Fundamentals of Electrical Engineering

Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 13
Methods of Circuit Analysis (Contd.)

So, we will we are back again. So, next we will take another example, right second example.

(Refer Slide Time: 00:21)

So, in this case you have to calculate the node voltages in the circuit shown in figure 3.4 a right. So, it is your what to call in this circuit you have to find out v 1, v 2 and v 3 before moving to the problem ah, first let me mark it this is your node 1, this you have to find out v 1, this is your node 2 you have to find out v 2, and this is your node 3, you have to find out v 3, right. And you have in this in this circuit you have one independent current source, that is this one this 3 ampere current source that is independent current source, then one dependent current source is here this is your 2 i x, right. And i x is here i x is here, right and you have to we have to find out.

That v 1, v 2, v 3 the node voltage is v 1, v 2 and v 3. So, let me clean it fast, right so, let us move to this what to call that problem. So, in this case, now the figure 3.4 we just shows the circle this is for the. Now the circuit is redrawn, this circuit actually is redrawn and your, this is your voltage v 1, I told you this is voltage v 2 and this is voltage v 3.

And currents are marked in different your, what you call node. So, this is node 1, this is node 2 and this is node 3, right. So, for example, further ah I mean before moving to that solution how we will do it for example.

Here you have to apply your, what you call KCL. So, if you apply KCL suppose that node 1, this is your node 1, and if you apply KCL, then this 3 ampere current, this 3 ampere current this is independent current source, this 3 ampere current is entering into this node, right.

(Refer Slide Time: 02:12)

Therefore, you can, right and other 2 current that i x and i 1 is leaving, your node 1 therefore, this one equation is 3 is equal to your i 1 plus i x right. So, this is your what you call that your i 1 plus i x, now if you take the direction of i 1, the direction of i 1 is like this in this current this i 1 actually flowing like this is like this here it is i 1, right.

Therefore, i 1 is equal to it will be your v 1 minus in this those voltage is v 3. So, minus v 3 divided by this resistance is 4 so, this is your i, right. Similarly,, your i 2 is equal to sorry i x is equal to this i x is equal to this moving from node 1 to node 2 you have to see the reaction. So, i x equal to your, it will be v 1 minus v 2 divided by this 2-ohm resistance divided by 2, right?

So, this is your expression for because it is the nodal analysis we have to apply KCL, and then you find out your what to call i 1 and i 2; that means, this equation actually it will be v 1, I mean if you put i 1 is equal to v 1 minus v 3 by 4, plus i x v 1 minus v 2 by 2 is equal to 3. Then with these equation this equation we will come like this it is i 1 is equal to your v 1 minus v 3 by 4, right. Plus, your v 1 minus v 2 by 2 is equal to 3, right. So, this will be your if you apply KCL then this will be the your what you call that equation in terms of voltage v 1, v 2, v 3, that is it that is your in equation 1, right.

Similarly, if you apply your what you call that KCL at node 2 so, let me clear it, right.

(Refer Slide Time: 04:23)

So, similarly if you apply KCL here at node 2, then i x is your what you call incoming current is i x, right. Incoming current is i x, and your that and other 2 current i 2 and i 3, the leaving that node 2 so, it is actually i 2 plus i 3, right. So, similar way, your i x is equal to you can write v 1 minus v 2 by 2, this is i x is equal to your i 2, right. i 2 will be is equal to v 2 minus v 3 divided by 8, because this resistance is 8 ohm, right. This is i 2 and i 3 your plus that i 3, i 3 will be is equal to be v 2 minus 0 divided by your 4.

Because this is 4 ohm, right? And this is your reference node. So, here reference voltage v 0 is equal to 0 so, v 2 minus 0 by 4. So, this way you can apply your what to call that your KCL at node 2, right. Later I will come to the solution, but just for the purpose of the explanation, let us try to your, what you call explain this.

Now next you apply KCL at node 3 so, first let me clean it then I I will write. So, second case the this case you apply your what you call, KCL at node 3, in this case this i 1 plus i

2 is equal to because this current is entering, i 2 also entering at a node 3 and 2 i x current is a dependent current source, it is leaving the terminal.

(Refer Slide Time: 05:55)

So, at node 3 if you apply so, it will be your i 1 plus i 1 plus i 2 is equal to your 2 i x, right.

Because this applied KCL that is your what you call KCL at node 3. So, i 1 is equal to your i 1 is equal to already you have seen it is v 1 minus v 3 by 4 because this is 4-ohm resistance, and this is voltage v 1 this is voltage v 3. So, i 1 will be is equal to v 1 minus v 3 by 4 plus i 2. So, i 2 is equal to your v 2 minus v 3 by 8.

So, v 2 minus v 3 by 8 is equal to 2 into that i x, if i x 2 into i x means it is 2 into this is a dependent current i x is equal to your v 1 minus v 2 by 2, because this is a this is i x i x is equal to v 1 minus v 2 upon 2. So, after some simplification you will get some relationship, right. So, this is actually your what you call that your KCL at your KCL at node 1 we have seen what is this node 2 you have seen and node 3, we have seen, right.

So, everywhere you will find only 3 branches here at node there are one branch 2 (Refer Time: 07:10) meeting up 3 branch actually, right. When according to reaction of the current, we are apply KCL here also, 3 braches here also 3 branches, right.

(Refer Slide Time: 07:21)

So, let me clean it. So, whatever I have shown, right now this at node 1 we are writing this equation ah, here one error is there that is your x 3 i here, here it is just hold on, right. This is 3 this 3 was missed, right. So, it will be v 1 i 1 I told you v 1 minus v 3 by 4 plus 1 minus v 2 by 2 if you apply the same thing it is actually 3 here actually I have missed is equal to, right. So, this is a correction now then upon simplification you will get 3 v 1 minus 2 v 2 minus v 3 is equal to 12 this is your equation 1, right.

So, next at node 2, at node 2 I told you i x is equal to i 2 plus i 3, right. So, let me clear it so, at node 2 I told you, this is at node 2, you are what you call i x equal to i 2 plus i 3, that I have wrote the equation written the equation also for you.

(Refer Slide Time: 08:23)

So, here also same thing so, at this is your i x is equal to v 1 minus v 2 upon 2. This is v 1 minus v 2 upon 2, this equal to v 2 minus v 3 upon 8 plus v 2 minus 0 4. So, upon simplification, it is minus 4 v 1 plus 7 v 2 minus v 3 equal to 0. So, this is equation 2, similarly, at node 3 also I have showed you how to write equation. So, same thing you writing that i 1 plus i 2 is equal to 2 i x. All this thing I have rewritten before, right before coming to this. So, here also upon simplification, your you get 2 v 1 minus 3 v 2 plus v 3 equal to 0 this equation 3.

Now, to use Cramers rule we put equation 1 2 and 3 in the matrix. So, let us solve it using Cramers rule, right. So, let me clean it so, this is equation 3 and other 3 equations one and 2 already given. So, let us clear it, right. So, next if you put the matrix form the equation is coming like this, that is your this is your A, this is your this is your A matrix, this is your A matrix write it 3 into 3 matter.

(Refer Slide Time: 09:26)

And this is your x means it is your state variable v voltage v 1, v 2, v 3 that is your x and this is your v mat your n into 1, that is your v matrix n into 1, right. Basically it is a column vector so, 12 0 0. So, based on that, suppose is your solution for this one, v 1 is equal to ah at the time of your explanation of Cramers rule, the solution of there we wrote know x 1 is equal to delta 1 by delta x equal to delta 2 by delta, till x n is equal to your delta n by delta.

So, in this case your x is nothing but your v so, here it is v 1, v 2 and v 3. So, v 1 is delta 1 by delta v 2 is equal to delta 2 by delta, and v 3 is equal to delta 3 by delta, right. So, we have to obtain delta 1, delta 2, delta 3 as well as delta the determinant of this matrix, right. So, let me clear it.

(Refer Slide Time: 10:36)

So, in this case, what we do that 3 into 3 into 3 this rule, this delta equal to this rule, I told you this is true early remember, this rule whatever we have made it here for computing the determinant, it is true only for 3 into 3 matrix. Only for 3 into 3 no for no other matrix, you cannot do this is only for 3 into 3 matrix. That is why if we have a 3 into 3 matrix, rather than finding out co factor other thing directly you can get the determinant quite easily, so, let me clear it.

So, so in this case, the 3, 3 minus 2 1, first you write down the matrix, the 3 minus 2 minus 1, then minus 4 7 minus 1, then 2 minus 3 and one after that you repeat the first 2 rows, that is first 2 rows, this is this first 2 row repeat the 3 minus 2 minus 1, then second repeat minus 4 7 minus 1 minus 4 7 minus 1 this you repeat.

(Refer Slide Time: 11:39)

Next this all this things make algebraic sum, right? So, it is 3 then how you will directly you are calculating the first one is look 3 into 7 into 1, right. So, this is this will have a plus sign before that so, it is a plus right? So, this is a plus sign.

Next your put plus like this, then you make it like this that minus 4, right. Then minus 3, then minus 1, right this is your, but plus sin, but all element should be taken whatever whatever sign they have minus 4 into minus 3 into minus 1, next is this one also. Say plus, so, it will be 2, right. Then minus 2, then you are what you call, this minus 1, right? So, 3 into 7 into 1 so, this part is 21 so, here it is 21, right. Then it is minus 4 into minus 3 into minus 1 so, it will be minus 12. So, that is why it is minus 12 plus minus so, minus 12.

Then this one 2 into minus into minus 1, it will be plus 4 so, this is plus 4. So, this part is done the right hand side is done, right. So, next we will come to the this side, this will be negative. So, I am putting somewhere here look it is minus, right. Then this side if you come like this then minus 2 minus 2 into 7 into minus 1 so, 2 into 7 into minus 1. So, if you multiply this, it will be actually plus 14, this will be actually plus 14, I mean, this one when you multiply this one, right so, that is why it is plus 14, right.

Next you multiply this 1 by your, what you call minus sign, will be there minus. So, minus sign there is a minus 4, right. Then it is minus 2 minus 2 then it is 1, then it is 1. So, it will be actually minus 8, right? So, this is here I have writing this is actually you are what you call that your minus 8, right? Sorry this is not minus 8 this is the third sorry, this is the just 1 minute just let me clear it let me clear it, I have, I have missed that third term this is the third term, it will be this thing just hold on second term is this one, which will be minus so, it is your 3 into minus 3 into your minus 3 into minus 1, right.

(Refer Slide Time: 13:54)

So, 3 this 3 into minus 3 into minus 1, it will be actually minus 9, right? So, that is why it is minus 9 is here, right minus 9 is here.

So, next is your this term minus 4 into 2, I mean next is this term so, it is minus then minus 4 minus 2 into 1 so, it will become minus 8. So, that is why it is minus 8, right. So, pervious where I am explaining I was overlooked as this thing right. So, that is why this determinant is 10 using this rule. So, it is very easy to compute just repeat, and just you take this algebraic sum plus and this side this side should be this is all minus left, right. Left hand side and right hand side all plus, it is very easy rather than finding out what already true for 3 into 3 matrix. So, let me clear it, right? So, with this delta will got 10.

(Refer Slide Time: 15:00)

Similarly, you are find out delta 1, delta 2 and your these thing so, in that case for finding out your delta 1 ah, that if you look into this that first row is replaced by this v matrix, v is 12, 0, 0, right. So, first row is replaced sorry, first column is replaced by 12, 0, 0. So, if you look into the previous matrix, only this column is replaced by 12, 0, 0, right?

So, and this is minus 2 7 as you, similarly following the same thing, you repeat this 2 rows, you repeat this 2 rows here at the end and this 12 minus 2 minus 1 0 7 minus 1 and 0 minus 3 1 all are there, but repeating the first 2 rows here. And following the same thing to find out what is the value of delta 1, right? So, let me clear it.

(Refer Slide Time: 15:59)

So, if you do so delta 1 you will get this is actually 48, this is actually 48, right. So, here it is sorry, here it is your delta 1 is equal to 42 48.

Similarly, for delta 2 also, that second row sorry second column of that matrix you replace by 12, 0, 0. So, if you look into that this is the second column replace this 2, 3, minus 1 minus 4 minus 1, 2, 1 all are there, only second column is replaced by 12, 0, 0, earlier for Cramers rule it has been explained, right. Only second row you sorry second column you replace.

So, and following the same thing you compute, then you will get delta 2 is equal to your 24 so, delta 2 is equal to 24. All these things for that dc circuit, we are trying to understand for dc circuit all these things in detail ah, same philosophy you will be applied for ac circuit single phase ac circuit, as well as 3 phase ac circuit at that time, I need not explain all these things because by that time we will go through everything.

So, it will be easier for me and at that time, time consumption will be less compared to present one. So, each and everything we are trying to understand right. So, this is delta 2 so, is equal to 24. So, clean it, so right.

(Refer Slide Time: 17:23)

Similarly, delta 3 also in similar way you compute so, that is delta your this that equal last column of the 3 into 3 matrix replaced by this rest of the elements of a matrix so, element same, right. And follow the, and repeat the first 2 rows here. So, first 2 rows repeated 3 minus 2, 12 and minus 4, 7, 0 that all this thing repeated and follow the same thing for compute computing delta 3. And you will get delta 3 is equal to minus 24, right?

So, once you got, then what you can do it now you know that, let me clear it.

(Refer Slide Time: 18:03)

So, v 1 will be delta 1 by delta also v 1 is equal to delta 1 by delta. So, it will be your 48 by 10 so, 4.8-volt v 2 will be delta 2 by delta. So, answer will be 2.4 volt, and v 3 your will be your minus 24 by 10 delta 3 by delta so, minus 2.4 volt. So, this is the solution right? So, this is simple one, I will how to solve by this rule what for computing determinants, actually it is true for 3 into 3 matrix right. So, let me clear it.

Now, then nodal analysis next we come to many more example, we will see nodal analysis with voltage sources, right.

(Refer Slide Time: 18:42)

So, now we will now consider how voltage sources affect nodal analysis for the purpose of explanation consider figure 3.5. So, suppose as soon as nodal analysis with voltage sources.

(Refer Slide Time: 19:01)

So, just for the purpose of this thing, let us this, this circuit we initially we have taken. Now question is that, here little bit let us try to understand that one reference node is there, right? This is at reference node voltage v 0 is equal to 0, right? And you have it is a nodal analysis, if you look into the circuit, right? These are non-reference node, this is v 1, this is v 2 and this is v 3. So, node 1, node 2, node 3 these are non-reference node, right?

Now, before coming here, before coming to the super node concept, before coming here, that this is the nodal voltage v 1 and it a 10-volt source is connected here ah; that means, then at this datum node reference node voltage is 0. So, basically here v 1 actually is equal to 10 volt, right? So, this is the only voltage source connected here, right. So, this is this is actually called this is actually v 1 itself will be 10 volt. Because whatever is there from this non reference node to your datum node or reference node that 10-volt source is already connected. So, v 1 will be is equal to 10 volt this you have to understand.

Second thing is that, if any voltage source is there between 2 non reference node so, this is one non reference node, and this is another non reference node. In between if you have any voltage source, like here we have a voltage source a 5 volt, then this 3 this combination is creating a super node we call super node, right. And in the super node both KCL and KVL, you can apply you have to apply, right.

So, just to for understanding, if any voltage source is there, between 2 non reference node, then that will be super node; that means, this whole thing together is a super node, right? And in this case you need KCL and KVL both, now question is if this combination itself is a super node then how you can apply KCL. So, in that case what you have to do is, so, this is the current i 1, this current i 1 entering into the super node upto this you will come, right. This whole thing is a super node, I have marked it here it is a super node, right.

So, this current i 1 is entering here, right. Similarly, if you look into that this i 4 current also entering into the super node, right. So, this is another thing and other thing is that current i 3 is leaving the super node and current i 2 leaving the super node. Because you have to you at that time we are not considering like this node or this node, as a whole it is actually super node, right.

So, you have to see in this; that means, how to remember, that if any voltage source is connected between 2 non reference node, that that actually combination it creates a super node, right? And in that case, you have to you have to see that this current i 1 is entering into this your super node, and current i 4 also entering the super node, because this is as a whole, right. As a whole this dash line whatever I have given it is a super node, and current i 3 also leaving the super node and current i 2 also leaving the super node.

So, in that case, that took that i 1, i 1 plus i 4 these are the incoming current to the super node, and i 3 and i 2 leaving the super node. So, is equal to i 3 plus your i 2, because these 2 are leaving the super node. So, this is this is you have what you call both KCL and KVL both you have to apply. So, first KCL so, when we apply KCL to super node. So, this equation is coming, right I hope you have understood this the concept of super node. So now, let me clear it.

So, that is why this dash line, it is there it is creating super node. So, current i 1 is coming here, entering into the super node; that means, entering into the dashed region, right. i 4 also and i 3 and i 2 these 2 are leaving. So, this is actually you have to consider as a super node.

(Refer Slide Time: 22:58)

Now, now you whatever I said, whatever I said that v 1 I told you 10 volt whatever (Refer Time: 22:59) is there, that is a different thing that already told.

(Refer Slide Time: 23:04)

And this is the this is your what to call i 1 plus i 4 is equal to i 2 plus i 3. This is your super node that at KCL when you make it, now in now in next if you if you try to write down all this i 1, i 2, i 3, i 4.

(Refer Slide Time: 23:22)

So, i 1 is equal to is as simple as before, i 1 is equal to your what you call that v 1 minus v 2 by 2, right. Because this is v 1, this is v 2, this voltage is v 2, this is v 1. So, i 1 is equal to v 1 minus v 2 by 2, right?

Similarly, your i 4, i 4 is equal to v 1, this resistance is 4 ohm, v 1 and this is v 3. So, it will be v 1 minus v 3 by 4, right? That is your i 4 and i 3 is equal to this i 3 writing here, i 3 is equal to this is v 3 and this is reference node it is voltage is 0. So, basically v 3 minus 0 by 6 so, v 3 minus 0 by 6, this is your i 3.

And similarly your i 2, this i 2 is equal to this voltage is v 2 the reference node voltage is 0 so, v 2 minus 0 by 8. This is your i 2. So, that means, your i 1 plus i 4 is equal to i 2 plus i 3 replace those I by all v v voltages v 1, v 2, v 3, then you will get one equation using KCL. So, that is why this clearly I am clear, let me clear it, right.

(Refer Slide Time: 24:37)

That is why, this equation we are writing v 1 minus v 2 by 2, plus v 1 minus v 3 by 4 is equal to v 2 minus 0 by 8 plus v 3 minus 0 by 6.

So, right so, this is next to next in the super node you have to apply KVL, right. 1 KCL because you have to whenever a super node is there a constraint equation will come, right. Whenever a super node is there a constraint equation will come. So, how to make it? So, I will make it like this expression is there later. So, question is that I have to apply your what to call that I have to apply your KVL also. So, forget about the circuit only this as it is v 3.

(Refer Slide Time: 25:23)

So, suppose this is for your understanding, suppose this is my your what to call node 3, and this is my node 2 node 2, right. And in between the voltage source is there minus plus, right. And this voltage is high volt. So, this is and this is connected, and here voltage is v 3 this is the reference node. So, basically this is actually ground, this is actually your ah actually it is ground. So, this voltage is v 3. So, this is plus this is minus, and similarly here also these voltage this node 2, right. It is it is actually voltage is v 2, v 2 this is plus this is minus; that means, this v 3 is actually that what to call the v 3 it is the voltage across the side.

So, basically v 3 will be is equal to 6 into i 3, right. Similarly, here also your v 2 will be actually 8 into i 2 from ohms law, right. So, rather than; that means, this in this circuit we will take the total voltage v 3. So, this is v 3 this is plus minus, right? This is plus minus, right now what you do in the clock wise you apply KVL in this equation because super node requires KCL and KVL both because of this voltage source one constraint equation will come. So, if you move like this.

So, it will be look minus 5, because it is encountering minus 5 volt, because it is minus sign, right plus this voltage is v 2. So, plus v 2 encountering plus sign first, then here it is encountering minus sign first minus v 3 is equal to 0 right; that means, that v 2 minus v 3 is equal to 5, this is the constraint equation coming because of the super node, I hope you have understood this, right.

Ah So, whenever super node comes you make this circuit and make it clear such that things will be very things will be totally clear, this is actually node 3, this is actually node 2. And anything whenever I make symbol like this, arrow like this so, arrow means it is always plus and nothing is mentioned means it is minus, right. So, any super node any non-reference node, you to take this should be plus sign, this should be plus sign, this should be plus sign, and this reference should be minus 1, right. There should not be any confusion so, this v 2 minus v 3 will be 5 so, let me clear this, right.

(Refer Slide Time: 27:50)

Therefore, this equation, this is the same thing is redrawn here, the same thing is redrawn here plus minus plus minus this only clock wise we are moving applying KVL whatever this is the dash lines show the other component of circuit, but our interest is only this much whatever I showed, right.

(Refer Slide Time: 28:08)

So, if you do so, you get that same question v 2 minus v 3 is equal to 5, whatever just now ah just now whatever we wrote v 2 minus v 3 is equal to 5, right ? So, let me clear it.

(Refer Slide Time: 28:29)

So, then equation 19 it is chapter 3, that is why 3.19, 3.1, 3.22. So, equation your 19, your just hold on equation 19, 21 and 22, this 19 sorry, this 19, 21 and 22, we obtain the node voltages therefore, a super node has the following properties a super node has the following properties number one a super node requires the application of both KCL and

KVL.

That I told a super node has no voltage of it is own, right. Super node has no voltage because, 2 node and one voltage all these things are combination is making a super node. So, super node has no voltage source of it is own, and the voltage source inside the super node voltage source provides a constraint equation needed to solve for the node voltages, right.

(Refer Slide Time: 29:36)

That means your let me clear, what is that constraint equation this one, this one, this v 2 minus v 3 is 5, this is constraint equation coming because of that super node, just I told you, right? Therefore, your what you call that it needed to solve for the node voltages, right?

So, thank you very much, we will be back again.