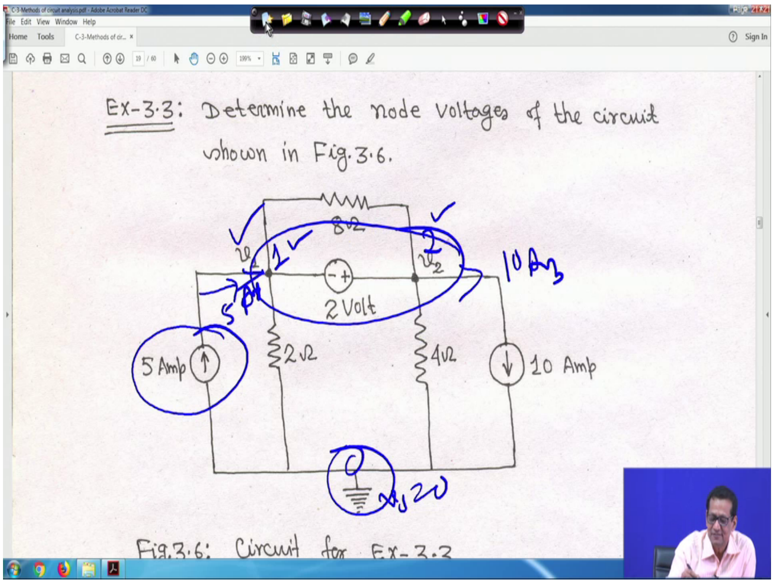
Fundamentals of Electrical Engineering
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
Methods of Circuit Analysis (Contd.)

So come to welcome again; so come to your another example of super node. Let me tell you once again that for DC circuit, everything is being explained in detail, right, as far as possible, let us try to make things you know fundamentals or fundamental should be as far as possible to clear; such that when same thing will apply for AC circuit and even in magnetic circuit or your couple circuit, right. So, you will find magnetic circuit also will be analogous to almost electrical circuit, right.

So, at that time, all these things I need not to explain because at that time for your all these concept will be clear, right. So, that is why for this circuit perhaps I was spending more time and more examples such that if varieties of examples such that it will help you a lot, right.

(Refer Slide Time: 01:09)

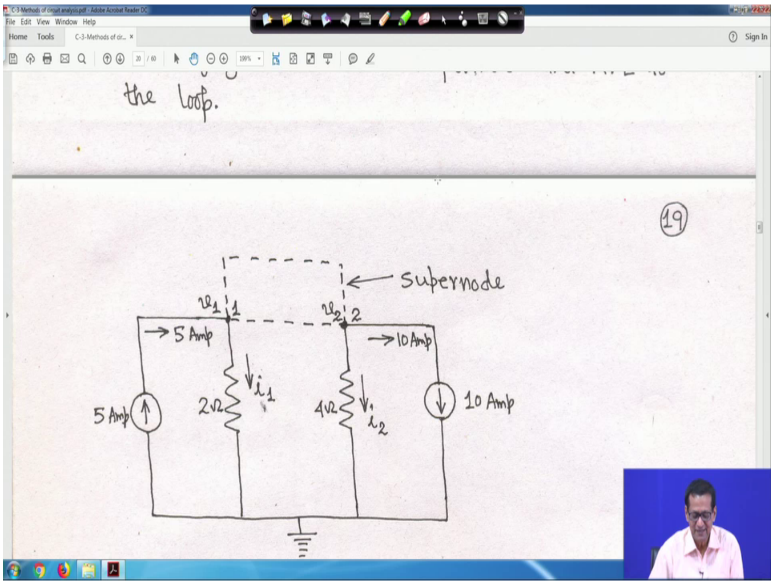


And so now, this example; your what you call this is actually taken, it is a supernode because this is your this is your reference node this row and its voltage is v 0 is equal to 0. So, in this case 1; 1 your what you call one independent current source is connected here at node 1 a 5 ampere current is actually, this 5 ampere means this current actually 5 ampere current is entering into this node, right.

Here is 5 amperes and another 10 ampere current source is there at node 2. So, this means this 10 ampere current is leaving this node 2 and this v 1 and this is node 1; this is actually node 1, this is actually node 2.; these 2 are non reference node and in v 2 in 1 voltage source is connected; that means, it is a it is forming a supernode, right. So, something like this because if a voltage source is there, it may be 1, it may be 2, does not matter; if voltage sources are there in between 2 reference non reference node, then its creates a supernode, right.

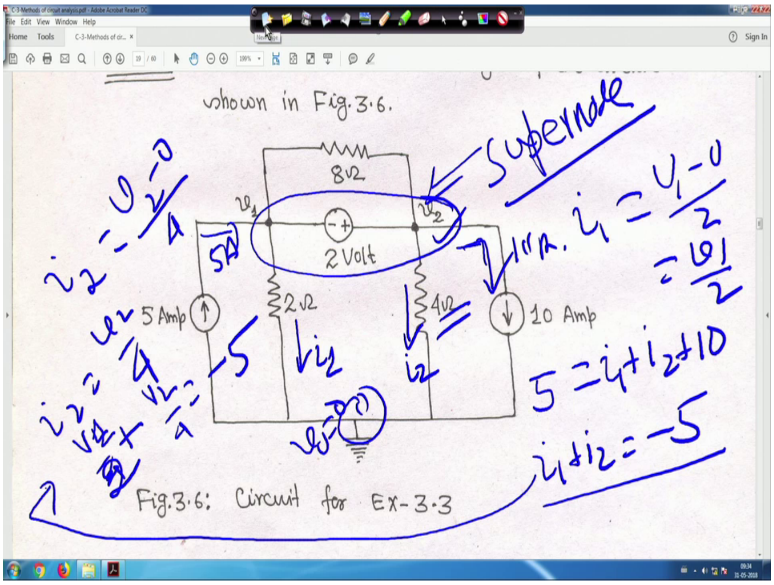
So, next move to the problem. So, you have to find out v 1 and v 2.

(Refer Slide Time: 02:19)



So, if you look here this is actually super node, right. So, voltage source is not there because we have to apply your what you call that your KCL. So, that is why I have drawn like this and this is actually supernode that which current is entering and which current is leaving, right. So, in this case, what we have done is before going to that before going to that. So, this is actually super node, right.

(Refer Slide Time: 02:42)



This is actually super node this is supernode because 2 1 voltage source is there in between 2 non reference node; so, in this case, a 5 ampere current this 5 ampere current actually entering the super node. And this 10 ampere current actually it is leaving the supernode and this is node 1, say this is also current leaving the supernode, say i 1 and this is the currents at node 2, this is the i 2, right.

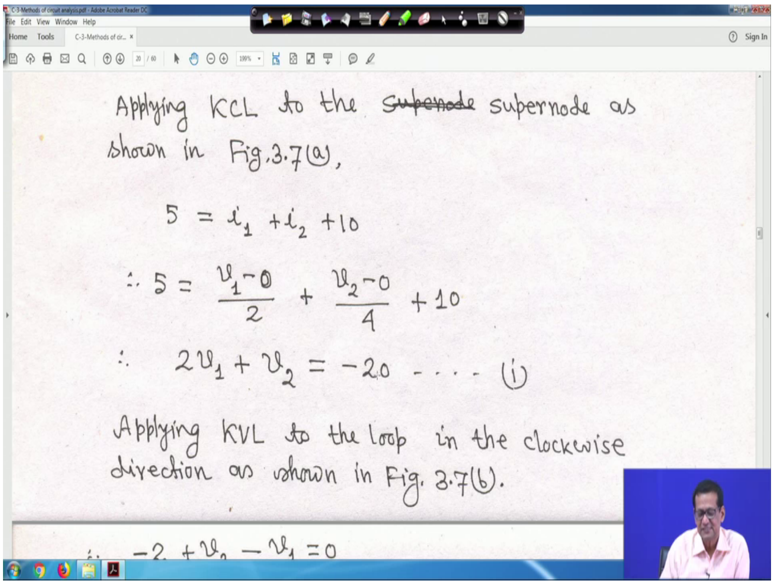
So, if you write down the equation KCL, if you apply KCL at the supernode, right, then this 5 ampere current is entering. So, 5 this current is entering to the supernode, but current i 1 i 2 and 10 ampere all are leaving the supernode. So, is equal to i 1 plus i 2 plus 10, right. So, this because this i 1 also leaving the supernode, i 2 also leaving the supernode and 10 ampere also leaving this because it is 10 ampere current source, right. So, it is leaving the supernode, right, this is arrow is showing downward and this is upward. So, it is entering therefore, your i 1 plus i 2 is equal to minus 5. So, this is what; this is your what to call that i 1 plus i 2 is equal to minus 5.

Now, what is your; what is your i 1 and what is your i 2? If you see i 1, here i 1 is equal to your basically v 1 minus 0 because this is a node that is referrence that is your reference node v 0 is 0. So, i 1 is equal to v 1 minus 0 by 2 v 1 minus 0 by 2 is equal to v 1 by 2 right that is your i 1, similarly, this i 2 also i 2 also this voltage is v 2; so, reference node.

So, voltage is 0 v 2 minus 0 by 4 right because this resistance is 4, therefore, your i 2 is equal to v 2 by 4, right; that means, that if you substitute here i 1 is equal to say here, here I am making it, I am making it, here, i 1 is equal to v 2 by 4 plus i 2 is sorry i 1 is equal to v 1 by 2, sorry, i 1 is equal to v 1 by 2 plus i 2 is equal to v 2 by 4 plus v 2 by 4 is equal to minus 5.

Accordingly, you can solve it that it will be 2 v 1 plus v 2 is equal to minus 20, right. So, let me clean it, right. So, all these things I wrote for you; so, if you look into this if you look into this that your KCL 1.

(Refer Slide Time: 05:22)

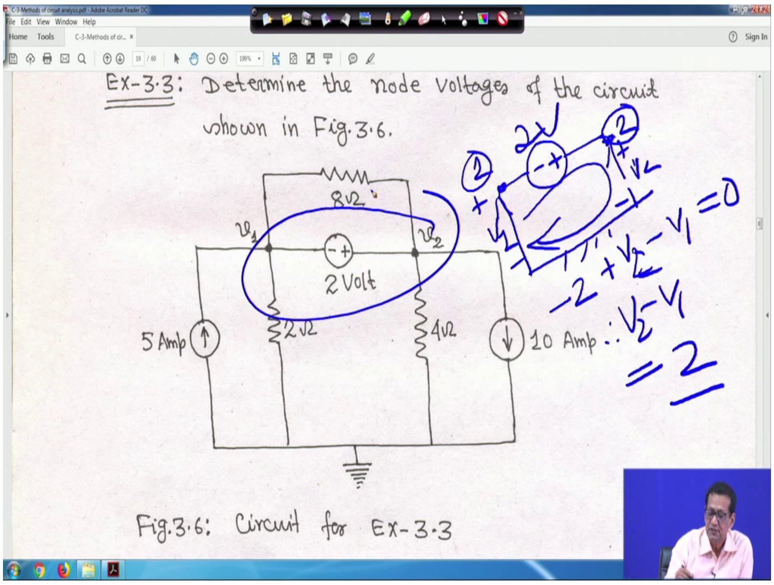


So, ultimately it is coming your 2 v 1 plus v 2 is equal to minus 20, right, all these things I wrote that 5 is equal to whatever I have mentioned there, whatever I have explain there same thing is written here.

So, if you simplify this it will be 2 v 1 plus v 2 is equal to minus 20, this is equation 1, this is for your what you call this is for your KCL; when you are applying KCL to the supernode when you are applying KCL to the supernode, right, this is 1 equation, now let me clear it second one is.

This better you come to this circuit second one is that you have to apply KVL also in the because of supernode that KVL is also.

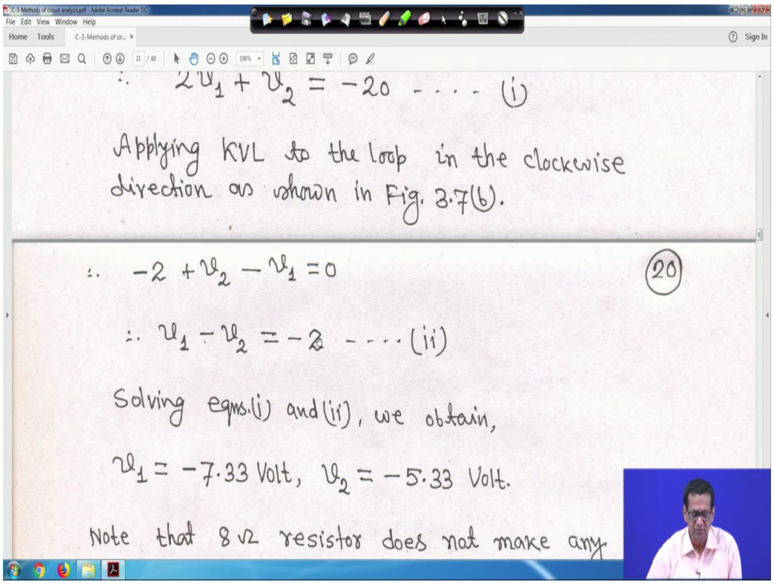
(Refer Slide Time: 06:20)



So, this is node 1 these I am making it I am making it for you, this is your node 1 and this is your node 2, right. So, hear you have the I told you that with respect to that reference nodes. So, this voltage; this voltage actually your what to call v 1 at node 1, this is your ground this is your ground whatever it is right and this voltage is your v 2, right, this is your v 2. So, this is plus in between that you have minus plus this voltage source is there right and this is your 2 volt this is your 2 volt, right and this is node 1 and this is node 2. So, here if you apply clockwise you take that is your KVL.

Right, then it will be minus 2 then plus this v 2, then, here it is here it is minus right, I told you everything then minus v 2 minus v 1 is equal to 0 because it is encountering minus terminal pass when you are moving like this. So, minus 2, then encountering plus terminal pass plus v 2 in encountering minus terminal pass. So, minus is equal to 0; that means, your v 2 minus v 1 is equal to 2, right, this is the constant equation coming because of this supernode, right.

(Refer Slide Time: 07:46)

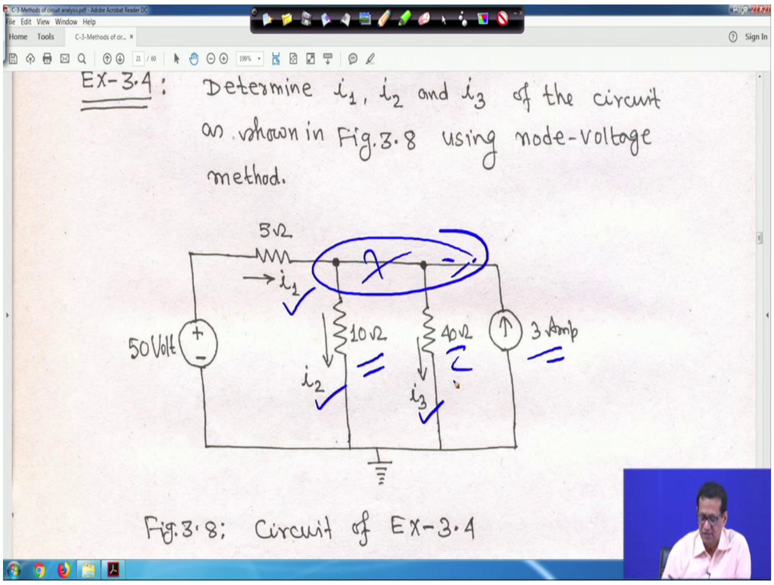


So, let me clear it. So, this is your v 1 minus v 2 is equal to minus 2 or v 2 minus v 1 is equal to 2, right. So, solve this 2 equation that equation 1 and equation 2, you solve it, then you will get that v 1 is equal to minus 7.33 volt and v 2 is equal to minus 5.33 volt note that 8; 8 ohm resistor; that means, this one you note that 8 ohm resistor does not make any difference because it is connected across the supernode.

So, this is interesting just go let us go back to the circuit, right. So, if you come to that this circuit that 1 8 ohm resistor is connected across the your what to call that supernode because this is actually this is actually this is actually your super node, right; so, 8 ohm resistors actually not making any difference of the voltage v 1, v 2, whatsoever, right.

So, in; that means, it has no effect on this your what to call in the supernode, right. So, that is why it has it is not making any impact. So, anyway; so, this is your what you call that one example of your supernode, right just hold on.

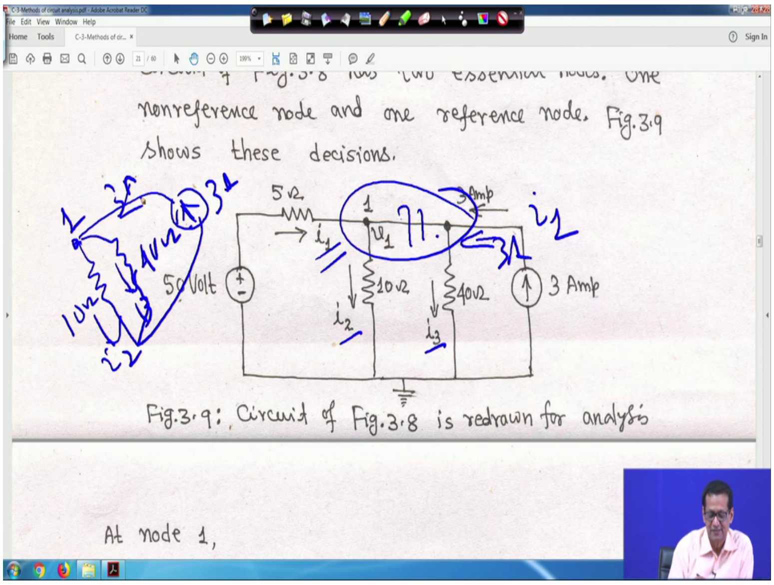
(Refer Slide Time: 09:25)



So, next one is next one is your example your 3.4. So, determine i 1, i 2 and i 3 of the circuit as shown in figure 3 by using node voltage method, right. So, you have to find out i 1 also i 3, but just let me tell you there is no element here. So, basically this is a this is a common node because no electrical element is here or here, no electrical element is here, it is a common node; that means, 10 ohm, 40 ohm and all 3 ampere, they all these all these resistors as well as the independent current source all actually connected in parallel, right.

So, 10 ohm 40 ohm because nothing when electrical element is there you have to find out i 1, i 2 and i 3, right. So, in that case; so, you have to solve this one very simple, it is. So, an 50 volt source is there this side 3 ampere.

(Refer Slide Time: 10:21)



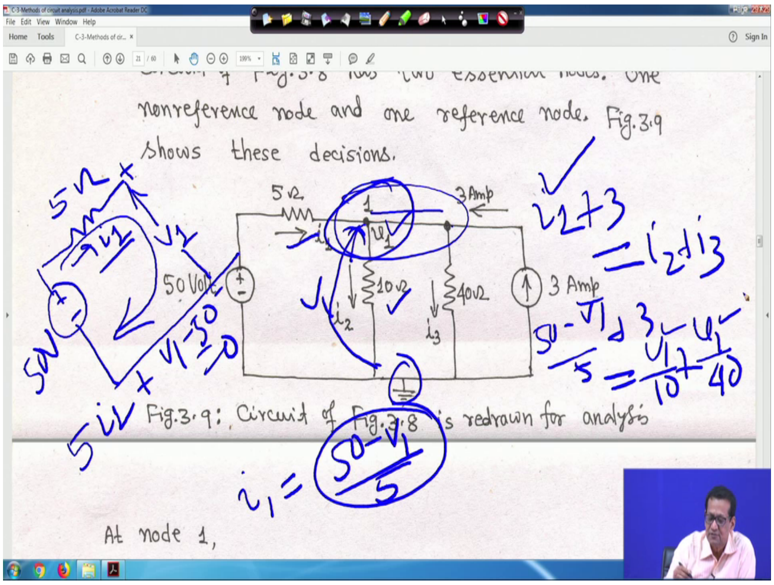
So, just for the purpose of this thing, this circuit is redrawn and voltage actually, this I told you this is actually a single node. So, basically this one because no electrical element is here no electrical element is so, basically it say node 1, right only v 1 and this side is plus minus 50 volt and this is your i 1, this current, we have taken i 2 and this current is i 3 and this is 3 ampere.

So, how to solve it? Right, how to solve it? So, first thing is that you have to find out that your what to call that what is the what is the expression of i 1 because you have to apply KCL at node 1. So, first you have to find out what is i 1. So, if you if you apply your what to call KCL at node 1, then it is i 1. The current i 1 is entering right and then this 3 ampere because it is a it is a it is a it is a single node, right.

So, this 3 ampere current also it entering, it is entering right this 3 ampere current is also entering because it is a single node, I am not writing this, I mean if you make it like these it will be something like these. So, this is the node this is your voltage source at 10 ohm, then across here also another 40 ohm is connected, right and here also same thing a current source is also connected a current source it is also 3 ampere. So, this is actually node 1 this current is your i 2, this current is your i 3 and this 3 ampere current is this thing, right. So, that is the that is the thing; that means is a single node, right, it is single node.

So, let me let me clear it, then again I will do it for you, right. So, in this case; that means, if you apply your KCL here.

(Refer Slide Time: 12:16)



Then it will be i 1, this current is entering at node 1, this is 3 ampere current, I mean this is the same thing this is same thing this 3 ampere current, I told you that all are connected at same point, then i 1 plus 3, these 2 are your this current are entering into the node 1 and i 2 and i 3 leaving.

So, is equal to i 2 plus i 3, right, now question is how to find out i q. So, just for finding out the i 1, I told you previous 2 example also take this your what you call this separately, I making it for you just for your understanding this is plus minus right, this is 50 volt, right and you have 5 ohm resistance, this 5 ohm resistance current is your i 1.

Right and this voltage is v 1 means. So, with respect to this reference this your reference; that means, v 1 actually v 1 actually 10 into i 2, right, this v 1 actually 10 into, but we will not write this, we will make it that this is your make it plus right I told you always the reference and non reference, you should take plus this is your v 1 voltage and this is your ground right circuit is like this. So, this is v 1, right. So, v 1 is equal to 10 into i 2, but we are taking directly, this v 1 like this, right. So, if you make, then now you what you do you apply KVL, right.

So, in this case, what will happen it will be 5 into i 1 this 5 into i 1 plus your v 1, then encountering minus terminal first. So, minus 50 is equal to 0, right; that means, I am writing here; that means, your i 1 will be is equal to 50 minus v 1.

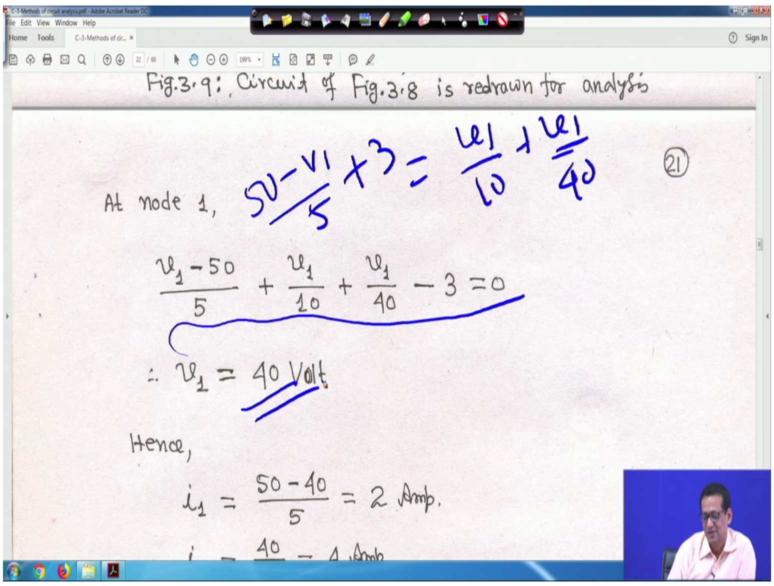
Divided by 5, this would be your i 1, right. So, this i 1, you have to substitute here 50 later I have given, but just for your understanding trying to write each and everything. So, i 1 will be 50 minus v 1 by 5 look so many parameters are there. So, many register values, then current source voltage source while I making write writing like this there is every possibility I can overlook or I can make some error also particular numerical value.

If it is anything is there, you just when you will go through this video, you please inform me sir, there it is a there is a error, but hope trying to look into every trying to look into each and every parameter there writing, but still there may be a probability. So, this is your i 1. So, here you put i 1, then that; that means, your 50 minus v 1 by 5 plus 3 is equal to i 2. So, i 2 is equal to your v 1 by 10 because this is v 1 voltage and this is 10 ohm resistance.

So, i 2 is equal to actually v 1 by 10 because reference voltage is 0 basically v 1 minus 0 by 10 that is v 1 by 10, right and similarly i 3 is equal to i 3 also it is same node 1, it is I told you it is node 1 i 3 is equal to also v 1 minus 0 v 1 divided by 40, right. So, this is your what you call this is your what to call KCL equation, then you replace it by v 1 and all these things.

So,; so, if this all equation this is this is v 1, this is v 1, this is v 1, you can easily solve for voltage your v 1, right. So, now, whatever I have written will go to that; right.

(Refer Slide Time: 15:47)

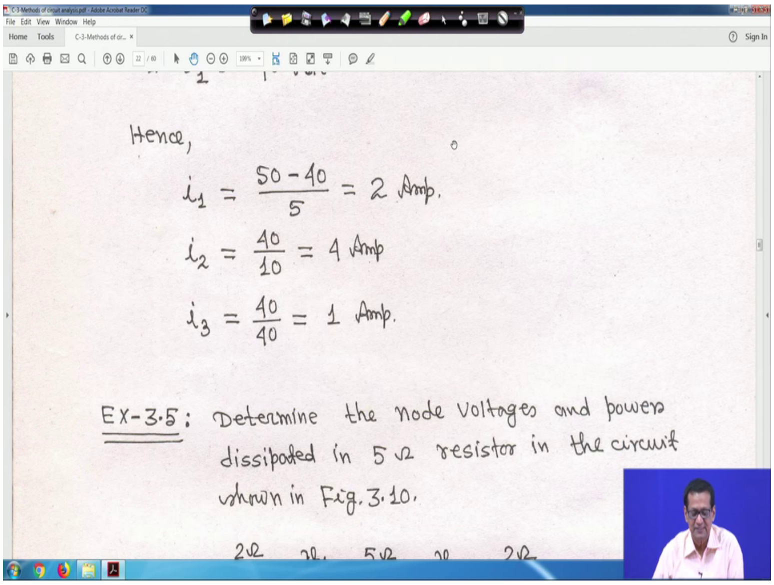


So, so, if you look into that that same equation whatever just I said that I am putting in this form whatever I written you just put it and you will get in the same form right just for the purpose of your explanation I explanation I wrote everything.

But you can write it because there it was writing 50 minus v 1, if you bring all this things to the left hand side like this, it will equation will be like this, right because your what you call there are what I wrote this v 1 by 10 plus v 1 by 41st equation, what I wrote actually to add 50 minus v 1 by 5 plus 3 is equal to I wrote v 1 upon 10 plus v 1 upon 40, right.

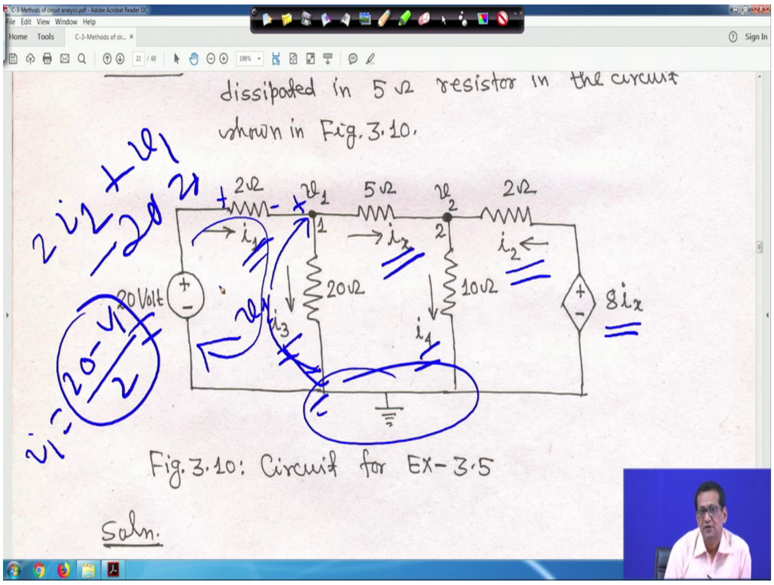
If you bring it to everything to 1 side say, right hand side, it will becoming like this same thing, right and solving this you will get v 1 is equal to 40 volt, right.

(Refer Slide Time: 16:44)



So, therefore, therefore, i 1, i 2, i 3 easily, you can calculate i 1 is equal to 50 minus your what v 1 by 5. So, these 2 ampere i 2 is equal to again v 1 by 10. So, 40 by 10; 4 amperes; so, i 3 is equal to 40 by 10; 1 ampere right. So, this is your so, this is your answer, right slowly and slowly we will take difficult problem.

(Refer Slide Time: 17:11)



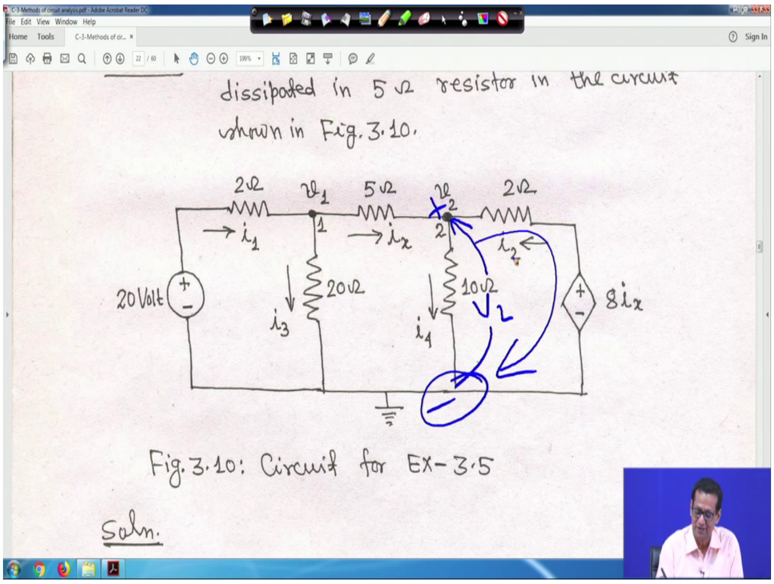
So, here also determine the node voltages and the power dissipated in 5 ohms resistor in the circuit shown in figure 3.10. So, this is the circuit and all the all the theory what you call current directions are marked, this is your i 1 this is i 1, right and this is your what you call this is your i 2, this is i 2 actually it is a its a DP or dependent voltage source, it is given it is 8 i x and this is actually i x and this is a dependent voltage source. So, it is 8 i x and this current is i 3 and this current is i 4. So, before proceeding further how to obtain i 1 here, here is the same thing here also same thing this is your plus this voltage actually v 1, right, this voltage is v 1; you make it.

Connection here this is your same thing it is a reference node, right. So, in this case also you have to apply KCLs at node 1, you have to obtain i 1 in terms of v 1 and this 20 volt, right. So, in that case anyway your v 1 is equal to actually 20 into i 3 that is your ohms law, but here we want in terms of your voltage. So, if you apply if you apply know in this loop whatever I told previously 2-3 cases, you apply KVL here.

Then in this case what you will get you will get 2 i 1 because 2 into i 1 plus this plus it encountering plus terminal first right here also if you mark, right, it is plus minus. So, 2 into i 1 understandable right 2 into i 1 plus v 1, then minus 20 equal to 0 because it is encountering that minus terminal first. So, this is equal to 0; that means, your i 1 is equal to 20 minus your v 1 divided by 2, this is the equation for i 1 this way you have to obtain.

Otherwise you cannot solve, right. So, anyway let me clear it, right. So, all this things are marked. So, only thing is that that you have to apply your at node 1 and node 2, you have to apply your KCL right and after that you make it. So, similarly your another thing is that this is 8 volt and here also here also before going to that after that directly I will tell you, right.

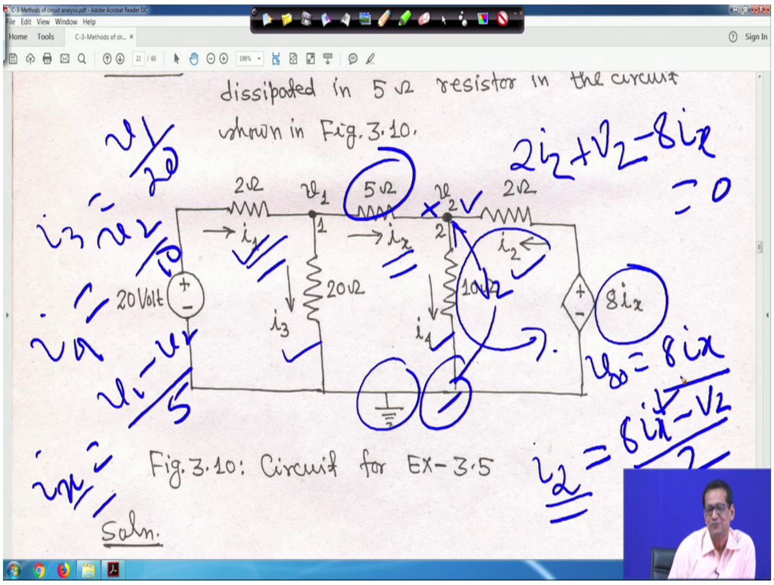
(Refer Slide Time: 19:32)



So, this is plus this voltage is v 2 and this is ground means it is minus this is ground minus, right. So, in this case, it is also if you say we move like this, if you move like this say clockwise direction, right, if you move like this or anticlockwise direction, the way you want, if you move like this just hold on I for your for simplicity, I will make things in the I will take the loop in the direction of this your what you call this current.

So, let me let me let me; sorry. So, take like this, then things will be easier for a right and these voltage these voltage is v 2, right.

(Refer Slide Time: 20:12)



So, this is minus and this is plus right. So, if you do. So, you are moving it is it is it is taken anticlockwise meaning is same right ultimate the result will remain same. So, it will be 2 into i 2 that 2 into i 2 right otherwise clockwise if you take sign has to be change thats all 2 into i 2 plus this v v 2 it is encountering plus terminal first.

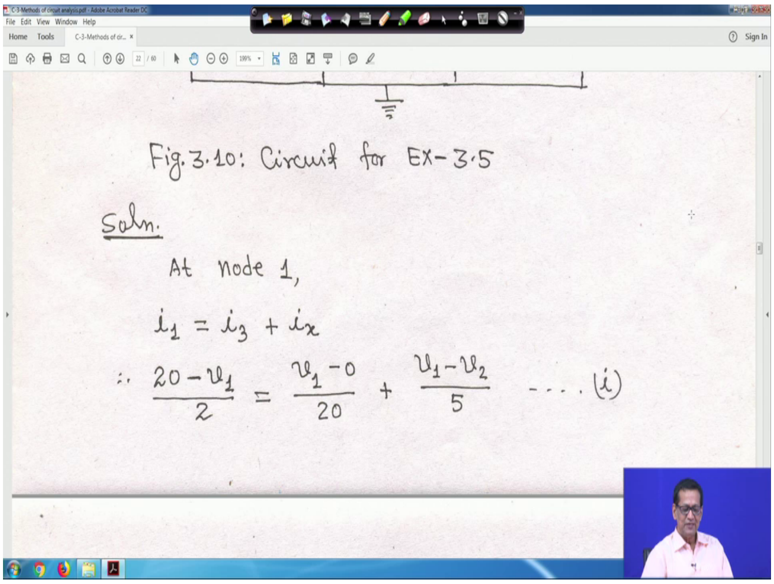
So, it is plus v 2 plus v 2 right and then your it is encountering minus terminal, first, it is it is a it is a what you call it is a dependent voltage source, right, basically is a dependent voltage source. So, if you make it v 0 0 is equal to say it 8 i x something like this is a dependent voltage source not current, right, there should not be any confusion.

So, it will be then minus your 8 i x is equal to 0 because encountering minus terminal first. So, 2 i 2 right 2 i 2 plus your this v 2 because it is node 2 v 2 minus 8 i x is equal to 0; that means, your i 2 will be is equal to right 8 into i x minus v 2 divided by 2, right.

So, in this case, what you can do is now next you have to find out what i i 1 also I told you that how to find out, i 2 also like this, then your i 3, this is this is actually reference load reference load voltage is directly, you can write i 3 is equal to v 1 by 20, right this is your i 3. Similarly, i 4 is equal to your v 2 by 10 v 2 by 10, right; that is you i 4. So, all this things i 1 then i 1, i 2, i 3, i 4, all how to get it in terms of v 1 and v 2 all this things are explained.

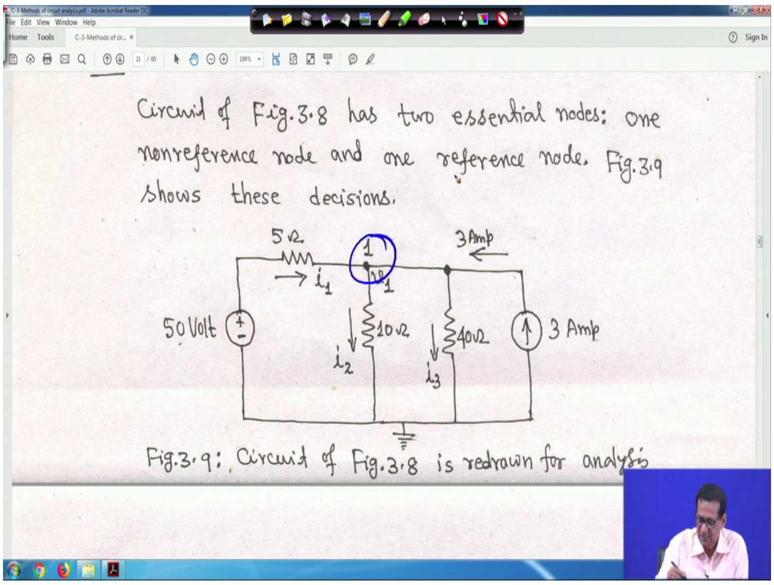
So, next is that you are what you call and I or what to call and i x this i x i x is equal your current moving from node 1 to 2. So, i x is equal to v 1 minus v 2 by 5 because this 5 ohm resistance is here right. So, so v 1 so, this i x has to be substitute here and you have to simplify it. So, rest it is given there. So, after that I just show you that step of equation and how to solve it, right.

(Refer Slide Time: 22:40)



So, at node 1 i 1 is equal to i 3 plus i x.

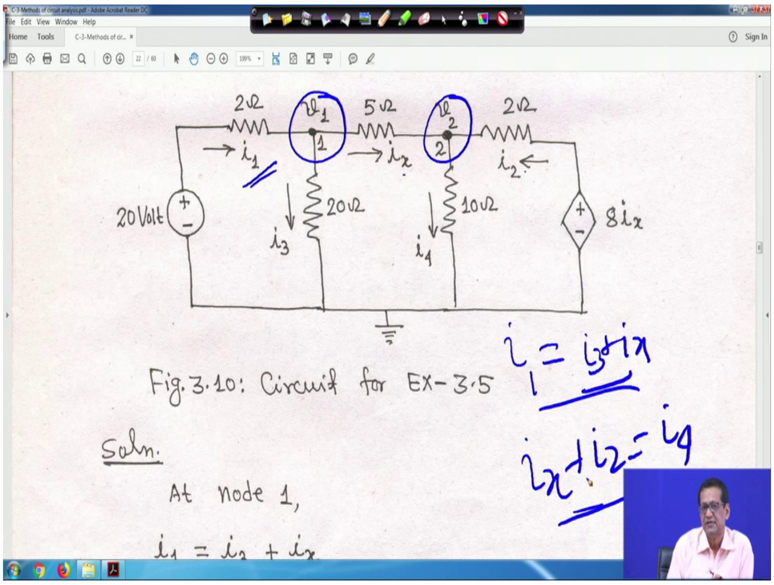
(Refer Slide Time: 22:45)



So, at node 1, if you look at node 1, this is at node 1 just your further this thing. So, at node 1 if you apply you what to call KCL here it is your i 1 is equal to your what to this thing.

Ah you have to make it this is actually node 1. So, just hold on this is just let me clear it first, right, let me clear it first. So, here, right; so, we go we went to that previous one by mistake; so, here; so, node 1 if you apply KCL here. So, it is i 1 is equal to your i 3 plus i x this is at node 1, right.

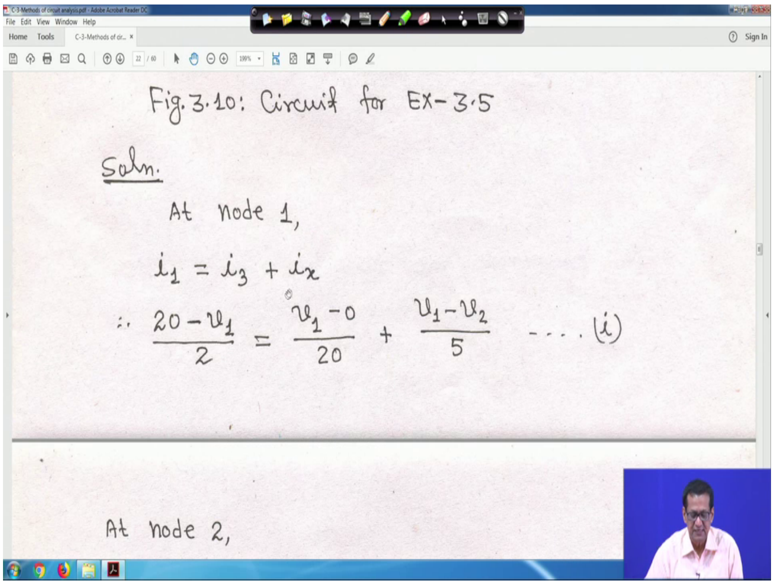
(Refer Slide Time: 23:32)



And similarly at node 2 if you apply KCL, then your because this current is incoming and these 2 are outgoing at and at node 2 your i 2; i 2 and i x entering into the node 2 and this i 4 is leaving. So, it is i x plus i 2 is equal to i 4 these 2 equations you have to write to after that you put in terms of v 1 and v 2.

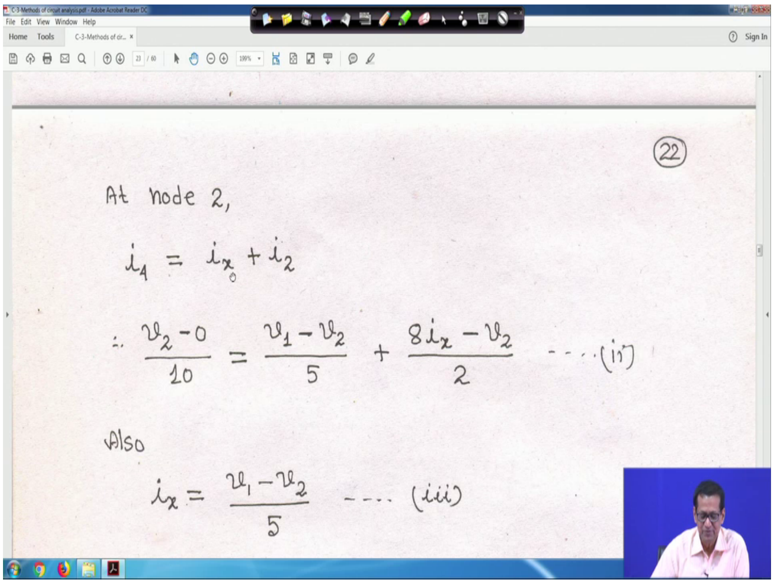
Whatever it can be explained, right; so, let me clear this.

(Refer Slide Time: 24:10)



That is why at node 1, it is i 1 is equal to i 3 plus i x; all this things have been told all this things have been told now you put it here now you put it here, right.

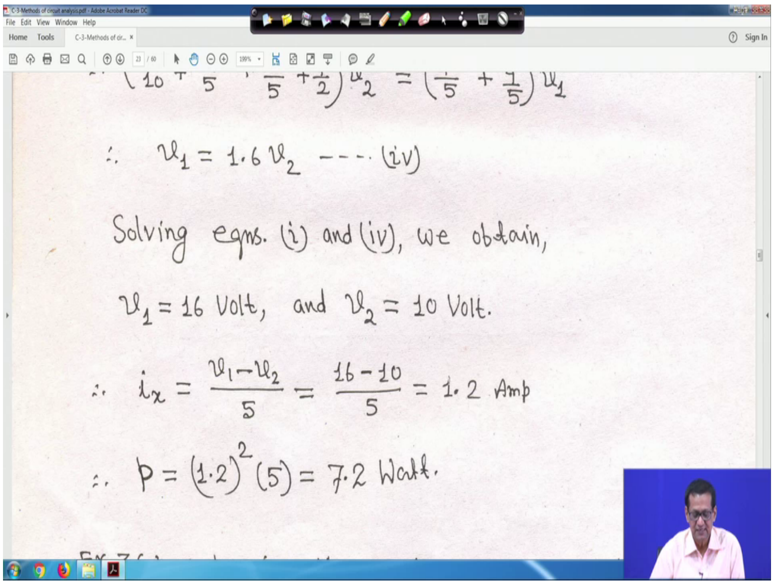
(Refer Slide Time: 24:21)



And similarly your at node 2 i 4 is equal to i x plus i 2. So, all these things have been explained and told by blue ink, right.

So, so, just you put it there and i x also I told you v 1 minus v 2 by 5 we have to put it here i x. So, from equation 2 and 3, we will get upon simplification all these things we get v 1 is equal to 1.6 v 2; this is equation 4. Now solve equation 1 and 4; this is your equation one right in terms of v 1 and v 2 solve equation 1 and 4, right.

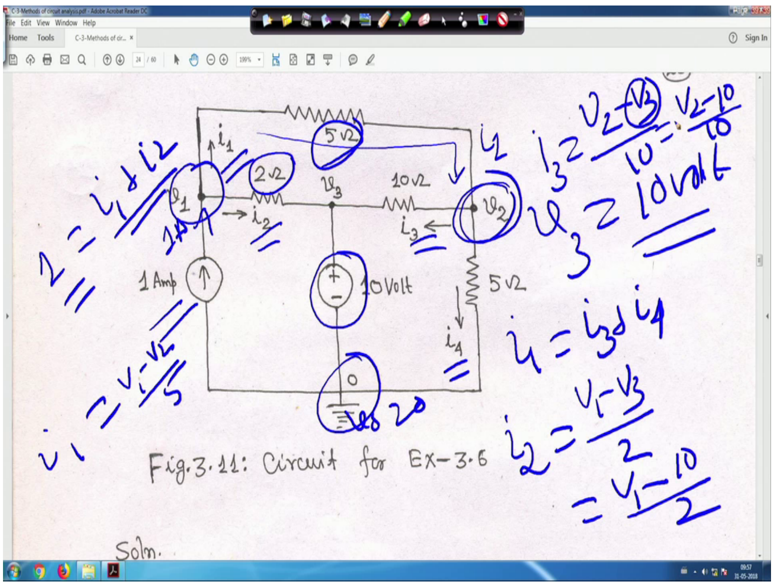
(Refer Slide Time: 24:54)



You will get your v 1 is equal to 16 volt and v 2 is equal to 10 volt, right and i x is equal to v 1 minus v 2 by 5.

From that circuit, you will get it is 1.2 ampere and power it is your one point, i square r 1.2 square into 5. So, 7.2 watt, right, this is the power i square r 7.2 watt. So, this is the answer, right, next is determine the node voltages of the circuit as shown in figure 3; 11; 3.11. So, you have to find out the node voltages of the circuit.

(Refer Slide Time: 25:29)



Look, in this case, in this case, you have one current source here for 1 ampere; that means, this current actually this current is 1 ampere this current is 1 ampere entering into the node, right and next if voltage source is there here 10 volt and this is your node 3 and this is your reference node. So, here voltage v 0 is 0, yes, again and again I told you; so, basically directly actually v 3 is equal to 10 volt, right; so, this actually 10 volt right.

So, v 3 is given, I mean this is that v 3 will be directly you can write 10 volt another is that v 2 right and these are the current. So, these are the current i 2 i 1 and your i 3 and i 4, right. So, you have to apply your; what you call that your if you apply KCLs at node 1 if you apply KCL; 1 ampere current is increasing. So, yeah sorry entering 1 is equal to i 1 and i 2 leaving the node.

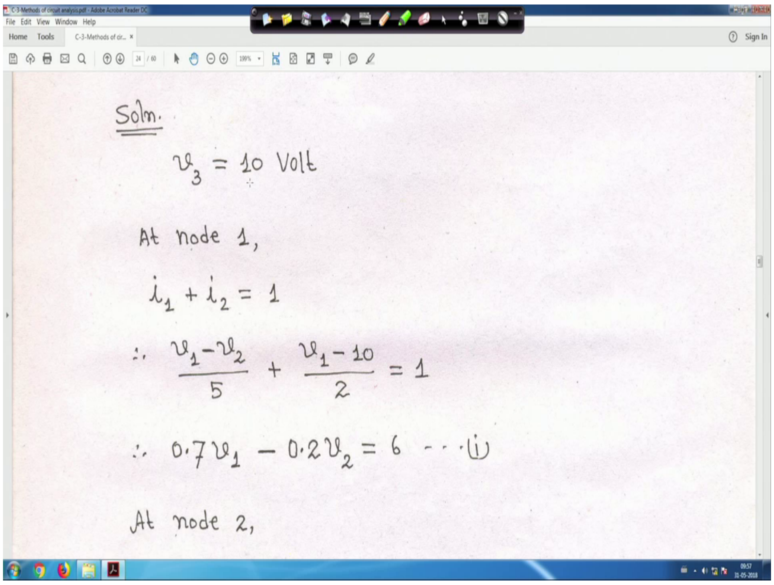
So, it will be i 1 plus i 2. So, i 1 plus i 2 is equal to 1; this is one equation will come, right, similarly, if you apply here that your this node that KCL, then i 1 current this i 1 current is flowing like this flowing like this flowing like this. So, this is your i 1; i 1 current is entering right into this into node 2 and i 3 and i 4 are leaving. So, i 1 is equal 2 i 3 plus i 4, right, this is your what you call.

And next is you have to find out what is your i 2; i 2 is if you look into your first i 1 i 1 is equal to this v 1 minus v 2 upon 5, right. So, it is v 1 minus v 2 upon 5, this is i 1 right and next one is your what you call i 2; i 2 will be is equal to here I am writing i 2 is equal to here, it is 2 ohm. So, it is v 1 minus v 3 upon 2 v 1 minus v 3 upon 2 that is actually v 1 minus v 3 is equal to 10 volt v 3 is equal to 10 volt minus 3 upon 2, right.

So, i 2 is equal to v 1 minus 10 upon 2, then i 3 i 3 is equal to it is flowing from 2 to 3. So, i 3 is equal to here to write here i 3 is equal to v 2 minus v 3 by 10. So, v 3 actually 10 volt; so, basically it will become v 2 minus 10 by 10, right.

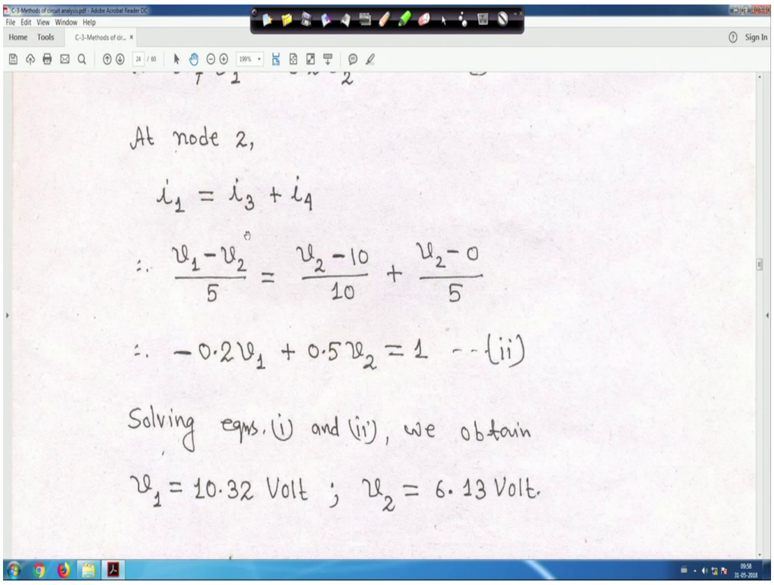
So, this will be your i 3. So, if you I mean these are all the equations, now go to the solution, all these things are explained, now go to the solution right whatever I said.

(Refer Slide Time: 28:15)



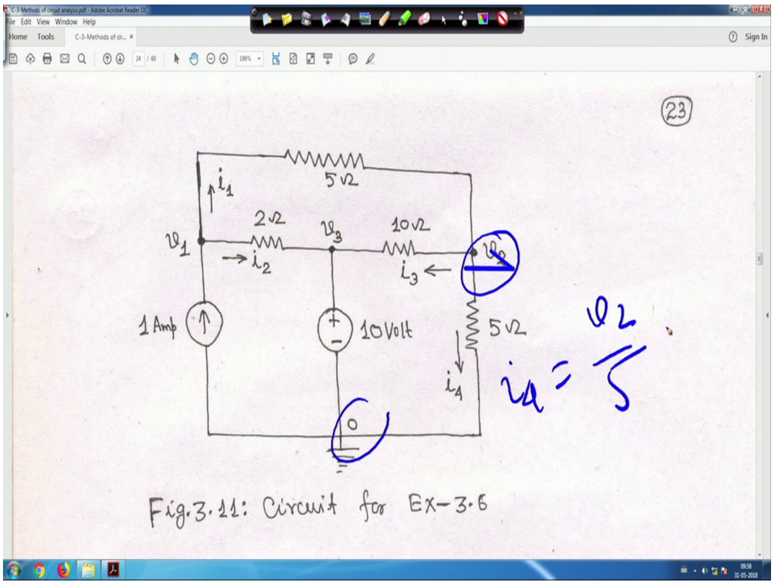
Now, v 3 is equal to 10 volt at node 1; i 1 plus i 2 is equal to 1, I told you substitute i 1, i 2; this i 2 also v 1 minus 10 by 2 I told you. So, you will get one equation in terms of here point seven v 1 minus point 2 v 2 is equal to 6.

(Refer Slide Time: 28:31)



This is equation 1 at node 2 also I told you i 1 is equal to i 3 plus I 4. So, i 1 is v 1 minus v 2 by 5 is equal to v 2 minus 10 by 10, I told you plus i 4 is equal to v 2 by 5, right.

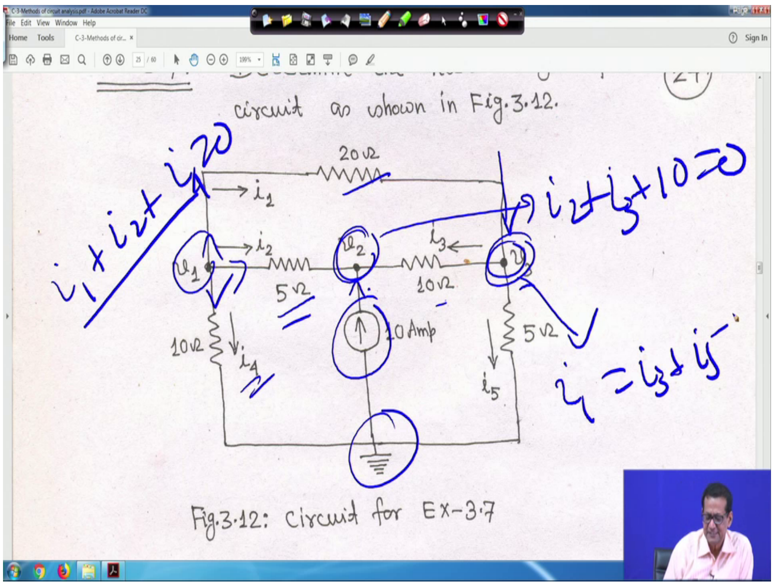
(Refer Slide Time: 28:45)



So, this is your this is your i 4. So, i this is your this is your sorry this is your v 2 this is your v 2 and this is reference node. So, i 4 is equal to v 2 by 5, right.

So, if you; that means, if you make it will minus point 2 v 1 plus minus 0.5 v 2 is equal to 1. So, solving equation 1 and 2, you will get v 1 is equal to 10.32 volt and v 2 is equal to 6.13 volt, right.

(Refer Slide Time: 29:19)



Now, next one is determine the node voltage of the circuit as shown in figure your 12 that 3.12 as shown in figure 12, right.

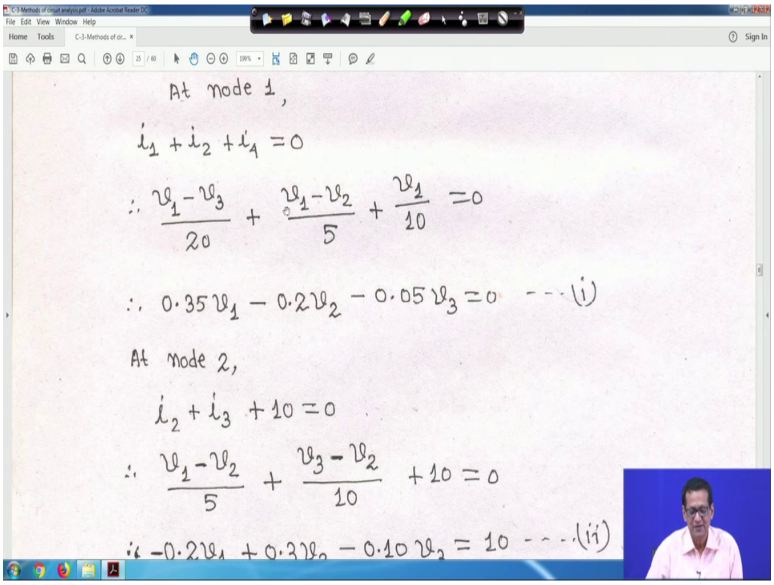
So, in this circuit in this circuit, you have a you have a current source, here you have a current sources of 10 ampere, right and this is this is v 1 this is v 2 and this is v 3, right. So, in this case also, you apply your at node 1 you apply your KCL right and similarly, i 1 is equal to v 1 minus v 3 by 20, you can find out i 2 is equal to v 1 minus v 2 by 5, you can do it, right, similarly i 3 moving for this direction to this direction. So, it will be v 3 minus v 2 by 10 that you can easily find.

Similarly, your i 5 will be v 3 by 5 right because this is reference node voltage is 0 i 5 will be v 3 by 5. Similarly, i 4 will be v 1 upon 10, easily, you can find out and this 10 ampere current source is entering. So, if you apply KCL at this point it will be your what you call this current is also leaving this current is also leaving this current is also leaving the node.

So, basically if you apply the KCL then i 1 plus i 2 plus i 4 is equal to 0, this is at node 1 and all i 1, i 2, i 3, easily, you can know in terms of v 1, v 2, v 3, you can substitute, similarly, at node 2, if you apply node 2, if you apply KCL. So, this all these 3 currents i 2, i 3 and 10 all are entering into the node; that means, here also i 2, there also leaving it was 0 or entering also it will be 0. So, some of the that thing are 0. So, i 2 is entering i 3 is entering and 10 is also entering. So, i 2 plus i 3 plus 10 is equal to 0 this is another KCL.

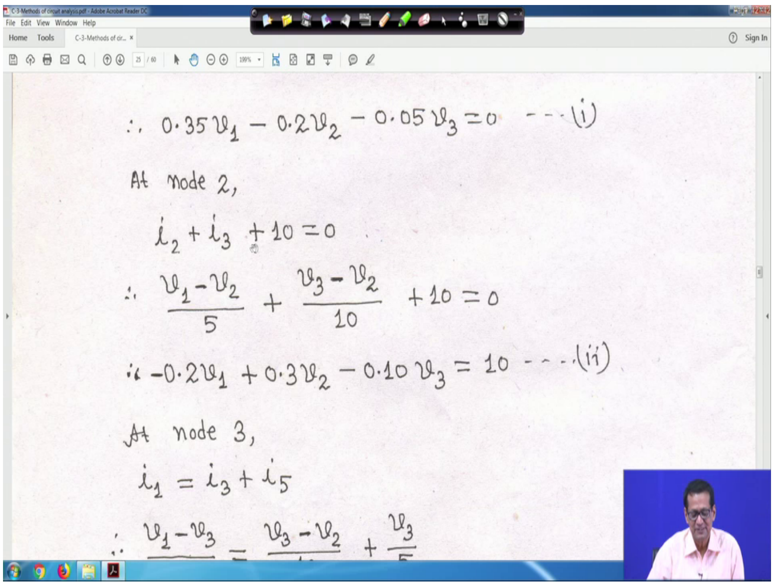
Then here if you to apply your KCL at node 3, here, if you apply KCL, then this i 1 current actually entering into this node right. So, i 1 is equal to other 2 currents are leaving; so, i 3 plus i 5, right. So, all these things, you will be knowing with this right with this; this is for exploration this is for the explanation now all these things, right.

(Refer Slide Time: 31:27)



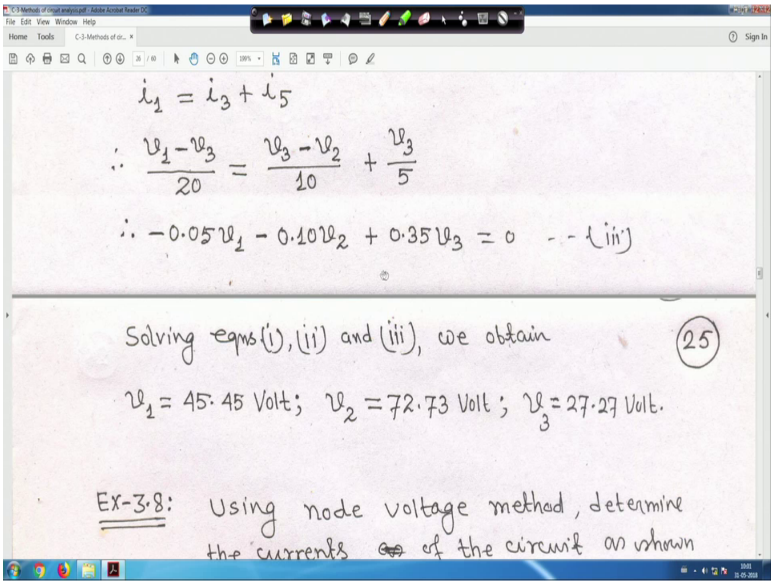
So, all this things, if you make it and this directly, you can write I told you i 1 plus i 2 plus i 4 is equal to 0, you now know i 1, i 2 and i 3 in terms of all this. So, this is one equation. Similarly, at node 2, I told you i 2 plus i 3 plus 10 is equal to 0 put all this things.

(Refer Slide Time: 31:37)



This is your what to call another equation after upon simplification at node 3 also, I told you i 1 is equal to i 3 plus i 5.

(Refer Slide Time: 31:46)



So, you know i 1, you know, i 3 in the all those things, you get another equations and then you solve equation 1, 2 and 3 the way you want Cramer's rule or anything, the way you want to solve it and you will get v 1 is equal to 45.45 volt.

V 2 is equal to 72.73 volt and v 3 is equal to 27.27 volt, I hope you are understanding this, right and same thing that we will be applied for single phase and 3 phases circuit right and so, here only if we make our understanding clear, then things will be will be very easy for your AC circuit analysis.

Thank you very much, we will be back again.