Fundamentals of Electrical Engineering
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Lecture – 12
Methods of Circuit Analysis (Contd.)

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So, come back again, right. So, that is why this your this 2 equations, you can write in the matrix form, right. So, it is 1 upon R 1 plus 1 upon R 2, and this is minus 1 upon R 2 and here it is minus 1 upon R 2 1 upon R 2 plus 1 upon R 3 v 1 v 2 is equal to 4 6. So, these 2 questions can be solved right, but here it is only 2 you are, what you call only 2 unknown. So, easily you can solve right, no even no matrix operation is required just you can substitute one after another and you can easily solve it. But if you are if it is your what you call, if it is more than 2 3 4 or 5 say something it count, then you have to follow certain techniques.

So, just Cramer's rule we will discuss here, and one small technique I will show you which is valid only for 3 into 3 matrix, right. So, this is simultaneous equations and Cramer's rule.

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So, little bit for circuit analysis as I thought little bit idea is required. So, that is why I have taken this thing the small mathematics, right. So, considered a set of simultaneous equations having distinct from, the equation is a 1 1 x 1, right. I am just making it like this, it is a 1 1, x 1, a 1 2, x 2 up to a 1, x n is equal to say b 1.

This equation 1 the one equation; similarly, a 2 1, x 1, a 2 2, x 2, a 2, n x n this is a b 2. Similarly, last one a n 1, x 1, a n 2, x 2 sorry a n 1, x 1, a n 2, x 2 up to a n n, x n it is a b n, right. So, you have n, n number of simultaneous equations, and b n you write. So, this equation, if an you put this equation in the matrix form, let me clean it.

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If you put this equation in the matrix form, then we can make it like this, that a 1 1 this is your this is your a matrix, this is your a matrix, it is a 1 1, then a 1 2, a 1 n these all this things little bit you have studied in your higher secondary, right in matrix chapter.

And this is a matrix, this is your x 1 x 2 x n, right. So, a a matrix is n into n matrix, right. And this is x is actually n into 1 vector right hand side this b 1 b 2 b n it is actually b or what you call n into 1 vector, right. So, this is actually given as equation 10. So, this equation it can be written as AX is equal to B. Or x is equal to directly solved x is equal to a inverse b of course, if a inverse exist, but any way sometimes we do not follow a inverse. So, you follow different method to solve such linear equations, right for computational efficiency.

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Now, this is actually your AX is equal to B. Now a is equal to your a matrix, this a 1 1, a 1 2, a 1 n, a 2 1, a 2 2, a 2 n then in a term a n 1, in throw a n 1, a n 2 a n n, right.

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Similarly, X is equal to your x 1 x 2 up to x n, and B is equal to b 1 b 2 b n. So, is a square matrix n into n matrix, where x and b are column vector that is n into 1 matrices, right. So, it is actually column n 1, n 2 we can write n 1 n into 1 matrix, right matrices.

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So, there are several methods for solving equation your this above equation AX is equal to B. That is your back substitution ah, Gaussian elimination, matrix inversion, Cramer's rule and numerical analysis. There may be many method for solving such this thing, but we have to apply which will be computationally efficient, but any way we will not do any we will not write any computer code or anything. So, only we have only we have to solve those things which can be solvable in the classroom as a classroom exercise accordingly we will see, right.

We have to see that classroom how we can quickly we can solve this one.

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So, Cramer's rule actually Cramer's rule can be used to solve the simultaneous equations, according to Cramer's rule the solution of equation 3.10 that is your AX is equal to B, right. That is your that is your if the equation is your AX is equal to B, right. When you are solving this equation; so, it can be easily solved x 1 is equal to delta, 1 by delta will see what is delta 1 or delta x 2 is equal to delta 2 by delta and x n up to the delta n by delta.

So, we have to find out delta 1, delta 2 up to delta n, and then delta delta delta, right. So, delta is a deltas are the determinants, but we have to know some procedure that how to solve it quickly, right. So, in this case what we will do that delta is the determinant, and then the delta 1 delta 2 all delta n also determinant, but how to obtain this?

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Where the deltas are the determinants given by this delta is equal to your this determinant you find out this is a basically it is the a 1 1, a 1 2, a 1 n, a 2 1, a 2 2, a 2 n, and a n 1, a n 2, a n n. This is the determinant delta, now for delta 1 delta 2 delta 3 what you will do, right. Look just you look that what you can do is for delta 1, for you have to find out x 1; x 1 is equal to say delta 1 upon delta, x 2 is equal to say delta 2 by delta and x n is equal to your delta n by delta, right.

So, how will do is that this is the first if you take the first column, this first column only you replace by b 1 b 2 and bn, rest will remain same. So, that is whatever determinant will come that will be delta; that is why b 1 b 2 b n the rest all the elements from here to here all the elements, this all the elements all the elements it is same.

Just for delta 1 you will replace the first column of this of a 1 1, a 2 1 up to the place by b 1, b 2, b n, just replace it and calculate delta 1. Similarly, just let me clean it, that will get delta 1.

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So, similarly your delta 2, in the case of delta 2 just replace your what you call the second column by b 1 b 2 I mean the second column of this matrix this is the second column, right. This is the second column, this is a second, this is that your second column, this one you replace by b 1, b 2 rest for a rest for all the a s element remain same.

All you replace that, then you will get the delta 2. Similarly, for the delta n the last one, the last one you replace by b 1, b 2 and b n, rest of the all a element will remain same. You will get delta n this way delta 1, delta 2, delta 3 all can be determinants can be computed, right. Then direct solution then you will get ones delta 1, delta 2 up to delta n known, then x 1 is equal to delta 1 y delta x 2 is equal to your delta 2 y delta, x 3 is equal to delta 3 y delta up to xl is equal to delta n y delta. So, this way you can easily solve, it just replace one column just shift it, right. So, let me clear it, right. So now, question is that ah how to make it?

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Now, if delta is the determinant of matrix and delta k is the determinant of the matrix formed by replacing the k th column of matrix A by B. This I told you all these things I told you, right. So, only thing is that it is evident that Cramer's rule applies only when you are what you call, that your delta not is equal to 0.

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Because it will be division by 0s so not possible; so, when delta is equal to 0, the set of equations has no unique solution, because the equations are linearly dependent, right. So, in that case you will find ah the equation has no unique solution; where equations are linearly dependent.

So, the value of the determinant delta can be obtained by expanding along the first row. These are underlined along the expanding along the first row. So, how will do it ?

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So, if it is given like little bit bigger I have written the a 1 1, a 1 2, a 1 3 up to a 1 n, then a 2 1, a 2 2, a 2 3 up to a 2 n, right similarly, a 3 1, a 3 2, a 3 3 up to a 3 n, and then a n 1, a n 2, a n 3 up to a n n.

Now, if you write like this, that then you have to write this thing.

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So, let us this first one we are writing a 1 1, M 1 1, right basically, can be obtained expanding along the first row; that means, along the first we are trying to explain, column wise also can be given both we will see, right. So, in that case a 1 1, M 1 1, minus a 1 2, M 2 plus a 1 3, M 1 3, plus dot, dot, dot plus minus 1 to the power 1 plus n, then a 1 n M 1 n, right.

So, where this determinant that will see later. So, look at that, this one this one this is a 1 our objective is have to find out M 1 1, M 1 2, M 1 3 and last one is that is your M 1 n. Particularly look at the last term, minus 1 to the power 1 plus n into a 1 n into M 1 n. So, if you know the M matrix m, then determinant this M 1, the determinant that M 1 1, M 1 2, M 1 3 up to M 1 n, then easily can be solved, right. So, how to get it so, let me clear it .

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So, the mat minor, this is actually this is the, where the minor M ij is an n minus 1 into n minus 1 determinant of the matrix form by striking out the i th row and the j th column, right?

So, later we will see that you just strike it of ith throw and jth column. For example, for example, because it will be a it will be your just 1 minute it will be your, it will be sorry it will be your n minus 1 into n minus 1 matrix, right it is the minor. Now suppose it is M I j, right so, it is I th row say i is equal to 2 and j is equal to 3; that means, second row and third column, right. So, if you replace that rest of suppose second row ith row suppose this row. This is gone say, this is not there ith row and jth column. So, that is j is equal to the 3 third column. So, just this 2 will go rest of the elements will be there, right.

So, accordingly it will be n minus 1 into n minus 1 matrix. So, rest of the elements this one, this one, this one all will be there, right. And this will not be there completely gone, but this one, this one, this side will be there, this will not be there. So, this rows columns you have to you have to just eliminate rest the matrix order minor will be M minor your n minus 1 into n minus 1, because one row and one column is completely eliminated, right? So, let me clean it, right.

So, similarly, your the value of delta may also be obtained by expanding along the first column. If you see that rather than your, rather than rows if you expand this along this first column, right that also will that also will get that your what you call the value of delta. So, just if you just let me clean it. So, if you make it, if you are what you call if you make this one, then it will be delta is equal to a 1 1 M 1 1 minus a 2 1 M 2 1 because we are moving towards the column, right?

So, expanding towards the column so, a 1 1, M 1 1, then a 2 minus a 2 1 M 2 1 plus minus plus, minus plus, then plus a 3 1 M 3 1 plus upto that minus 1 to the power n plus 1 a n 1, M n 1 here also while I am explaining this plus minus plus minus, right. And then it is coming minus 1 the power 1 plus n 1 n this is expanding your what you call expanding along the first row, but the same thing will get along the first column. So, second equation is your along the your what to you call the first column, right? So, this is a 1 1, M 1 1 minus a 2 1 this thing.

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Suppose for a 2 into 2 matrix, right for a 2 into 2 matrix determinant is simple a 1 1, a 2 2 minus a 1 2, a 2 1 this you know. For a 3 into 3 matrix it is suppose a 1 1, a 1 2, a 1 3 look in the classroom for pass, I think maximum we will solve upto 3 into 3. Will not think about 4 into 4 because easily it can be solve I know, but it will be more time consuming, but will solve only upto your what you call that 3 into 3 matrix, right. That way we will formulate the problem in the assignment, right. So, this one this is a simple your delta is equal to a 1 1, a 1 2, a 1 3, then a 2 1, a 2 2, a 2 3, then a 3 1, a 3 2, a 3 3.

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Now, if you if you expand this, if you expand this, you are in the in the direction of the your column a 1 1, a 2 1, a 3 1 and look it is if the if you see the last formula. This last one that nth term nth term is your what you call it is your minus if you see the nth term, it is just for just hold on if you see the nth term, then it is your minus 1 to the power n plus 1, then a n 1 then M n 1, right?

This is your now what you do is you put n is equal to 1, n is equal to 2 and n is equal to 3. If you put n is equal to if you put n is equal to 1, then it will be minus this one first term, right. So, it is minus 1 square, then a 1 1 then M 1 1, right. Similarly, if you put n is equal to 2, it will be minus 1 cube. So, minus term will come then minus a 2 1, M 2 1 if you put n is equal to 3, then it will be minus 1 to the power 4.

So, it will be plus then a 3 1 M 3 1, right. So, let me let me clean it, right. So, that is why this when you expand this it is coming like this, right. Suppose a 1 1 first one, then minus 1 to the power n plus 1 that is this thing you are, what you call then you are this if you go to this formula a n m, n 1 just you just you see this one. For example, everything will remain same, that is your say M n 1, right then you are writing a n 1, then minus 1 to the power say n plus 1. Now if you look into that, that n n is equal to 1 means it is, a 1 1 is there n is equal to 1 means this is minus 1 square.

So, it will be plus, then this term a 1 1 coming then M 1 1. So, ith row and M 1 1 means here i is equal to 1 and j is equal to 1 . Ith row and jth column you eliminate, right. So, ith row means first one first row column you eliminate. So, this is the first row and this is the first column. If you eliminate remaining will be a 2 2, a 2 3, a 3 2, a 3 3 so, a 2 2, a 2 3, a 3 2, a 3 3. So, this is the first one. So, it is understandable to, right? No problem?

The second one, let me clean it, I will come second one that, right. Second one again it is again your say sorry, it is again your just hold on, let me clean it . Just hold on, right so, in second one it is again the same term, I am writing minus n plus 1, then a n 1 then M n 1, right.

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Now in this case second term n is equal to 2; that means, it is minus 1 to the power 3, then a 2 1 then M 2 1, right. So, minus 1 to the power 3 means it is minus, right. So, this is actually minus 1 cube so, it will become minus a 2 1 is there, and what is M 2 1?

So, here M I j, i is equal to 2 and j is equal to 1 right; that means, you second row and the first column. So, second row means this one will go, right. And i is equal to 2 means ith row. So, second one j is equal to 1 first column and this one will your what you call it will go. So, what will be there? A 1 2, a 1 3, a 1 2, a 1 3 and a 3 2, a 3 3, a 3 2, a 3 3, right; so, this way minus you can easily obtain. So, I hope this things is understandable to you, right?

So, let me let me your what to you call clean it, right.

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Similarly, last one also, last one shall I tell you I think last one also you will be able to understand, because n is equals to 3 so, it will be a 3 1 minus 1 4, right. So, i is equal to 3; that means, third row and your first column, right so, third row and first column will be eliminated.

This is the third row so, a 1 2, a 1 3 and a 1 2, a 1 3 and a 2 2, a 2 3 will be remaining, right. So, this way you can find out so, this is a 3 into 3 matrix. So, not will not go beyond 3 into 3, because it will take more time, but this is a methodology that how to find out. This I have explained in the your what to call in that your column side, right. (Refer Time: 18:22) of the column side not in the rows, but same thing.

Row wise also you can do it, you will get the same result .

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So, if you multiplied this all this things if you simplify you know how to simplify it. So, you will get this is my delta, right? So, another thing is that this method just true for 3 into 3 matrix, remember this is this is what technique is there that is simple method, a simple method for obtaining the determinant of a 3 into 3 matrix is by repeating the first 2 rows and multiplying the terms diagonally as follows, right. It is a very it is a very simple thing, but remember it is only true for 3 into 3 matrix.

Not 2 into 2 not 4 into 4, but this will be very useful for you and you will see how quickly you can obtain it, right so, that me clean it, right. How you do what you will do it?

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Just when you are very easy to understand, what you do it? That this is a 1 1 just hold on, this is a 1 1, a 1 2, a 1 3, a 2 1, a 2 2, a 2 3, a 3 1, a 3 2, a 3 3 this cross I have made it just how to do it this, right? Then what we will do? This first 2 row you repeat further, right. First 2 row you repeat. That is a 1 1, a 1 2, that is a 1 1, this is a 1 1, a 1 2, a 1 3 repeat.

And a 2 1, a 2 2, a 2 3, a 2 1, a 2 2 you repeat, you make it like this. So, first thing is your 3 into 3 matrix, this is your 3 into 3 matrix, and then what you do? You repeat the first 2 rows a 1 1, this is a 1 1, this is a 1 1, a 1 2, a 1 3, a 2 1, a 2 2, a 2 3 you repeat. Then you see how quickly you will get it. Let me clean it, then what you do ?

Then your ah, then what you do that you go for your multiplication algebraic sum algebraic sum means this one a 1 1, a 2 2, a 3 3, this is plus sign. So, a 1 1, a 2 2, a 3 3, this one whatever it come.

This a 1 1, a 2 2, a 3 3 may be negative positive, it does not matter, but whatever it is you have to take plus sign then this this one, that is all this things I have taken then plus your plus your a 2 1, then a 3 2 happen then a 1 3, right. This is a 1 3 then plus this one, plus this one. This will be your plus, then a 3 1, then your a 1 2, then your a 2 3. This is your this is your what you call these are all plus sign I have made this is plus, this is plus, this is plus, this side will be plus, and this side this side you take minus, right.

So, minus means that your this elements will be plus minus, it does not matter the sign should be minus then minus you take the a 1 3, this is a a 1 3, a 2 2, a 3 1 so, it is a 1 3 a 2 2 a 3 1, right? Then minus then this one your a 2 3, a 2 3, a 3 2 then a 1 1, right. And then this is 2 3 a 3 2 and this is a 1 1, and then minus this another one, that your a 3 3, then a you are what you call 1 1 1 to a 1 to then your a 2 1. So, all this things plus, plus, plus all this things minus, minus, minus.

And whatever you will get that will be your determinant, right? So, just just copy the first 2 rows, and this side you take a 1 1 plus, it is plus, this is plus and this side you take minus, and just simplify you will get the determinant. This is a true only for 3 into 3 into 3 matrix, do not try for other thing it will never be, but for 3 into 3 matrix, it is easy to compute rather than computing your minor another thing state forward we will get it, this is a simple simple procedure, right.

So, let me clean it, right that is why I have make it like this and finally, if you the way I told whatever, I wrote same thing is written here same thing is written here, right .

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So, this is your delta, same thing is written here, next we will take the example we will solve it is a it is a basic fundamentals of electrical in course. So, we have to solve a several numericals varieties type of numerical, such that your you know your idea the thoughts will be more or less your thoughts will be clear, right?

So, a little bit practice is necessary for such kind of thing. So, we will take different type of problems, and just let me tell you one thing that this this whatever I am making it, it is a scanned copy that notes which I prepare for something, right.

And if you have any, if you find any error or any any any error any calculation error or anything please let me know, then I can rectify myself because so many problems are there, there is possibility there is possibility that there may be an error in calculation and other thing I to hope it is correct, but may be somewhere something has gone something might have gone wrong also, I do not know. But I want to get feedback from you ah, right. So, if any anything anything you this thing you want, right. Anything you find that wrong calculation or anything then I will rectify it.

But verities of problem will try to solve I have collected problem from several places and try to put it here . Such that you will have also you will have you know, our thoughts and other things ideas and concepts will try to clear, right.

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So, first so, for take a simple example, right; so, determine your what you call the node voltage is a circuit in figure 3.3, a because chapter 3, that is why marking chapter 3 sometimes in the later stage, I do not ah, but let us see that you have to find out the node voltages, that is a unit circuit. So, just just this one this is your node 1 this is your node 2, right. And this is your reference node datum node, and this voltage v 0 is equal to 0, right.

So, this is your datum node, it it will in that new assignments or exam whatever you get this thing will not be mark, but you know that it it is v 0 is equal to 0 . So, this is my node 1, this is my node 2; that means, my this voltage is v 1 this voltage v 2 with respect to this reference node, this is my reference node. And we have to find out, right node volt v you have to obtain v 1 is equal to how much? And v 2 is equal to how much? V 1 is equal to how much? V 2 is equal to how much, right?

And 2 current sources are there 2.5 and 5 details, I made it in the next diagram, but before that I just told you that this is the thing. So, so even if it is v 1 v 2 will not mark in the circuit, if you are ask that what are the node voltages, right? Then you have to assume this is v 1 and this is v 2, right so, let me clean it .

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So, this is the detail circuit so, this is your this is your reference node, this is your reference node, right. This is marker o, and I told you that v 0 is equal to (Refer Time: 25:58) this is your voltage v 1, this is v 2, now act node 1, this is your node 1 and this is your node 2. So, here you have to apply KCL, and here at node 2 you have to apply KCL. So, if you do so, look this 2.5 this is a current source 2.5 ampere independent current source, this is also independent current source.

This current is entering, right 2.5 ampere is entering. So, it is your incoming current. So, is equal to, right is equal to that i 2, i i 2 and i 3, these are leaving this terminal node 1, this node 1 it is leaving. So, it is actually is equal to i 2 plus i 3, right. So, this is this is actually node 1, but when you come to node 2, node 2 then what will happen? This i 2 current actually is your what to call entering into this node. So, this current is entering into the node. So, this is i 2 then this 5 ampere current also here, this is current source, this 5 ampere current also entering into node 2.

So, it will be i 2 plus 5, though this 2 are incoming current, these are entering into the node and is equal to your this 2 point this is a 2.5 ampere current. So, I mark it here that is actually leaving the node, because direction is this way it is given, right is equal to 2.5, right. And plus your plus the right hand side, this i 1, this is also your i 1.

So, these 2 equation this is one equation i 2 plus 5 is equal to 2.5 plus i 1, and this side is 2.5, this is incoming current and 2 whether i 2 plus i 3. Hope I have not missed anything, because so many short problems are there so many branches are there. So, during this lecture, there is a p possibility occasionally I also may miss something. So, if by chance it is miss during this lecture, you will also let me know that I have missed that hopefully hopefully things are next equations, right.

So, this is how will, right; the equation, right. And hope this is understandable to. So, let me clean it, right.

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So, that node 1 so, wrote you that 2.5 is equal to i 2 plus i 3 is equal to that i 2 and i 3 equations we have to write. So, here let me again me, let me this thing I let me mark it.

So, this this voltage this your what you call that i 2 and i 3 we have to write and i 1 also

So, this this just hold on so, this i 2 actually if you right i 2 this voltages is v 1 this voltage is v 2.

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So, it is v 1 minus v 2 divided by 8 this is i 2, right. So, this current is flowing from 1 to 2. So, v 1 minus v 2, similarly this is your datum node O, this reference voltage is 0, right. So, i 3 is equal to writing here, i 3 is equal to this is b 1 by 4, actually b 1 minus 0 by 4 that is your b 1 by 4, because this 4 ohm resistances here, right. This is v 1 minus v 0, but v 0 is 0.

So, v 1 minus 0 by 4 so, v 1 by this thing; and then your i 1 so, i 1 I am writing here that i 1 is equal to this is datum node reference 0. So, it is voltage here is v 2 so, it will basically v 2 minus 0 by 12, so, that means, it is actually v 2 by 12 this is your i 1. So, i 2 you have got, i 3 you have got, and i 1 also you have, right so, v 2 by 12. So, all these things will put in those KCL equations then we will get the equations in terms of v 1 and v 2 so, hope this is understandable; so, cleaning it, right. .

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So, that we have (Refer Time: 29:42) i 2 plus i 3 we are writing here, right. So, this is your i 2 and this is your i 3, whatever we have explain. So, after simplification we will get this equations 3 v 1 minus v 3 is equal to 20. Similarly, other thing also this equation also we wrote at that time of explanation, and i 2 is equal to than v 1 minus v to upon 8, and i 1 is equal to b 2 by 12 this also you have written, right so, clean it right.

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So, then upon simplification we got that your minus 3 v 1 plus 5 v 2 is equal to 60. Now solve you what you call you solve equation 1 and equation 2 this 2 question, we solve you will get v 1 is equal to 13.3 3 volt and v 2 is equal to volt, right.

So, this is the answer, but let me clean it one few things, I can tell that you, please solve it of your own here problem it is not mentioned, that you you are you find out you find out how much power is supplied by this current source and this current source. This is an exercise for you whether power how much power they are supplied, right. It may accordingly we will find out; it may be you know that is your 2 current sources are there. How much power by this one this current source, and how much by this current source, this is an exercise for you.

Thank you very much, we will be back again.