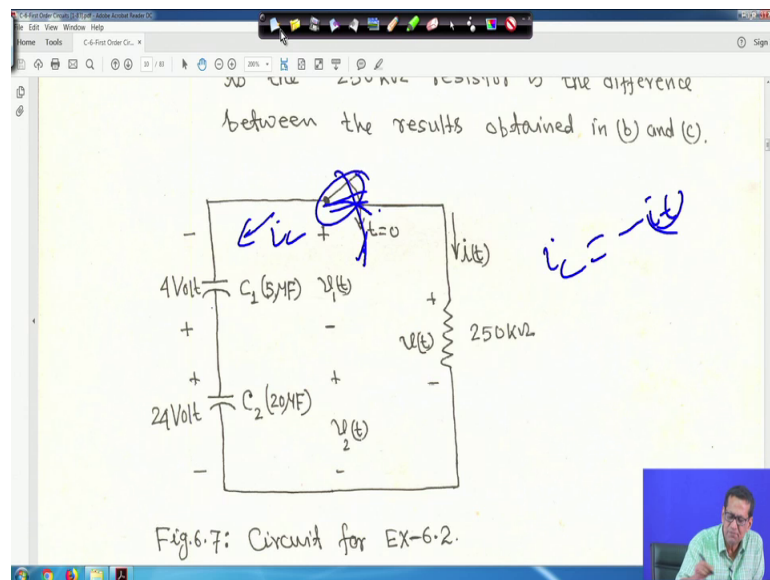
So, by definition we can calculate the expression for $v_1(t)$ and $v_2(t)$. So, this we know $v_1(t)$ is equal to look at the polarity of v_1 and v_2 right everything. So, just a just we have we

have made it know your what you call that i_C plus i_R is equal to 0 something we have given no. So, our i_C plus here what you call that your i_t instead of i_R you can make it is i_t right.

So that means, i_C is equal to your, this is the general thing right. So, you have to find out v_1 and v_2 right and this current i_C is showing to both to your for C_1 and C_2 both are in series so, both C_1 and C_2 . So, what you have to do is, just we have to see just you have to see that we write the equation right in i_t is given. So, this is your just hold on, let me let me clear it let me clear it. So, this if you come to this circuit equivalent circuit, this is your i_t this is your i_t and this is suppose this is my i_C right somewhere if you put the way you solve that i_C plus then i_t is equal to 0 right or my i_C is equal to minus i_t right.

So, we know i_C is equal to c into dv by dt in general, this is your what you call this is your equivalent circuit, but if you if you come to your this circuit original circuit right original circuit, when c is closed when this switch is closed switch is closed this is closed.

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So, this is the i_t and say this is current is flowing i_c . So, here also i_C is equal to i_C is equal to then minus it because i_C plus i_t is equal to 0, this is closed apply case some point at KCL right. So, i_C is equal to minus i_t right so; that means, i_C is equal to one is

your what you call that your C 1 dv by dt another will be dv 1 by dt another will be C 2 dv by separately we have to do it.

(Refer Slide Time: 32:17)

By definition, we can calculate the expressions for (12) $v_1(t)$ and $v_2(t)$:

$$v_1(t) = -\frac{1}{C_1} \int_0^t 80 \times 10^{-6} e^{-t} dt - 4$$

$$\therefore v_1(t) = -\frac{10^6}{5} \int_0^t 80 \times 10^{-6} e^{-t} dt - 4$$

$$\therefore v_1(t) = (16 e^{-t} - 20) \text{ Volt}$$

$$v_2(t) = -\frac{10^6}{20} \int_0^t 80 \times 10^{-6} e^{-t} dt + 24$$

So, if you if you come to now this expression, this equation if you come to this equation we are writing now after you just do the integration. So, it will be minus 1 upon C 1 0 to t this i t expression is there, this i t expression is there when we have got this expression 80 e to the power minus t micro ampere right and 0 to t. And this is we are making it minus 4, that is initial condition v c 0 is given 4 volt, but here we are putting minus 4. So, why we are writing minus 4, just to this is a this is your what you call across your a problem to your you find out that why you are doing it, try to understand try while doing it right. Instead of plus 4 here it is minus 4 right just try to understand this. So, this is a problem to you right.

And otherwise if you cannot do it answer in that forum right do not worry, but first you think. Then similarly v after simplifying this v 1 t is equal to 16 e to the power minus t minus 20 volt, similarly v 2 t is equal to here same thing that i t is there and your whatever capacitors value was there everything is given there right everything is ok. But, here we will putting that initial value v 2 0 24 volt here it is plus sign right do not minus sign look at the polarity there and just think right why we have made it like this. So; that means, if we do it v 2 t is equal to 4 e to the power minus t plus 20 volt right we also can obtain v 2 t by using KVL.

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The screenshot shows a digital whiteboard with the following handwritten text:

$$\therefore v_2(t) = (4e^{-t} + 20) \text{ Volt.}$$

We also can obtain $v_2(t)$ using KVL,

$$v(t) = v_1(t) + v_2(t)$$
$$\therefore v_2(t) = v(t) - v_1(t)$$
$$\therefore v_2(t) = 20e^{-t} - 16e^{-t} + 20$$
$$\therefore v_2(t) = (4e^{-t} + 20) \text{ Volt.}$$

If you apply KVL $v(t)$ is equal to $v_1(t)$ plus $v_2(t)$ or $v_2(t)$ is equal to $v(t)$ minus $v_1(t)$, $v_2(t)$ you got $20e^{-t}$ to the power minus t therefore, $v_1(t)$ minus $16e^{-t}$ to the power minus t plus 20 . So, if you simplify you will get the same thing $v_2(t)$ is equal to $4e^{-t}$ to the power minus t plus 20 volt right.

(Refer Slide Time: 34:08)

The screenshot shows a digital whiteboard with the following handwritten text:

$$\therefore v_2(t) = 20e^{-t} - 16e^{-t} + 20$$
$$\therefore v_2(t) = (4e^{-t} + 20) \text{ Volt.}$$

(b) The initial energy stored in C_1 ,

$$w_{C_1}(0) = \frac{1}{2} C_1 v_{C_1}^2(0) = \frac{1}{2} \times 5 \times 10^{-6} \times (4)^2 = 40 \mu\text{J}$$
$$w_{C_2}(0) = \frac{1}{2} C_2 v_{C_2}^2(0) = \frac{1}{2} \times 20 \times 10^{-6} \times (24)^2 = 5760 \mu\text{J}$$

Now b part the initial energy stored in C_1 . So, this question is given there where it is minus 4 and here it is plus 24. Initial voltage 4 volt here 24 volt, but this is a question to you right. So, that second part is initial energy stored in C_1 it is $w_{C_1}(0)$ is equal to half

$C_1 v_1^2 = 10$ square, you put all the values you will get 40 micro joule. Similarly for $w_2 = 20$ if you make half $C_2 v_2^2 = 20$ square substitute all the values right you will get your 5760 micro joule right.

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The total energy stored in the two capacitors is (13)

$$w_c = 40 + 5760 = 5800 \mu\text{J}$$

(c) As $t \rightarrow \infty$

$$v_1 = v_1(\infty) = -20 \text{ Volt}$$

$$v_2 = v_2(\infty) = 20 \text{ Volt}$$

Now, total energy stored in the 2 capacitor you add 40 micro joule plus 5760 micro joule so, 5800 micro joule right. Now, v_1 as t tends to infinity, v_1 is equal to v_1 infinity will get minus 20 volt as t tends to infinity right and v_2 is equal to v_2 infinity you will get 20 volt right. So, all v_1 , v_2 expressions are there just see that t tends to infinity how much it is therefore, the energy stored in the 2 capacitors is that is w_c is equal to half, that is your $5 + 20$ into 10 to the power minus 6 into 400 ; 400 means this 20 square right.

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Therefore the energy stored in the two capacitors is

$$w_c(\infty) = \frac{1}{2} (5+20) \times 10^{-6} \times (400)$$

$$w_c(\infty) = \frac{1}{2} (5+20) \times 10^{-6} \times (400) = 5000 \mu\text{J}$$

(d) The total energy delivered to the 250 k Ω resistor is

$$w_R(\infty) = \int_0^{\infty} p(t) dt = \int_0^{\infty} 20e^{-t} \times 80e^{-t} dt \quad \mu\text{J}$$

So, minus 20 square is 400 plus 20 square is also 400 right. So, and it is your half C 1 plus C 2 that is 25 micro farad actually, that is 25 into 10 to the power minus 6 400 so, 5000 micro joule but here it is 5800 micro joule. So, there is a difference of this 2 is your how much 800 micro joule.

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$$\therefore w_R(\infty) = \int_0^{\infty} 1600 e^{-2t} dt \quad \mu\text{J} = 800 \mu\text{J}$$

Comparing the results obtained in (b) and (c) shows that

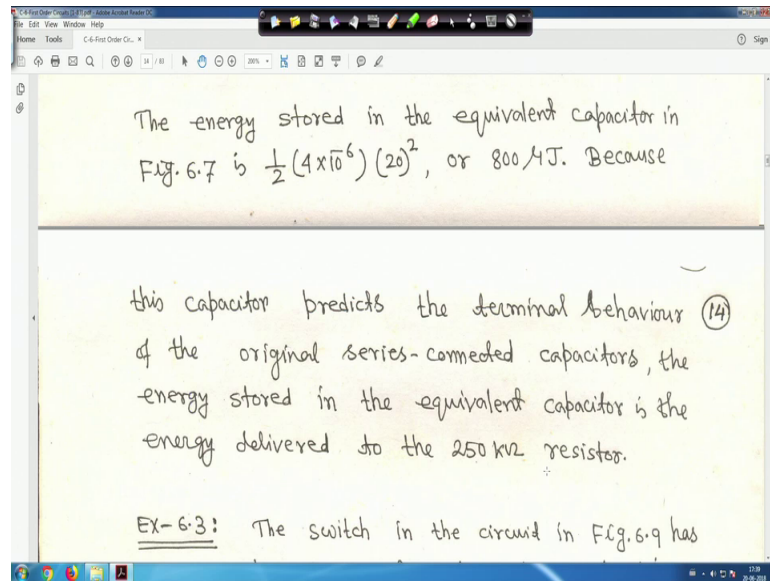
$$800 \mu\text{J} = (5800 - 5000) \mu\text{J}$$

The energy stored in the equivalent capacitor in Fig. 6.7 is $\frac{1}{2} (4 \times 10^{-6}) (20)^2$, or 800 μJ . Because

So, the total energy delivered to that 250 kilo ohm resistor is w r infinity is equal to 0 to infinity p t dt. That is 0 to infinity to just p t power is equal to v into i, v is this much i is equal to this much this is micro joule right.

Therefore, 0 to infinity 1600 it is 10 to the power minus 2 t micro joule 800 micro joule right; that means, comparing the result obtained in b and c shows, 800 micro joule is equal to 5800 minus 5000 joule is equal to 800 micro joule. This energy stored in the equivalent capacitor is half equivalent was 4 microfarad half v this square it is 800 micro joule right. So, this 800 micro joule actually dissipated in the your resistor.

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So, because this capacitor predicts the terminal behavior of the original series connected capacitor. The energy stored in the equivalent capacitor is the energy delivered to the 250 kilo ohm resistor. So, this problem this is a good problem for you. So, just when we will get this your video just have a look that what has been done. Then if you understand all these then you will not face any difficulties of solving this kind of problem.

Thank you very much, we will be back again.