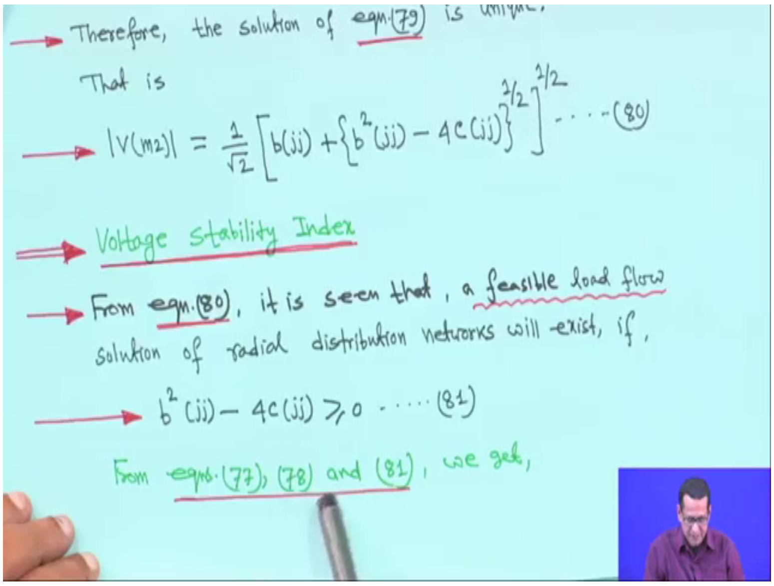
Power System Engineering
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 38
Voltage stability of distribution network (Contd.) & approximate method

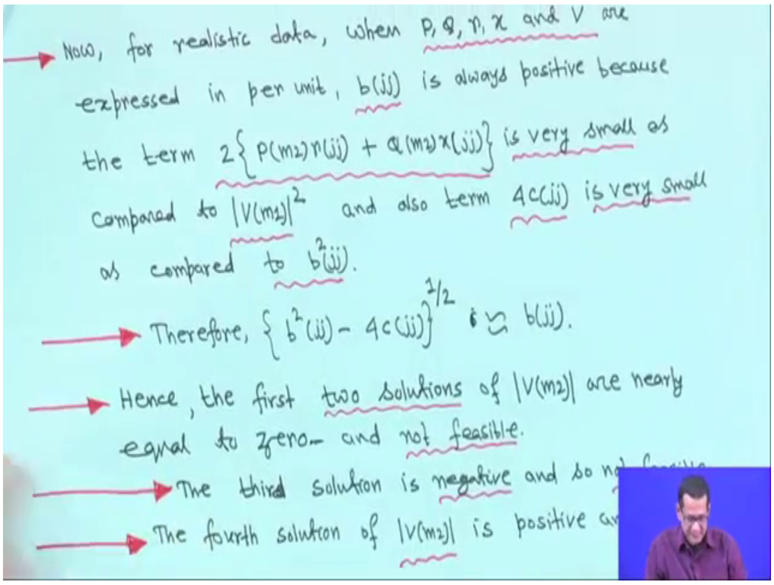
Next, you please come to that voltage stability index right.

(Refer Slide Time: 00:23)



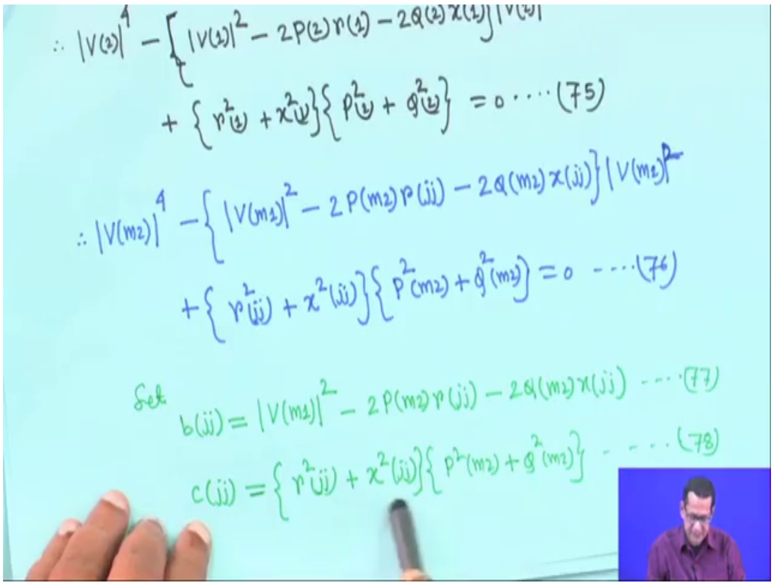
So, from this is equation 80. So, a feasible solution is possible if B square jj minus 4 C jj greater than equal to 0, because this is B square jj minus 4 C jj it is to the power half therefore, B square jj minus 4 C jj greater than equal to 0 right so; that means, B it has to be this condition. So, and B jj and C c jj all have been defined before right.

(Refer Slide Time: 01:04)



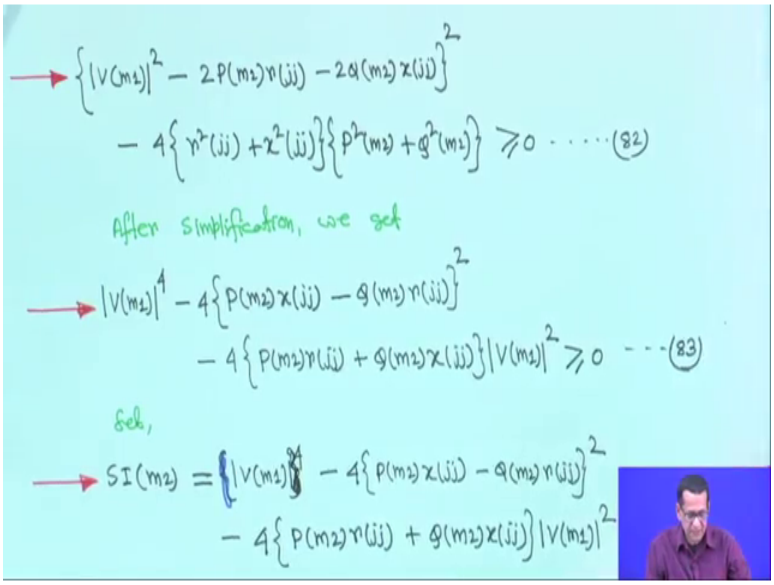
Therefore, from equation 77, 78 and 81 we get; that means, equation 77 and 78 is B jj and C jj; that means, here it is this is your this is your 77 is B jj and 76 your C jj these 2 equations right.

(Refer Slide Time: 01:12)



So, if you put those things B and C j B jj and C jj value here.

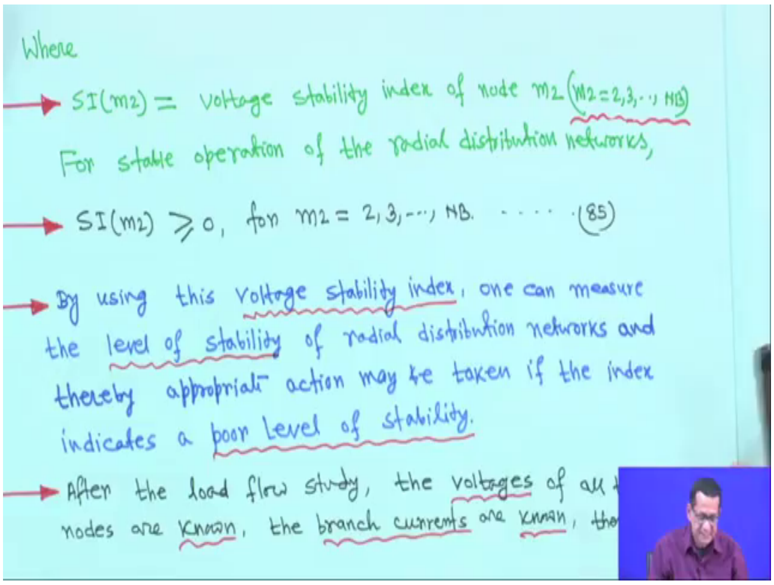
(Refer Slide Time: 01:24)



Then it will be your V m square magnitude of course, minus 2 P m 2 r jj minus 2 Q m 2 x jj right this whole square minus your 4 into r square jj plus x square jj into P square m 2 plus Q square m 2 greater than equal to 0 this is equation 80. Now you simplify this equation I am writing only the final form of after simplification these you can easily do it this square term you expand. These two terms you multiplication you just multiply and simplify right you will get V m 2 1 to the power 4 minus 4 into P m 2 x jj minus Q m 2 r jj whole square minus 4 into P m 2 r jj plus Q m 2 x jj multiplied by magnitude V m square greater than equal to 0 this is equation 83, right.

Then you assume let us assume S I m 2 is equal to this whole term. So, it is V m one to the power 4 actually this is V m 1 to the power 4 minus 4 into this term right this term minus 4 into this term that this is greater than equal to 0 means S I m 2 value always will be greater than equal to 0 and this S I m 2 we are defining as a stability index. So, this is equation 84.

(Refer Slide Time: 02:48)

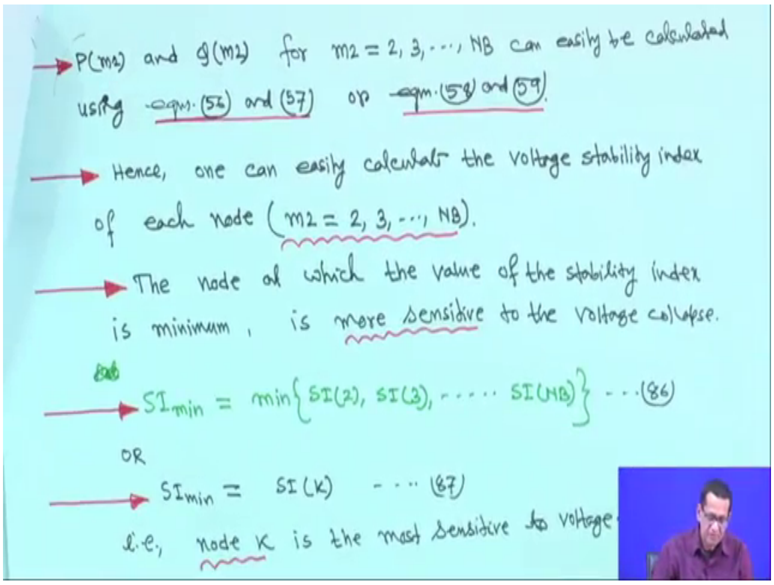


Now, this S I m 2 actually voltage stability index of node m 2 right. So, this what because this is we are taking that your what you call that quadric from that quadratic equation only. So, this is actually stability index for node m 2 this whole thing we are defining as a stability index voltage stability index right.

So, that is for of node m 2 that is m 2 is equal to 2 3 up to node N B right N B is the total number of nodes of the network. For stable operation of the radial distribution networks S I m 2 must be greater than equal to 0 for m 2 is equal to 2 3 up to N B. So, by using this voltage stability index one can measure the level of stability of radial distribution networks and thereby appropriate action may be taken if the index indicates a poor level of stability. I mean it has to be greater than equal to 0, but by chance if you find it is very close to 0 or may be much less than 0.4, 0.3, 0.5, something like that then you have to take some appropriate measure such that it will remain voltage stable.

So, after the load flow study suppose you are running the load flow then all the load hold all the voltages of all the nodes are known, voltage magnitudes are known, branch currents also are branch currents are also known after the load flow studies.

(Refer Slide Time: 04:14)



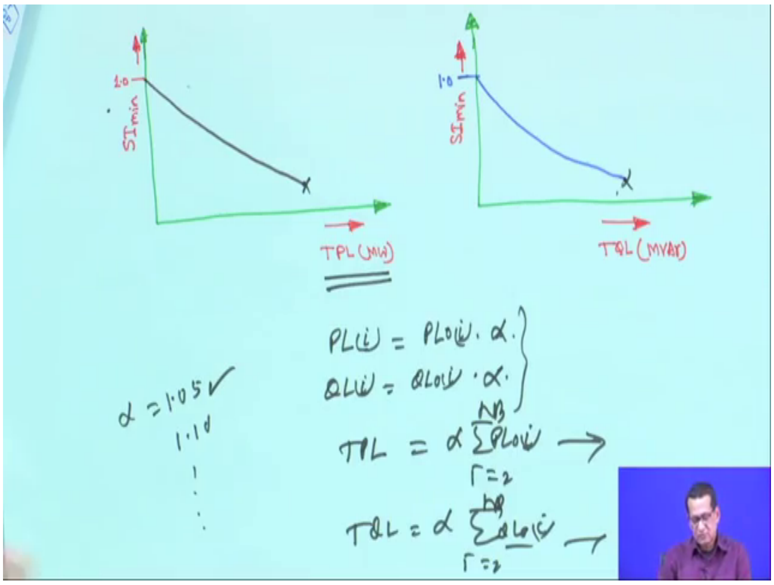
So, and P m 2 P m 1 and Q your P m 2 and Q m 2 this is P m 2 sorry this is actually P m 2 P m 2 and Q m 2 right for m is equal to 2 3 N B can easily be computed using equation 56 and 57 or equation 58 and 59 this is this all this expression is given for the second load flow case for equation 56 and 57 at that time will branch losses are there right, but in 58 or 59 when is a last node there was no branch losses right so only one term I explain also all those sigma symbol is there in that or is equal to it is only one term that also I have explained right.

So, all this can be computed using that load flow technique; that means, that load flow is require because after load flow only you have to you will you have to gain you will get all this things right. Similarly for the first load flow also it is possible right to get P m 2 and Q m 2, but we will stick to it that only right. Hence one can easily calculate the voltage stability index of each node that is m 2 is equal after load flow studies all the results are known voltage magnitude and P m 2 and Q m 2 all are known for m 2 is equal to 2 3 up to N B n B is the total number of nodes.

Therefore you can calculate S I 2, S I 3 and so on up to S I N B right. Therefore, node at which the value of the sensitivity sensitivity index is minimum that node is more sensitive the voltage collapse. For example, here it is S I m 2 is this one m 2 S I 2 S I 3 all we will calculate up to S I N B and your condition is that S I m 2 will be greater than equal to 0 right, but node at which this voltage value will be minimum that node actually is more sensing to the voltage collapse right.

And therefore, S I min that is yours is equal to minimum of S I 2, S I 3 and S I N B right for example, S I min is equal to say S I K that K-th K-th S I value right where it is minimum where, node K is the most sensitive to voltage collapse and this is because this one is giving the same minimum value say this is equation 86, this is equation 87. And you have to compute for S I values for all the nodes after that you take the minimum of that and that corresponding to that node that is why this is s minimum is equal to S I K K is that K-th node and this S I K value is the minimum one.

(Refer Slide Time: 06:39)



And for example, in distribution system you have just for the purpose of this thing for a distribution system suppose total load is this one and TQL suppose you have that total- your P L, say P L I is equal to say nominal load that mean initially load was P L i multiplied by some factor alpha.

Similarly, say Q L i is equal to say initial load Q L i into alpha so; that means, total when you when you find out the T P L T P L actually it is given in megawatt T P L is equal to alpha time P L 0 I i is equal to 2 to N B and TQL is equal to alpha times i is equal to 2 N B it is Q L 0 I. So, this total this is, this side T P L is the total load know for assuming that all the loads of all the nodes are increased uniformly.

For example, alpha is equal to say 1.05, 1.10, this way you go on increasing right. So, in that case every time you find out every time you increase this take this value for all the loads, for all loads you load is increasing you increase uniformly to all the loads assuming that load growth load growth is there just to test it right.

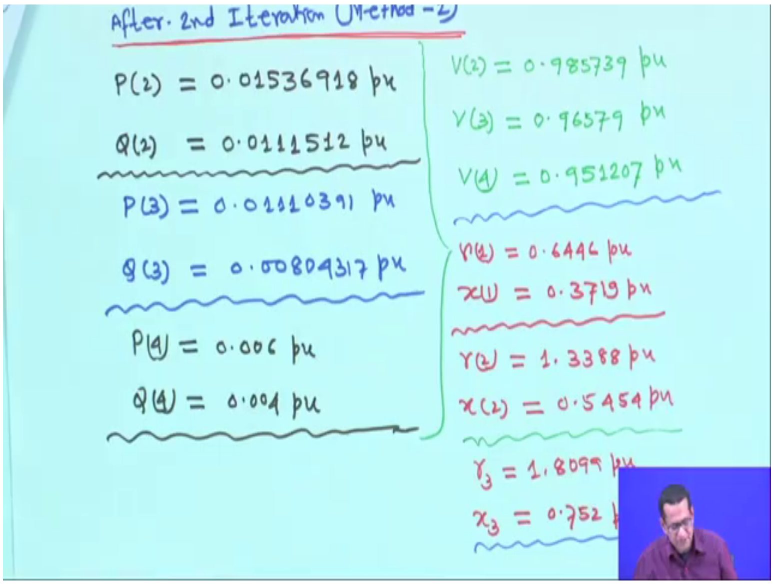
And once you are doing it that you will find this S I min value gradually will decrease this is the maximum value 1, right and gradually it will decrease and here also S I mean gradually decreasing maximum value is say 1 right therefore, this is TQL if you plot somewhere it will come somewhere you will find after that there will be no solution for your that voltage magnitude.

So, just before that that it will that you what you call the system voltage will collapse for example, here substation voltage is always 1, right suppose if you take for the m 2 is equal to 2 and m one is equal to1 for branch 1, these voltage is always 1 suppose these are not there P m 2 x x jj r jj is there suppose P m 2 Q m 2 P m your at node 2 suppose there was total load in the distribution networks and nothing is there. Suppose all load is 0.

So, if you use this expression you will find S I value will be 1 right that is why this value is starting from 1 right, that is why it is starting from 1 assuming that substation voltage is 1 right if it is 1.0 to 5 say it will go up it will go up somewhere here right it will be more than that because it is V m 1 to the power 4.

So, in anyway so but that is why it is starting from 1, so if you go on its almost not linear exactly curvilinear, but gradually will find at some point there will be no solution and system voltage will collapse right.

(Refer Slide Time: 09:27)



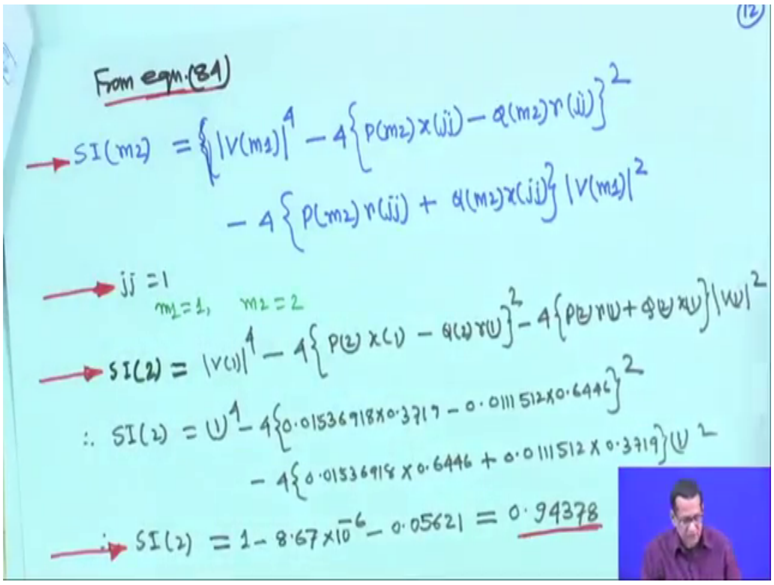
So, will take an example suppose just to show you that we will use that second method second what you call after second iteration whatever you have done that method 2 data load flow data. So, after second iteration we got P 2 is equal to 0.01536918 per unit Q, this all data taking after the load flow studies from method 2 second iteration right everything has been explained all this so I given just have a look.

But for rather than going to those pages everything I have rewritten right similarly Q 2 is equal to 0.0 triple 1512 per unit P 3 is equal to 0.0 triple 10391 per unit and Q 3 is equal to 0.00804317 per unit, similarly, P 4 0.006, Q 4 0.004 this data are available.

after second iteration these are the voltage magnitude V 2 are 0.985739 per unit V 3 was 0.96579 per unit and V 4 0.951207 per unit and all are r all r value and x value in per unit they are all per unit they are same all r 1, x 1, r 2, x 2, r 3, x 3, here everywhere thing I am putting it bracket so I should put it here in bracket right.

So, all these things r 1, x 1, r 2, x 2, r 3, x 3, all are in per unit these data we have used earlier, but in per unit value rewritten here. So, using this data we will calculate S I 2, S I 3, and S I 4 because it is a 4 node problem.

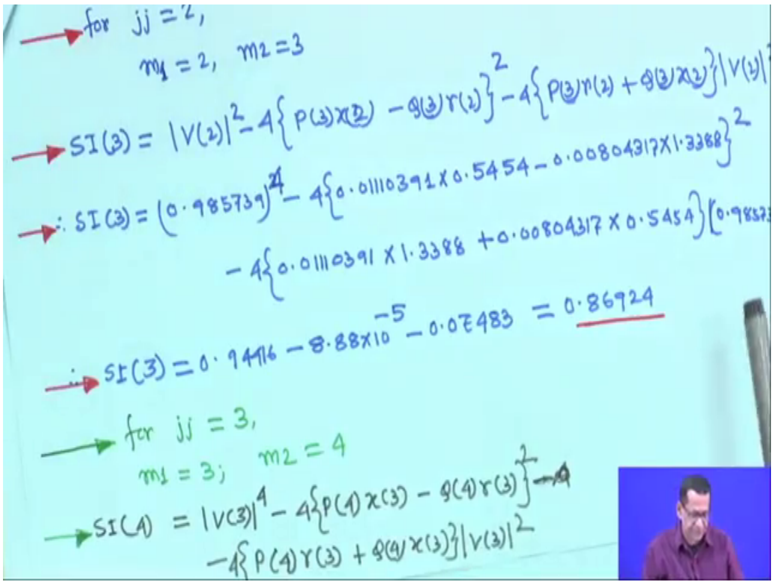
(Refer Slide Time: 10:56)



So, using equation 84 this is my expression for S I m 2 this is expression. So, when jj is equal to 1, m 1 is equal to 1, m 2 is equal to 2; so what you just to give a flavour of the whole testability in x. Therefore, S I 2 is equal to V 1 to the power 4 minus whatever is given here you put P 2 x 1 minus Q 2 r 1 whole square minus 4 into P 2 r 1 plus Q 2 x 1 magnitude, magnitude V 1 square.

So, put all these value because all these all these data are given, all these data are given, all these data are given after second method 2, after second iteration method. So, if you put all these values you will get it will come actually it is 1. So, 1 minus 8.67 into 10 to the power minus 6 actually minus 0.05621 and so it is approximately 0.94378 that is the voltage stability index value at node 2. Similarly find out node 3 and node 4.

(Refer Slide Time: 11:54)

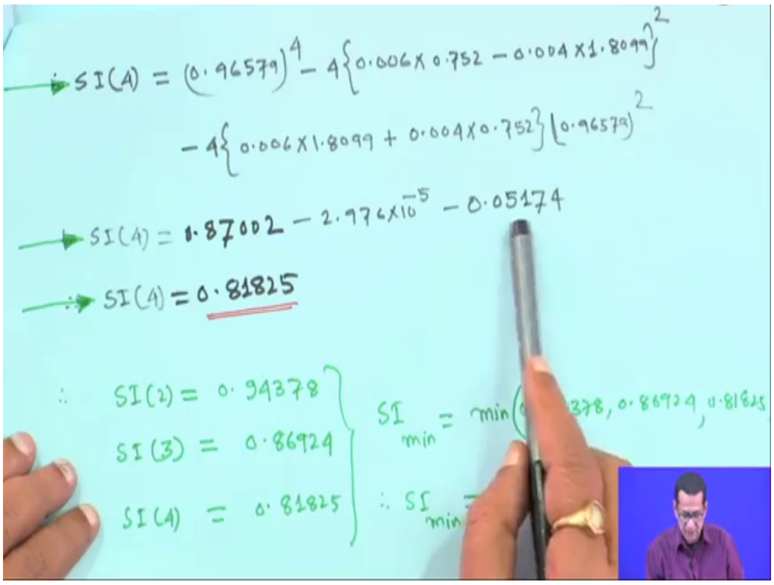


Similarly, when jj is equal to 2, m 1 is equal to 2, m 2 is equal to 3 right therefore, S I 3 is equal to V 2 2 square minus 4 into this term minus 4 into this term into V 2 square all these voltage V 1, V 2 everything is given all these all these values are available voltage V 3 a every here it is everything is available right.

After second iteration so put all these values if you do so and substitute and compute it is actually now V 2 square so 0.985739 minus this one, whole square minus this one into your V V 2 square so it is 0.985739 square. So, you simplify it is S I 3 is equal to 0.94416 minus 8.88 into 10 to the power minus 5 minus 0.07483 this coming 0.86924 right.

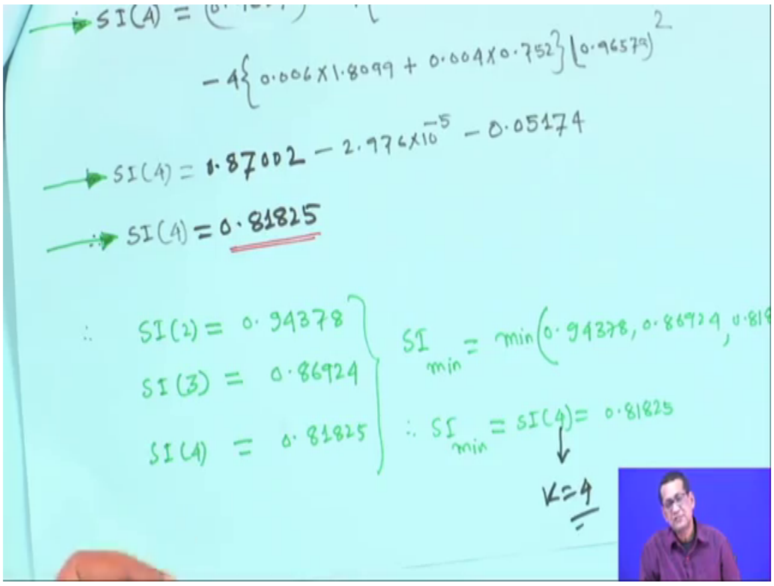
Similarly, for jj is equal to 3 m 1 is equal to 3, m 2 is equal to 4, put this thing and get this expression right in the expression that is same expression of S I m 2 you put this thing get this expression.

(Refer Slide Time: 13:03)



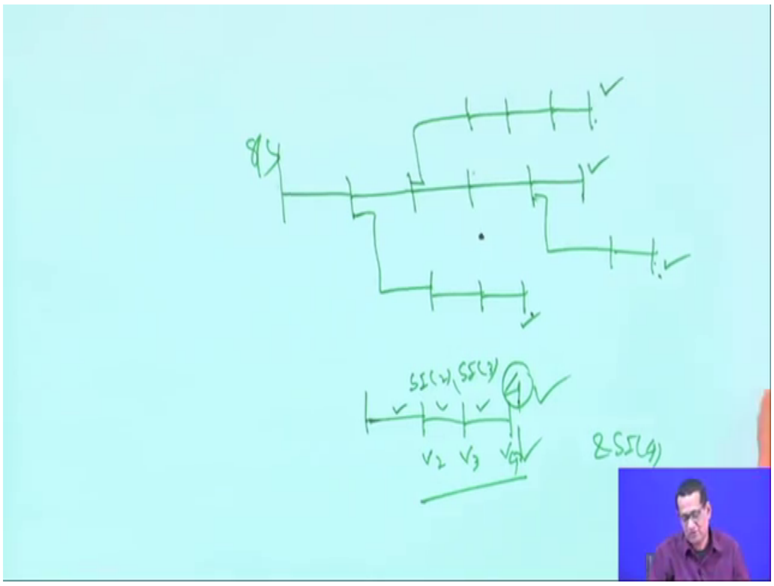
And put all these value this is 0.596579 that V 3 values right so put all these values all these value have been substituted, all these values have been substituted here right substituted. So, S I 4 is equal to then after simplification 0.87002 minus 2.976 into 10 to the power minus 5 minus 0.05174, S I 4 is coming 0.81825 that is the voltage stability index at node 4.

(Refer Slide Time: 13:37)



So, this is S I 2 0.94378, S I 3 0.86924, S I 4 is 0.81825 that means you minimum of all these is 0.81825 that is S I 4 that means, K is equal to 4, here it is your K S I K, K is equal to node 4. So, node 4 is the more sensitive to the voltage collapse this is of course, 2 because it is a small radial network main (Refer Time: 13:56) type there is no your what you call no lateral one such other thing.

(Refer Slide Time: 14:12)



Otherwise suppose if you have a distribution network like this, if you have a distribution network like this say this one you have, you have a network like this and suppose you have a lateral branches, lateral branches.

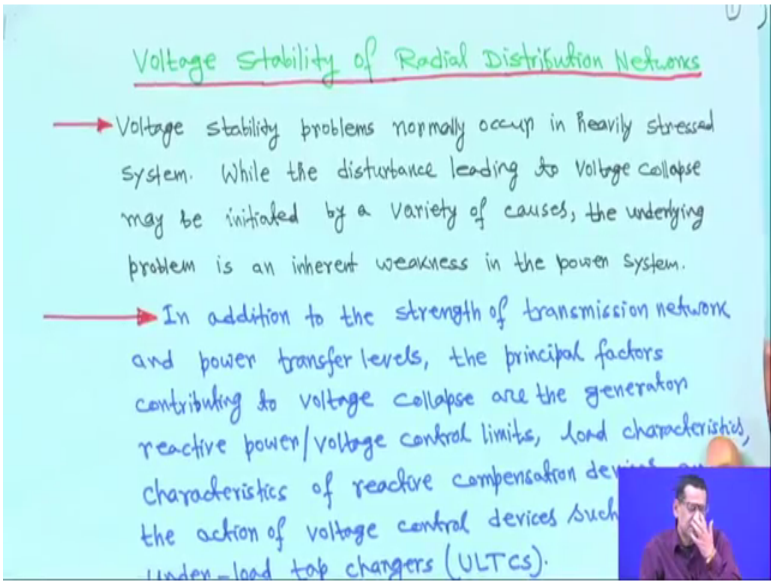
Suppose you have a lateral branches like this, like this, like this, suppose you have lateral branches there is it is no node the your guarantee that this node last node or this last node or this or this will out of this 4 one will be more sensitive voltage collapse no there is no guarantee that right this is of course, substation.

So, you have to find out actually your what you call that each node S I value and increase the load on every node you may find these are these are the then this node this node or this node or this node last node may not be your one of them may not be sensitive voltage collapse it is may be come some other node depend on the load characteristic r x values.

And your load values how it is so it is not that you what you call the example we took it will be small example it will be only 4 nodes. So, this node is coming actually sensitive of voltage collapse reality it you may have many laterals, but does not mean any of these lateral main figure n node will be sensitive of voltage collapse no it depends on the load r x values of the your what you call r x value of the your each branch load and at the same time also substation voltage level many many factors are there.

So, but for these example it is showing that this node is more sensitive of voltage collapse, but which load it will be collapsing that you have to increase the load and you have to see when there will be no solution of the voltage magnitude for node 4 at that time it will be collapsing.

(Refer Slide Time: 15:56)

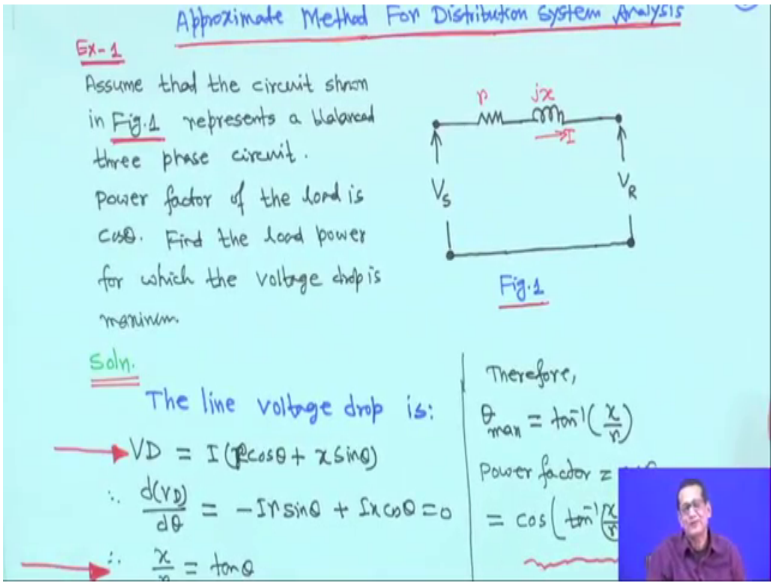


So, this voltage stability index that is voltage stability of radial distribution network and whether it is for a your what you call if you there are many many other versions are there for voltage stability index right of distributed real distribution networks, but here only as per the classroom purpose we want to solve that is why for this thing suppose a load flow will be given I will suggest one thing to you.

For this example for that 4 bus example, for 4 bus example what you can do is that just check just check what you can do we neglect the losses. So, if you neglect the losses we have computed V 2, V 3 and V 4 in the second method by neglecting the losses. Suppose loss is neglected you just yourself compute what is the value of S I 2, S I 3 and your S I 4, right just neglect the losses this results are available up to in the very first iteration when branch losses are neglected right.

And with loss neglected we just try to find out what are this what are what are the value of the stability index and which one will be more sensitive voltage collapse you will find again it is node 4, because it is simple 4 bus problem. In the class we cannot go beyond that right and as for a classroom exercise is concerned with these distribution load flow and your voltage stability your what you call index now it is complete right.

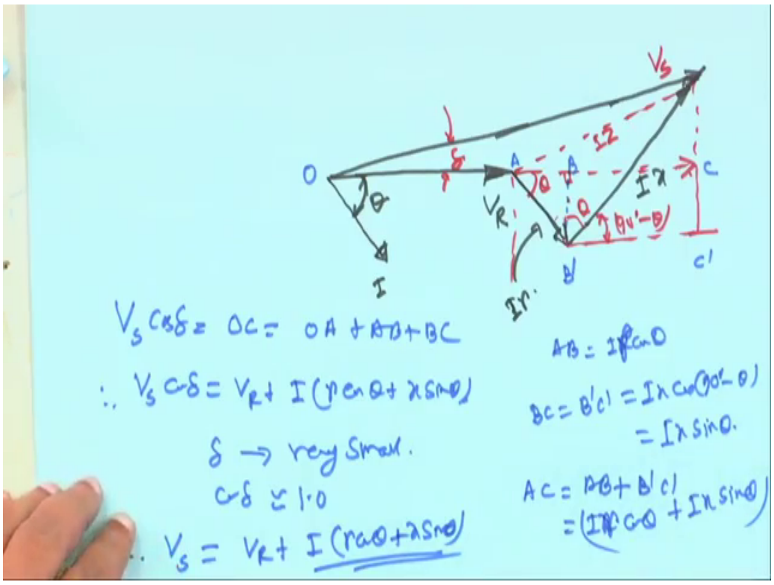
(Refer Slide Time: 17:31)



Only thing is that in the assignment another thing you have to do very seriously and good problems will be given in the assignment and just your job will be to your what you call to solve those things right and compare to the your past system analysis course where also heavy competition was there. Here also huge competition is there, but we will see that we will think of all this things I have thought over that the problem will be given in such a fashion such that you can do it quickly right without calculating too much right, but these are the basic thing.

Now, another thing is that approximate method for distribution system analysis approximate method means suppose this diagram is there this diagram is there r this is forget about this thing you forget time being you forget about this thing. Suppose this is r this is sending end voltage this is receiving end voltage and this is V r right and load is lagging so current is lagging.

(Refer Slide Time: 18:32)



Now if you your what you call if you draw the your that your long line short your short line, medium line, long line those phasor diagram go to that short line phasor diagram then if you take this is reference, this is reference, this is my V r, and current is lagging so this is my current i, right these angle is theta.

Therefore, this is your same thing parallel to this line somehow it will be parallel to this line. So, this term is will be your I r, because I have taken small r, small x resistance reactants I r and this is 90 degree with this right, and this is will be just somewhere here it is. So, this is your I x, I is the magnitude of the current is the magnitude of the current right and this is your I z right and this is your V S these angle is delta right and this angle is theta.

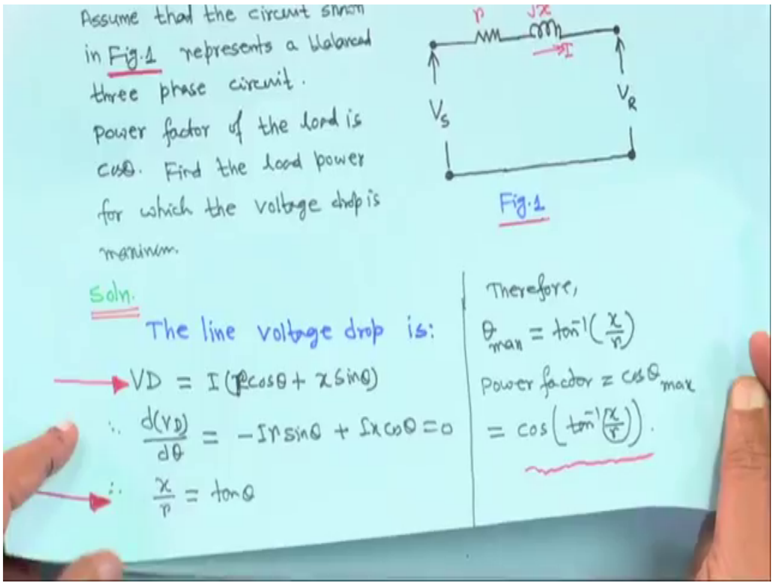
So, if you draw a horizontal line and this one is this is again this one is again theta right. So, this is your 90 degree, this is your 90 degree, this is your 90 degree minus theta; that means, this angle is theta; that means, this angle is 90 degree minus theta right and if you draw a vertical line and this thing right and then your let me write down here this equation so this is this point is A, this point is suppose I am drawing a vertical line from here this point is B, and this point is C; that means, this is your say B dash this is your C dash right.

Therefore, what is A B, A B is equal to your this portion is I r, so I r cos theta then what is B C B C actually is equal to B dash C dash right is equal to your this is I x cos 90 degree minus theta right I x cos 90 degree minus theta is equal to I x sin theta. Then your V S, then your A C first you find out A C A C is equal to A B plus your B dash, C dash B dash C dash and B C same right so that is your that is your I r small r no it should be small r so I small r cos theta plus I small x sin theta right.

Therefore, your V S cos delta you take this is your delta. So, take the projection on that. So, V S cos delta actually if it is here O, so V S cos delta will be actually O C is equal to O A that is your V r plus A B plus your B C; that means, V S cos delta will be is equal to O A V r this O A V r right this O A V r. Therefore, it should be V r plus I you take common these two term it will be r cos theta plus x sin theta right.

But this angle delta actually is very small right delta actually is very small. So, delta actually very small right; that means cos delta approximately 1.0 right; that means, this approximate equation of V S will be V r plus I then r cos theta plus x sin theta right that means, this is actually; that means, V S minus V r is equal to I r cos theta plus x sin theta. So, this is actually voltage drop right this is the approximate way of calculation right because we have taken delta is very small so cos delta is 1.

(Refer Slide Time: 22:43)



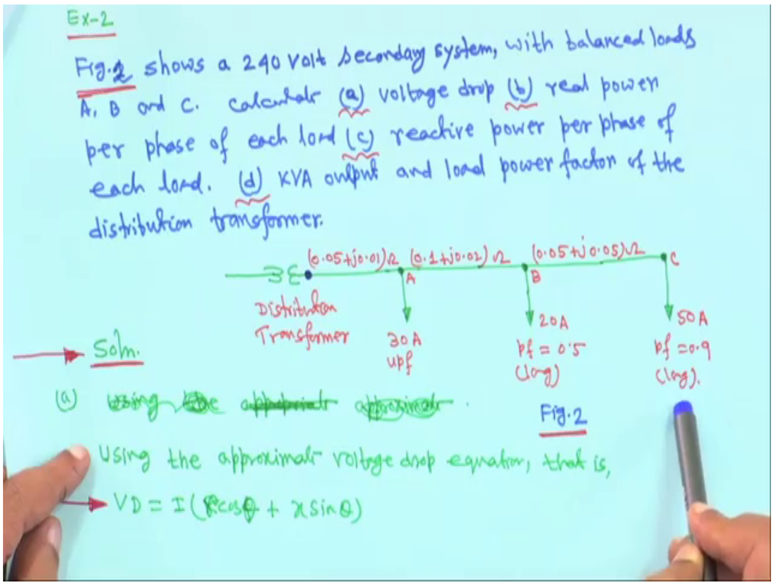
So, it will be V S is equal to V r plus I r cos theta plus I x sin theta right so that is why that is why in approximate method right in approximate method you will use these approximate this voltage drop; that means, voltage drop then will this one is actually I r cos theta plus I x sin theta right.

So, so starting with an numerical type of thing problem assume there is example assume that the circuit shown in figure 1 this figure. Represent a balanced three phase circuit power factor of the load is cos theta right that is that current power factor load is load current angle also this is theta right so power factor is cos theta current is lagging with respect to V r right.

So, find the load power factor for which the voltage drop is maximum right so the line voltage drop V d is equal to just now we have seen voltage drop will be I r cos theta plus x sin theta this approximate one. So, I r cos theta plus x sin theta right just now we have seen so; that means, if you take that derivative with respect to theta so d voltage drop d theta is equal to minus I r sin theta plus I x cos theta is equal to 0.

Therefore, x by r will become tan theta right this is the relationship therefore, the theta max that is will be tan inverse x by r right so here it is tan theta. So, theta max from here you will get theta is equal to theta is equal to tan inverse x by r that is actually theta max right. So, power factor is equal to cos theta max that is cos tan inverse x by r this means that if you if the your what you call if that your power factor of the load if it is like same as the your same as your that impedance angle then voltage drop will be maximum right.

(Refer Slide Time: 24:30)



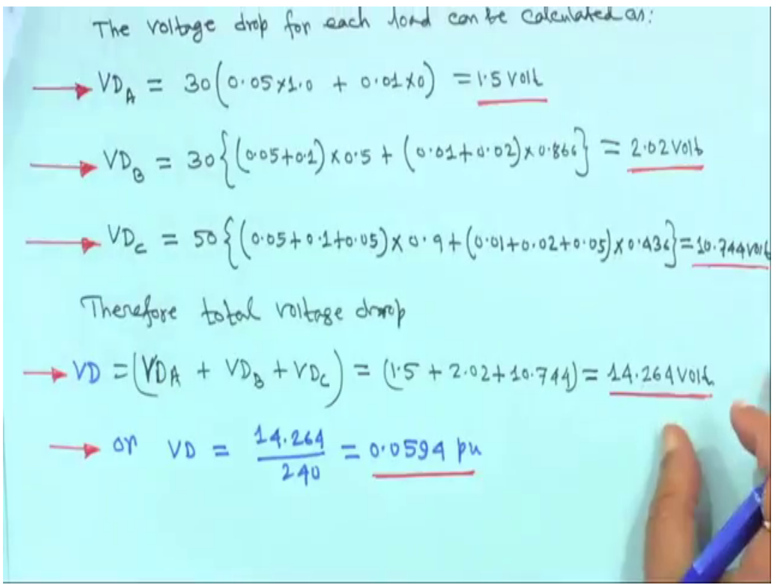
So, next one is say example 2. So, this is a simple one I took the next one is example 2, suppose in example 2 that figure 2, this is actually figure 2, right this is figure 2 we have a we have a distribution transformer here and here that there are 3 tapping points that is 3 nodes are there node A, node B, and node C, the 3 nodes are there right.

So, in that case that branch impedance are given for actually here not marked anywhere so this you also marking 1 node by a point only by a dot right. So, this is 0.05 plus j 0.01 ohm, this is 0.01 plus j 0.02 ohm right and this one 0.05 plus j 0.05 ohm and here in this case everything is given that here no question of iteration or anything because data results are given.

Here it is drawing 30 ampere at unity power factor this is given, here it is drawing 20 amp ampere, but power factor is 0.5 right lagging and here 0.55 ampere, power factor is 0.9 lagging right. So, first use the approximate voltage drop equation that is voltage drop approximately is equal to I r cos theta plus x sin theta this is the voltage drop equation just now we have seen just now we have.

So, first I will so you have to find out voltage drop the real power per phase of each load and then c reactive power per phase of each load and number d K V output and load power factor of the distribution transformer. This is actually distribution transformer here I by bar I did not show the node, but later I will show you right. So, first thing is how to calculate approximate method right so, this is the formula. So, what you do the current is given at at point A 30 ampere unity power factor right.

(Refer Slide Time: 26:29)



So, the voltage drop for each load can be calculated as 30 ampere is the current right I is the there then in bracket it is r cos theta plus x sin theta. So, r for this branch is 0.05 ohm right so and cos theta is unity so cos theta is 1 so r into 1 plus your x 0.01 for this branch, x is 0.01 into that is unity power factor so sin theta is 0, so sin theta is 0 so that is 1.5 volt.

But thing is that directly I am making it, but how I am going getting it I will come later right. Now that now the voltage drop due to your what you call load at B that is load at B. So, power factor here it is 0.5 lagging, power factor is 0.5 lagging, and for this branch 0.1, and j 0.02 ohm right this I is this 1.

So, for this as V d r cos theta so at that time please take from these point to these point what is resistance and what is reactance right so that is 0.05 plus 0.1 0.05 plus 0.1 cos theta is 0.5 right it is given right. Similarly from here it is 0.02 plus 0.01 0.002 plus 0.01 or 0.01 plus 0.02 into it is cos theta so sin theta is equal to 0.866 that due to load B multiplied of course, 30 because here also your this thing this is actually 20, I have I this is actually 20 because here it is 20 amperes, 20 ampere right it is 20.

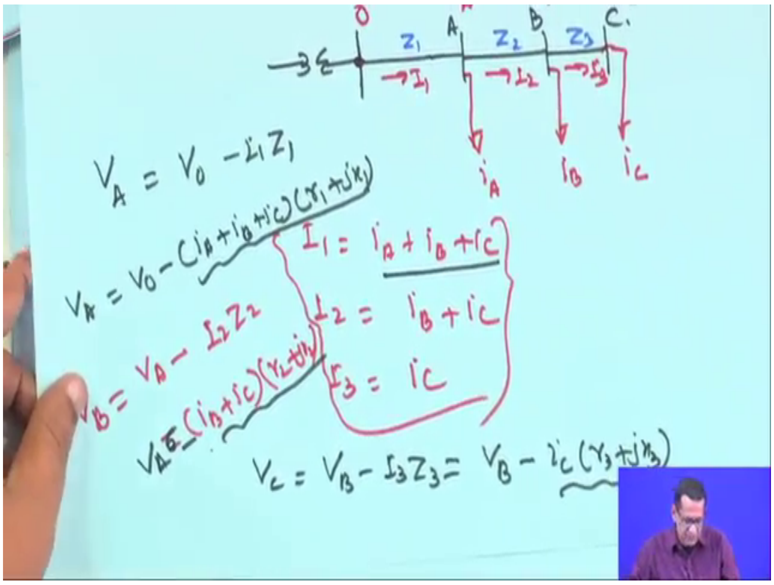
So, this is actually 20 into that your I r I into r cos theta plus x sin theta so 2.02 volt. Similarly due to this one due to this one 5 ampere it is there power factor is 0.9 lagging right. So, in this case you forget about this add r 1, r 2, and r 3 right 0.05 plus 0.1 plus 0.05 into cos theta. So, r 1, plus r 2 plus r 3 into 0.9 right 50 is there, I is there.

Then similarly x 1 plus x 2 plus x 3 from here you calculate all the reactors 0.0 0.01 plus 0.02 plus 0.05. So, into your what you call cos theta is 0.9 so sin theta will become 0.436 right I am not calculating here, but power factor cos theta is given so sin theta 0.436 so, that is 10.744 volt.

If you now add all these things therefore, total voltage drop V D A plus V D B plus V D C it becomes 1.5 plus 2.02 plus 10.744 that is 14.264 volt right. So, but voltage of this figure 240 volts secondary your trans system I mean this point suppose this point, this point that is blue colour this point voltage is 240 volt right; that means, if you drop you compute in per unit it will be 14.264 upon 240 that is 0.0594 per unit right. So, this is the voltage drop.

Now question is how we are computing this approximate one. So, if you if you if you recall the load flow things. So, what I will do I draw this one this circuit again and I will show you, but this is this one I showed directly such that quickly you can compute after that exact calculation also will show.

(Refer Slide Time: 29:57)



So, in this case suppose you have this is a distribution transformer is a this is your node. So, this is suppose instead of one as suppose this is your node A, node B, and node C. This is actually your node A, node B, and node C here also node A same thing I am drawing, but putting bar just to just to show this node right these are the node.

So, everywhere current everywhere current is known everywhere current is known this currents are known. So, this current is i A then this load current is i B this is i c. So, i A, i B, i C at load point a tapping point A B C these are known right with their power factor with a your power factor also known right.

So, there one is unity, another is lagging or lagging power factor are the 12 lagging power factor right, and this is your substation this node this is substation, and this is your distribution transformer say this node is O right. So, then what is this current what is this current flowing through this branch suppose this is your I 1, this is your I 2, and this is your I 3. There are 3 branches right that means, I one is equal to your i A plus i B plus i C right here.

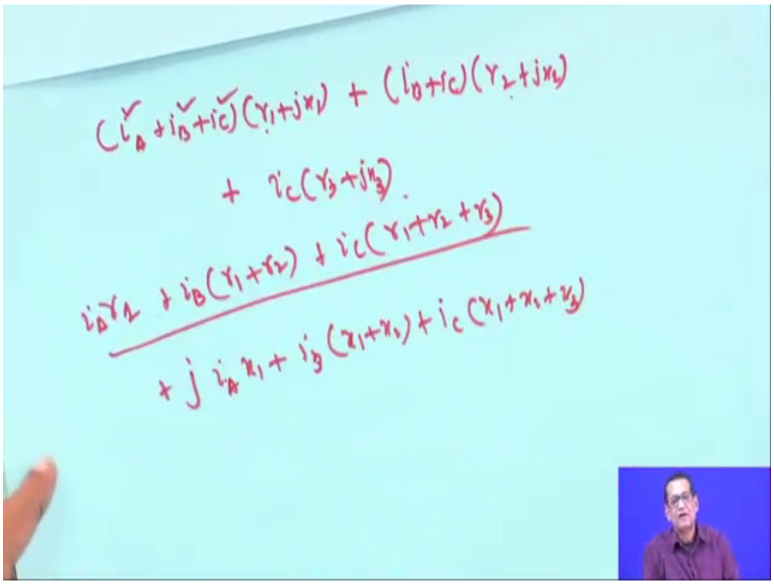
Similarly, I 2 is equal to i B plus i C and I 3 is equal to your i C this same current is going through your node I 3 is equal to i C. So, I 1, I 2 everything you know. Then when you calculate the your this voltage is say V O, this voltage say V A, this voltage say V B, this voltage is V C right. Therefore, and impedance of this branch also also given all this branch given so impedance is z 1, it is z 2, it is z 3 this is given therefore, if you if when you write the equation V A is equal to V O ; that means, these voltage these voltage V O minus I 1, z 1 right.

So, if you if you thing like that therefore, what is my your what you call that I one z one that voltage drop right. So, I 1 is equal to i a plus i B plus i C right. So, if you write V A is equal to your V 0 minus I 1 is i A plus i B plus i C into z 1, z 1 means r 1 plus j x 1 this branch one right this is your first voltage drop I A plus I B plus I C r 1 plus j x 1 right.

Similarly that is what you call in terms of I A plus I B plus I C. Similarly if you write V B, V B is equal to V A minus I 2 z 2, right. So, that will become actually this one become I 2 is I B plus I C into r 2 plus j x 2 right. So, this is your V B similarly your V C will be from this graph from this figure only from this figure only V C will be V B minus I 3 into z 3 is equal to V B minus I 3 into z 3 that is I 3 is nothing, but I C I C into your z 3 is equal to r 3 plus j x 3 right.

So, what we are doing is that total voltage drop and this that so this is here for the branch 1, voltage drop this is your branch 2, this actually it is actually V A minus this 1. So, this is branch 2 voltage drop and this is your branch 3, voltage drop right if you add all these voltage drop if you add all these voltage drop just for the sake of understanding.

(Refer Slide Time: 33:54)



So, this is your if you add all I A plus I B plus I C into r 1, plus j x 1 plus then other voltage drop is I B plus I C into r 2 j x 2 and last one is I C into r 3 plus j x 3 right. This way if you make then you will see that first is i A this i A into r 1. So, it is your i A into r 1 right this term. Now when to take I B I B into r 1 I B into r 2 that means, your I B your r 1 plus r 2 right.

Now when you take I C I C into r one then your, I C into r 2 and I C into r 3 so I C r 1 plus r 2 plus r 3 this is this is the real part. Similarly the reactive part it will be j right it will be j then it will be i A x 1 plus i B x 1 plus x 2 plus i C x 1 plus x 2 plus x 3.

Thank you we will be back.