**Power System Engineering**
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**Lecture - 48**
**Load frequency control (Contd.)**

So this equation we have seen this is equation 8. Now another thing is that it is assumed that change in load is sensitive with the speed at frequency variation, we are assuming the load is frequency sensitive it is not insensitive to the frequency, we are assuming that load is frequency sensitive right. However, for small changes in system frequency delta f because, delta f change is very small right that. The rate of change of load with respect to frequency that is your del P L upon delta f can be regarded as constant right, then delta P L delta p f will regarded as a it can be regarded as a constant.

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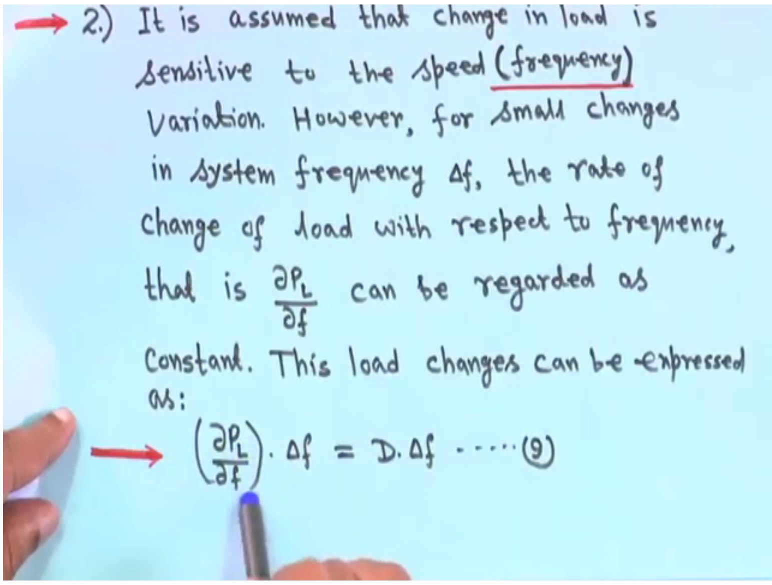


Although we are, but we are assuming that load is sensitive to the change in frequency therefore, this load changes can be expressed as your delta P L upon delta f into delta f this we are taking as a constant and, we are assuming that due to this is due to the your very small change in delta f, this is very small into delta f and, these we define actually d into delta f this is equation 9. If you do not consider that D is equal to 0 later we will see, but we will assume that load is sensitive. So, change in frequency.

So, in that case delta P L upon delta p f can be regarded as a constant right how we will take that later will come later, we will take numerical small example will come right.

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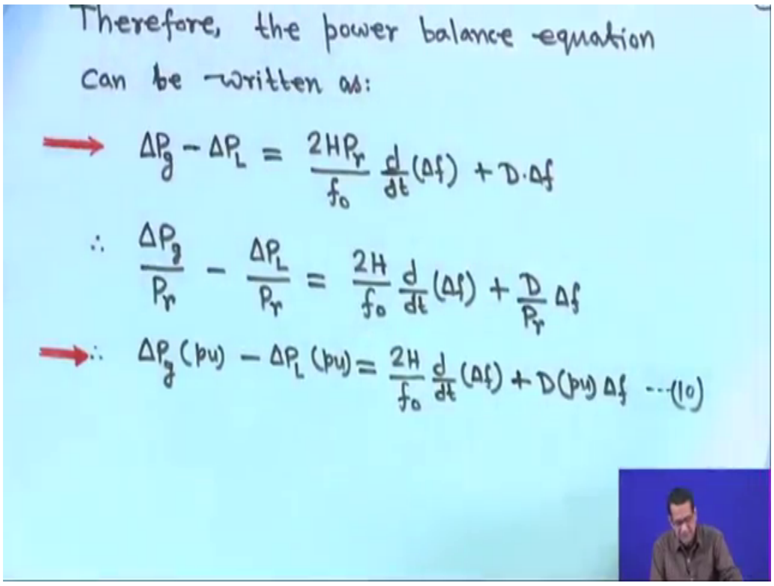


So, this is your D into delta f therefore, the power balance equation therefore, the power balance equation this one delta P g minus delta it has 2 terms, 1 is the rate of change of kinetic energy that is 2 H, you know rate of change of kinetic energy means pi or what we will call the power. So, 2 H P R upon f 0 d dt upon delta f plus that d d into delta f term the previous equation; that means, this term this term right.

So, look we will take Laplace transform on both sides later, but except using S in that your (Refer Time: 02:07) time constant. Nowhere I will put delta P g S or delta P L S or your delta f s, it is understandable to you see. If we put everywhere S it will be you know little bit clumsy; so, I do not want that. Now what do you do this equation both side you divide by P R right both side P r. So, this is delta P g upon R minus delta P L upon R this P R you divide is gone. So, it is 2 H upon f 0 d dt upon delta f plus D upon P R into delta f.

So, it if you assume this is a base value then delta P g upon it is megawatt of course, delta P g upon delta it is in per unit minus delta P L it is also per unit then 2 H upon f 0 d d upon this delta f is in hertz right this delta remember hertz f 0 is hertz 18 second here no unit change it is in hertz. And D will be your what you call in per unit because this D is equal to your delta P L upon delta f and you are dividing it by P r.

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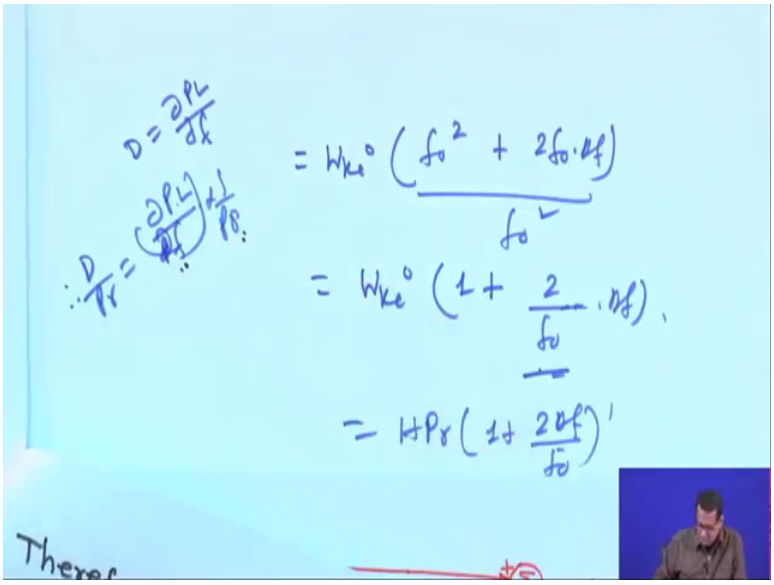

So, delta P R will be delta P L upon your P R delta P L upon delta f right into 1 upon P r. So, delta P L upon this P L delta P L P R it will become per unit that is why this and this, but this will remain hertz frequency will be hertz right; that means, unit of D will be per unit megawatt per hertz. So, that is why this D here is written in per unit; so, d is also per unit into delta f this is written right this is equation 10.

So, next is your that; that means, your next one just hold on therefore, if you if you take the Laplace transform both this after this; this delta P g, delta P L D all is in per unit. So, again and again this per unit will not be writternable, but equation 10 onwards; all these values unless and until I mention all are in per unit only right.

But again and again I will not write per unit then things will become clumsy; I do not want that, but equation 10 onwards, everything is in per unit and frequency of course, is in hertz, H will remain second, f 0 also will remain in hertz right, but this delta P g value delta delta P L value D all are in per unit right. So, again and again I will not write, but understandable.

So, if you and every time I will not put delta P g S delta P L S or delta fs right because it is Laplace transform understandable; such that in mathematics writing will be easier right so, but it is understandable. So, you take the Laplace transform of this equation both side you take the Laplace transform; if you can simplify you take the Laplace transform and then you simplify if you if you if you do so.

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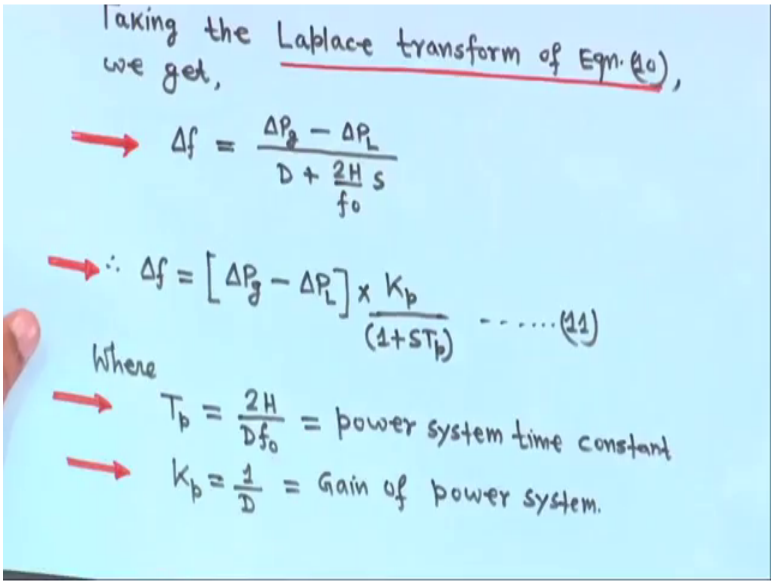


If you do so, then you will get delta f is equal to delta P g minus delta P L upon D plus 2 H upon f 0 into S this you know now right. So, because if you take that this equation that this is a Laplace transform it will be S into initial conditions are 0 you assume right; S into your delta f s, but S I mean bracket you are what to call not considering then again and again for all the variables I have to put; so, you do not want that right. So, it is your 2 H upon f 0 S into delta your f and this will be D d into this thing all these things in take the Laplace transform and simplify right.

So, if you do so, it will become del f will become delta P g minus delta P L divided by d plus 2 H upon f 0 into S; this way it will come. Or you can what you can do is this one you further simplify; that means, this how this thing is coming that delta f is equal to delta P g minus delta P L into K p upon 1 plus S T p how things are coming from this equation. Numerator and denominator you divide it by D right if you I mean let me write for you.

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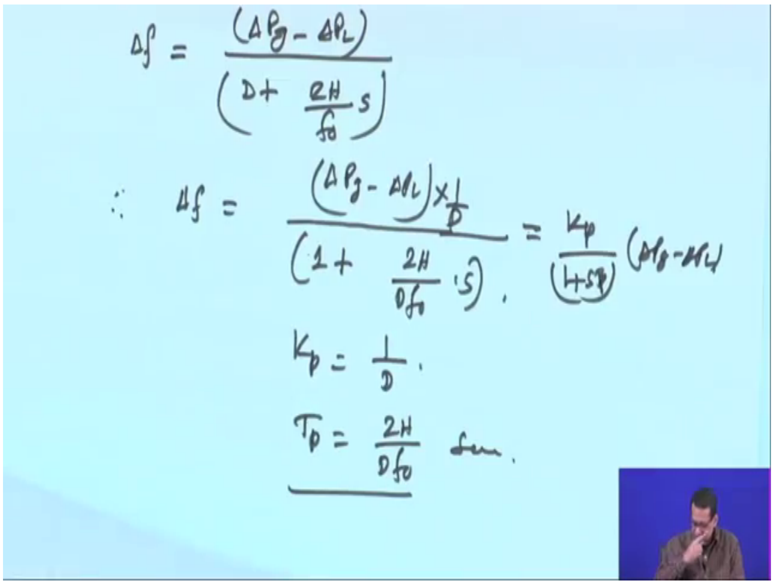

This is actually the equation delta f is equal to delta P g minus delta P L divided by D plus 2 H upon f 0 S this is the equation. Numerator and denominator you divided by D then delta f will be is equal to delta P g minus delta P L into 1 by d right divided by D divide it will be 1 plus 2 H divided by your D f 0 into S right. You define K p is equal to 1 upon D and T p, you call T p is the power system time constant it is 2 H upon d f 0 this will be in second right.

So, if you define this then this equation will become K p upon 1 plus S T p into delta P g minus delta P L; just hold on this is your K p upon 1 plus S T p into delta P g minus delta P L right. So, same thing just I have written left hand side here it is your right hand side. So, this delta f is equal to delta P g minus delta P L into K p upon 1 plus S T p; this is equation 11.

So, here T p is equal to your power system time constant 2 H upon D f 0 and K p is equal to 1 upon is equal to we call sometimes gain of power system. So, with this governor transfer function we got, turbine got and this generator we got right. This is actually sometimes we call this is power system transfer function or generator transfer function right.

Once you get it; then you combine governor turbine and generator because here we have also related everything delta P g we have related to delta f of course, delta P L is there right. So, this one; this one if you represent in a small block diagram; so, this is delta P g.

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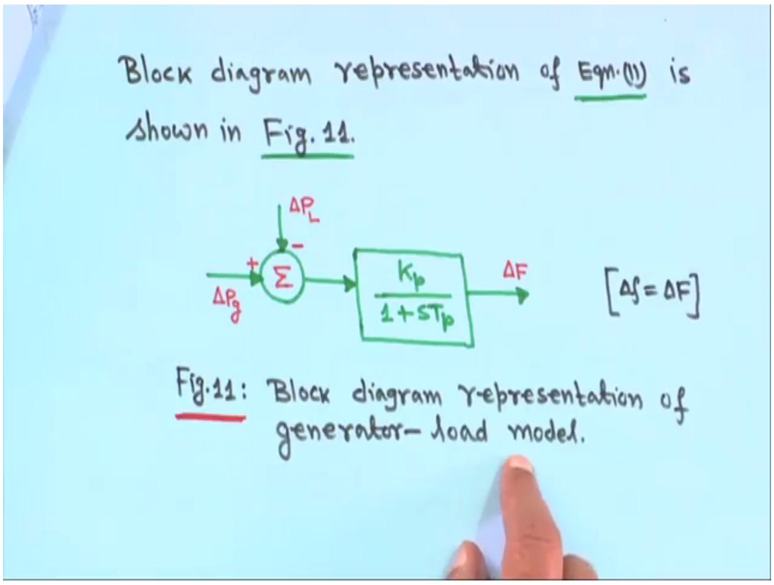


And this is delta P L delta P L is minus sign delta P g plus because delta P is minus into this one then the block diagram K K p upon 1 plus S T p that is delta f; small delta f capital they are same; I have written here right.

This is the block diagram represent our generator load model; sometimes we call this part we call sometimes power system model also right; so, this part. So, delta P g we have related with the turbine your what you call turbine power output right delta P g; then that one also related to delta E governor output. And then finally, delta f was the input and u write; so, all these things we have made it.

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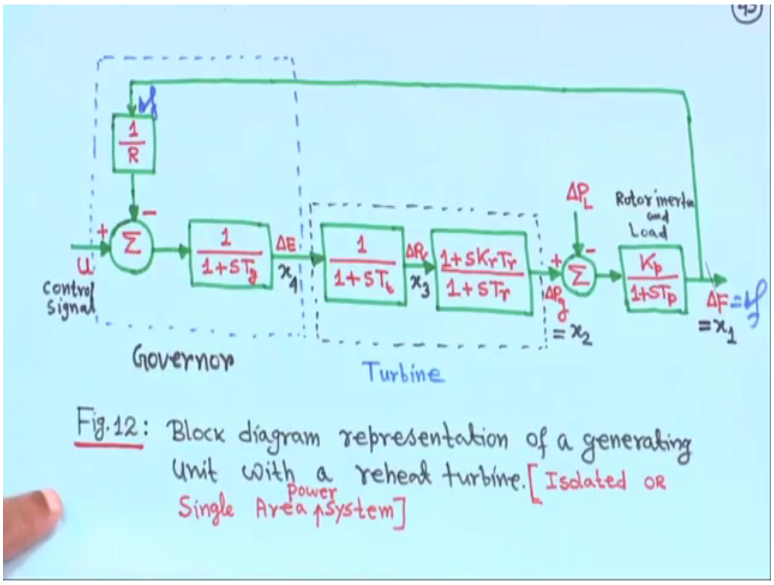


Now if you combine; now, it will be like this because this block this governor one you got it know u minus del f R upon this one this is delta E. Let me just one minute just you go now one after another right. Then just see this part this is the governor part, this we have made it just a hold on I will show you I will show you just hold on.

This is this is your speed regulation parameter R just hold on these such that you will have a good feeling about this that how we actually develop the model right just hold on if just if I can make it once again here right. So, this is your turbine part first let me show you, this is your turbine part this is your turbine part 1 plus S K S K r upon 1 plus S T T into 1 plus S T r delta P g and delta E E is the input delta P g is the output. So, this is delta P g is the output, delta E is the input and this term is added here right.

So, these 2 terms are there; so, you have broken it right because of that state variable form. So, this is actually in between one variable is added delta P g right, but this is the 2 and delta and this one just now we have made it no delta f smaller or capital does not matter delta P g minus delta P L into this one just now we have made it. And this one already we have made it no delta E into u minus 1 upon R 1 upon 1 plus S T g and this is input was delta f also in that block diagram it was shown you write all these things you just make it and just add it no it is completing a closed loop frequency or closed loop your what you call transfer function.

So, just 1 minute if I get it to immediately I will show you just 1 minute if it is here or not just 1 minute if I get it I will show you this is your turbine model; this is just hold on if I get it I will show you again just hold on. Here this speed this one we took that speed changer was not shown were speed changer was not shown here.

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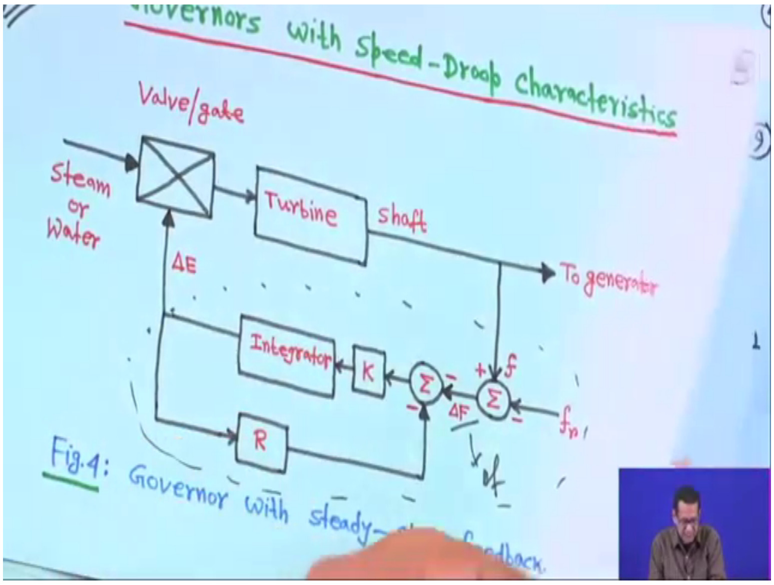


But somewhere I made a speed changer also and this one is a final block diagram; somehow it has been missed somewhere here right.

But this already we have derived u minus I derived for u also here I derived for u this one this one I have derived for u delta E is equal to u minus delta f it is in front of me actually right into 1 upon this one also derived this one also derived. So, this was also clubbed together and its input was delta f now, in the input was delta f.

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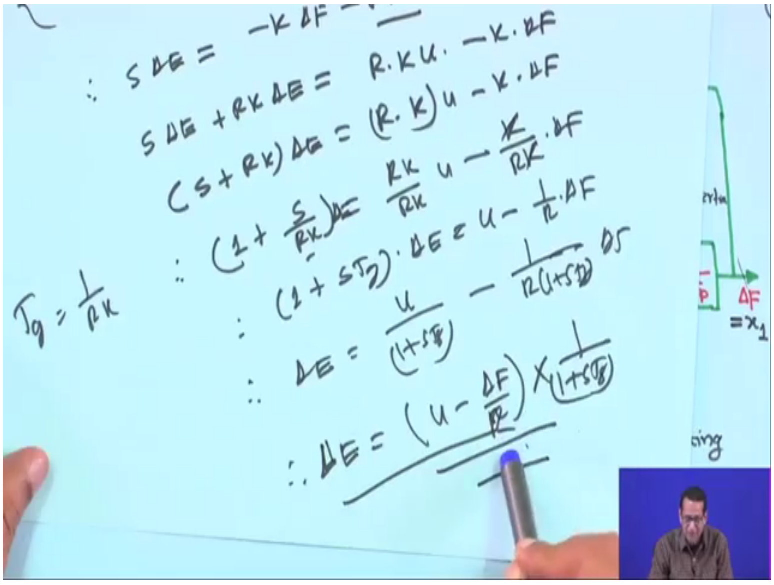


So, this delta f; this delta f is equal to same thing small delta f right. So, this is your delta f same thing; so, this is coming here this delta f actually coming from here. So, this is a closed loop system; so, with this is a block diagram representation of a generating unit with a reheat turbine it is isolated or single area power system what is area will come later.

So, this is actually what you call that isolated power system; this is governor, this is turbine and this is your power system. So, this is actually I have written rotor inertia load right, this is this dashed portion is turbine and this portion is governor and this is u is your control signal actually. So, now, what we will do this with this one we have got this block diagram. Now what we will do is that we have to what you call we have to go for state press representation and then we will see the steady state error and other things.

So, what you do? This we have to make it in the form of x dot is equal to ax plus bu right that way you have to make it. So, this one we have taken out this output is taken as x 1 right, this delta P g is taken as x 2 in between because you have to make it a state variable this output of this one you have taken as x 3 and this one output of this governor time (Refer Time: 13:15) this is x 4 right and this is u this thing. So, we have to write down and assuming that all the initial conditions are 0. So, we have to write down x 1 dot, x 2 dot, x 3 dot and x 4 dot.

And I have to put we have to put it in the state variable form right. So, what we will do is that in the state place representation this all these things are shown here. Delta has been taken as x 1, delta P g taken as x 2 this I have given a name delta P V x 3 and delta E as x 4 this 4 state variables right.

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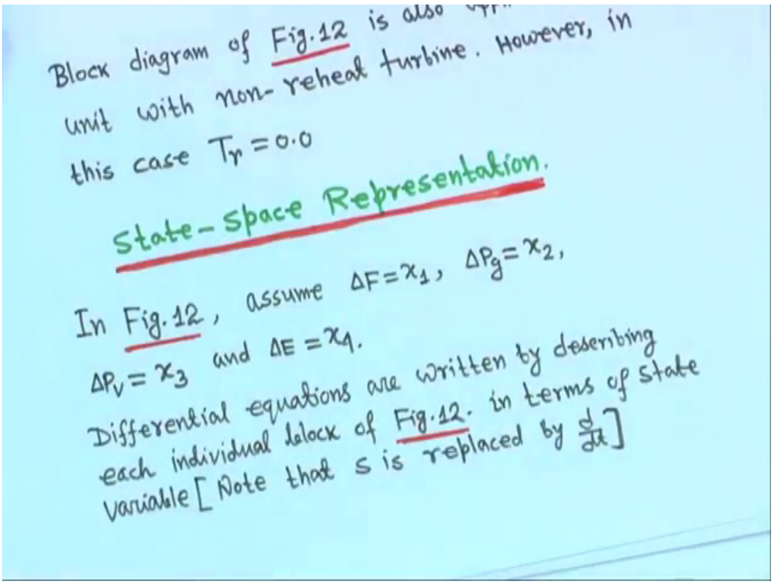


So, all these things are given here delta f x 1, delta P g is actually x 2 delta P V x 3 and delta E x 4. So, differential equation are written by your describing each individual block of figure 12, this is figure 12, this is figure 12 right in terms of state variable note that S is replaced by d dt because it is in a Laplace transform. So, what will do that you write all the equations x 1 dot, x 2 dot all I have written here x 1 dot, x 2 dot, x 3 dot, x 4 dot.

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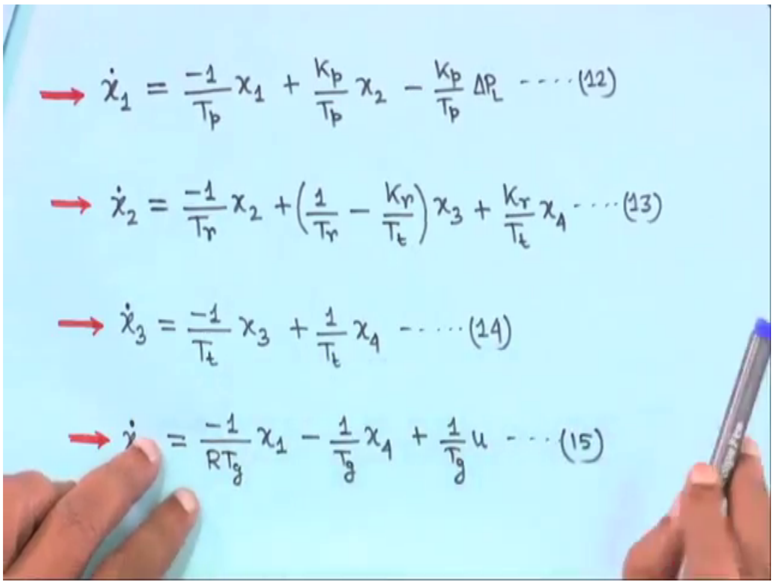


But that is I have written here, but I am writing here for you write such that things will be understandable. So, just see how we are writing first you write down the equation for your x 1; this is your x 1 right. So, how we will do this? Just 1 minute here one or two small thing you have what you call you have to understand right first you see this x 1; this x 1.

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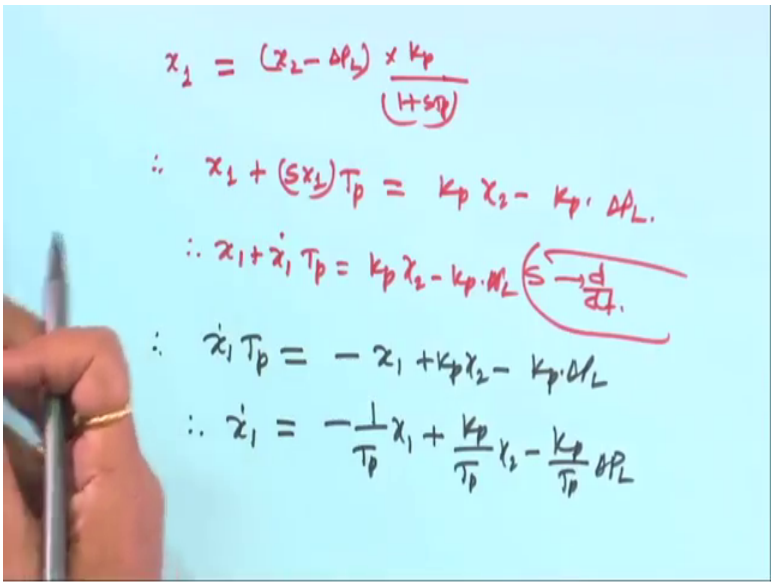


So, x 1 is equal to I will write little bit slowly such that handwriting will be better right because this is a pen and paper. So, this is your x 1 is equal to this delta P g actually your x 2. So, basically it will be x 2 minus say delta P L because delta P g you have taken x 2 that is your x 2 minus delta P L right into K p upon 1 plus S T p right.

So, that is your x 1 you cross multiply you cross multiply; that means, it will be x 1 then S x 1 T p we are tooking x and x 1 together is equal to K p x 2 minus K p into delta P L right; that means, this S x 1 actually S is ddt. So, S x 1 will be x 1 dot. Therefore, because S actually is ddt therefore, it will be x 1 dot; that means, your x 1 x 1 dot into T p is equal to K p x 2 minus K p into delta P L right.

That means, just hold on; that means, this x 1 dot T p is equal to minus x 1 plus K p x 2 minus K p into delta P L. Therefore, x 1 dot is equal to minus 1 upon T p x 1 plus K p upon T p x 2 minus K p upon T p delta P L. So, this is actually this is what I have written here this equation; this is equation 12 minus 1 upon T p x 1 plus K p upon T p x 2 minus K p upon T p delta P L; this is equation 12.

So, this is what I have written here this is actually your equation 12. So, this I have written here directly instead of writing so, many step, but here I showed you what is x 1 dot is equal to right. Next we will write your other equations now before writing x 2 dot; I will write x 3 dot for you; why? Now we will know right first I will write x 3 dot. So, you come to the come to your diagram these x 3 first I write x 3 dot and then I will come to x 2 dot.

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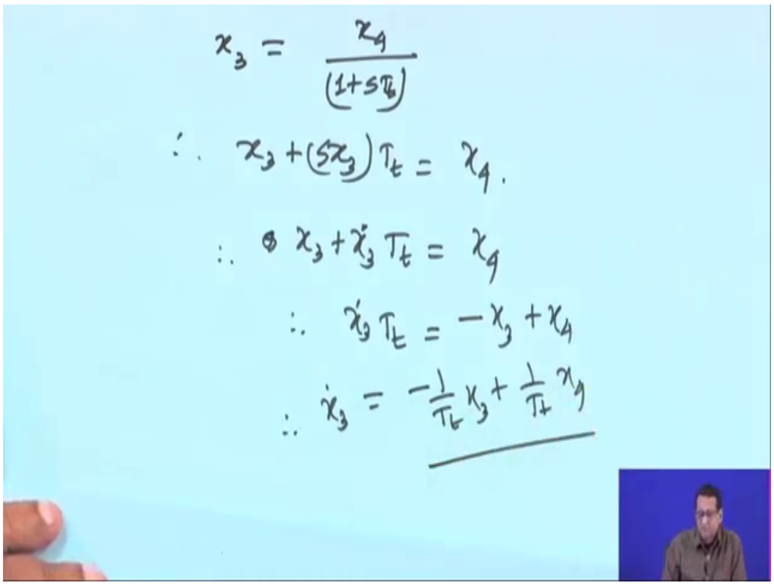

So, if you look at the diagram that x 3 will be is equal to x 4 in 1 upon 1 plus S T t. So, from this diagram only x 3 is equal to x 4 divided by 1 plus S T t; then go for cross multiplication again it will be x 3 plus your S x 3 into T t is equal to x 4 right; that means, x 3 is x 3 dot. So, it is x 3 plus x 3 dot T t is equal to x 4.

That means, your x 3 dot T t is equal to your minus x 3 plus x 4; that means, your x 3 dot is equal to minus 1 upon T t x 3 plus 1 upon T t x 4. So; that means, this equation your this equation x 3 it is x 3 dot is equal to minus 1 upon T t x 3 plus 1 upon T t x 4 this one we write next right why I wrote this one first you know now.

This is actually equation 14; this is your equation x 3 dot. Now we will come to x 2 because in the x 2 dot equation this x 3 dot is required; that is why x 3 dot is obtained first right. Next you come to your x 2 dot right this x 2 dot; so, this x 2 is equal to x 3 into whole all of this thing.

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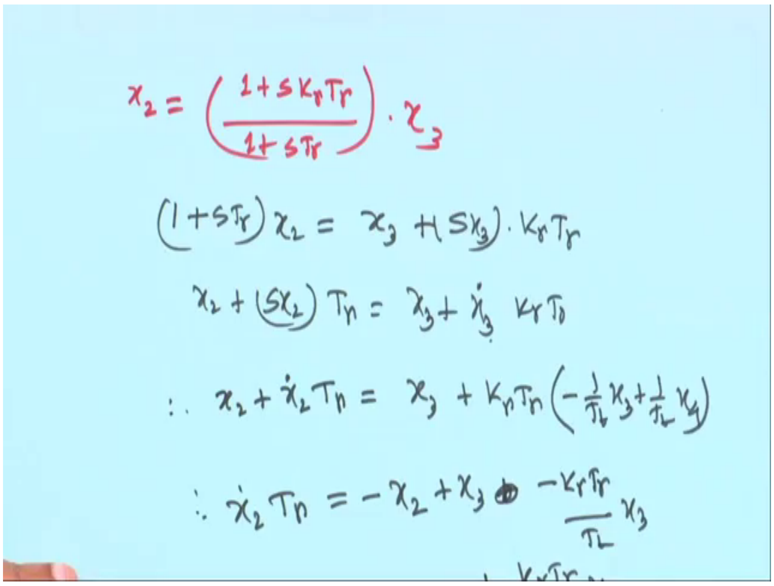

Therefore, your x 2 is equal to this is x 2; this is x 2 is equal to 1 plus S K r t r divided by 1 plus S T r into your this x 3 this x 3 right. Now when you now multiply with this one cross multiplication; so 1 plus S T r x 2 is equal to multiply this one; it will be x 3 plus S x 3 because S is here into K r into T r right.

So, if you multiply this one then it will be x 2 plus S x 2 T r is equal to x 3 it is S x 3 means x 3 dot k R T r because x 3 dot is coming because x 3 dot you have to substitute from that equation. This x 3 dot you have to this S x dot value you have to substitute here otherwise you cannot that is why I wrote that equation 3 first right.

. So, the way I have chosen the state variable; so, this one actually is x 2 is x 2 dot. So, it is x 2 plus x 2 dot into T r is equal to x 3 plus K r into T r and this x 3 dot this 1 minus 1 upon T t x 3 plus 1 upon T t x 4; you substitute it will minus 1 upon T t x 3 plus 1 upon T tx 4 u substitute right; that means, my x 2 dot T r is equal to minus x 2 right, plus x 3 plus you multiply sorry it will be minus minus K r T r upon T t x 3 right.

Then it is plus K r T r upon T t x 4 right just hold on I have to go a little bit up otherwise you cannot see this right this one right. Then both side you divide by T r and simplify then you will get this equation then will get equation 13 then you will get equation 13 right the both side you divide by T r and simplify then you will get your x 2 dot is equal to minus 1 upon T r x 2 plus 1 upon T r minus K r upon T t x 3 plus K r upon T t x 4 this is equation 13 right.

So, x 1 dot, x 3 dot, x 2 dot all I showed just to make it right just to save some time I did not do further, but you make this thing. Next your x 4 dot right just see the x 4 dot; now for x 4 look at this diagram; look at this diagram right this is your x 4; this is your x 4. So, you can write x 4 is equal to x 4 is equal to this is u and this delta f means x 1 this means x 1.

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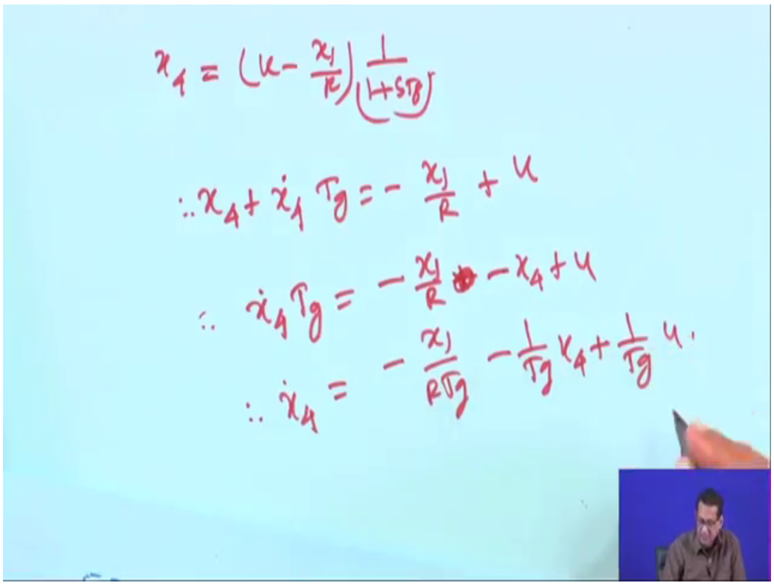

So, it is plus u minus x 1 in by all R right. So, it will be u minus x 1 by R right into 1 by 1 plus S T g right that is the x 4 is equal to right. So, let cross multiplication again; so, writing directly it is x 4 and S x 4 T g x S 4 means x 4 dot. So, it is x 4 dot then T g is equal to this term writing first minus x 1 upon R plus u or x 4 dot T g is equal to this we are writing first minus x 1 R right.

Minus x 4 plus u write or x 4 dot is equal to minus x 1 upon R T g minus 1 upon T g x 4 plus 1 upon T g u write; this is actually your equation number 15 this is equation 15 that is x 4 dot minus 1 upon R T g x 1 minus 1 upon T g x 4 plus 1 upon T g u; this is equation 15. I think up to this it is your what you call it is understandable that how to obtain all state variable equation x 1 dot, x 2 dot, x 3 dot they are 4 state variables right.

So, I have taken state variable like this you are what you call here x 1, x 1, x 2, x 3, x 4. If you can take this is x 1, x 2, x 3, x 4 also no problem the way what you can take this way right. So, this is all this thing after that we have to put this equation in x dot is equal to x plus B u form right.

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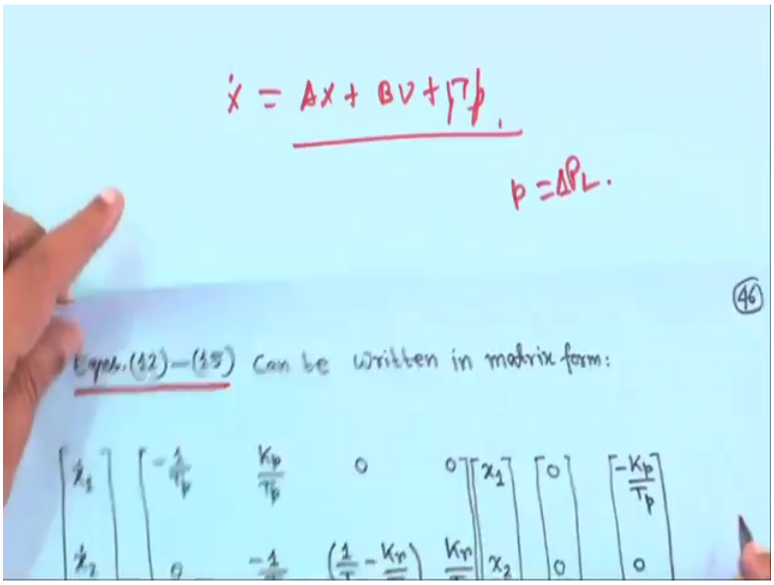


That means, this equation we have to write that you are in this form x dot is equal to A x plus B u plus gamma p this gamma p from this way you have to write this equation. Now if you do; so, then what we will get right.

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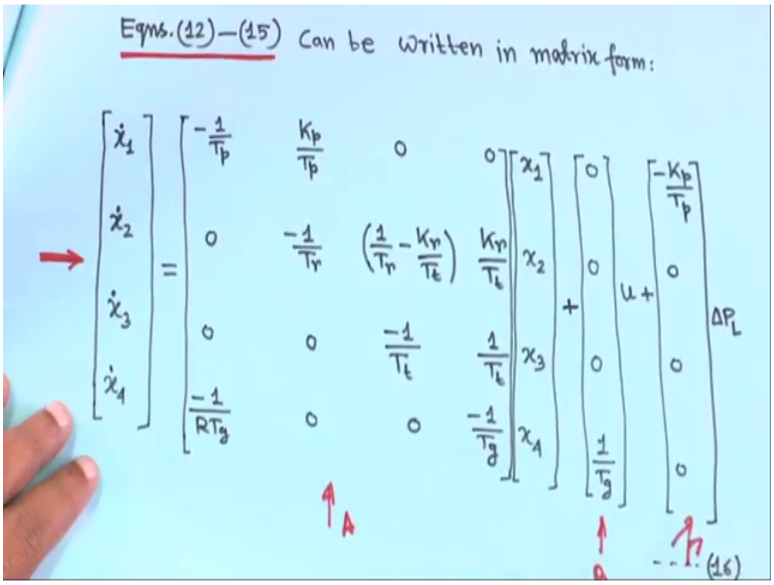


So, when you make it x dot is equal to x plus B u plus gamma p right. So, this is actually your this one your a matrix right and B u this is u this is u this is actually this one will be B and this is delta P L. Gamma p means it is your what you call gamma p means this p actually is equal to your P L right I made delta P L because change in the load; so, delta P L.

So, u and delta P L, but if you look at this equation just hold on just hold on this is also we have taken delta P L. So, this is I have also written delta P L. So, p is equal to actually delta P L right. So, if you put in this form x dot is equal to A x plus B u plus gamma p just hold on right just hold on.

So, this A is actually state transition matrix right B is equal to your control transition matrix and gamma is equal to disturbance transition matrix. This is actually this one this one actually your gamma right this is actually is your gamma this is gamma and this is B; this is A; A B and gamma right an x 1, x 2, x 3, x 4 are the state variables u is the control variable and delta P L basically is a disturbance in the system right.

So, this one if you write then if you look at this those equations that equation 12 13, 14 and 16 this x 1 dot is equal to actually minus 1 upon T p x 1. So, minus 1 upon T p x 1 plus K p upon T p x 2 plus K p upon T p x 2 right; no x 3, x 4 term is involved in this equation. So, their coefficients are 0; so, it is 0 0 right because x 1, x 2, x 3, x 4 no u term is your involved in this equation.

Therefore, all this is 0 right and then disturbance term is there minus K p upon delta P L. So, in a very first row this is minus K p upon T p into delta P L is here; this is x 1 dot is written I think you know this. Similarly x 2 dot x 2 dot is equal to your x 1 is not involved here. So, it is 0 right then this one your x 2 dot that is minus 1 upon T r; so minus 1 upon T r is here right. Then this one 1 upon T r minus K r upon T t is x 3 so 1 upon T r minus K r minus T t x 3 in the third term and fourth term K r upon T t x 4. So K r upon T t x 4 so K r upon T t x 4 right and there is no u term, no disturbance term so corresponding this is 0, this is 0.

Now third term only minus x 3 and x 4 are involved others everything is 0 so x 1 is 0, x 2 is 0 it is minus 1 upon T t and it 1 upon T t is x 3, this is 0, this is 0 right. And last term x 4; x 1 and x 4 is involved x 2, x 3 is not there both are minus sign so minus 1 upon R T g 0 0 and x 4 is this minus 1 upon T g this one x 4 and it is u 1 upon T gu so 1 upon T g is here because this is u but here it is 0 right. So, this is actually written in this form x dot that is x dot is equal to actually x 1 dot, x 2 dot, x 3 dot, x 4 dot capital X dot any one is equal to A into x plus B into u plus gamma into P this way we have written this equation.

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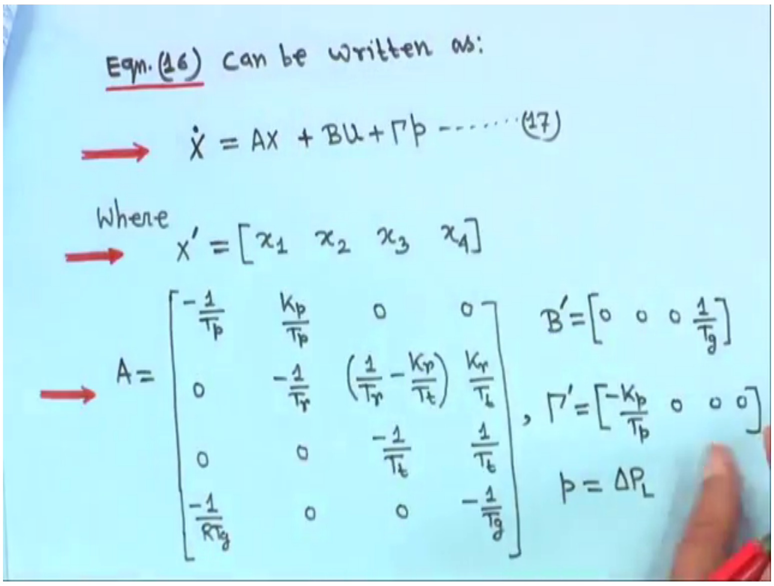


So, it can be written as X dot is equal to A x plus B u plus gamma p where X transpose I have taken state variable x 1, x 2, x 3, x 4 A matrix is this matrix, this matrix is A matrix right. And it is B matrix 0 0 0 1 T g, but I have written be transpose dash is transpose as a space can be accommodated. So, 0 0 0 1 upon this is B transpose that is why it is written that this is A actually and similarly that gamma actually minus K p upon T p 0 0 0 0, but taken as a transpose as that space can be saved here. So, gamma dash is equal to minus K p upon T p 0 0 0 and this p is the disturbance that is your delta P L right. I hope you have understood this state variable.

Thank you very much.