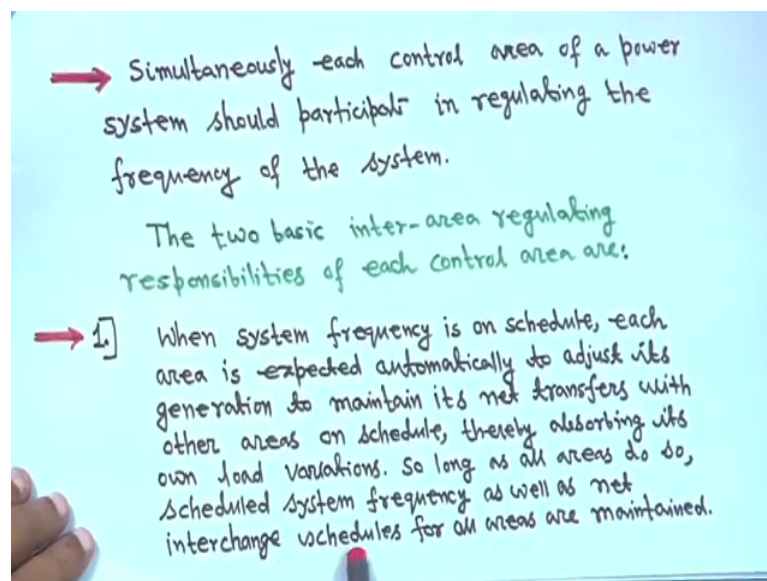


Power System Engineering
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Lecture - 53
Load frequency control (Contd.)

So that means, in a normal steady state operation is controlling of a power is also strive to meet is old demand just I told you right.

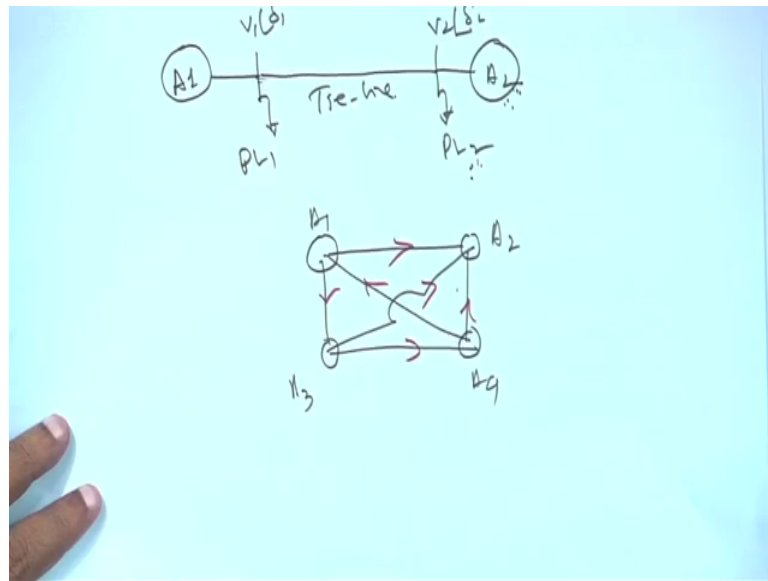
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Next is that simultaneously your each control area of a power system should participate in regulating the frequency of the system because you have to regulate the frequency. So, therefore, the two basic inter area regulating responsibilities of each control area are just its a big one. So, I have I just am explaining what exactly it is for in system frequency frequencies on schedule right each area is expected automatically to adjust its generation to maintain its net transfer with other areas on schedule.

Thereby absorbing its own load and very over load variation. So, long as all areas do so, schedule system frequencies as well as net interchange schedule for all areas are maintained. That means suppose you have.

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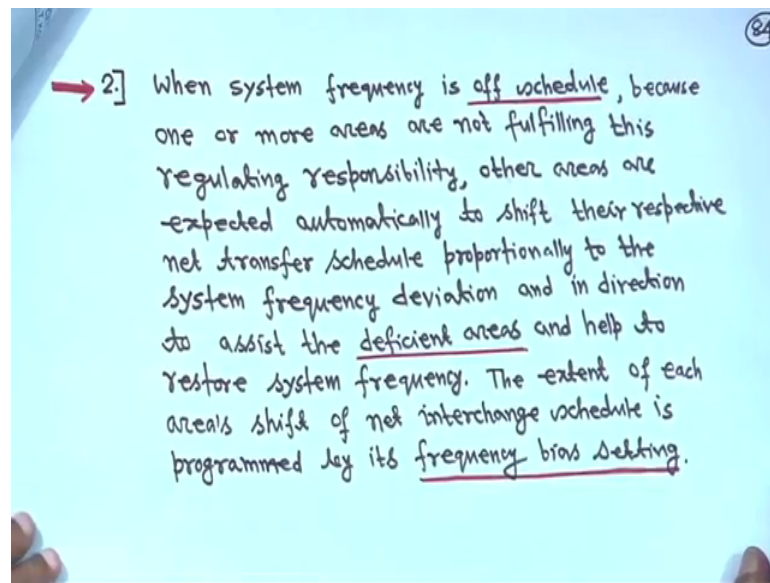


Suppose you have 4 different power system area. So, every areas have some contact with other area such that this much power one can I mean in general actually buying and selling power something like this. So, it is your what you call areas they have some buying or selling power like this right some area that is your what you call.

So, in that case if suppose at a particular time suppose when I am recording this one here time is and in front of my watch, it is 12:25 pm right at this at this at this time if I have this some power you know some contact with every as everything and if the same thing is maintained right then frequency is also maintained in schedule value right. So, as long as it is maintained, but all of a sudden if suppose suddenly whatever I had that what you call that scheduled power flow and load demand I know this one by chance if it is not suppose load demand has increase or decrease or something has happened in the system of the system is a stable system no fault nothing we are not considering any abnormal condition we are considering only normal operating condition right.

So, in that case what can happen that because of this because of this you are what you call that different things that load another things it known. So, all the tie line power flow is also known to us right, but if there is a change if there is a change in the load right then what will happen this power this frequency also will not be on its a schedule value whatever it is given same is for this power flow to the tie line right.

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So, in that case what will happen that second one is when system frequency off schedule right I mean it is not because one or more areas are not fulfilling this regulating responsibilities other areas are expected automatically to shift their respective net transfer schedule proportionally to the system frequency deviation.

And in direction to assist the deficient areas and help to restore system frequency the extent of each areas shift of net interchange schedule is programmed by its frequency bias setting that is you have we will it will see later that is your $b f c$ node d plus 1 upon r frequency response characteristic that adaptive equal b this is the frequency bias setting right. So, detail of detail of this thing cannot be your all these things cannot be done in the class as per as your classroom exercise is concerned I will see that, but this is you have to you are what you call regulating responsibilities of this you are what you call basic inter area regulating responsibilities of each control area right.

So, this is the idea. So, whatever these two things are shown here that we cannot show you where without what you call this thing as plus which size, but we go for simulation or coding at that time those things are easily be checked right the through simulation only.

(Refer Slide Time: 04:12)

(85)

→ Cooperative assistance between areas is one of the planned benefits of interconnected operation.

Ex-2:

A power system has a total load of 1260 MW at 50 Hz. The load varies 1.5% for every 1% change in frequency. Find the steady-state frequency deviation when a 60 MW load is suddenly tripped, if

(a) there is no speed control.

Next is now I told you many times the cooperative assistance between areas is one of the plan benefits of interconnected operation right. Now, we will take a take an example. We will come again that two area modeling other thing, but first look at this example.

A power system as a total load of 1260 megawatt 1260 megawatt at 50 hertz, the load varies 1.5 percent for every 1 percent change in frequency find the steady state frequency deviation when a 60 megawatt load is suddenly tripped if number one there is no speed control right.

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(b) The system has 240 MW of spinning reserve evenly spread among 500 MW generation capacity with 5% regulation based on this capacity. Assume that the effect of governor dead bands is such that only 80% of the governors respond to the reduction in system load.

Soln.

Total remaining load is $(1260 - 60) = 1200$ MW.

The damping constant of remaining load is

→ $D = \left(\frac{1.5}{100} \times 1200 \right) \times \left(\frac{100}{50 \times 1} \right) = \underline{36 \text{ MW/Hz.}}$

And second thing is the system has 240 megawatt of spinning reserve evenly spread among 500 megawatt generation capacity with 5 percent regulation based on this capacity right assume that the effect of governor dead band is such that only 80 percent of the governor respond to the reduction in system load right.

So, total remaining load will be 1260 minus 60 because your here it is given that your there is a your what you call 1 percent 5 when there is load suddenly tripped right, there is no speed or there is no the this 1260 total load was 1260 megawatt and system at 50 hertz load varies 1.5 percent for every 1 percent change in frequency and this is your find the steady state frequency deviation when a 60 megawatt load is suddenly tripped right; that means, your shift means that load has switched off right something like this. So, load is isolated; that means, load is load decrease, first thing is there is no speed your speed control first one and second one is the system as 240 megawatt of spinning reserve.

Evenly spread among 500 megawatt generation capacity with 5 percent regulation right. So, based on this assumed that the effect of governor dead band is such that only 80 percent of the governor respond to the reduction in system load therefore, total remaining load is 1260 minus 60. So, 1200 megawatt right this one the damping constants of remaining load is d is equal to your 1.5 upon 100 your into 1200 because it is given the load varies for 1.5 percent for every 1 percent change in frequency.

So, this is 1.5 upon your what you call 100 into 1200 into 100 upon 50 into 1 for simplification everything I have written for you right. So, basically it will be 1.5 by your 1 percent of frequency right. So, just written I mean all this thing in detail. So, it is this type of this type of problem we have seen before also. So, it is 36 megawatt per hertz right.

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→ (a) With no speed control, the resulting increase in steady-state frequency is

$$\rightarrow \Delta F_{ss} = \frac{-\Delta P_L}{D} = \frac{-(-60)}{36} = \underline{1.667 \text{ Hz.}}$$

→ (b) The total spinning generation capacity is equal to Load + Reserve = (1260 + 240) = 1500 MW.

Generation contributing to regulation is $0.8 \times 1500 = 1200 \text{ MW.}$

So, number with no speed control means the resulting increase in steady state frequency is $\Delta F_{ss} = \frac{-\Delta P_L}{D}$ because your no speed control means the speed regulation parameter was not there it is it was actually it was something like your oh if you ask you like you know open loop system something like this.

So, in that case r term will never come r term will never come. So, it will be minus ΔP_L upon D . So, it will be minus of minus 60 upon 36 because a load has decreased to 60 megawatt. That is why this minus sign is taken; that means, speed is frequency will increased speed will increase so minus of minus 60 upon 36. So, that is actually 1.667 hertz and second thing is the total spinning generation capacity is equal to the load plus reserve this should be in your mind right.

So, total spinning generation capacity is equal to load plus reserve. So, 1260 plus 240 that is 1500 megawatt right because it is given know 240 megawatts spinning reserve to your evenly speed among 500 megawatt generation capacity right. So, the total spinning generation capacity is equal to load plus reserve. So, load is 1260 plus 240; so 1500 megawatt. So, generation contributing to regulation it is given by governor your governor is responding to this thing that your governor dead band is such that dead band will not cover in this, but for numerical 6 we have taken this.

Only 80 percent of the governors respond to the reduction in system load right. So, generation contributing to regulation is 0.8 into because 80 percent it is so 0.8 to the 1500; so 1200 megawatt right.

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A regulation of 5% means that a 5% change in frequency causes a 100% change in power generation. Therefore,

$$\rightarrow \therefore R = \frac{0.05 \times 50}{1200} \text{ Hz/MW} = \frac{2.5}{1200} \text{ Hz/MW} = \frac{1}{480} \text{ Hz/MW}$$

$$\rightarrow \therefore \frac{1}{R} = 480 \text{ MW/Hz.}$$

The composite system frequency response characteristic is

$$\rightarrow \beta = D + \frac{1}{R} = 36 + 480 = 516 \text{ MW/Hz.}$$

Now, it is the problem in the given regulation of 5 percent means that a 5 percent change in frequency causes a 100 percent change in power generation therefore, R is equal to regulation parameter it is 5 percent into 50 divided by 1200 right this is your because generation contributed regulation is 1200 megawatt.

So, in that case R is equal to 0.05 into 50 that is 5 percent of the frequency is 50 hertz divided by 1200 hertz for megawatt that is why it is actually R will become 1 upon 480 mega hertz per megawatt this is actually 480 hertz per megawatt therefore, 1 upon R is equal to your sorry it is it is actually 1 upon 480 right hertz per megawatt therefore, 1 upon R is equal to 480 megawatt per hertz just thus near your reciprocal.

Therefore the composite system frequency response characteristic we know beta is equal to D plus 1 upon R. So, D we computed 36 just now here it is here it is your this thing or D value right D is a 36 megawatt per hertz here you have computed right and 1 upon R is 480 megawatt per hertz; so 36 plus 480, so 516 megawatts per hertz right. The compound is composite system frequency response characteristic is right.

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Steady-state increase in frequency is

$$\rightarrow \Delta F_{ss} = \frac{-\Delta P_L}{\beta} = \frac{-\Delta P_L}{(D+1/R)} = \frac{-(-60)}{516} \text{ Hz}$$
$$\rightarrow \therefore \Delta F_{ss} = 0.1162 \text{ Hz.}$$

Ex-3:

A 200 MVA generator operates on full load at a frequency of 60 Hz. The load is suddenly reduced to 20 MW. Due to time lag in governor system, the steam valve begins to close after 0.22 sec. Determine the change in frequency that occurs in this time. Given $H = 10 \text{ kW-sec/kVA}$ of generator capacity.

Now, steady state increase in frequencies ΔF_{ss} minus ΔP upon β actually minus ΔP about $D + 1$ upon R .

So, β we have calculated 516 so minus of minus 60 by 516 hertz. So, ΔF_{ss} will be 0.1162 hertz right. So, what you have seen once one you see there is no speed control right. So, in that case it was 1.667 hertz right when you consider everything at that time it is 0.1162 hertz right. So, this is another problem. So, example three, a 200 m v a is 200 m v a generator operates on full load at frequency of 60 hertz.

The load is suddenly reduced to 20 megawatt load has decreased due to the time lag in governor system the steam valve begins to close after 0.22 second determine the change in frequency that occurs in this time given H is equal to given 10 kilowatt second per k v a; that means, H is equal to 10 second actually because kilowatt and k v a it will be dimensionless kilowatt upon k v a will not be there. So, kilowatt upon k v a means it is a dimensionless quantity only second will be remaining H actually nothing, but 10 second if it is given like that also does not matter. It is 10 second right of generating generator capacity.

So, you have to value the change in frequency that occurs in this time and these are the data given.

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$\rightarrow$ Stored Kinetic energy = $10 \times 200 \times 1000 \text{ kW-sec}$
 $= 2 \times 10^6 \text{ kW-sec.}$

$\rightarrow$ Excess power input to generator before the steam valve begins to close = 20 MW.

$\rightarrow$ Excess energy input to rotating parts in 0.22 sec = $20 \times 1000 \times 0.22 = 4400 \text{ kW-sec.}$

$\rightarrow$ Stored Kinetic energy is proportional to the square of frequency.
Frequency at the end of 0.22 sec

$\rightarrow = 60 \times \left(\frac{2 \times 10^6 + 4400}{2 \times 10^6} \right)^{1/2} = 60.066 \text{ Hz.}$

So, stored kinetic energy you know 10 into 200 into 1000 kilowatt. The second right it was given that it is 200 m v a generator operation full load at friction load is suddenly used 20 megawatt due to time lag and governor system. So, this everything is converted to your what you call kilowatt seconds. So, it is stained into 200 into 1000 is kilowatt second right stored kinetic energy it is equal to 2 into 10 to the power 6 kilowatt second right.

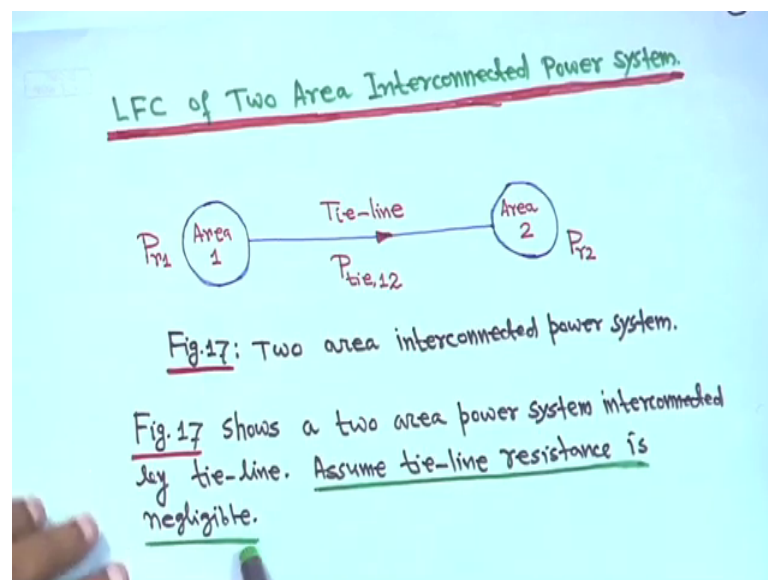
So, second thing is your this thing your what you call if excess power input to the generator before the steam valve begins to close 20 megawatt. So, this is that 20 megawatt data is given here right therefore, excess energy is input to the rotating parts in 0.22 second is 20 into 1000; so converted to kilowatt into 0.22. So, 4400 kilowatt second right therefore, store kinetic energy is proportional to the square of frequency therefore, frequency at the end of 0.22 second will be 60 into 2 into 10 to the power 6 2 into 10 to the power 6 plus 4400 divided by 2 into 10 to the power 6 to the power half right.

Because it is called frequency at the n square of you are what you call kinetic energy proportional to the square of the frequency. So, frequency at the end of this one it will be these 60 into divided 60 is the frequency right. So, it is coming actually 60.066 hertz right. So, this is your idea right. So, this one it is actually you take for your there should not be any confusion you take 200 megawatt generator right. So, such that this 200 into 1000 kilowatt right and h is given 10 second h is 10 second. So, it is p r into h. So, rated

capacity actually it is rated capacity right, so you this 200 megawatt not mva; so it is 10 into 200 into 1000 kilowatt second.

So, this is a simple thing I mean as far as this what you call this problem is concerned. So, frequency is increase 2.0 0.066 hertz right; so slightly above your nominal frequency. So, more problems we will see let me finish this chapter after that 3 4 problems.

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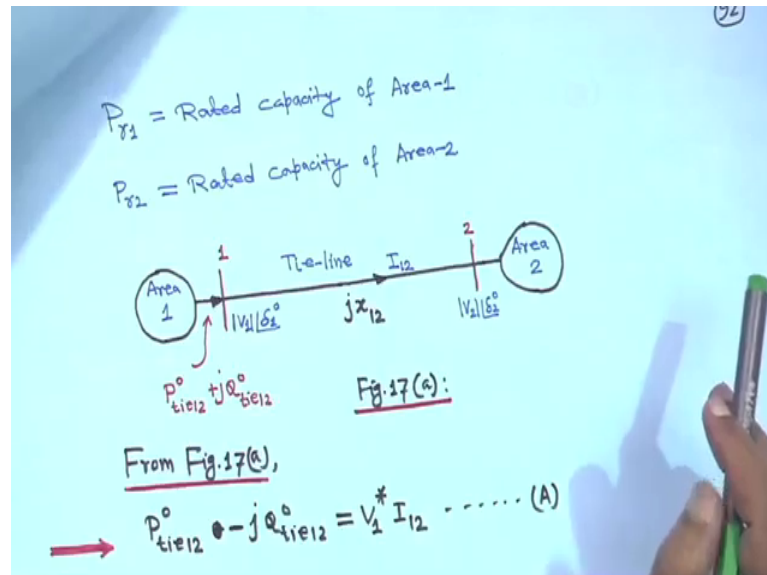
I will take for you right. So, next one is the L F C of two area interconnected power system right. Suppose you have a two area interconnected power system.

So, in this case just I told you suppose this is area one this is area two and what is control area I will say if it is the tie line, tie line is nothing, but a when I have to call three phase tie lines and the real power flow in $P_{tie,12}$ our inter system of real power we assume later we will see that this tie line in the resistance is less I mean negligible compared to the reactance. So, resistance part will neglect right. So, a rated area capacity of this one real power in terms of the real power P_{r1} an area rated capacity of this one is P_{r2} right and these two power system actually are interconnected equal area one and area two.

So, it is L F C of two area interconnected power system right. So, figure 17 this is figure 17 it shows a two area power system interconnected by tie line assume tie line resistance

is negligible. We are not considering the tie line resistance right. So, this one this like this is a schematic diagram right this one we can write in little bit better way right.

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Suppose if P_{r1} is rated capacity of area one and P_{r2} rated capacity of area two, now this is two. So, this side to bar bars are there right, this is one, this is two. This tie line current flowing from one to two, say for example, say current is flowing from 1 2, it may be two to one also no problem right what just we have taken here I_{12} this is the current.

A resistance of this tie line it is tie line means you are assuming the balancing and it is a three phase transmission line and you know for transmission line resistance is much much smaller than the reaction. So, r is neglected, but if you consider r also model can be developed let me tell you if you consider also. So, in that case whatever model will show you to be slightly different, but will not consider that in this course and another thing is that as soon as you considered that actually you will find response is slightly better right and in terms of your what you call settling time because as soon as you consider r resistance in line and you know resistance actually add damping when you are you will not consider the resistance r .

So, in this case in this case sub suppose here load maybe there I am no I am not shown here, but here load is p_{l1} one here load is p_{l2} right anyway this barber voltage is magnitude v_1 initial angle was δ_1 and it is barber voltage magnitude V_2 angle is δ_2 right. So, and this and this power coming into this point into this point it is

initially $P_{12}^0 + jQ_{12}^0$ right that is that schedule power that is initially right. Your real power flowing from 1 to 2 and this is a reactive power in this direction say 1 to 2 if it is not, it will reverse a resistance is neglected.

So, from your figure, this is your mask as a figure 17 a right therefore, you know that from load flow studies you know in general $P - jQ$ is equal to V conjugate I . So, same equation we will right here P_{12}^0 it is a superscript 0 means that in nominal value or schedule value right minus jQ_{12}^0 is equal to V_1 conjugate I_{12} because this is the current flowing to this is I_{12} and this is a voltage your V_1 actually in general phasor quantity.

So, magnitude is V_1 magnitude V_1 angle δ_1^0 we are writing $P_{12}^0 - jQ_{12}^0$ is equal to V_1 conjugate I_{12} this is equation a I have given this equation a right now I_{12} the current flowing from this end to this end is equal to $V_1 - V_2$ upon jX_{12} r is neglected r_{12} is neglected; that means, I_{12} is equal to $V_1 - V_2$ upon jX_{12} .

(Refer Slide Time: 17:46)

and

$$\rightarrow I_{12} = \frac{V_1 - V_2}{jX_{12}} = \frac{|V_1|\angle\delta_1^0 - |V_2|\angle\delta_2^0}{jX_{12}} \dots (B)$$

From Eqm. (A) and (B), we get

$$\rightarrow P_{12}^0 - jQ_{12}^0 = |V_1|\angle\delta_1^0 \left[\frac{|V_1|\angle\delta_1^0 - |V_2|\angle\delta_2^0}{jX_{12}} \right]$$

$$\rightarrow \therefore P_{12}^0 - jQ_{12}^0 = \frac{|V_1|^2 - |V_1||V_2|\angle\delta_2^0 - \delta_1^0}{jX_{12}}$$

So, V_1 is a magnitude V_1 angle δ_1^0 minus magnitude V_2 angle δ_2^0 divided by jX_{12} this is equation b from equation a and b we get. So, equation a and b what we will get that $P_{12}^0 - jQ_{12}^0$ is equal to V_1 conjugate into I_{12} this I_{12} here you substitute this I_{12} value this one you substitute we are doing it and multiplied.

If you multiply you will get $P_{tie12} - jQ_{tie12}$ equal to magnitude this one you have substituted. So, it is V_1 conjugate. So, it is magnitude V_1 angle minus delta 10 the magnitude V_1 angle delta 10 minus magnitude V_2 angle delta 20 divided by $j \times 12$ then you multiply if you multiply it will become magnitude V_1 square this delta 10 minus delta 10 it will become angle 0. So, anyway V_1 square minus magnitude $V_1 V_2$ angle delta 20 minus delta 10 upon $j \times 2$ right.

Then what you do you numerator and denominator you have to you know multiplied by j and then you separately an imaginary part. So, what we will do is that.

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$$\begin{aligned} \Rightarrow P_{tie12} - jQ_{tie12} &= \frac{|V_1|^2 - |V_1||V_2|\cos(\delta_2 - \delta_1) - j|V_1||V_2|\sin(\delta_2 - \delta_1)}{jX_{12}} \\ \Rightarrow P_{tie12} - jQ_{tie12} &= \frac{j[|V_1|^2 - |V_1||V_2|\cos(\delta_2 - \delta_1)] + |V_1||V_2|\sin(\delta_2 - \delta_1)}{-X_{12}} \\ \Rightarrow P_{tie12} - jQ_{tie12} &= \frac{|V_1||V_2|\sin(\delta_1 - \delta_2)}{X_{12}} - j\left[\frac{|V_1|^2 - |V_1||V_2|\cos(\delta_2 - \delta_1)}{X_{12}}\right] \quad \dots (c) \\ \Rightarrow P_{tie12} &= \frac{|V_1||V_2|\sin(\delta_1 - \delta_2)}{X_{12}} \quad \dots (d) \end{aligned}$$

This one $P_{tie12} - jQ_{tie12}$ is equal to V_1 square minus magnitude V_1 magnitude V_2 then this one you make it cost your angle theta is equal to cos theta plus j sin theta. So, same way you make it here right minus magnitude V_1 magnitude V_2 cosine delta 2 minus delta 10 then minus j magnitude beyond magnitude V_2 sin delta 20 minus delta 10 right or you multiply numerator and denominator by j . So, $P_{tie12} - jQ_{tie12}$ multiply numerators denominators. So, $j \times j$ square. So, it will be minus and here j is here rest of the thing in the bracket is the same thing here same thing here.

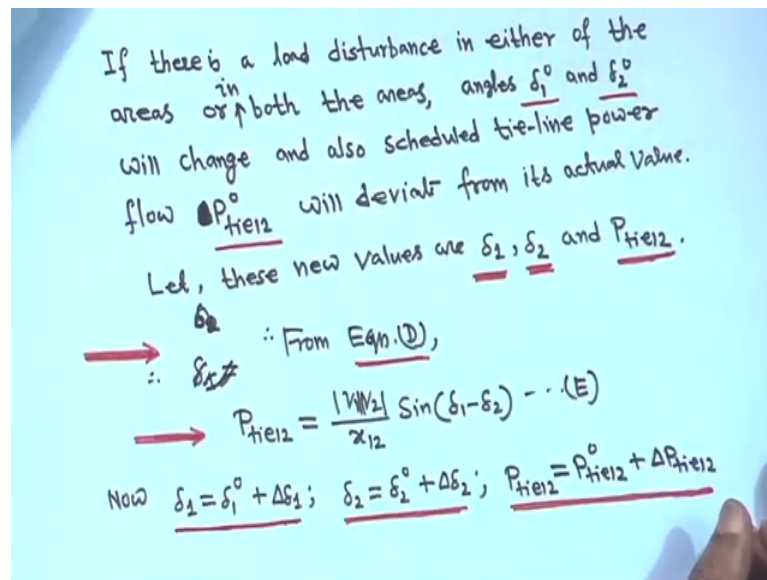
Now, this now this equation you multiplied by j then it will be like this $P_{tie12} - jQ_{tie12}$ here is equal to magnitude V_1 upon man magnitude V_1 into magnitude V_2 upon X_{12} sin delta 10 minus delta 20 right minus j V_1 square in bracket minus magnitude V_1 minus magnitude will cos delta 21 delta 20 minus delta 10 upon X_{12} ; so

this one as soon as you multiply by this equation by j as soon as you multiply this equation by j . So, it will become your what you call that your j is this $V_1 V_2 \sin \delta_2 - \delta_2 \sin \delta_1$ right now if you just if you just numerator and denominator by your you call minus minus divide by minus 1 say then in that case this will minus, but then instead of $\delta_2 - \delta_1$ we have taken that $\sin \delta_1 - \sin \delta_2$ ultimately look plus that why this is plus right and this, but; that means, our of our thing is not q flow or maybe only real power flow. So, P_{12} actually $V_1 V_2 \sin \delta_1 - \delta_2$ this is equation c and this is equation d right.

Now, this is that nominal power flow; that means, P_{12} this P_{12} which is flowing it is magnitude V_1 magnitude $V_2 \sin \delta_1 - \delta_2$ right this is the power q . We are not interested our interest is only the real power right and let me tell you one thing as the line resistance is neglected; that means, whatever power is here P_{12} . So, assuming that power is flowing in this direction whatever P_{12} is here it will be same as here because assuming line is lossless if line is lost in the line is a lossy line I mean if you consider r then whatever power will be here it will not be the same power here because that that loss will be there according to the direction of the power flow that this loss has to be added are your what you call subtracted from this value from P_{12} .

But in this case resistance r is neglected for this line therefore, whatever power will be there P_{12} you want to be written in same in here right I think it is understandable to you. So, in this case your P_{12} is equal to this one.

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So, next what we will do suppose there is some disturbance in the load some load disturbance is there because load is here I have not shown in the diagram, but load is here load is here for example, I am making it here suppose that load is here load is here load is here.

So, in that case your what you call this side this side P 11 and this side P 12. So, suppose there is some disturbance in the load it may be either of a either a barrier if the load disturbance is there then what will happen they then there will be change in the tie line power flow there will be change in the voltage angle also, but in a real power change we are assuming the load real power change and earlier you told real power changes then frequency hence the voltage angle may change right will change, but question is that a voltage magnitude will not change it will remain as easy as it is. So, based on this based on this our this philosophy right what we can do is.

Now, suppose if there is a load disturbance either of the areas are in both the area I mean this terminal may be here or maybe here or both the area simultaneous it can happen right. So, in that case the angles δ_1 superscript is 0 and δ_2 is 0 will change because real power changes we are assuming load disturbance means if you are assuming only real power not reactive power in this case suppose real power changes right in reality both can change, but we are assuming small perturbation only real power change

in that case the δ_{10} and δ_2 also change also first tie line power flow I told you this one earlier it was schedule powerful P tie 120.

Now, we will deviate from its actual value because earlier some load whether some load whether some power was flowing to a tie line, but as soon as load disturbance happen this tie line power flow also will change along with that δ_{20} δ_{10} will change because δ_{10} δ_2 the frequency this earlier this your what you call I told you several time right. So, δ_{10} and δ_{20} also later we will see how it can be relate to the frequency later we will see right.

So, this will change if it is. So, above change in small changes if it is so; that means, now the new values are δ_1 δ_2 and say P tie 12 these are new values; that means, from this one from equation d that is $p_{tie\ 12} = V_1 \text{ magnitude} \times V_2 \text{ magnitude} \times \sin(\delta_1 - \delta_2)$, earlier we are writing earlier we are writing this one that whatever you got that $P_{tie\ 120}$ rather is equal to magnitude V_1 magnitude we have upon $X_{12} \sin(\delta_{10} - \delta_{20})$, but because of these load changes now the new value has become that P tie 12 is equal to magnitude V_1 magnitude by X_{12} because V_1 V_2 will be unaffected $\sin(\delta_1 - \delta_2)$ this equation e right.

Because this is new value δ_1 δ_2 and P tie 12 because of your what you call load changes therefore, now δ_1 actually is equal to δ_{10} plus $\Delta\delta_1$ initially it was δ_{10} now because of the $\Delta\delta_1$ change this is δ_1 similarly δ_2 initial it was δ_{20} , but because of this $\Delta\delta_2$ change δ_2 is equal to δ_{20} plus $\Delta\delta_2$ right initially the P tie 120. Now because of this load disturbance it has happened. So, P tie 12 is equal to x P tie 120 because this change $\Delta p_{tie\ 12}$.

(Refer Slide Time: 25:25)

$$\begin{aligned} \therefore P_{tie12}^0 + \Delta P_{tie12} &= \frac{|V_1||V_2|}{X_{12}} \sin(\delta_1^0 + \Delta\delta_1 - \delta_2^0 - \Delta\delta_2) \\ \therefore P_{tie12}^0 + \Delta P_{tie12} &= \frac{|V_1||V_2|}{X_{12}} \sin((\delta_1^0 - \delta_2^0) + (\Delta\delta_1 - \Delta\delta_2)) \\ \therefore P_{tie12}^0 + \Delta P_{tie12} &= \frac{|V_1||V_2|}{X_{12}} \left\{ \sin(\delta_1^0 - \delta_2^0) \cos(\Delta\delta_1 - \Delta\delta_2) \right. \\ &\quad \left. + \cos(\delta_1^0 - \delta_2^0) \sin(\Delta\delta_1 - \Delta\delta_2) \right\} \quad \dots (F) \end{aligned}$$

Assumptions:

- $\Delta\delta_1 - \Delta\delta_2 \Rightarrow \text{very very small}$
- $\cos(\Delta\delta_1 - \Delta\delta_2) \approx 1.0$ and $\sin(\Delta\delta_1 - \Delta\delta_2) \approx \Delta\delta_1 - \Delta\delta_2$

All this thing rather than taking direct perturbation for your understanding what I have done I have made it details such that will understand all these things you put it here put in this equation; that means, delta 1 you substitute delta 10 plus delta delta 2 you substitute delta the 0 plus delta delta and P tie 12 is equal to P tie 120 plus del you put it here you put it here then you can write P tie 120 plus delta P tie 2 is equal to V 1 magnitude V 1 V 2 upon X 12 a sin of delta 10 plus delta delta 1 minus delta 20 minus delta delta 2 or you can write left hand side will remain same magnitude V 1 V 2 upon X 12 sin delta 10 minus delta 20 plus delta delta 1 minus delta delta 2.

Like or you can write P tie 120 plus delta P tie 12 is equal to magnitude V 1 magnitude V 2 upon X 12 then it is your sin a plus b formula. So, sin a cos b plus cos a sin b this way you write right. So, it is sin delta 10 minus delta 20 into cos delta delta 1 minus delta delta 2 plus cos delta 10 minus delta 20 into sin delta delta 1 minus delta delta 2 this is equation f now you have to make some simple assumption, assumptions are delta delta 1 minus delta delta 2 is very very small right.

So, we can write then cosine of delta delta 1 minus delta delta 2 approximate equal to 1 and you know for small theta if theta is very small then sin theta is equal to theta we can write right. So, here sin of delta delta 1 minus delta delta 2 you can write directly delta delta 1 minus delta delta 2 these 2 are assumptions right because our objective is that we have to make it a linearized model right.

So, if it is. So, then here you put cost this cos delta delta 1 minus delta delta is 1 and sin delta delta 1 minus delta delta 2 is equal to delta delta 1 minus delta delta 2 to put it here all this you put it here.

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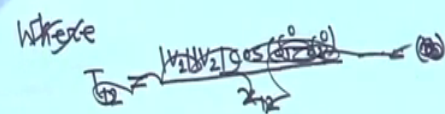
$$\begin{aligned} \Rightarrow P_{tie12}^0 + \Delta P_{tie12} &= \frac{|V_1||V_2|}{X_{12}} \left[\sin(\delta_1^0 - \delta_2^0) + (\Delta\delta_1 - \Delta\delta_2) \cos(\delta_1^0 - \delta_2^0) \right] \\ \Rightarrow P_{tie12}^0 + \Delta P_{tie12} &= \frac{|V_1||V_2|}{X_{12}} \sin(\delta_1^0 - \delta_2^0) + \frac{|V_1||V_2| \cos(\delta_1^0 - \delta_2^0)}{X_{12}} (\Delta\delta_1 - \Delta\delta_2) \\ \Rightarrow P_{tie12}^0 + \Delta P_{tie12} &= P_{tie12}^0 + \frac{|V_1||V_2| \cos(\delta_1^0 - \delta_2^0)}{X_{12}} (\Delta\delta_1 - \Delta\delta_2) \\ \Rightarrow \Delta P_{tie12} &= \frac{|V_1||V_2| \cos(\delta_1^0 - \delta_2^0)}{X_{12}} (\Delta\delta_1 - \Delta\delta_2) \end{aligned}$$

If you do. So, $P_{tie12}^0 + \Delta P_{tie12}$ is equal to $V_1 V_2$ upon X_{12} sin of delta 10 minus delta 20 plus delta delta 1 minus delta delta 2 into cosine delta 10 minus delta 20 or you can write $P_{tie12}^0 + \Delta P_{tie12}$ is equal to magnitude V_1 magnitude V_2 upon X_{12} into this term sin delta 10 minus delta 20 plus your magnitude V_1 magnitude V_2 upon X_{12} into this term cos delta 10 minus delta 20 into delta delta 1 minus delta delta 2 or $P_{tie12}^0 + \Delta P_{tie12}$ this term earlier we have written know P_{tie12}^0 actually magnitude V_1 magnitude V_2 upon expand 2 sin delta 10 minus delta 20.

This term this term is equal to P_{tie12}^0 plus this additional term has come magnitude beyond magnitude V_2 cosine delta 10 minus delta 20 upon X_{12} into delta delta 1 minus delta delta 2. So, $P_{tie12}^0 + \Delta P_{tie12}$ both side will be cancelled and we will get small perturbation the expression of delta p I p_{tie12} is equal to this one this one this one will be canceled right.

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where



$$\Delta P_{tie12} = \frac{|V_1||V_2|\cos(\delta_1^0 - \delta_2^0)}{x_{12}} (\Delta\delta_1 - \Delta\delta_2) \dots (22)$$

$$\therefore \frac{\Delta P_{tie12}}{P_{r1}} = \frac{|V_1||V_2|\cos(\delta_1^0 - \delta_2^0)}{P_{r1} x_{12}} (\Delta\delta_1 - \Delta\delta_2)$$

$$\therefore \Delta P_{tie12} (pu) = T_{12} (\Delta\delta_1 - \Delta\delta_2) \dots (23)$$

That means delta P tie 12 right is equal to V 1 V 2 cosine delta 10 minus delta 2 upon x 12 into delta delta 1 minus delta delta 2 right.

Next is both sides because this is power flow in from 1 to 2. So, both side what you can do is this is your related capacity of area on P r 1. So, both sides that in power is flowing from your this thing from 1 to 2 and rated capacity of this side is P r one this is your P r 1. So, both sides the divided by P r 1 if you divide by P r 1 delta P tie 2 upon P r one is equal to V 1 V 2 cosine delta 10 minus delta 20 upon Q r on P r 1 into x 12 into delta delta 1 minus delta 2 or this is a converted to per unit delta P tie 12 per unit this term we are defining as whole thing as T 12 into delta.

We will come to that what is this into delta delta 1 minus delta delta 2 this is equation 23.

Thank you.