Power System Engineering
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Lecture – 04
Overhead Line Insulators (Contd.)

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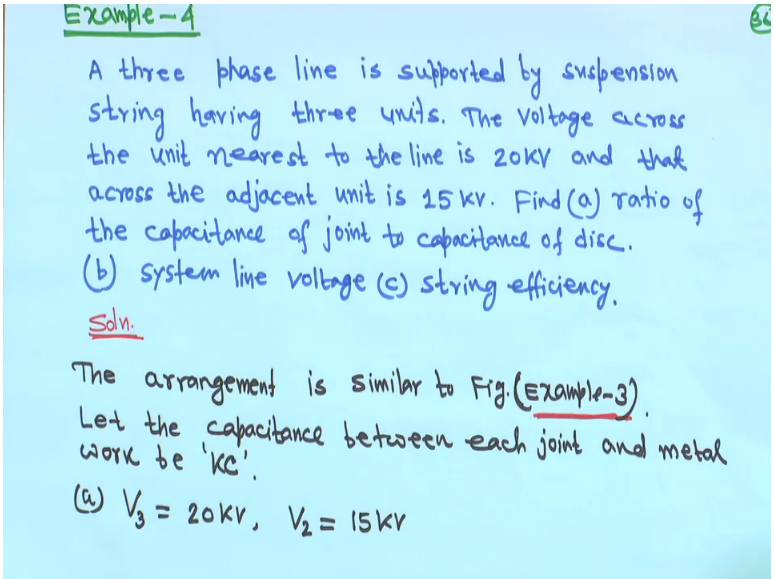

So, come to your example four. So three-phase line is supplied by suspension string having three units. The voltage across the units nearest to the line is 20 KV and that across the adjacent unit is 15 KV. Find, a - ratio of the capacitance of joint to capacitance of disc; b - system line voltage, and c - string efficiency. So, it has given support suspension string that three phase line three unit. So, the diagram is similar to example three.

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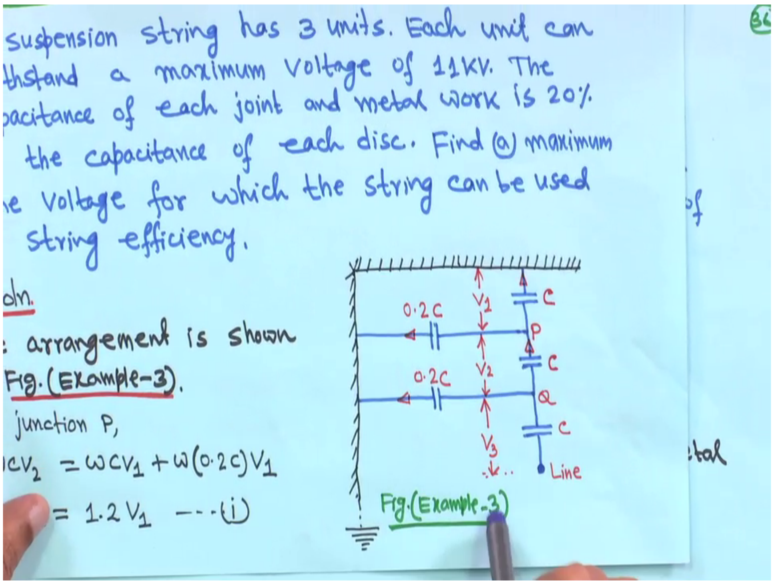

Diagram is similar, but numerical data other things are different, but this is similar diagram for this example also. So, that is why the arrangement is similar to figure this example 3. Now, let the capacitance between each joint and a metal work be KC. So, this is this we know that it is we have to find out the K first right KC. Now, first part is you have to find out the ratio of the capacitance of joint to capacitance of the disc that is K you have to find out. So, V 3 is equal to 20 KV that is given here - 20 KV, because the voltage across the unit nearest to the line is 20 KV; that means, diagram is same. So, this V 3 must be 20 KV nearest to the line is 20 KV. And adjacent units that this one it is given 15 KV and that across the adjacent will be 15 KV. So, V 2 will be 15 KV. So, V 3 20 KV, V 2 15 KV, these data are given.

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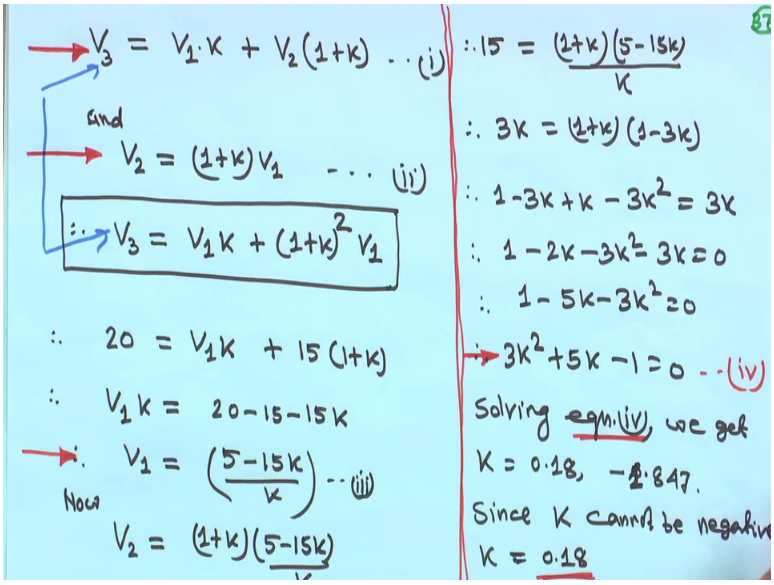

Now, following the same thing you please write the equations right the way we have all the way as written that that same thing P and Q apply your KCL I am not doing for you all these things the same thing you please do it. If you do so, then you will get your what you call for this problem your this one V 3 actually you will find V 1 K plus V 2 into 1 plus K. You will get V 3 is equal to your V 1 in to K plus V 2 into 1 plus same way you put a KCL at point Q same way, then you will get because omega C omega C will be cancel on both side. So, nothing is written here if I write omega C V 3 is equal to your I mean it is hold on, let me make it for you.

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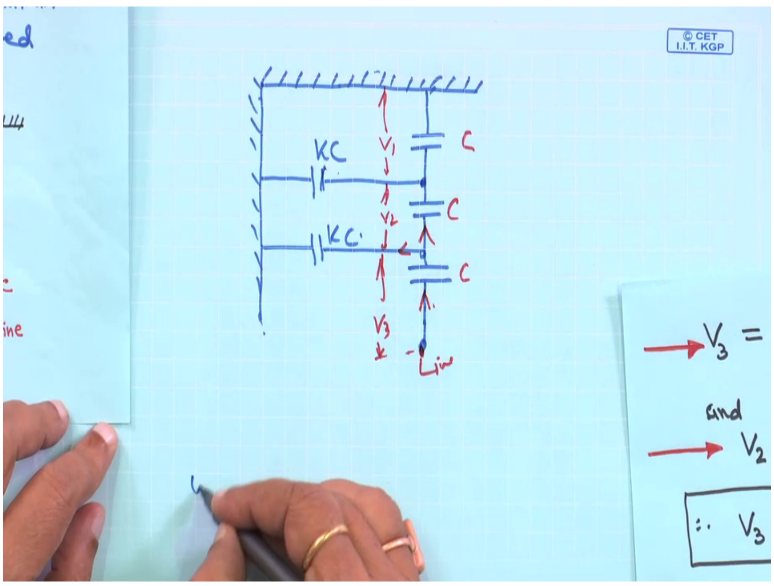

Suppose you have a this one your same diagram I am drawing for you, but one i will show other one you can do it. This is C, this is also C, this is also C, and this one and here this is your KC, here also KC. And this voltage is V 1, this is V 2, and this is line, and this is V 3, this is your line, and this is C, C, C; and current is going here going here going here. So, when you write the equation, so current through the voltage across this is V 3.

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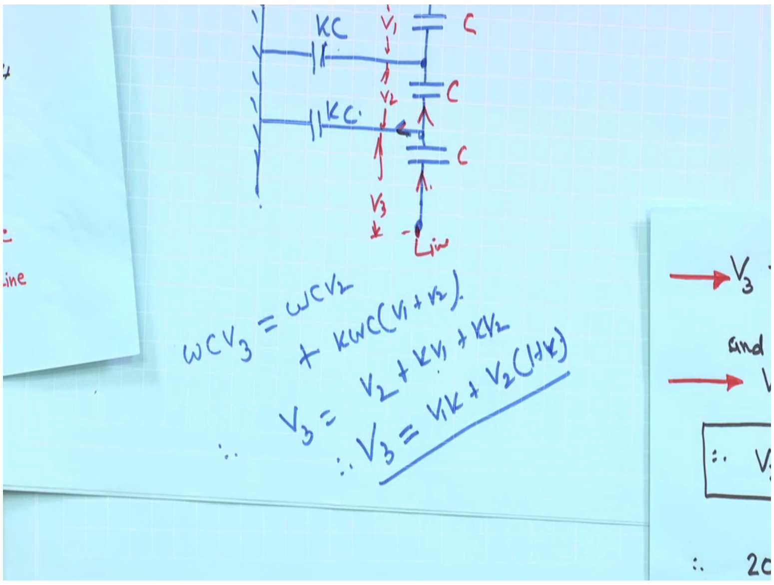

So, this current this current is omega C V 3 is equal to this current plus this current this current is omega C V 2 plus now this current, so this that means, this current is this voltage here means this common point this one this voltage is V 1 plus V 2 that means, plus your K omega C V 1 plus V 2. That means, omega C omega C will be cancel on both side, therefore, V 3 is equal to V 2 plus K V 1 plus K V 2 that means, V 3 sorry let me make it up; that means, V 3 is equal to your what I am writing that V 1 K. So, for this term, I am writing first V 1 K plus you take V 2 common then 1 plus K, this is the equation right, so that way you can find out that that is why I have written that is why I have it for you that is why first one you can easily write V 3 is equal to V 1 K plus V 2 into 1 plus K. This is equation one.

Similarly, you please apply the way I showed you. Similarly, you apply here KCL at P point also in terms of K then you will get in that case V 2 is equal to 1 plus K V 1 you will get. So, this is actually equation 2. Therefore, this V 2 you substitute here, V 2 you substitute in equation 1, V 2 is equal to your 1 plus K V 1 you substitute here. So, if you do so then this V 3 is coming here. So, it will be V 1 K and you are substituting V 2 is equal to 1 plus K V 1 it will be 1 plus K whole square into V 1. Now, V 3 is given 20, V 2 is equal to V 1 K, then your this V 3 is given 20 KV, this is given. And V 1 K putting in this equation V 1 K plus this V 2 is 15 into 1 plus K this one we are putting.

Now, V 1 K is equal to 20 minus 15 minus 15 KV this equation, therefore, V 1 is equal to we are writing 5 minus 15 K upon K, this is equation 3.

Now, it is same thing here also you should have, but i have written it, but V 2 is equal to 1 plus K V 1, so 1 plus K and V 1 is equal to 5 minus 15 K upon K. So, 1 plus K into 5 minus 15 upon K, that means, this your what you call this V 1, we are substituting in this equation 2. Therefore, it is a quadratic equation. So, 15 is equal to because V 2 is equal to 15 is given, adjacent because lowest unit voltage is 20 KV and adjacent to that is 15 KV, therefore this your V 2 is equal to 15 KV. Therefore, this is a quadratic equation you simplify, so you will get 1 minus 5 K minus three K square is equal to 0 or other way I am writing 3 K square plus 5 K minus 1 is equal to 0, this is equation four. Therefore, you solve this equation, you will get two values, one will be K is equal to 0.18, another is minus 1.847, but this is K cannot be negative. So, this value ignore you will get K is equal to 0.18, this is the answer for the part one.

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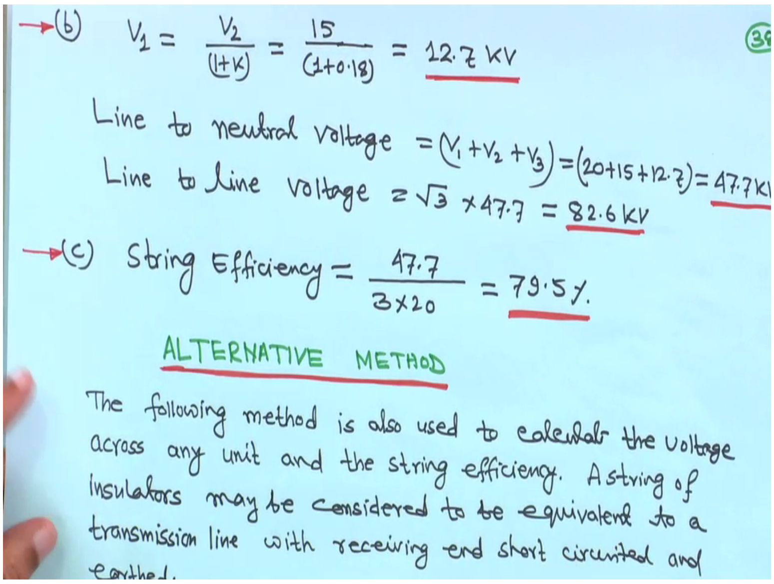

Now, b, it is given that where is that problem here; b it is given that you have to find out not this one, example 4 - this one, b actually you have to find out system line voltage system line voltage this is b. So, in this case, b is equal to V 1 is equal to V 2 upon K plus 1, because we have seen this one, V 2 is equal to 1 plus K into V 1. So, it is 15 upon 1 plus 0.18. So, it is coming 12.7 KV. A line to neutral voltage V 1 plus V 2 plus V 3, you sum it up all, V 1 is your 12.7, this is actually V 1 we are writing at the end V 3 writing first 20 plus V 2 15 plus this V 1 12.7 is coming 47.7 KV. Line to line voltage multiply by root 3, it will become 82.6 KV; and C is the string efficiency. So, line to neutral voltage is 47.7 divided by 3 into 20 because lowest unit voltage is 20 KV and there are three disk - three units, so 3 into 20, so it will come around 79.5 percent. So, this is what you call that you are string efficiency.

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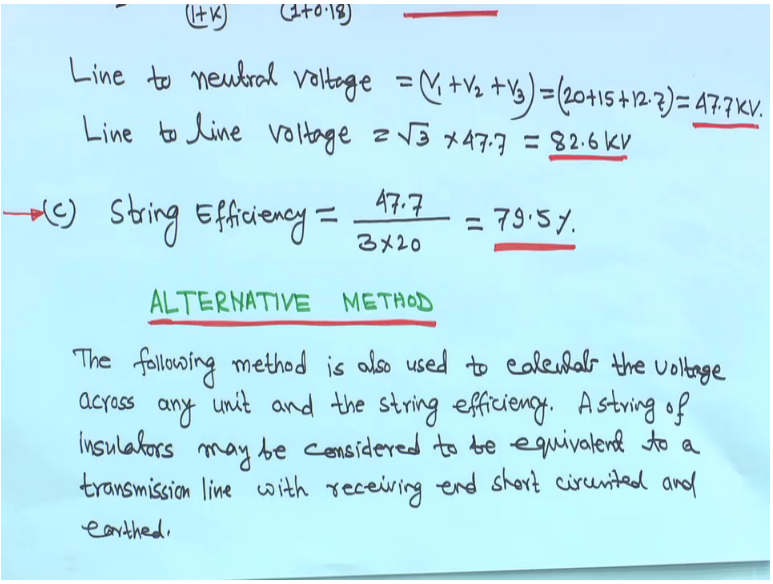

Now, next is alternative method. Now, the following method can also be used to calculate the voltage across any unit and the string efficiency. Actually a string of insulators may be considered to be equivalent to a transmission line with receiving end and short circuited what you call receiving and short circuited and earthed. That means, when you rather than taking this conception you consider that it is a series of capacitance in a transmission line connected and short circuited at the your receiving end and the shunt capacitance are also there. As if it is something like your line to your ground capacitance, this way it can be imagine.

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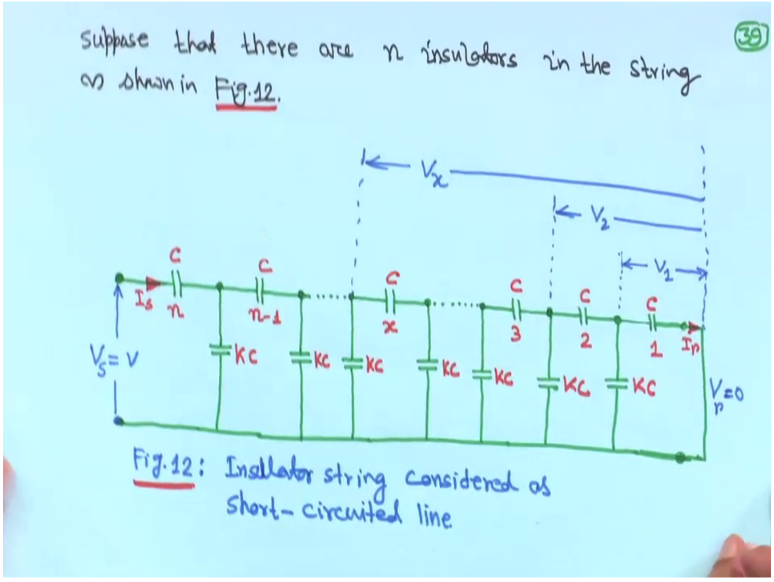

So, if you do so suppose you have a n insulator in the string as shown in figure 12 it is made like this. For example, say for example, imagine this is a sending end then this is a receiving end. Sending end means the conductor is here actually and this is your n-th, unit in the string insulator. And this is n minus 1 then some value x some number x there then it is that your near the your what you call the tower end, it is 1, as if it is a short circuited earth and these are all KC capacitance as if your shunt capacitor this way you imagine. And voltage here it is from the top of the tower, we are going like this say it this side actually short circuited earth means as if it is a receiving end; that means, it is on the tower side and this is on the line side. So, this voltage is V 1 from here to here. From here to here, it is V 2.

From here to now, one thing is there you should not be confuse with the previous thing previously across each thing we have taken V 1, this we have taken V 2, this we have taken V 3 like this, but for this kind this thing it is V 1. And here total is V 2, form here to here, it is V 2; and from the x number from here to here, it is V x; it is not across each one. First one is across the first one when it is this voltage; that means, across this two. So, this is V 2 and so on. So, do not be confused with the previous example, so previous derivation. So, these are all KC, KC, KC value.

Now, insulator one just I told you is that top most unit connected to the cross arm that is why I told you it is a top most unit connected to the cross arm and your that is earth. And insulator n is the lowest unit attached to the line conductor that means, this one this is conductor this is conductor actually we imagine that is a long transmission line the way we have done that your sending and receiving voltage calculation for the long transmission line this is the same thing.

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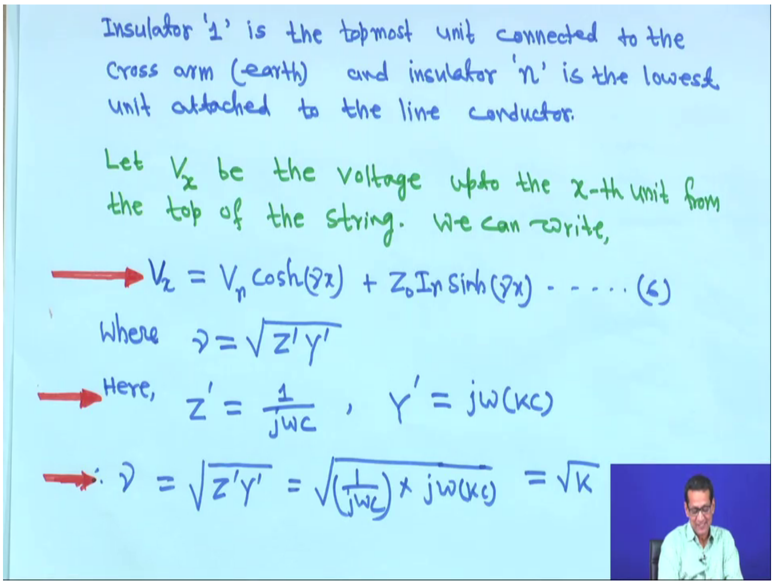

So, you V x is the voltage from here to here. As if for a transmission line you have also done no find out the suppose that voltage V x at a distance x, here similar way let V x is the voltage up to the x-th unit there it was at distance x here it is at up to the x th unit from the top of the string. I mean this is the top of the string means this is a tower side from here to here. So, from that we can write that this expression you know that from your transmission line expression, I am not deriving here these are known to you because for long transmission, line this expression that hyperbolic expression we have made it right for distributed line parameters. So, this already we have made it, those who have taken the course or even you have not taken the course, you have studied in your inductance sorry that long line, short line and medium line calculations there you have made it.

So, V x is equal to v r cos hyperbolic gamma x plus Z 0 i r sin hyperbolic gamma x right this is equation 6. This is actually known to us because from transmission line voltage calculation, we have derived it for long transmission line. So, where gamma is equal to your say we make under root Z dash Y dash, there also we have seen this right gamma is equal general gamma is equal to root over Y Z. Now, Z dash is equal to one upon j omega C and Y dash is equal to j omega KC; only thing is here actually you do not have what you call that inductance it is a capacitance, but philosophy remains same. There you are using something else in terms of inductance, but here it is capacitance, but philosophy is same. Therefore, gamma is equal to root over Z dash Y dash. So, if you this one 1 upon j omega C into j omega KC because Y dash is your j omega KC.

And Z dash is 1 upon j omega C. Here inductance is not there, lined inductance is not there, but philosophy remains same. So, gamma is equal to finally, it will come root over K actually. And another thing is that your end is short circuited that is the tower end actually that is short circuited that V r is equal to 0; that means, this as it short circuited tower end we have think where thinking then receiving in voltage V r is equal to 0. So, if it is short circuited zero that means, put that thing in equation six; that means, in this equation you please put V r is equal to 0 in this equation you please put it.

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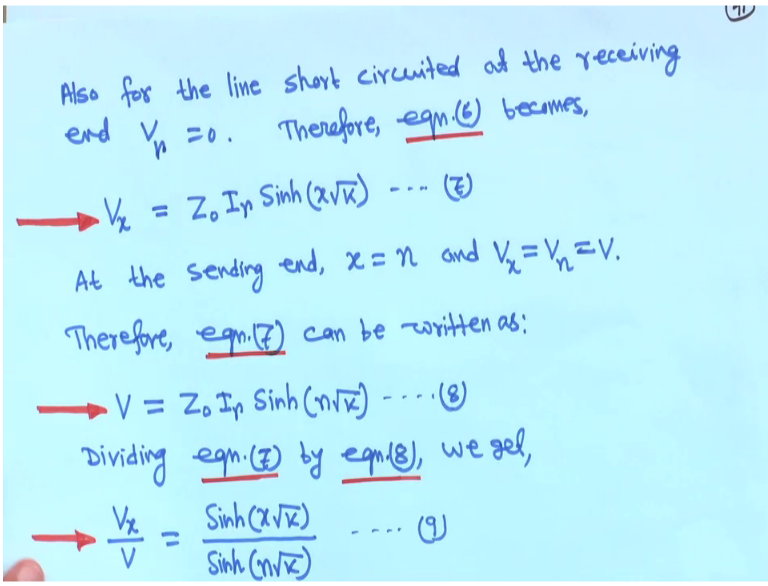

If you do so then V x will become Z 0 I r sin hyperbolic say x root k. So, in this case v r is equal to 0, so this term will not be there. And here it is gamma is equal to root K you put root k, so it ultimately V x will become Z 0 i r sin hyperbolic x root k, that is why V x is equal to Z 0 i r sin hyperbolic x root K. This is equation 7. Therefore, at the sending end, x is equal to n, look at the diagram at the sending end, I mean this one the same way we do not know at the transmission line at the end x is equal to l. So, here also at the same thing at that your what you call at the sending end x is equal to n, because it is n number of disc is there. So, x is a number not a length here, not a length here.

So, x is equal to n in that case you will find V n is equal to V x is equal to V n is equal to v because V x is equal to n means it will be V n, and V n will be is equal to V the total voltage. V n is equal to v mean from here to here the total voltage total voltage V n is equal to V that is V 1 plus V 2 up to plus sum up all V 1 plus V 2 are plus plus plus up to your V n right all this thing. So, V x is equal to V n is equal to V, therefore, equation 7 can be written as V is equal to Z 0 I r sin because x is equal to n therefore, sin hyperbolic n root K this is equation 8.

Therefore, dividing equation 7 by equation 8, you divide equation 7 by equation 8, you will get V x upon V is equal to sin hyperbolic x root over K divided by sin hyperbolic n root over K. This is equation 9. From for m-th disk if x is equal to m I mean for m-th disk look at the diagram you have n number of disk n-th disk which is called as m-th disk. It is coming from 1, 2, 3 up to n in between some x number is there, for n-th disk from here say some m-th disk. You substitute in that case x is equal to m right. So, for m-th disk in this equation you replace x by m, it will be V m upon V.

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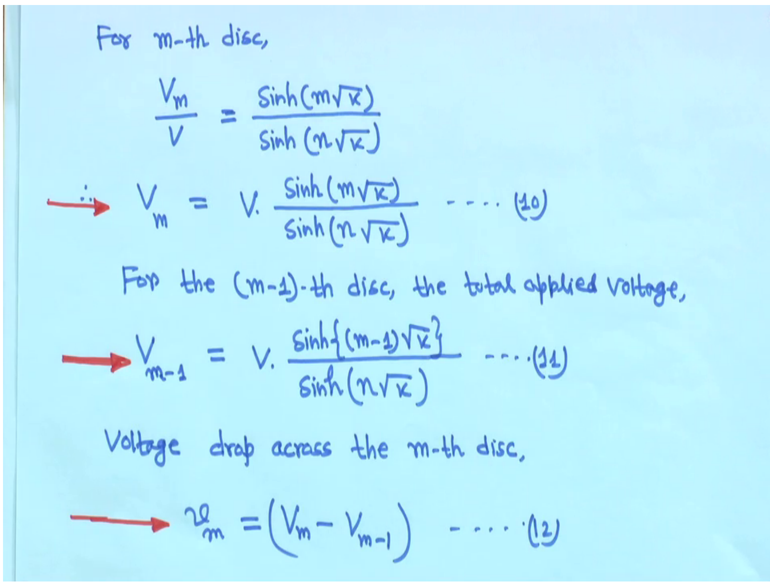

If it is V m upon V then for m-th disk, it will become V m upon V is equal to sin hyperbolic m root K by sin hyperbolic n root K, because here we have put x is equal to m. So, V m upon V is equal to sin hyperbolic m root K by sin hyperbolic n root K. So, V m upon V is equal to sin hyperbolic m root K divided by sin hyperbolic n root K or V m is equal to this cross multiplication V into sin hyperbolic m root K by sin hyperbolic n root K. This is equation 10. And for that m-th m minus 1 th disk I mean just the previous one if this is the m-th disk then previous disk that applied voltage replace m by m minus 1. In that case, it will be v m minus 1 replace this m by m minus 1 is equal to V into sin hyperbolic m minus 1 into root K divided by sin hyperbolic n root K. This is equation number 11.

Look the here the equation is totally change earlier it was omega C then what you call then omega KC all this things are there, but here it is in terms it is a most generalized one and it is in terms of that sin hyperbolic. So, voltage drop across the m-th disk then if v m will be capital do not confuse this derivation with the previous one right here the voltage drop across the m-th disk it will be capital V m minus capital V m minus 1. This capital V and do not confuse do not mix up with the previous one here that nomenclature is just different. So, do not mix up when will stay. When you will listen to this video or when you will solve numerical. So, do not mix up.

So, voltage across the m-th disk that small v m is equal to difference between your v m and v m minus 1. So, capital V m minus V m minus 1, similarly if m is equal to 2, 3, 4 like this. So, all the voltage across the each unit you will get I mean across each using this formula equation two help across each unit you will get the your or across the each disk you will get the voltage.

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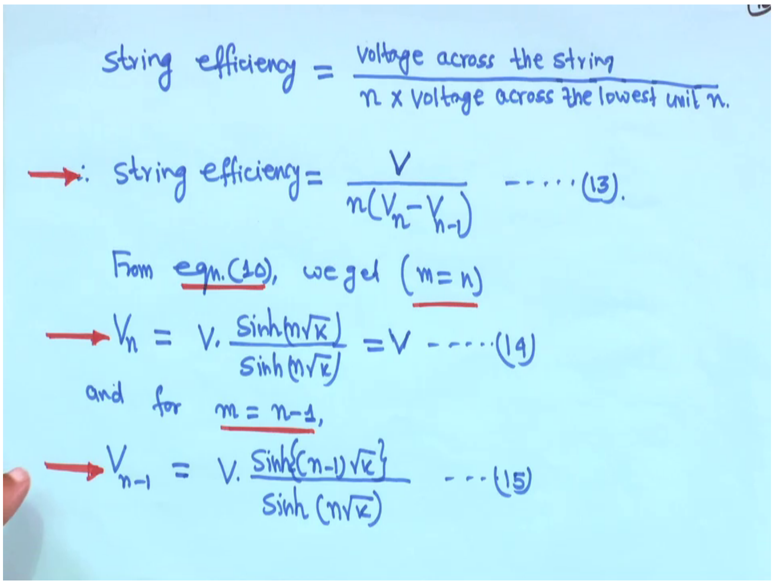

So, now, in this case the string efficiency is equal to voltage across the string divided by n into voltage across the lowest unit n. So, voltage total voltage is V and the string efficiency lowest voltage V, it is n and voltage across the lowest unit will be V n minus V n minus 1; that means, same thing. That means, in this case if you replace m by n suppose if m is equal to n, the n-th disk the last one, then V n will be is equal to V n minus V n minus 1. So, that is why it is written string efficiency is v upon n into V n minus V n minus 1. This is equation 13. From equation 10, we get for m is equal to n. So, this is equation 10. This is your equation 10.

In that case if you put m is equal to n here then v m will become V m actually V m will become v that m is equal to n means V n, V n will become v because sin hyperbolic if m is equal to m root K and sin hyperbolic n root K, it will cancel and V n will be is equal to V. Therefore, from equation 10, if we get m is equal to n, then V n is equal to V then sin hyperbolic m root K sin hyperbolic n root K is equal to V that is equation 14. And for m is equal to n minus 1, if m is equal to n minus 1 then this same equation m is equal to n minus 1. If you do then V n minus 1 is equal to v m is you replace this m by n minus 1 in equation 10, it will be V n minus 1 is equals to v then sin hyperbolic then n minus 1 root K by sin hyperbolic n root K. This is equation 15.

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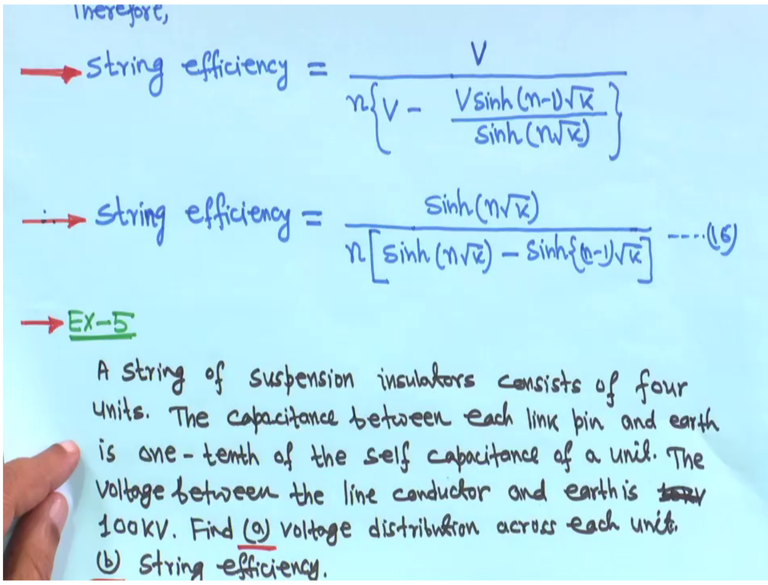

Therefore, string efficiency will be V, I mean this is the string efficiency V upon n into V n minus V n minus 1. So, string efficiency will be your V upon n into V minus V sin hyperbolic n minus 1 root K divided by sin hyperbolic n root K. They are just equal to just simplify sin hyperbolic n root K, because V V would take common V V will be cancel sin hyperbolic n root K by n into sin hyperbolic n root K minus sin hyperbolic n minus 1 into root K 16. This equation is independent of the voltage V string efficiency it is it is independent of this one.

So, this is actually if you use what you call that that most I mean if you imagine the line that is string insulator in each disk if you imagine that it is a long line and no inductance is there, but capacitance is there in the line that line capacitance and shunt capacitance KC is there. Therefore, receiving end is earths that is a tower side, it is a earth and short circuit end. So, receiving and voltage is 0, and using the same your what you call that long transmission line voltage expression, you can get the all the voltages as well as the string efficiency, and this calculation of course, must be very correct one compared to this one. But difference will be too small later we will see the example and other thing.

So, now, come to the one example. A string of suspension insulator consists of four units there are four units or four disks. The capacitance between each link pin and earth is one-tenth of the self-capacitance of unit; that means, K is equal to actually 0.1, because it is one-tenth of the self-capacitance of unit. The voltage between the line conductor and a earth is 100 KV that is also given right between the line conductor and the earth. Find, a - voltage distribution across each unit; b is take it is given that voltage between the line conductor in the earth; that means what you call the total voltage. You have to find out two things, voltage distribution across each unit and string efficiency.

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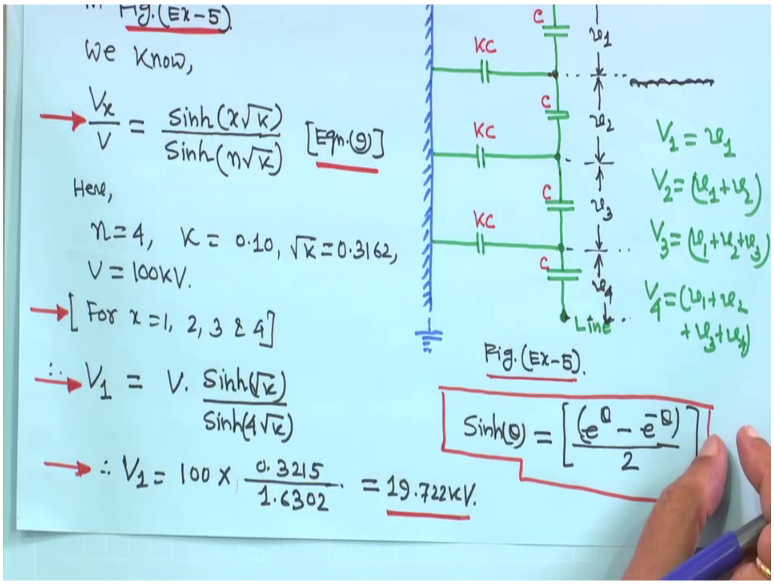

Now, so this is the circuit diagram V 1, V 2, V 3, V 4. And because four units are there 1, 2, 3, 4, then we have KC KC KC. V 1 just let me tell you one thing please do not confuse with other examples and other thing because it is have taken as a long line thing. So, please do not this thing confuse with the previous one because here nomenclature is different than the previous one these are all small v 1, small v 2, small v 3 small v 4. So, it is capital V 1 is equal to small v 1, capital V 2 that is small v 1 plus v 2; that means, this is actually capital V 2. Capital V 3 means this one is equal to small v 1 plus small v 2 pus small v 3; and capital V 4 that is total is equal to your small v 1 plus v 2 plus v 3 plus v 4. So, do not confuse with the previous one that should not be no place you have your what you call confusion everything is clear right. So, arrangement is shown in figure.

So, we know that V x upon V sin hyperbolic x root K upon sin hyperbolic n root K this is from equation nine we have seen here n is equal to 4, K is equal to 0.1 is given. So, square root of K is 0.3162. And total voltage I told you it is given that is of line conductor to the earth; that means, the top of the tower that is this thing where insulators is hung from. So, V is equal to 100 KV. And x is equal to there are four units, x is equal to 1, 2, 3 and 4. Therefore, V 1 when x is equal to 1 right this same using equation nine in v into sin hyperbolic root K by sin hyperbolic 4 root K. It is given that it general formula sin hyperbolic theta is equal to e to the power theta minus e to the power minus theta upon 2. So, whatever it is you just make V 1 is equal to 100 into 0.3215 by 1.6302 that is 19.722 KV, directly you can get it from calculator, no need forget about forget about this thing it directly you will get it, so 19.722 KV.

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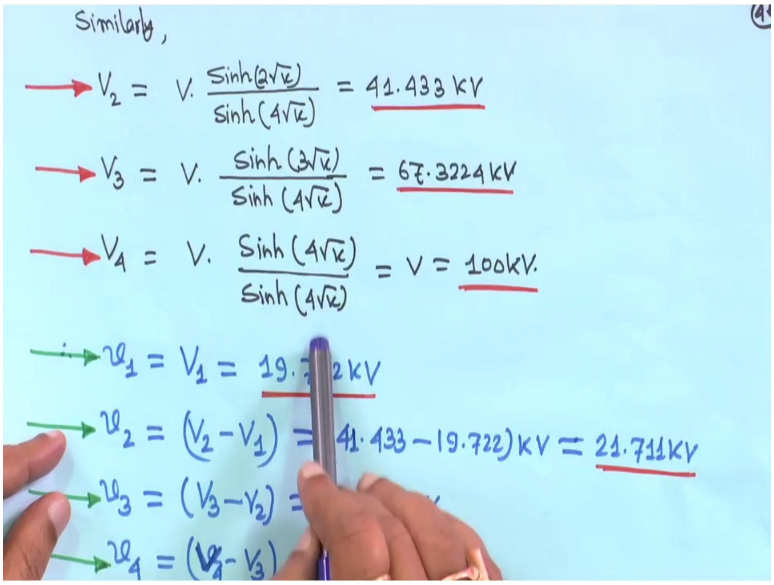

Similarly using the same example I show the same expression this one put x is equal to 2, 3, 4, so you will get V 2 is equal to V into sin hyperbolic 2 root K by sin hyperbolic 4 root K, you will get 41.433KV. Similarly, V 3 you will get sin hyperbolic 3 root K by sin hyperbolic 4 root K you will get 67.3224 KV. Similarly, V 4 is equal to V sin hyperbolic 4 root K, this is same, so it will V 100 KV.

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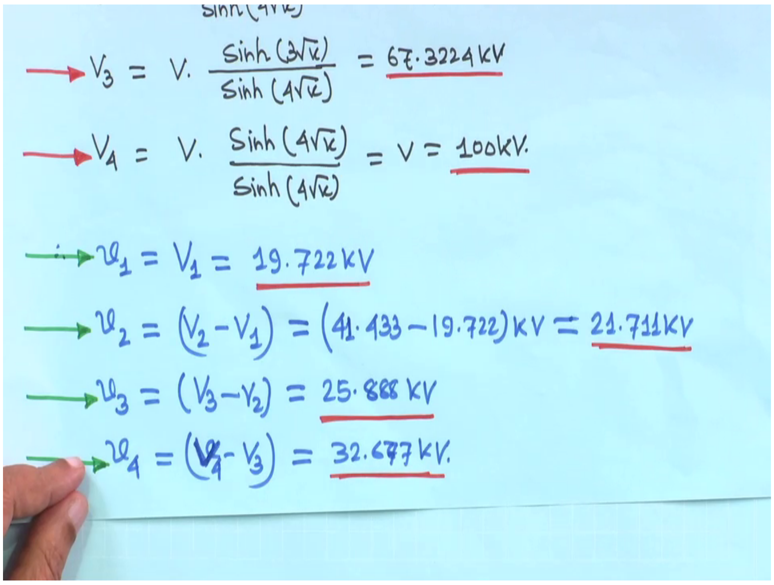

Therefore, small v 1 is equal to V 1 19.722 KV. This was small v 2, it will be V 2 minus V 1 because from this expression I told you [FL], this is V 1 this is v 2, v 3, v 4; this is capital V 1, this is capital V 2, this is capital V 3 and this is capital V 4. Therefore, you have to take the differences. So, v 2 is equal to small v 2 is equal to V 2 minus V 1 you will get 21.711 KV. Small v 3 is equal to capital V 3 minus V 2 you will get 25.88 KV and small v 4 is equal to V 4 minus V 3 you will get 32.677 KV that means this expression this expression you can use. If you put n, it will be V m minus V m minus 1 this expression you can use right that is equation 12. So, with that we are getting v 1, v 2, v 3, v 4, all small v 1, v 2, v 3, v 4, you will get using equation 12; and this is your all the voltages.

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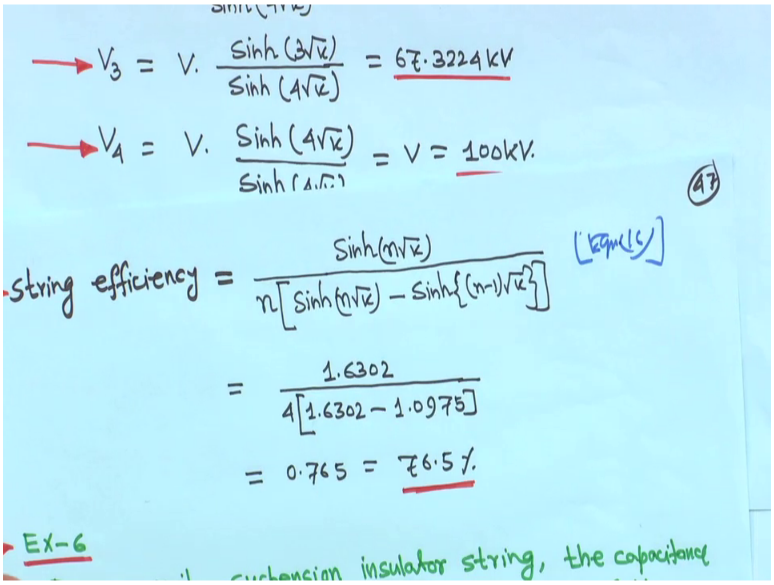

Now, string efficiency this formula we know, this formula we have derived now, we have from this equation. This is from equation 16.

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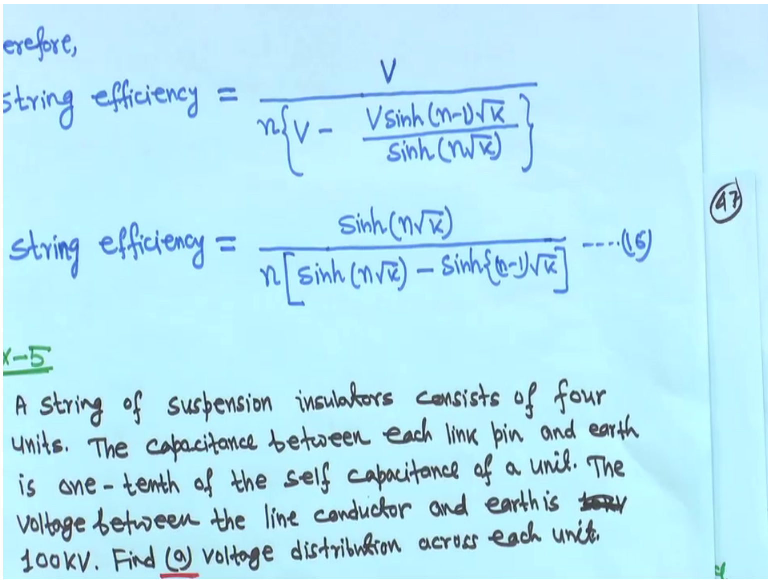

So, this is actually in bracket I am write down for you this is actually equation 16. So, string efficiency it will be sin this sin hyperbolic n root over K by n into sin hyperbolic n root over K minus sin hyperbolic n minus root over k. So, you know n is equal to 4 root K is known n is known is 4, you substitute all these value. So, you will get approximately 76.5 percent the efficiency is. So, this is using the generalized formula using a long transmission line.

I have one exercise for you when you will do this that same example, the example you find out your string efficiency and voltage across each disk using that your general method. And just compare the result. You will compare this result with the general thing whatever we have done before this is a general generalized one and that is why in terms of KC whatever we have got. You compare the result and see the differences, from your exam purpose from your assignment purpose please see the differences whether you get any differences or not string efficiency and this one. This is an exercise for you when you will do this thing. We will come more examples and other things in insulator.

Thank you now.