Power System Engineering
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Lecture – 37
Load flow of radial distribution networks (Contd.) & Voltage stability of distribution network

So next is loss of branch 22 sorry branch-2, right? So, similarly PLoss 2 will be r 2 into P 3 square plus Q 3 square upon V 3 square, this already I have told you right, that I mean; if you just that electrical equivalent of branch-2 if you take just hold down I will show you once again.

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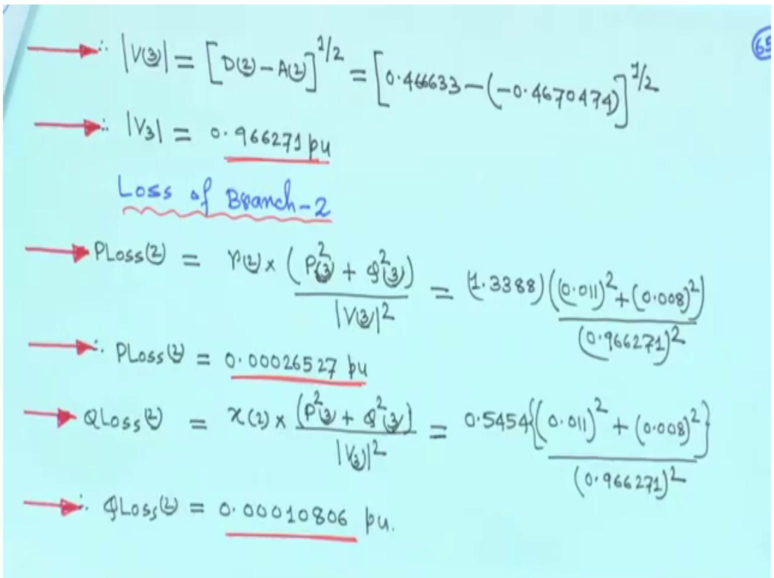

So, power loss of branch-2 will be PLoss 2 will be r 2 into P3 square plus Q 3 square upon V 3 square. So, r 2 values in per unit you substitute and P 3 Q 3 you substitute voltage V 3 just now, you have computed here, this one this one you substitute you will get PLoss 2 will be 0.00026527 per unit. Similarly, for QLoss 2 you have to take x 2 into P 3 square plus Q 3 square upon V 3 square. So, only r x changing this term is same right once you compute this term separately, once you multiplied by r once again multiply by x right. So, it is also QLoss 2 will be 0.00010806 per unit, this is the loss of branch-2 because, in the next iteration loss has to be added right.

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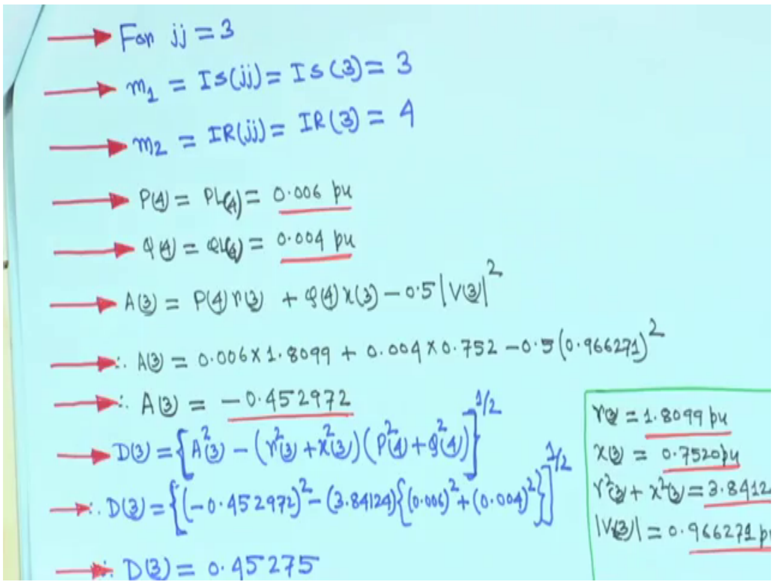

Similarly, for branch 3 when j j is equal to 3 sending end node will be 3 and receiving end node m 2 will be 4. So, at that time P 4 will be PL 4, QL Q 4 will be QL 4 because, at 4 this is your is there here, it is load is your PL 4 plus j QL 4, right? This is the load same and if you take electrical of electrical equivalent of this branch of this branch. Suppose this is your branch 3 and this is your branch 4. So, here you have a load that is your P4 plus j Q 4, right? But it is the last node.

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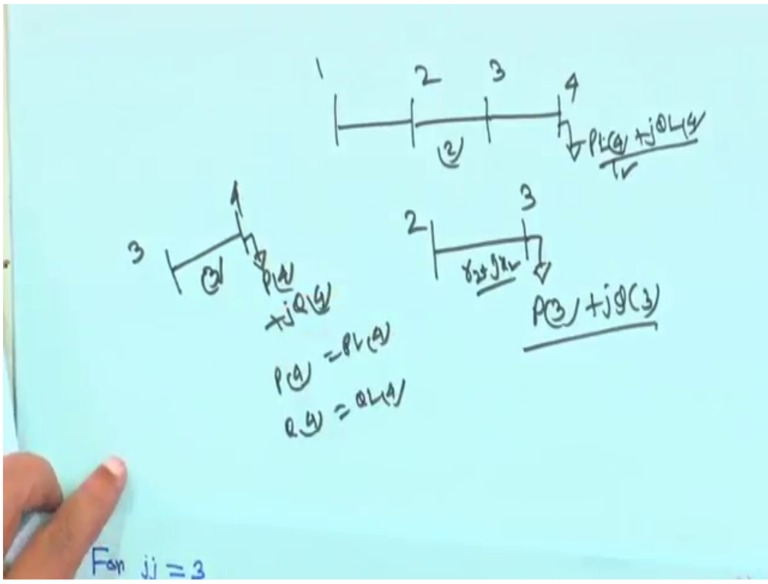

So, beyond that there is no other branches nothing is there. So, naturally P4 will be is equal to this one PL 4 similarly, Q 4 will be is equal to QL 4 and this is for branch 3 this is the electrical equivalent of branch 3, right? So, that is why that P4 is equal to PL 4 here, I should I should make if you want I can make it bracket also no problem, but I told you whether it is a suffix or whether, I am putting in bracket they are same, right? So, P 4 will be 0.006 per unit and Q 4 will be 0.006 per unit.

So, again from this equation A 3 A j j and D j j. So, A 3 will be P 4 r 3 plus Q 4 x 3 minus 0.5 V 3 square from the same equation of A j j I mean this equation. This equation that is your 68 equation 68 from this equation only right here, you put m 2 is equal to 4, m 1 is equal to 3 right, same equation 68. So, it is your then put all this value P 4 r 3 Q 4 x 3 and V 3 square, this V 3 you have computed just now you have computed, right? So, that is you have computed V 3 is equal to 0.966271 just in the previous slide you have computed. So, put everything here all this thing everything here. So, you will get A 3 is equal to minus 0.452972.

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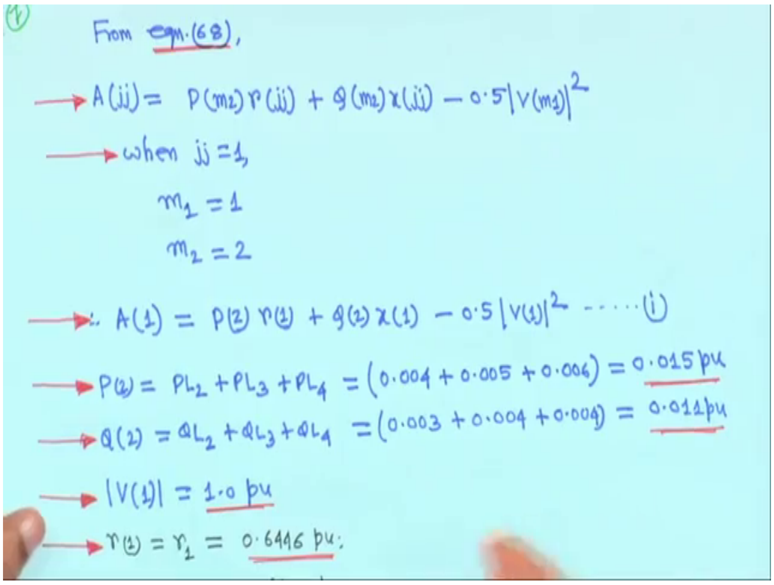

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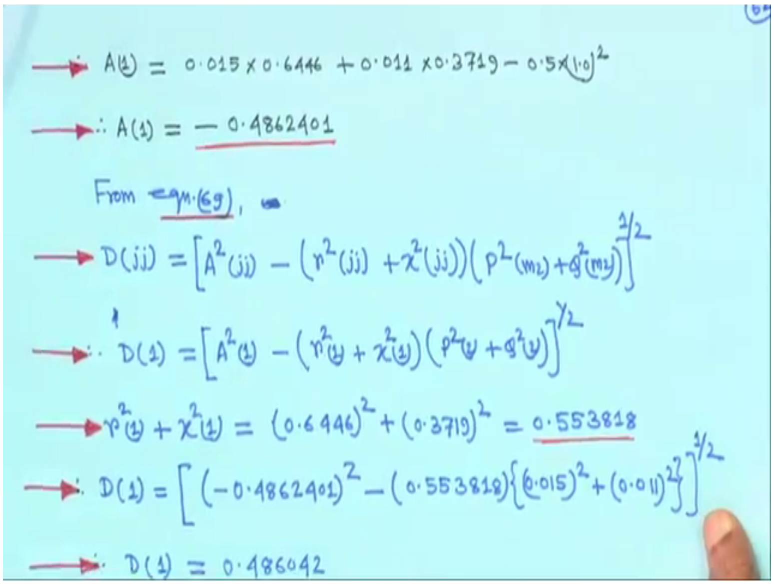

Similarly, from equation your this from equation 69, right? From equation 69 it expression of D j j here you put j j is equal to 3, m 2 is equal to 4, m1 is equal to 3 so; that means, in this equation D3 will get a square 3 minus r square 3 plus x square 3 into P square 4 plus Q square 4 to the power half, put all these value. So, just now you have computed this A 3 you have a computed. So, it is minus 0.452972 square minus r 3 square plus x 3 square I have computed here.

So, 3.84124 right into that P 4 square plus Q4 square. So, 0.006 square plus 0.004 square whole to the power half. So, it is coming actually, 0.452 every time I told you this and this they are very close in magnitude, but this is only here is a minus sign, right? Therefore, from that voltage equation that, V j j is equal to your D j j minus A j j that, will be your equation your 66.

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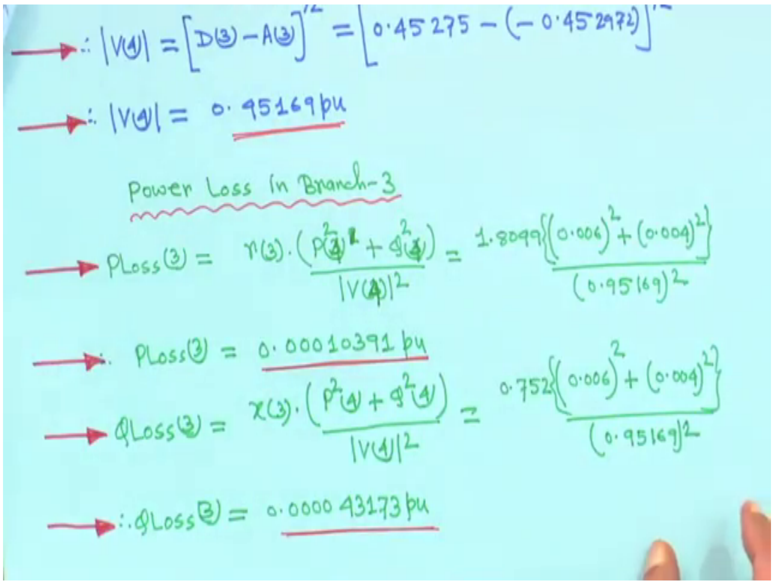

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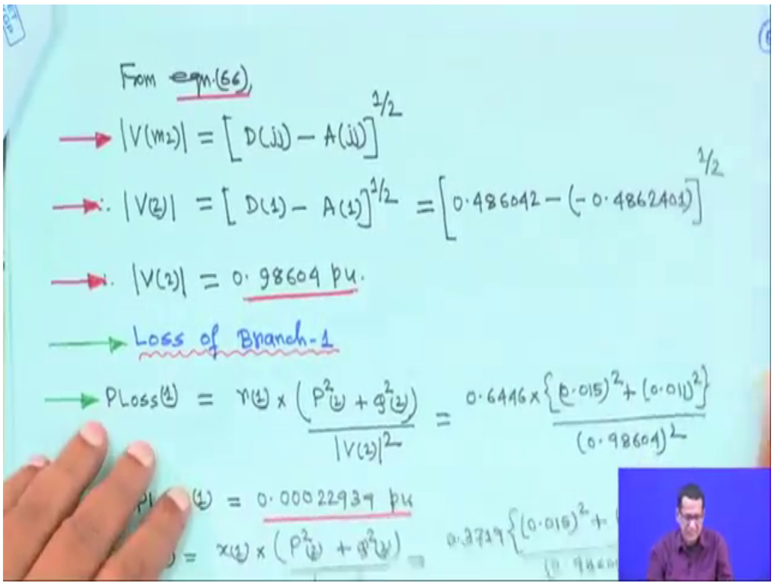

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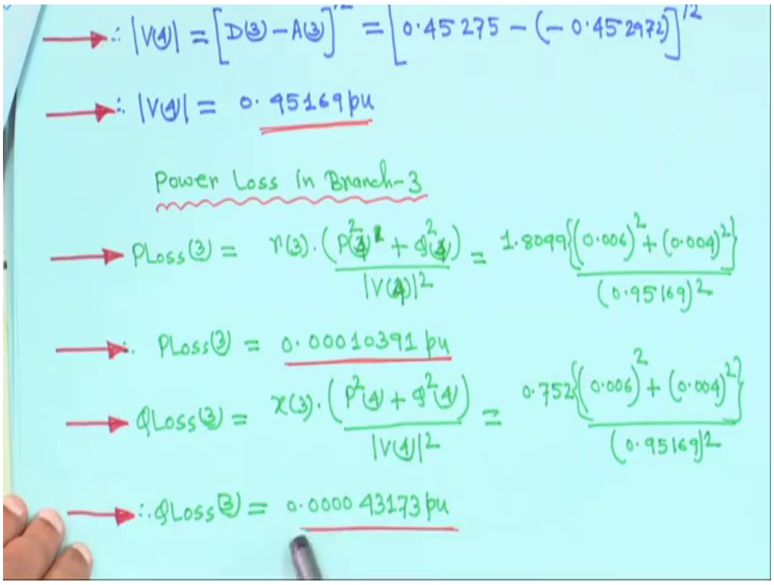

This equation this equation, right? So, here you put it all this 2 values 0.45275 minus in bracket minus 0.452972 to the power half. So, minus will be plus, right? So, you simplify, so it will be 0.95169 per unit.

So, this is this is the value your voltage magnitude in the first iteration now, power loss in branch 3 similar way power loss in branch 3 will be r 3 into P square 4 plus Q square 4 upon V your magnitude V 4 square. So, r 3 is 1.8099 then P 4 Q 4 you substitute square divided by voltage 0.95169 it is square. So, branch 3 power loss is coming 0.00010391 per unit similarly, for QLoss for branch 3 it will be x 3 into P square 4 plus Q square 4 upon V 4 square, right? Substitute again only here this term this term here, it is same only here it is r here it is x, right? If you simplify, it will be QLoss branch 0.000043173 per unit right. So, this is your branch 3 power loss now. Next is iteration 2 I am not showing that, convergence criteria difference of all that is because, only 2 iterations are shown.

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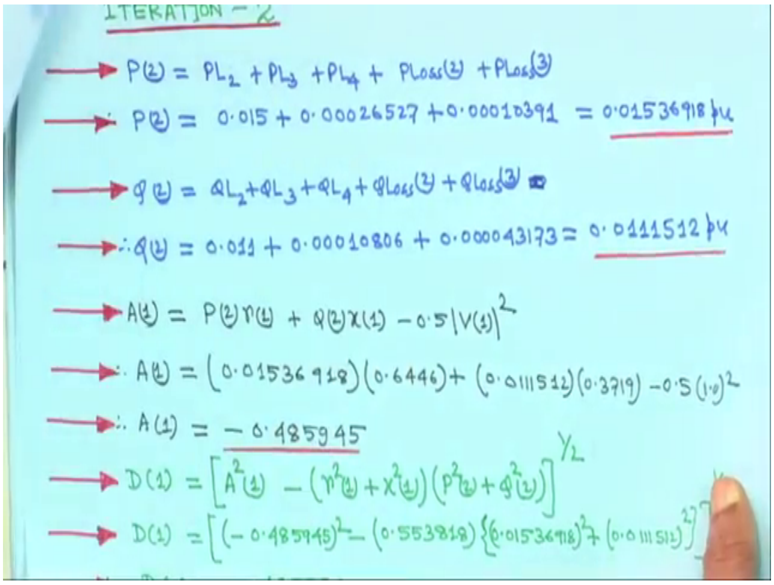

So, just what will be there, right? Now, iteration 2 P 2 will be total load PL 2 plus PL 3 plus PL 4, but in front of this I mean beyond this node only 2 branches are there, branch 2 and branch 3, right? This is branch 1, right? And this is your branch 4 sorry branch 3, right? So, it is 1 2 3. So, beyond this node only 2 branches are there. So, PLoss 2 and PLoss 3.

Similarly, for Q case it will be QLoss 2 and QLoss 3, right? So; that means, your that your just hold on; that means, iteration 2 P 2 will be then PL2 plus PL 3 plus PL 4 plus PL loss 2 plus PLoss 3. So, this loss we have computed just now. So, sum it up all it will be 0.01536918 per unit this is the value similarly, Q 2 will be all the Qs this all the QL 2, QL 3, QL 4 plus QLoss 2 plus QLoss 3 QLoss this QL 2 plus QL 3 plus QL 4 already you have computed previously 0.011 here, also 0.015 this is 3 actually this one and summation of these actually this one, right? So, plus QLoss 2 plus QLoss 3.

So, it is coming 0.0111512 per unit, right? Similarly, again from the same equation you calculate A1 is equal to P 2 r 1 plus Q 2 x 1 minus 0.5 V1 square V1 is always one because, it is slack bus it is voltage magnitude is one. So, a one is equal to P 2 now, you have got this much Q 2 you got this much, right? So, put everything here all this things you substitute, right? Everything is here. So, you will get A1 is equal to minus point 8 sorry minus 0.485945.

Similarly, again similar way you calculate D1, right? So, you have got new value of A1 you put it here, and r 1 square x 1 square is constant it was there already 0.553818 and here, P 2 square plus Q 2 square. So, this one square plus this one square to the power half. So, D1 will come 0.485738, right?

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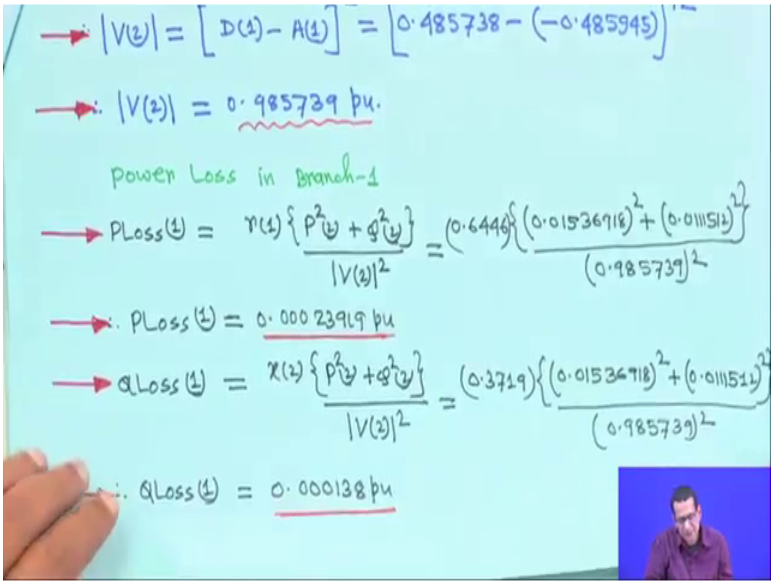

Therefore, that voltage equation that V j j equation is equal to D j j minus A j j to the power half, right? So, you put here D1 value and A1 value. So, this 1 minus bracket minus of this one, this will become actually 0.985739 per unit. This is voltage magnitude now, power loss in branch one in the second iteration same formula PLoss 1 r 1 into P 2 square plus Q 2 square upon V 2 square. So, r 1 then substitute new values of P 2 new values of Q 2 and new value of these voltage V 2, right? So, if you compute then PLoss one will be 0.00023919 per unit similarly, QLoss 1 also same thing right only x 2 value will be change changing 0.3719 into the same thing same thing right sorry. So, QLoss 1 will be 0.000138 per unit right. So, this is loss of branch one.

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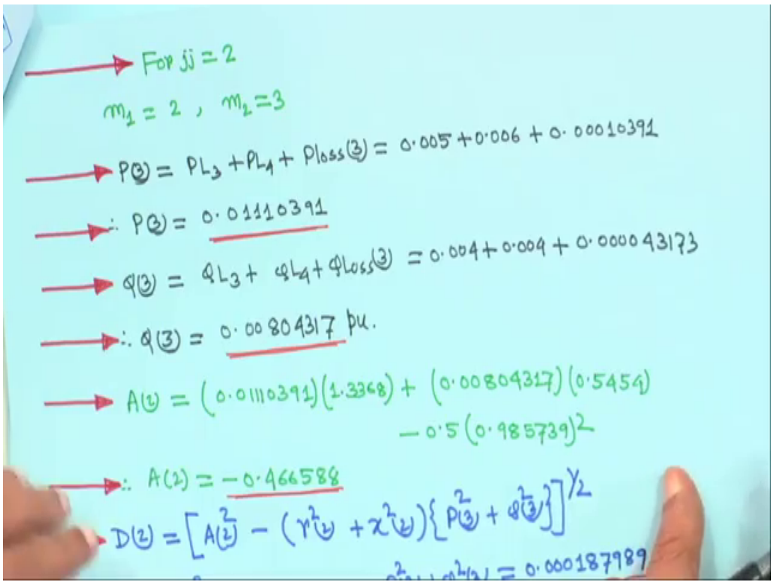

Similarly, for j j is equal to 2 m1 2 m 2 is equal to 3 now, everything is clear to you right every step all mathematical relationship everything is have been explained. So, everything is clear to you it is P because, beyond node 3 including the node 3 itself it is PL 3 plus PL 4 load, and beyond node 3 only one branch is there PLoss 3; that means, this is your what you call this is your node 3 beyond this only one branch is there, that is branch 4, right? So, sorry branch 3 this is one branch 2 this is branch 3. So, 4 node problem actually it is, right? So, it is PLoss 3.

So, PL 3 plus PL 4 plus PLoss 3 you have computed this one so, P 3 will be getting this much 0.01110391. Similarly, Q 3 be QL 3 plus QL 4 plus QLoss 3. So, this is your QL 3 plus QL 4 plus this one. So, Q 3 will get 0.00804317 per unit, right? Therefore, you calculate a your this thing A 2 is equal to. So, the way it is you are what you call put all these values in the same expression of A j j is equal to that expression and you compute A 2 is equal to minus 0.466588. Similarly, expression for D j j already we have seen you put all these values.

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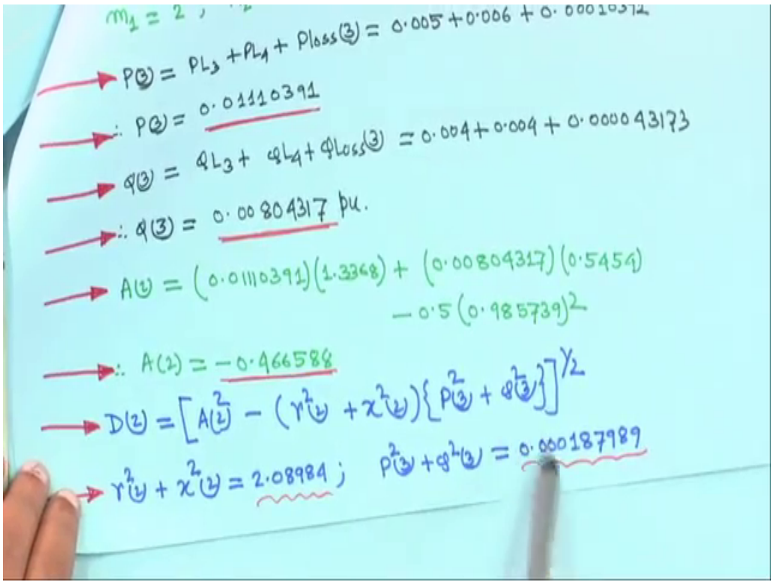

If you put all these value r 2 square plus x 2 square I have given P 3 square plus Q 3 square I have computed; that means, this P 3 square plus Q 3 square I have computed here, it is this much, right? Therefore, your this thing. This D 2 will be minus 0.466588 square minus in your what you call it will come 2, that is your r 3 square plus x 3 square into P 3 square plus Q 3 square to the power half it comes actually 0.4661668, right?

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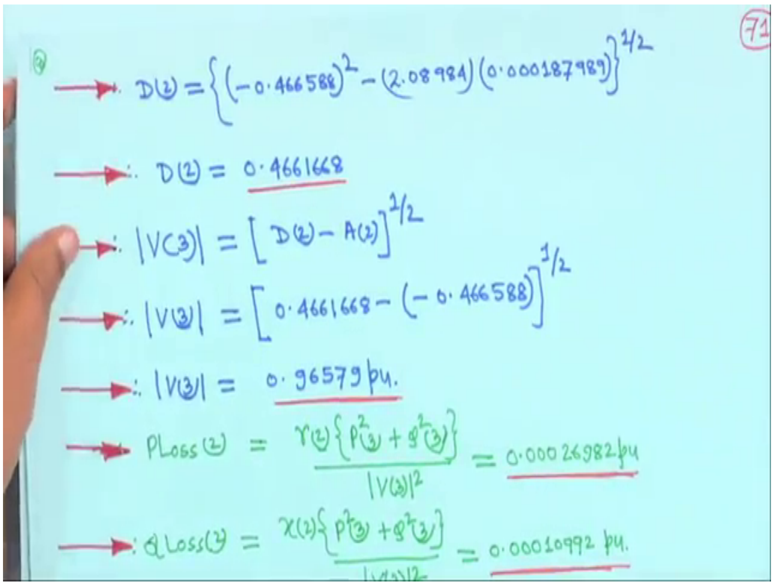

Therefore, magnitude of V 3 is equal to D2 minus A 2 to the power half therefore, V 3 is equal to this much, right? So, it is coming 0.96579 therefore, power loss in branch 2 it is r 2 into this formula it is coming 0.00026982 per unit, right? Similarly, QLoss 2 will be x 2 into P 3 square plus Q 3 square upon V 3 square substitute all this value you will get for 0.00010992. I did not put and showed it again because, every data is known just you put it and see that whether results are correct or not, right?

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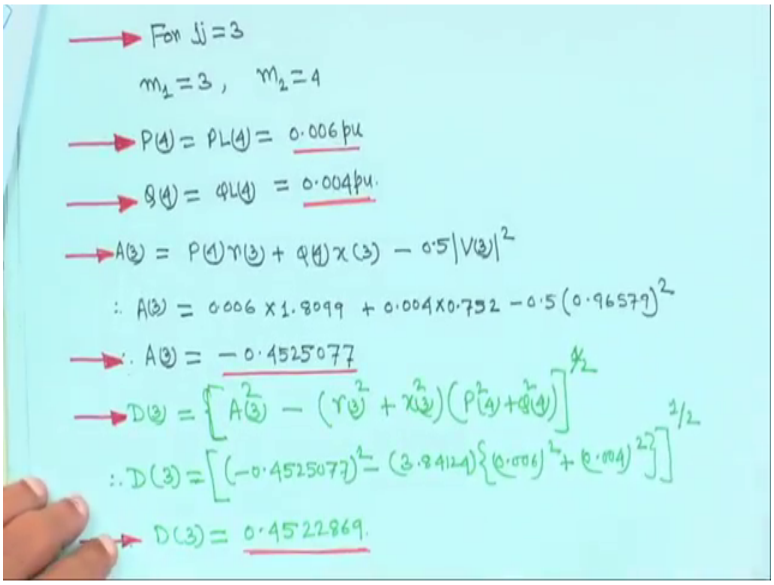

Therefore similarly, for j j is equal to 3 that is branch 3 m1 is equal to 3 m 2 is equal to 4 same way. So, P 4 is PL 4 is equal to 0.006 per unit and Q 4 is equal to QL 4 0.004 per unit. So, calculate A3 now, for this similar same expression for A j j you put get this equation and put all these values you will get A3 will be minus 0.4525077. Similarly, from equation of D j j there you put all this things you will get after substituting all these values you will get D 3 will be 0.4522869, right? You put all these values, that means; magnitude of V 4 that expression of V j your what you call V m 2 is equal to D j j minus A j j to the power half. So, 0.4522869 minus in bracket minus of 0.4525077 to the power half, magnitude of V 4 is equal to 0.951207 per unit, right?

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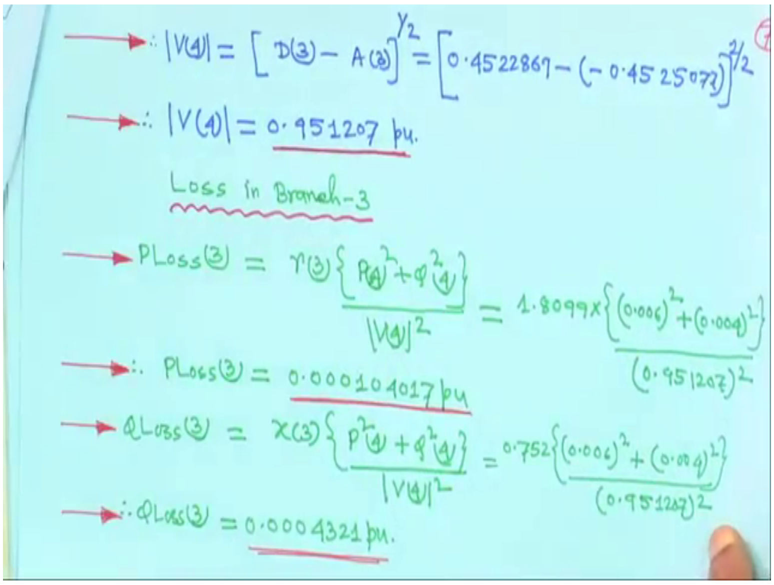

So, loss in branch 3. So, same formula PLoss 3 in the r 3 into P 4 square plus Q square your Q4 square upon V 4 square put all this things you will get PLoss 3 is equal to 0.000104017 per unit. Similarly, for QLoss 3 it is x 3 into the same term here same term here, you will get 0.0004321 per unit. So, these are your, what you call your P branch loss in branch 3, right?

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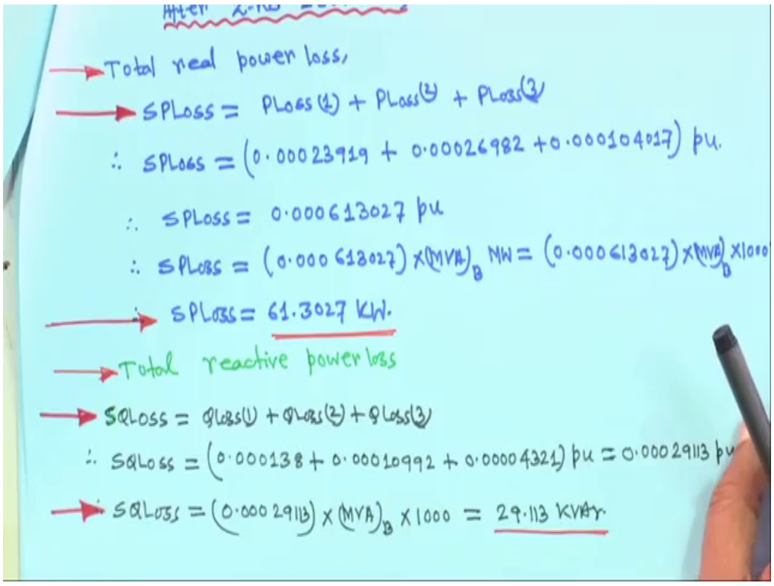

Now, so all the branch losses you have calculated if you compute total real power loss then, it will be PLoss 1 plus PLoss 2 plus PLoss 3 if you sum it up it will be

0.000613027 per unit. So, if you multiply by MVA base into 1000, that is kilowatt you convert, right? So, a total power loss is coming 61.3027 kilowatt. Similarly, for QLoss right it was coming actually you QLoss 1 plus QLoss 2 plus QLoss 3 sum it up and then you multiply by same MVA base into 1000 your MVA base is 100. So, it will become 29.113 kilo hour. So, in this case and if you compare if you compare the they result for your P method 1 and method 2, that just what difference we can observe after your what you call after second iteration, right?

So, in this method, that your when you take your I square and loss and this thing your here, when you are taking this injected power method that substation it is coming 57.48 I told you the difference is because, solution is not completely converged, right? But, if we do I square r loss here it is coming 60.94 kilowatt, right? But in this method 2 also it is coming actually, 61.3027, but I square r method here it is coming 60.94, right?

Similarly, here it is 29.113 and here, it is 28.94 so; that means, and if you look at the you just see yourself if you see the voltage magnitude of both the methods you will find for this example second method is converging faster than the first one, but one advantage of this second method is that, throughout this computation there is no trigonometric term or complex number is involved, but in the second method that expression of angle delta.

It can be also obtained using the same initial relationship, that 2 equations we wrote one equation we wrote I is equal to your V m 1 minus Vm 2 upon Z j j that is your I j j is equal to another one equation P m 2 minus j P m 2 is equal to your V conjugate m 2 into I j j using those 2 expression, that we can also get that your angle delta your what to call angle expression we can get it. But, as in the distribution system voltage angle is not that important therefore, it because we have seen the variation of angle is very, very small, that is why magnitude of voltage is sufficient.

So, second method will be better in terms of your what to call the first one because, we if we need the voltage magnitude only, but there are many many methods are available in the literature, but some advantages some disadvantages we will find in every method, here also for this example second method is better than first method, right? Compared to it will compare the voltage magnitude yourself after second iteration of both then will know, but different methods I have different advantage advantages, sometimes we need real power and your what to call real and reactive component of the current also for some other purpose.

But in this case, you will get the magnitude of current, right? But delta can also be obtained, but that you have to write another equation for delta, but not obtained for the second method so, but for this example as far as magnitude is concern this is require I mean this is your what to call voltage magnitude and your power losses, right? If this 2 things are required then second method one can move, but still it depends on the complexity of the network, that which one will be I mean sufficient, right? So, those things are not to be discussed here. So, 2; that means, for load flow studies we have seen the 2 different 2 different methods, right for load flow studies method 1 and method 2 one thing.

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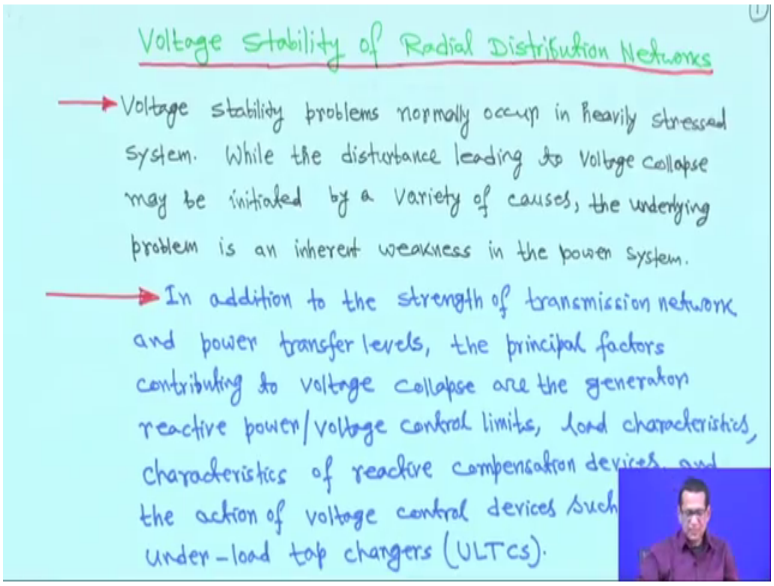

We have noticed that, you have not we have not done it through Newton Raphson method, but couple Newton Raphson method also works equally OL, right? But, in that case computation will efficiency may be less in couple Newton Raphson method, but here we are exploiting the radiality of the network and based on that, we are developing the algorithm. So, you should you should study OL up to 4 bus or 5 bus problem and just see how things can be solved.

Now, another thing is, that voltage stability of radial distribution network, right? So, voltage this is actually some feeling I give you regarding that voltage stability, right? So, the distribution system. So, voltage stability problem normally occur in heavily stressed system, right? Only heavily loaded system while that disturbance leading to voltage collapse may be initiated by variety of causes there are, different type of causes the underlying problem within inherent weakness is the power system.

So, in addition to the strength of transmission network and power transfer level the principle factor contributing to voltage collapse are the generator reactive power voltage control limits this sometimes create problem, load characteristics of course, whatever load flow you have studied we have the load as a constant power, but loads have different characteristics, right? And the characteristics of reactive power compensation reactive compensation devices we use different reactive power compensation devices, right? Including facts devices of course and the action of voltage control devices, such as your under-load tap changers, right?

So, suppose voltage is going down you have to change the tap the transformer such that, voltage can be raised to certain level, right? So, there are many factors the associated, that with voltage stability of the network.

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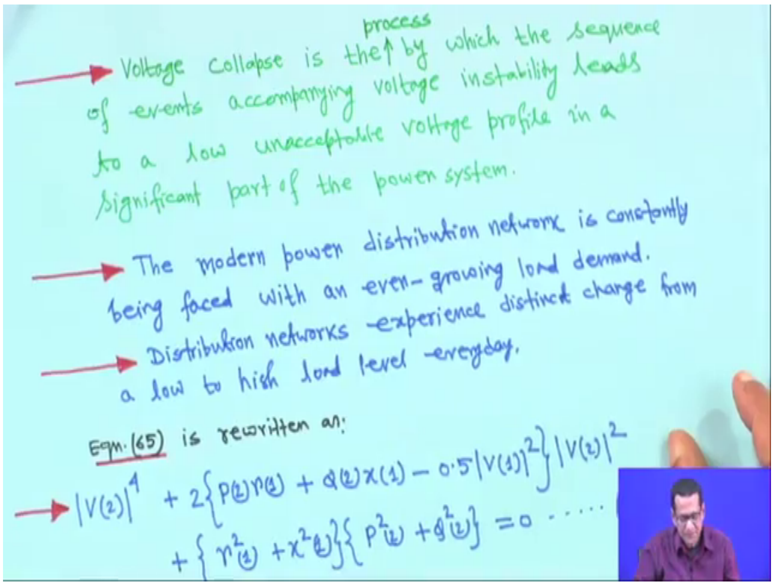

So, in general, that voltage collapse actually the process, right? By which the sequence of events accompanying voltage instability leads to a low unacceptable voltage profile in a significant part of the power system, right? So, part of the power system you find, that your voltage your what you call voltages are unacceptable, because it is very low, right?

So, it is the process by which the sequence of events accompanying voltage instability leads to a low unacceptable voltage profile in a significant part of the power system. There are many examples of voltage collapse in the volt, but I did not bring that list there are many so, but the modern power distribution network is constantly being forced with an ever-growing load demand because, load demand in power system is increasing; that means, on the distribution side that load demand is increasing, right?

So, distribution network also experience distinct change from a low to high load level everyday, right? So, that is what your, what you call that load is increasing. So, if load increases then naturally voltage magnitude will be low. So, voltage will gradually decrease. So, that then we will try to see something, but look this is an iterative process. So, just I will give you some idea about the voltage collapse, right?

And that; that means, same equation that equation 65 for method 2 only we will use here, because are all will try to find out in terms of your voltage magnitude, right? So, in that case that equation 65 can be written as V 2 to the power 4 right, plus 2 into P 2 r 1 plus 2 2 x 1 minus 0.5 V1 is magnitude V1 square into the V2 square plus r 1 square plus x 1 square into P 2 square plus Q 2 square is equal to 0, this rewriting equation what this number is 74, right? This is your what you call that your that method 2 load flow equation, right?

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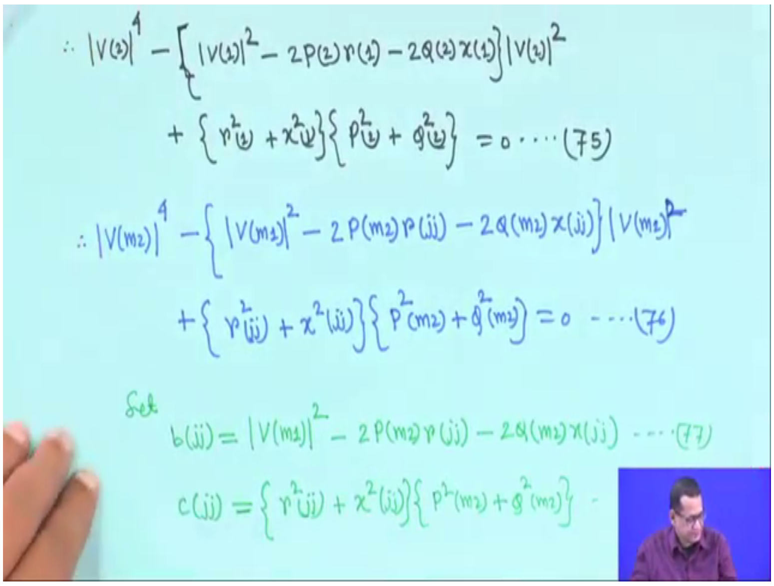

So, in that case or this equation this equation I read what you call this or V2 4 this equation, right? All this equation we are take what we are doing is that you multiply by this 2 and take minus common here, in this term you multiply this 2 and take minus common, right? So, these equation can be written as your V to the power 4 minus V1 square minus 2 P 2 r 1 minus 2 Q 2 x 1 bracket close magnitude V 2 square plus this term r 1 square plus x 1 square into P 2 square plus Q 2 square is equal to 0 this is equation 75.

Now, in general this equation can be general I. So, instead of 2 we will m 2 because, this is the this equation has come because, of the electrically equivalent of branch one of the distribution network. So, instead of 2 we will make m 2 because, m 1 is 1 and m 2 is 2 for branch one. So, in generalise we will make we will instead of 1 or 2 we will make V m 2 to the power 4 minus this already we have seen right for the load flow method 2 only thing is that here, we rewriting this equation in different way minus 2 P m 2 r j j minus 2.

Q m 2 x j j bracket closed into V m 2 square plus r square j j plus x square j j into P square m 2 plus Q square m 2 I equal to 0, this is equation is 76, right? Now, you assume that, let b j j is equal to this term v m square magnitude of course, minus 2 P m 2 r j j minus 2 Q m 2 x j j this equation 77 and c j j is equal to this term r square j j plus x square j j into P square m 2 plus Q square m 2 this is equation 78, right?

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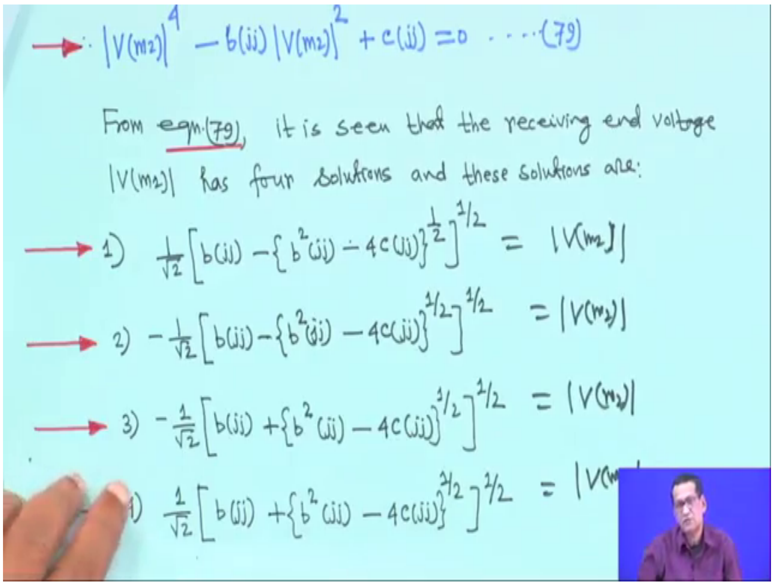

If you do so, then this equation becomes this equation. This equation become actually V m 2 to the power minus b j j V m 2 square plus c j j is equal to 0. So, from this equation you can see this equation actually had 4 roots it is seen that, receiving end voltage we have 4 solutions and these solutions are these 4 actually, this equation suppose V m 2 to the power minus b j j v m 2 square plus c j j is equal to 0.

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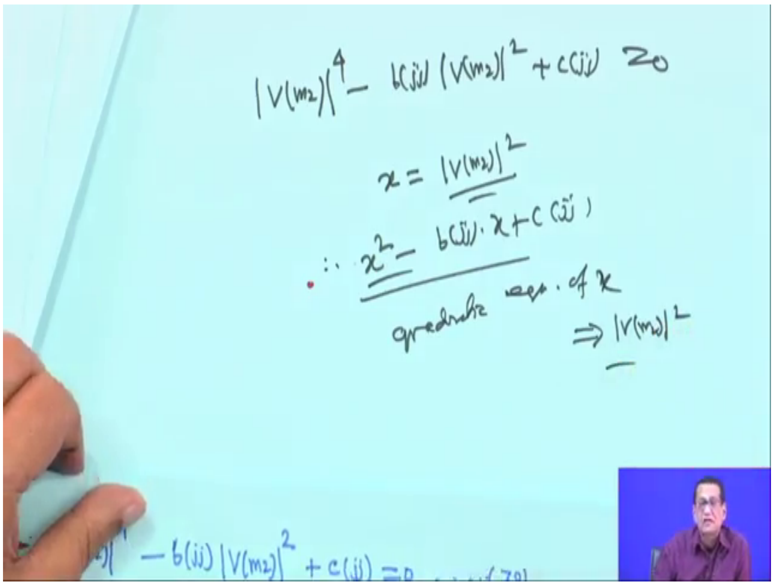

So, this is actually quadratic equation of V m 2 square for example, if you assume x is equal to say V m 2 square, then these equation V m to the power 4. So, this can be write as x square minus say b j j into x plus c j j, right? So, this equation actually quadratic equation of x this equation, but x is equal to V m 2 squares; that means, that is equal to it is your quadratic equation of V m 2 square, right? So, these equation actually quadratic equation of V m 2 square, if you solve this equation you will get 4 different type of roots first root is 1 upon root 2, this not shown during the load flow studied we have taken only the feasible solutions here, I will explain why other 3 solutions are not feasible solution right, 1 upon root 2.

Please solve this quadratic equation and get all these value I am writing directly all the solution, but you get it is a quadratic because of v m 2 square you get it, right? So, 1 by root 2 b j j minus in bracket b square j j minus 4 c j j to the power half then whole to the power half this is one another solution is minus 1 upon root 2 b j j minus b square j j minus 4 c j j at to the power half then whole to the power half another solution is minus 1 upon root 2 b j j plus bracket b square j j minus 4 c j j to the power half to the power half and another solution 4th one this is actually feasible solution whatever, equation we have put it in different form, but these equation we have used for the second method of the load flow study, right? Therefore, it is 1 upon root 2 b j j plus b square j j minus 4 c j j to the power half to the power half, right? So; that means, then this first 3 solutions actually in feasible solution in feasible solution it is not a it has your what you call, that only this one is the this thing feasible solution, why?

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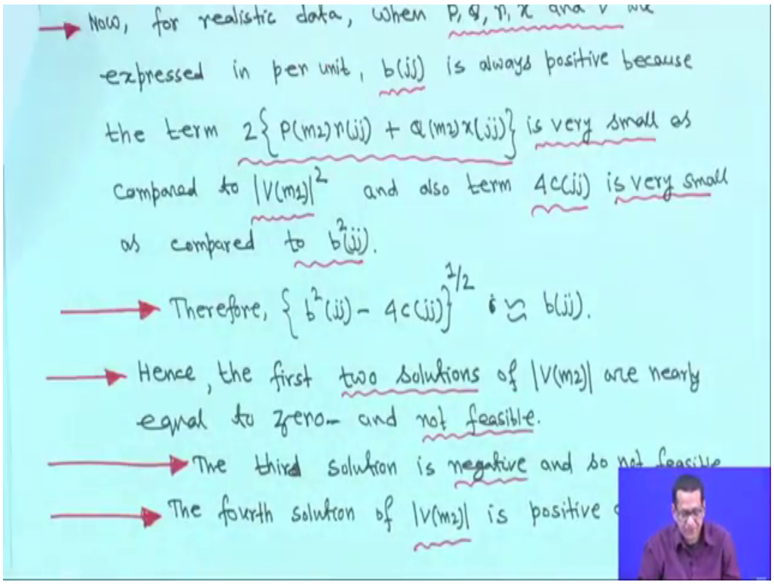

Look now, for realistic data when you take the data and when P Q r x and voltage all are expressed in per unit, that b j j is always positive, right? That means, your b j j will be always your positive compare to your this one hold down, that b j j is this term b j j is because, this voltage b j j why it will be positive because, this voltage V1 will be unity, right? But, this term in per unit product of this term P m 2 in per unit r j j also per unit, they are very small their product is much smaller here, also 2 Q m 2 x j j in per unit A x I Q m 2 is small x j j is smaller.

So, their product is smaller therefore, the and these value will be it is square V m square it will be around one I mean voltage is whatever, we get that is we slack bus voltage we assume one and that voltage is around that one may be little less than 1.969798 something like that, I mean this term is always positive this term is always positive when you are representing everything in per unit this term equation 77 this term b j j is always positive, right? So, that that is why I am writing is always positive because, the term 2 into that is this term if you take 2 common 2 into in bracket P m 2 r j j plus Q m 2 x j j. So, 2 into m 2 r j j is very small as compared to V m 1 square because, this term is nearly one, right?

And also the term 4 c j j is very small as compared to b square j j because, this second term this one actually this is per unit they are square r square j j plus x square j j into this P square and Q square also per unit, their product is also much smaller product is also much smaller. So, that is why equation it is equation 78 therefore, your what you call this very small therefore, if you compare b square j j minus 4 c j j to the power half, that is your this equation b square j j this term b square j j minus everywhere this b square j j minus 4 c j j everywhere it is there.

So, b square j j minus 4 c j j is approximately equal to b j j, right? Because, this term is very small so; that means, if this is approximately this is b square j j is approximately your equal b j j because, it is to the power half b square j j minus this term is neglected. So, b square j j to the power half. So, approximately b j j. So, right so; that means, if this term b j j if this term is b j j then you are what you call this solution this one and this one solution will be approximately 0, right? Also minus sign is there in feasible, right? But, still it is the first one and 2 it is becoming 0, in the case of third term minus sign is there.

So, voltage magnitude cannot be negative. So, this is also discarded only the last one 4th one is in feasible solution your, what you call feasible solution, right? So, this term that is why when you are computing know 2 term A j j D j j 1 was negative another was your what you call magnitude wise A j j D j j were very close to that, here also it is logic is same, right? So, this term and this term more or less this term and this term more or less it is same, right? But anyway, so this 4th solution is the feasible solution, right? Therefore, here the first 2 solutions of V m 2 are nearly equal to 0 and not feasible; that means, this solution actually this V m 2 this is a solution of this V m 2 right, this is the solution of the V m 2 receiving end voltage right these are solution of V m 2.

So, there should not be any confusion right this actually, this is also one solution this is also another solution these all are V m 2 4 solution. I am writing there right hand side because left hand side there is no space right. So, b m 2. So, only this solution 4th one is the feasible solution, right? So, in the third solution i told you it is negative. So, not feasible and the 4th solution of V m 2 is positive and feasible right.

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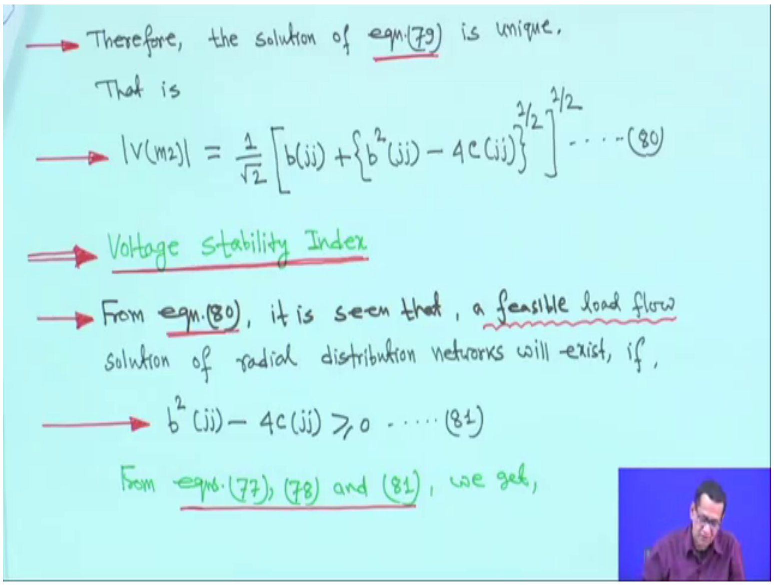

So, this is actually, your what you call that solution of equation 17th therefore, the solution of 79 is unique; that means, if this whatever solution we got that equation 79, there is unique solution because, it has only one solution this equation has the one feasible solution the solution is unique, right? Therefore, this actually I have put it in this version, but only this version I have used for the second method of the load flow studies. So, this is equation is 80.

Thank you.