Power System Engineering
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Lecture - 34
Load flow of radial distribution networks (Contd.)

Ok so next one that after giving all these explanation you that equation 38.

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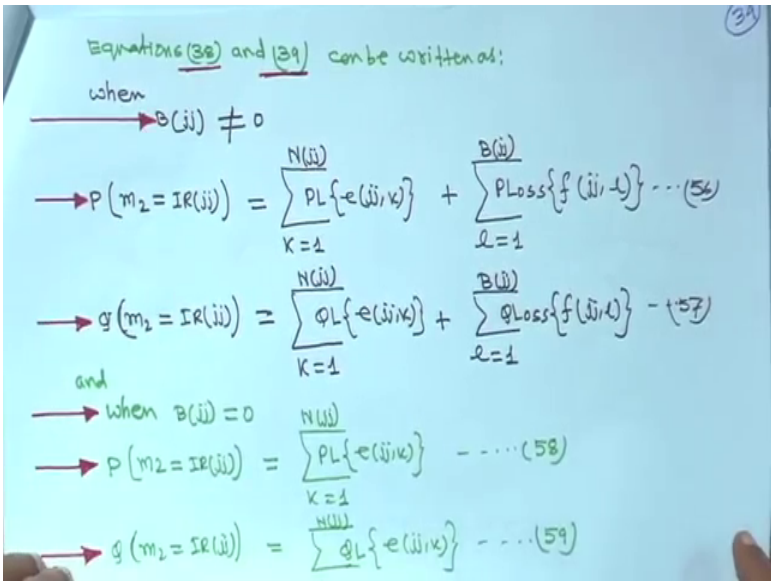

And 39 we will just split it, I mean just I mean idea is that; whenever last branch is coming there is no other branches beyond that so there is no question of adding branch losses. So, this equation when Bjj not is equal to 0 right? I mean from this table only right whatever your previously you have seen right from the table only, that with Bjj we have seen that your this thing that when not is equal to 0 you take because some branches will be there beyond that right and when your just see that if you if you have this diagram again look at that.

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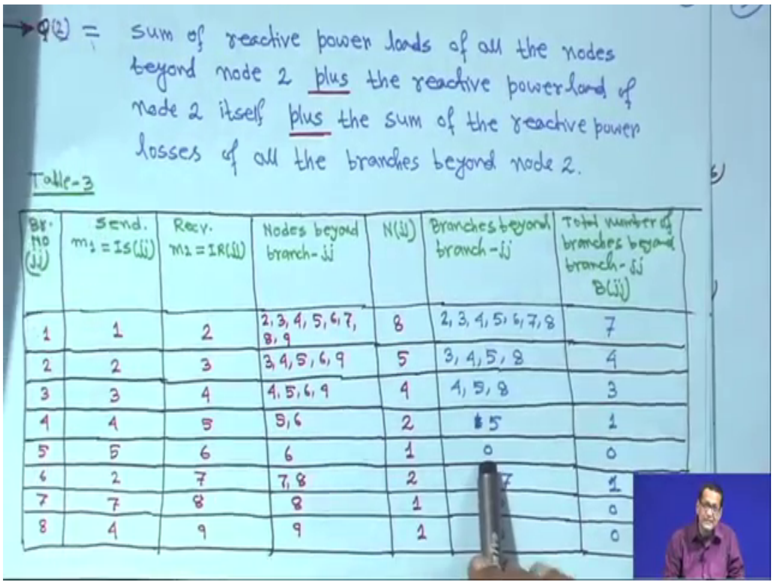

Br. No (jj)	Send. $m_1 = IS(jj)$	Recv. $m_2 = IR(jj)$	Nodes beyond branch-jj	N(jj)	Branches beyond branch-jj	Total number of branches beyond branch-jj B(jj)
1	1	2	2, 3, 4, 5, 6, 7, 8, 9	8	2, 3, 4, 5, 6, 7, 8	7
2	2	3	3, 4, 5, 6, 9	5	3, 4, 5, 8	4
3	3	4	4, 5, 6, 9	4	4, 5, 8	3
4	4	5	5, 6	2	5	1
5	5	6	6	1	0	0
6	2	7	7, 8	2	7	1
7	7	8	8	1		0
8	4	9	9	1		0

When Bjj is equal to this thing your 0 that mean no branches are there; that means, load of the load of that node fed through that a node itself right?

So, that also you have seen, and your similarly that your previous diagram if you look at that just hold on right just hold on that the that diagram right just see that or take any diagram it does not matter just hold on, now where that diagram I have kept it may be here just hold on right.

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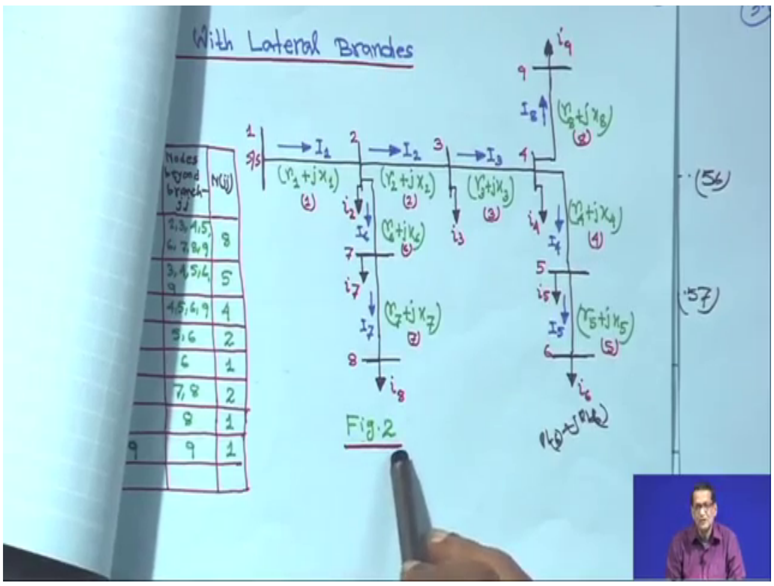

This diagram that whenever any node is coming a branch is coming in the last branch beyond this branch there is no node; that means, total load say from example 2 naught 9 P 9 will be PL 9 and Q 9 will be QL 9. Similarly, here also P 6 will be PL 6 Q 6 will be QL 6.

Similarly, you have also P 8 will be PL 8 and Q 8 will be QL 8 right. So, no branches. So, in that case the loss term in that a mathematical expression should is not existing so that is why in this case that is why here your if it is your Bjj if it is not is equal to 0 then you use this expression right this equation 38 and 39, but when Bjj is equal to 0 loss term does not exist although it is a single term first I am writing Pm 2 is equal to irjj when Bjj is equal to 0 right, it is k is equal to 1 to Njj PL ejj k similarly qm 2 I is equal to m 2 is equal to irjj k is equal to 1 to n is a qlej it is a single term; that means, this 58, 59 can be further written as for example.

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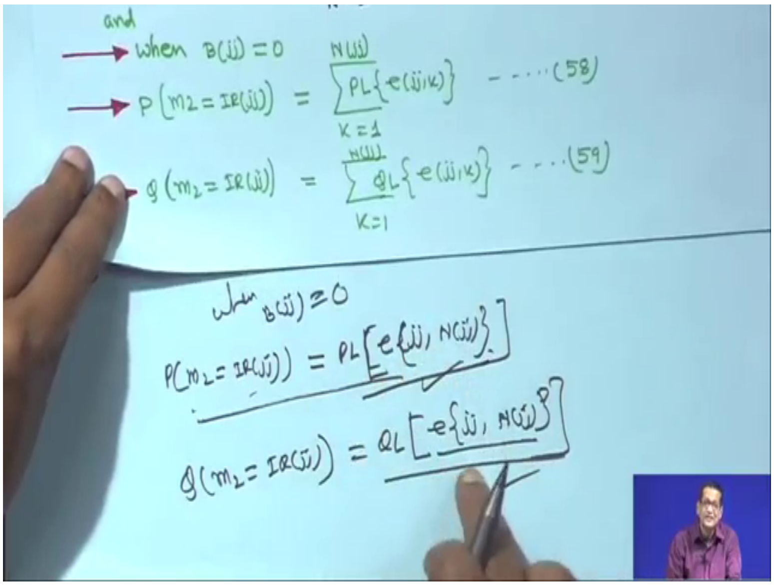

P say m 2 equal to irjj right, whatever maybe the branch number it is a single term because if you look at the circuit total load fed through this node 9 means it is PL 9 QL is 8 is a Q 8 is equal to QL 9 only single term right; that means, this instead of sigma I have written just for mathematical explanation so instead of sigma this Pm 2 irjj can be written as because only one term jj. So, it should be PL then it will be e your jj will be there and k is equal to 1 to n jj for some j.

So, it will be Njj right. So, I can make it a bracket right and then here you can make it right similarly qm 2 is equal to irjj right is equal to QL right. So, you make e then this is jj and Njj, this is a this is a single term ejj Njj is a single term because there is no branches beyond that so instead of sigma directly you can use this these 2 equations, but as for the sake of your mathematics I have just made this term dropped and this 2 sigma I have brought this 2 terms I have brought it here right, but it is a single term it is only one element it is also one element. So, that is why I have written this is a generalize thing for Pm 2 for only when Bjj is equal to 0 this is only when Bjj equal to 0; that means, no branches beyond any branch right.

So, it is a single element, but that means, sigma should not be any confusion so it will be directly you can use it right it is a single term; that means, whatever this one I have written for you here. So, there should not be any confusion it hope up to this all of you have understood right? So, this is the idea because when you write the algorithm we have to we have to use this equations right where load flow algorithm for second method.

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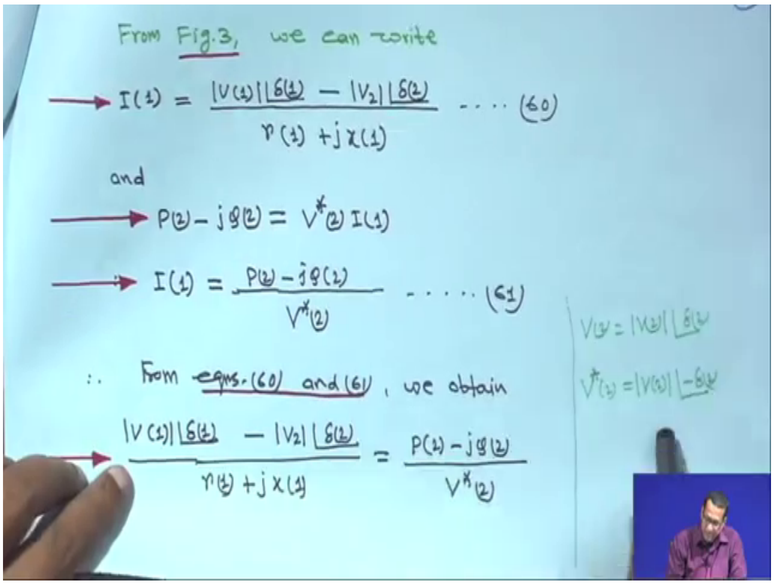

Now, from figure 3 we can write; that means, this is the figure right just hold on right and I will I am drawing I am drawing for you for figure 3 this is your figure 3 right this is your node 1.

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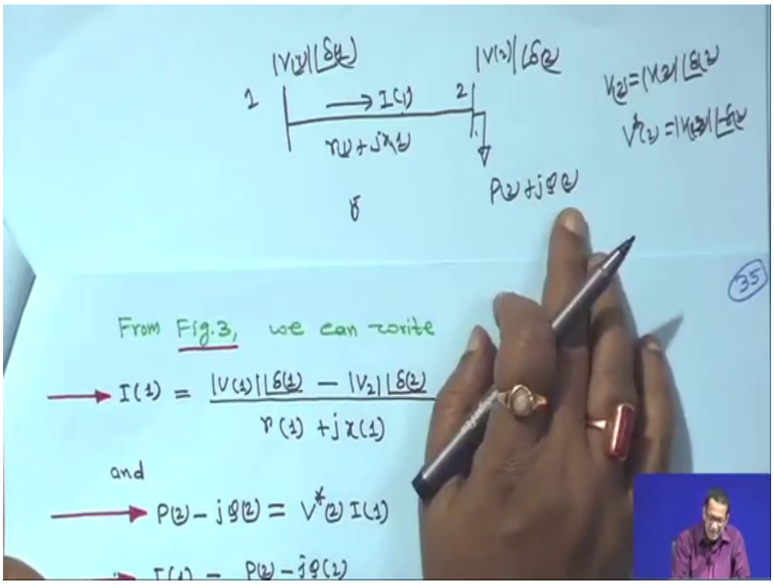

And this is your node 2. So, this is node one it is voltage was your V 1 magnitude angle delta 1 this is node 2 it is voltage was V 2 then angle delta 2 and I should keep it in bracket this is the total load fed through this node 2 P 2 plus jQ 2 this is the current flowing through this branch I 1 this is the impedance of the branch this is actually figure 3.

Somewhere I have placed it here, but this is figure 3, right? So, then from this equation we can write I 1 is equal to it will be V 1 magnitude angle delta 1 minus V 2 magnitude angle delta 2 divided by the impedance r 1 plus jx 1, divided by the impedance r 1 plus jx 1 this is equation 60. Now another thing is from the load flow studies you have studied know that anywhere that any bus power injection P minus jQ is equal to V conjugate I. So, this load P 2 lagging load it is P 2 and Q 2 it is here fed through this node 2 and current actually here going through I 1 right and voltage of this node is V2 actually your what you call this V2 conjugate.

when you write V 2 this is a pressure quantity when you write this way it is pressure quantity; that means, voltage angle is V 2 is equal to your V 2 I put it in like this right, V 2 angle your delta 2; that means, V 2 conjugate is equal to magnitude voltage V 2 right this is V 2 and angle minus delta 2 right therefore, we can write P 2 minus jQ 2 is equal to V conjugate 2 into I 1 because this total load fed through this node is P 2 Q 2. So, in that load flow equation you know P 2 minus jQ 2 is equal to V 2 conjugate I 1. So, V because current flowing through this branch is I 1.

So, it is P 2 minus jQ 2 is equal to V 2 conjugate I 1 or I 1 is equal to this equation P 2 minus jQ 2 upon V conjugate V 2 conjugate this is equation 61; that means, this 2 equation you equate 60 and 61 you equate therefore, you are writing from equation 60 and 61 that V1 magnitude angle delta 1 minus V 2 magnitude angle delta 2 divided by r 1 plus jx one is equal to P 2 minus jQ 2 upon V conjugate V 2 conjugate now you go for cross multiplication.

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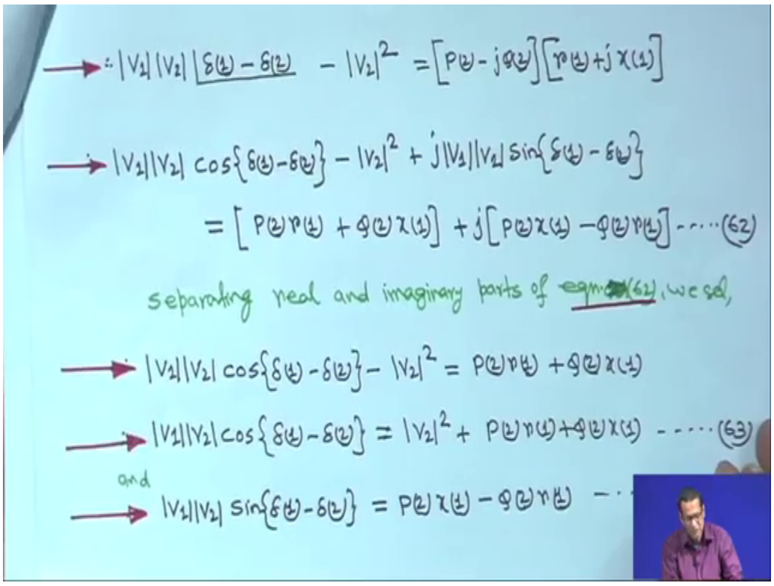

If you do so if you go for cross multiplication this here I am writing for your understanding that just now I wrote also that V 2 is equal to V 2 angle delta 2 therefore, V 2 conjugate is equal to V 2 angle minus delta 2 right. Therefore, this V 2 conjugate you make it here this is V 2 angle minus go for cross multiplication then what will happen? If you cross multiply this it will be then your V 2 V 1 angle delta 1 minus delta 2 that is why it is coming that V 2 V 1 or V 1 V 2 angle delta 1 minus delta 2 and this is magnitude V 2 angle delta 2 this is a magnitude V 2 angle minus delta 2 right.

So, this delta 2 plus and minus delta 2 will not be there it will be angle 0 degree anywhere right. So, it will be magnitude V 2 square therefore, minus magnitude V 2 square is equal to P 2 minus jQ 2 into r 1 plus jx 1 that cross multiplication, right? Then you just multiply this one you expand cosine and sine cos theta plus j sin theta. So, this is

V 1 V 2 cos delta 1 minus delta 2 then imaginary part putting here first real part I am putting that is why minus V 2 square I am putting here then plus j the related to that only plus j V 1 V 2 sin delta 1 minus delta 2 and if you cross multiply this one is equal to P 2 r 1 plus Q 2 x 1 plus j it will be P 2 x 1 minus Q 2 r 1 this is equation 62 right.

So, the just you what you do you separate real and imaginary part from equation 62 if you do. So, then real part is V 1 V 2 cos delta 1 minus delta 2 minus V 2 square is equal to P 2 r 1 plus Q 2 x 1 this one or take this term to the right hand side that V 1 V 2 cos delta 1 minus delta 2 is equal to V 2 square plus P 2 r 1 plus Q 2 x 1 this is equation 63, just because you have to eliminate that delta terms similarly is that imaginary part V 1 V 2 sin delta 1 minus delta 2 V 1 V 2 sin delta 1 minus delta 2 is equal to P 2 x 1 minus Q 2 r 1 P 2 x 1 minus Q 2 r 1 this is equation 64.

So, what you do you square equation 63 and 64 and you add it then what will happen V 1 V 2 common cos and sin. So, cos square delta 1 minus delta 2 plus sin square delta 1 minus, minus delta 1 minus delta 2 will be 1. So, left hand side also gives square and add it will be V 1 square magnitude, magnitude V 1 square magnitude V 2 square and you add it and right-hand side whatever it comes.

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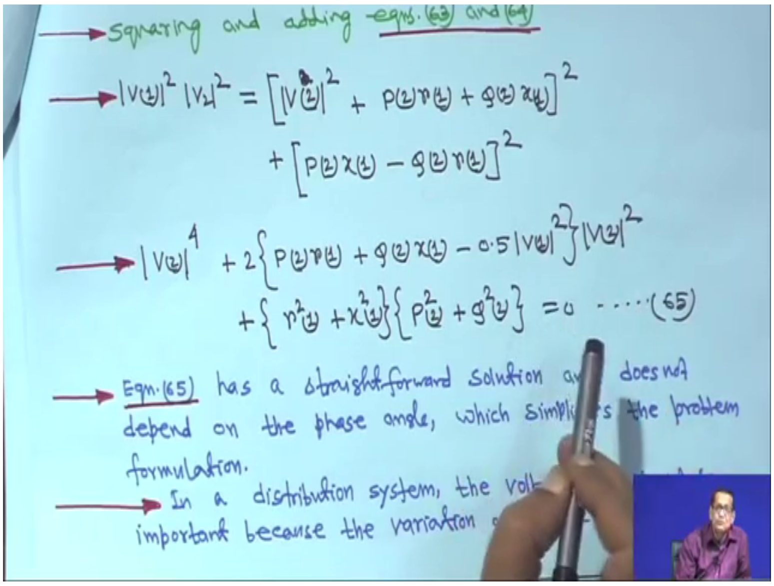

So, squaring and adding equation 63 and 64 then that it is becoming cos square delta 1 minus delta 2 plus sin square delta 1 minus delta 2 so; that means, it is one therefore, this is coming V 2 square magnitude V 2 square plus P 2 r 1 plus Q 2 x 1 whole square plus P

2 x 1 minus Q 2 r 1 whole square you expand this and simplify if you expand and simplify this you please do it right.

This you please do it if you do it will be magnitude V 2 to the power 4 plus 2 in bracket 2 in bracket P 2 r 1 plus Q 2 x 1 minus 0.5 that is half into V 1 square bracket close then multiplied by magnitude V 2 square plus r 1 r 1 square plus x 1 square into P 2 square plus Q 2 square is equal to 0; that means, this equation actually has 4 solutions because it is 4th power equation and if you look into that that is V 2 to the power 4 and V 2 square; that means, this equation is a basically a quadratic equation of V 2 square.

Because for example, if you take if you assume V 2 square is equal to x, then it will become x square because it is v to the power 4. So, V 2 square 2 square. So, it is x square plus 2 into this term into x plus this one and it is equation will come like this that ax square plus bx plus c is equal to 0 in this form, if you assume x is equal to V 2 square because it is a quadratic equation of V 2 square because it is 4 is here 2 is here that is power right. So that is why I have written equation 65 as a when will little bit in this course little bit we will study about the voltage stability of distribution system little bit.

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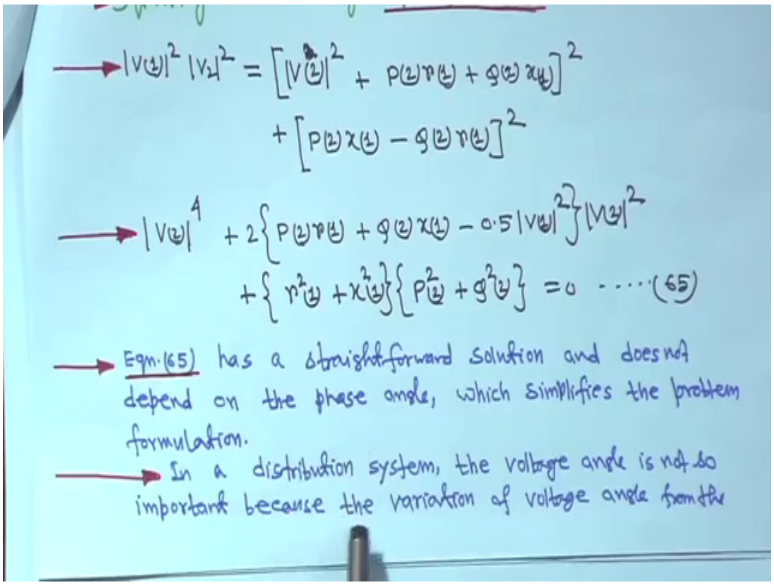

So, at that time we need this equation further it will be explained after compute it in the load flow right. So, equation 65 has a straight forward solution and does not depend on the phase angle, this equation is independent of the phase angle right and which simplifies the problem formulation, as per as trigonometric term is not there and it is magnitude only things are easier right in a distribution system the I told you earlier also, but here I have written in a distribution system the voltage angle is not. So, important because the variation of voltage angle actually from the substation to the tail end of the figure actually is very few degrees it will be 2 to 3 degrees not more than that I mean very small right.

And also note that from the 2 solutions of V 2 square because I told you these equation actually is a quadratic equation

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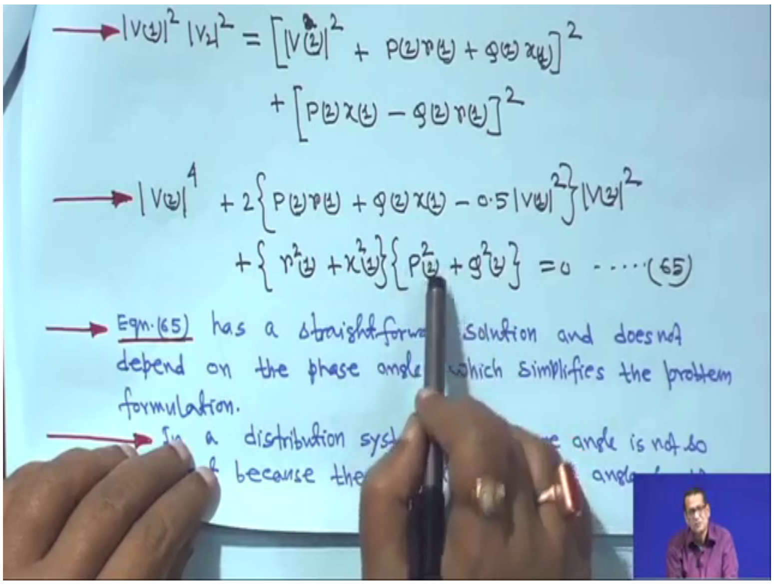

of V 2 square because it is V to the power 4 it is V 2 square, that it is a quadratic equation of V 2 square little bit more will be x then during voltage stability study right. So, that no 2 solutions of V 2 square only the only the one considering the positive sign of the square root of the solution of the quadratic equation gives a realistic value, the same is applicable when solving for magnitude V 2.

So, therefore, this equation has 4 solutions out of this 3 solutions are not feasible right only one solution is feasible realistic and this is this one we have given.

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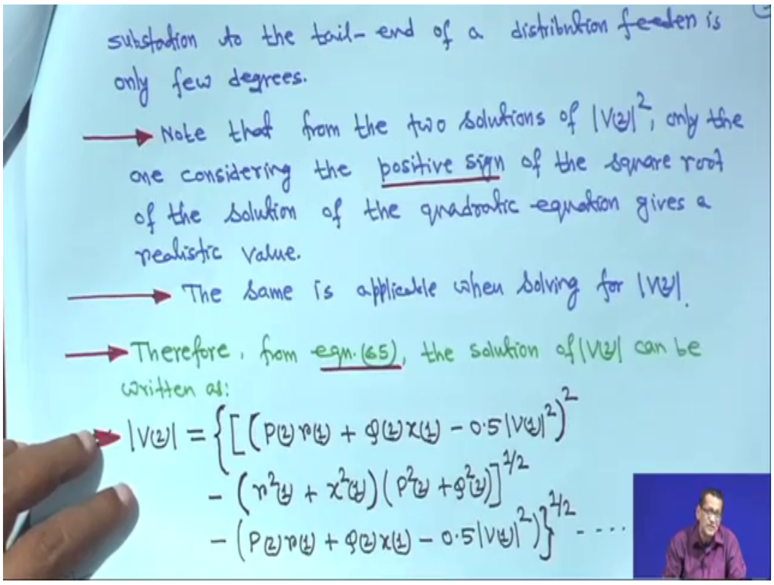

So, here it is something like this V 2 is equal to put this bracket then this bracket then P 2 r 1 plus Q Q 2 x 1 minus 0.5 magnitude V 1 square whole square, now minus r 1 square plus x 1 square into P 2 square plus Q Q 2 square this bracket is closed to the power half that is square root minus P 2 r 1 plus Q Q 2 x 1 minus 0.5 again magnitude V 1 square this bracket is close again to the power half, I suggest I this is I have written the final expression everything is correct right.

I suggest you please from this equation you place please get this solution right for this this one only, you will get 4, but you get this one this is the only realistic one and I have written here also I have written here also. So, this is for V 2 similarly that this is electrical equivalent of branch 1 V 2 will get similarly V you know V 2 then V 3 also similar expression we will get. So, what you will do from this only we will try to make it the what you call it is a your generalize equation general equation means suppose this is for branch 1 jj is equal to 1 jj is equal to 1 means m 2 is equal to irjj is equal to ir 1 is equal to 2 right that is this is actually will be replaced by m 2 receiving end right.

And this is your branch 1 all this 1 r 1 x 1 these are actually your rjj it is a branch and whenever 2 is there you will put m 2 m 2 is equal to irjj and one will be there V, but whenever your what you call when voltage V 1 will be there we will make V m 1 because sending end receiving end right, but when will take P square to Q square to where will be replaced by m 2

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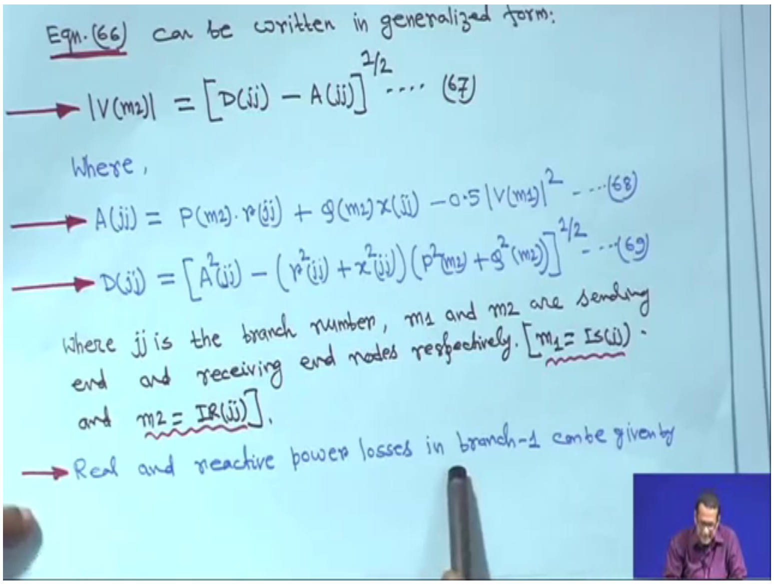

So if you do so what you are writing actually first it can be general it can be retaining generalize form say magnitude V m 2 is equal to bracket Djj minus Ajj to the power half this is equation 67; that means, this equation this equation if you I am I am just making it just see how is it.

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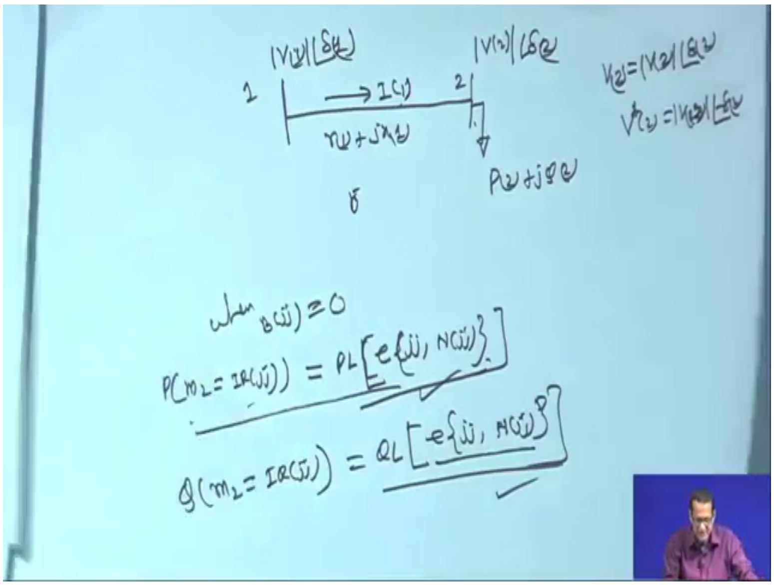

First this equation your a this one right.

For example, this term this term suppose if you write suppose a,

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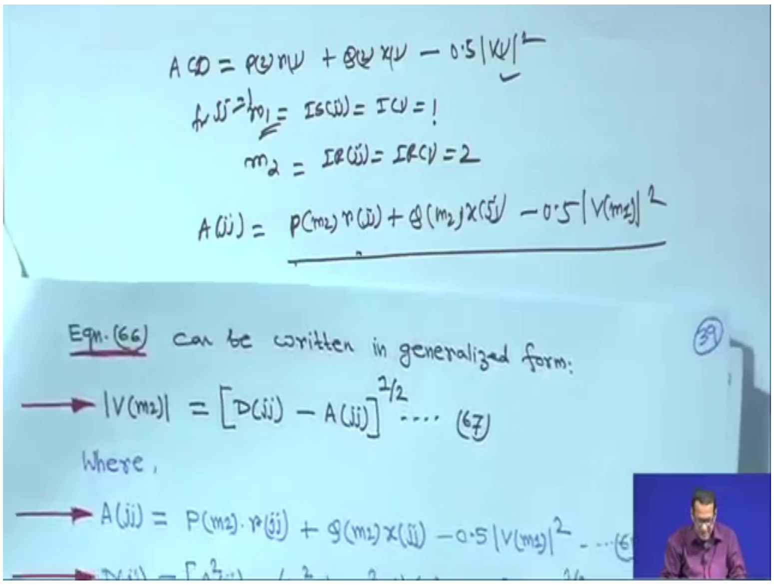

Say branch 1 A 1 is equal to first you write P 2 r 1 plus Q 2 x 1 minus 0.5 V 1 square, only this term this term means this term also P 2 r 1 plus Q 2 x 1 minus 0.5 V 1 square this, but anyway it is to the power bracket close separate thing, but this 2 terms are same actually right. So, if you define say for branch 1 it is A 1 P 2 r 1 plus Q 2 x 1 minus 0.5 your V 1 square that in this case what you can do is you replace because we know a one is equal to sending end node isjj basically it is I 1 is equal to 1; that means, this V 1 we are taking right.

And m 2 is equal to your irjj that is branch number is equal to ir 1 is equal to 2 right. So, jj is equal to this is for this is for jj equal to 1 right, for jj is equal to 1 we are getting that right; that means, all this one if you replace by your what you call jj; that means, A jj will be is equal to 2 that is this one for the power we will make it is a Pm 2, that is; Pm 2 then r jj again Q 2 we will make qm 2 because it is m instead of 2 we will make it is generalize one m 2 is equal to 2 then xjj minus 0.5 is m one is sending end node right we will make 0.5 V m 1 square this is generalize one, that is why I what that is why here I am writing assuming that A jj is equal to P m 2 r jj plus qm 2 xjj minus point 5 V m one square equation 68; that means, whatever I showed you here that how I am writing right.

Similarly, if it is so, then this term will be there r 1 square then Djj; that means, this equation we are writing here Djj we are writing V m 2 is equal to Djj minus Ajj Djj we are writing a square j minus this term right. So, how Djj is coming so in that case Djj is your A square j this is your this generalize one this one if you assume I mean I will put it later this one if you assume this is your Ajj it is square so A square j minus your what you call this r here it is just here it is that your r square jj plus x square jj into Pm 2 square plus qm 2 square to the power half; that means, this is your Djj.

So, that is why the I mean the way we have taken this is your Ajj square minus this one I mean Djj is equal to actually this term Ajj it is this is your A 1 it you take this is your a one then this is a 1 square minus this term minus A 1; that means, for your understanding say D 1 here.

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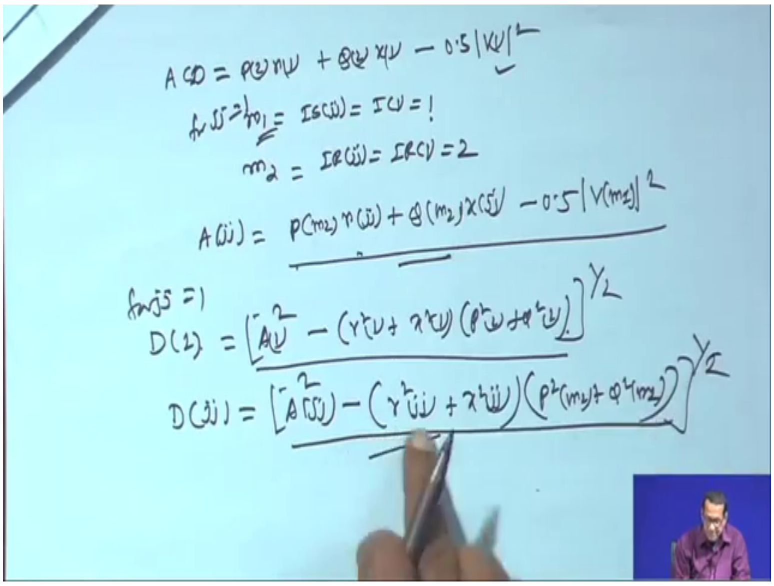

We are writing here we are writing D 1 is Djj general one first I let me write on D 1. So, look at this expression D 1 if you assume this is a one then D 1 is equal to this is square this is everything to it is it is whole thing it has been assumed A 1 which is square.

So, it is A 1 square right, minus this term will be there r square 1 plus x square 1 into P square 2 plus Q square 2 this will be there right? So, this thing your this thing actually Djj when you are writing here you look here the term is this term. Now question is that it is for branch 1 this is for the branch 1 jj equal to 1 if we now make it generalize it. So, instead of 1 we make jj and instead of 1 this is Ajj square minus this is actually r square jj plus x square jj right. So, this is the thing and this is P square m 2 plus Q square m 2 right. So, this is my Djj.

So, that is why this is for Ajj and this is for Djj that is why here we are writing Djj is equal to general one a square jj minus r square jj plus x square jj into P square m 2 plus Q square m 2 to the power half. So, that is in whole this is actually a whole thing we have taken will be power half right. So, there will be a this thing, half, right? This will be half this is because in this expression that your this half is there right. So, this is your actually a square minus this one is Dj that is power half is there that is why this half should be there right.

So, that is why your V m that is why V m 2 is equal to first term is Djj from here to here this is Djj minus Ajj right your this thing. So, Djj minus Ajj and whole to the power half. So, that is why this is whole to the power half this is equation 67 there is no confusion right there will be no confusion. So, that is why is general one we write Ajj is equal to Pm 2 rjj plus qm 2 xjj minus 0.5 magnitude V m 1 square and Djj is equal to A square jj minus r square jj plus x square jj into P square m 2 plus Q square m 2 to the power half this is equation 69.

Where jj is the branch number m 1 m 2 you know all the tables is given are the sending and receiving end nodes right respectively, where m one is equal to isjj and m 2 is equal to irjj next is a real and a reactive power losses in branch one can be given by right. So, real P loss 1 for branch 1 just now I will give you that diagram.

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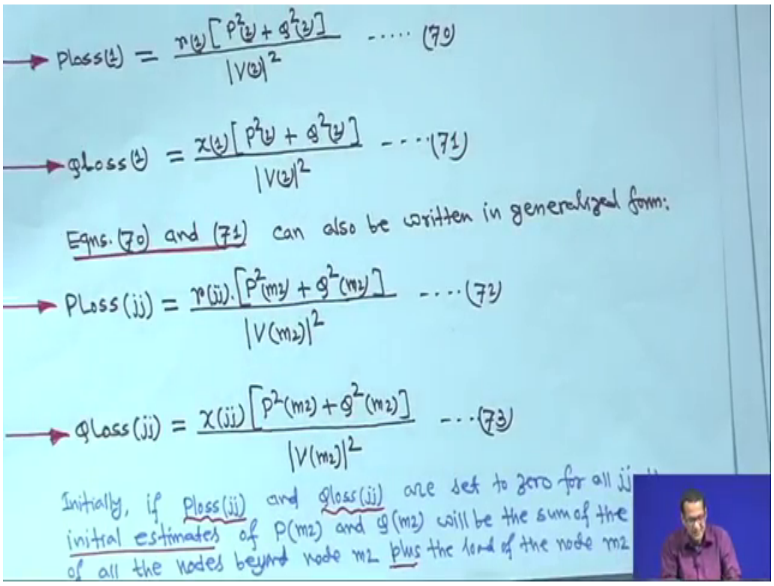

that r 1 into P 2 square plus Q 2 square upon V 2 square similarly Q loss 1 is equal to x 1 into P 2 square plus Q 2 square upon V 2 square this equation 70 71. How this loss expression is coming again you, again you consider that same branch 1 diagram let me draw it for you right.

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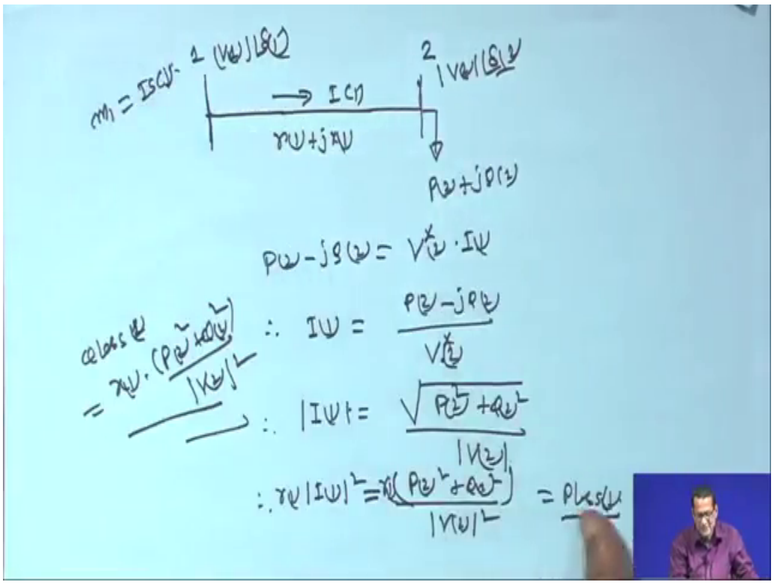

This is again I am drawing the same electrical equivalent of branch 1.

Suppose this is your branch this is 1 this is 2 total load fed through this node is P 2 plus j Q 2 right impedance of this branch r 1 plus jx right, and current flowing through this branch is your I 1 that we have seen right and voltage of this one it is V 1 angle delta 1 and this is V 2 magnitude of course, angle I will putting in bracket angle delta 2, right? Now question is that you know what is the expression of current you have seen know P 2 minus jQ 2 is equal to V 2 conjugate into I 1 right.

So that means, I 1 is equal to P 2 minus jQ 2 divided by your V 2 conjugate, right? If you take the magnitude of these both side if you take mod then it will be I 1 is equal to root over P 2 square plus Q 2 square divided by your magnitude V 2. Now it is if you if you take the square of this both side then it will be I 1 square is equal to P 2 square plus Q 2 square divided by V 2 square right both side this is your this resistance of this branch is r 1 if you multiply it this equation both sides by r 1, this will be your this thing both side this one also you multiplied by r 1 this will be your P loss 1 real power loss I square I 1 square r 1 right from branch loss this one.

Similarly, if you multiply this equation both side by you take it is square on both side then Q loss 1 is equal to your x 1 into P 2 square plus Q 2 square divided by V 2 square. So, this is real power loss P loss 1 and this is reactive power loss. So, first you right down these equation this is the current equation take the magnitude of the current root over P 2 square plus Q 2 square upon your magnitude V 2 then you square it. So, mod ir square will be P 2 square plus Q 2 square upon V 2 square multiply both side by r 1.

So, it is basically I 1 square r is equal to r 1 into P 2 square Q 2 plus Q 2 square upon V 2 square that is P loss 1 means loss of real power loss of branch 1 and similarly Q loss 1 it is real power loss of reactive power loss of branch 1 is equal to x 1 into P 2 square plus Q 2 square upon V 2 square, that is why; this equation is written 70 and 71 that P loss 1 is equal to r 1 into P 2 square plus Q 2 square upon V 2 square and Q loss 1 is equal to x 1 into P 2 square plus Q 2 square upon V 2 square this is equation 70 is real power loss of branch in reactive power.

Now, this can be generalized in instead of it is branch 1. So, you know the receiving end node is m 2 right is equal to irjj and a jj is one, but make it generalize one. So, put is replace 1 by jj and replace 2 by m 2 therefore, P loss jj will be rjj into P square m 2 plus Q square m 2 upon V m 2 square magnitude, right? Similarly, Q loss jj will be xjj into P square m 2 plus Q square m 2 upon V m 2 magnitude of course, magnitude V m 2 square this is equation 73. Now if you take for jj is equal to your one 2 number of node branches and if set m one is equal to ma your isjj and m 2 is irjj.

So, all the all the nodes all the branch classes you can easily compute if your all your V m 2 Pm 2 Qm 2 all are known to you. So, this is actually loss calculation that how one can do it right? So, that is why whenever we make such thing. So, this sending and receiving end. So, actually m one is equal to write isjj right, in general in general if you consider an electrical equivalent of any branch jj say I making it for you

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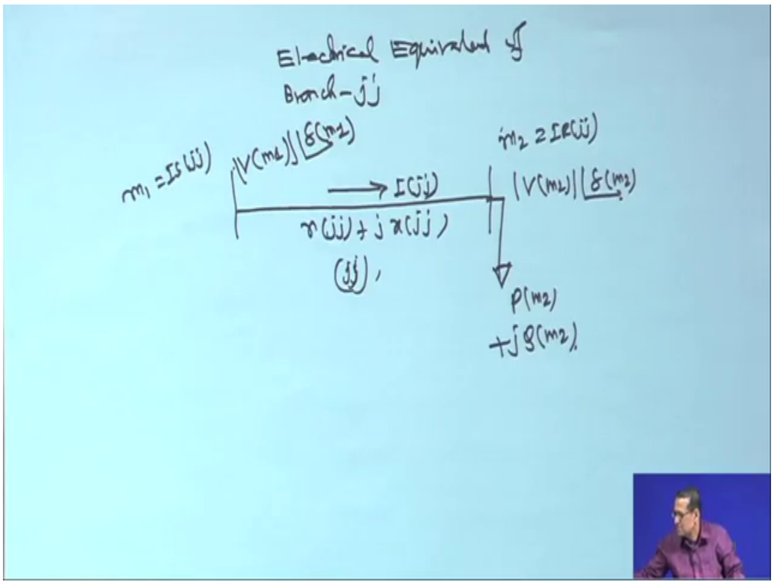

Say electrical equivalent electrical equivalent of say instead of 1 or 2 or anything it is a branch jj if it is. So, then you make this diagram right this side this is your branch number this is your branch jj this branch is jj no question of one 2 3 general jj.

So, this side is sending end node is ipm 1 is equal to is jj, and receiving end node m 2 is equal to irjj right, sending end side voltage will be then a V m 1 angle delta m 1 and receiving end side voltage will be V m 2 right angle delta m 2 and current flowing through this branch it is branch jj it will be ijj, and impendence of this branch then it will be rjj plus j xjj and total load fed through this node is node m 2 so it will be Pm 2 plus j qm 2 right; that means, this loss equals this is the general thing this is actually electrical equivalent of branch jj when it will said then you write m 1 sending end side isjj voltage is V m 1 angle delta m 1 receiving end m 2 is equal to irjj V m 2 im 2 Pm 2 plus jQm 2 and this is the thing.

And you know for j all the branch number you are making then sending end node table receiving end table. So, for each value of jj you will be knowing which branch and how to compute this one right? So, that is why this P loss jj and Q loss jj is given so in this case instead of P 2 and Q 2 you make Pm 2 minus jQm 2 is equal to your V m 2 conjugate into ijj then you take this one loss you will get this one this is generalize one. So, instead of that 1 2 I showed and this is the loss expression the general expression and this is actually your electrical equivalent of branch jj for any radial distribution get 1 I hope it is understandable.

If you have any when you will go through this lecture if you have any clarification you can put the question in the forum or you can send with a mail also you will get the answer right?

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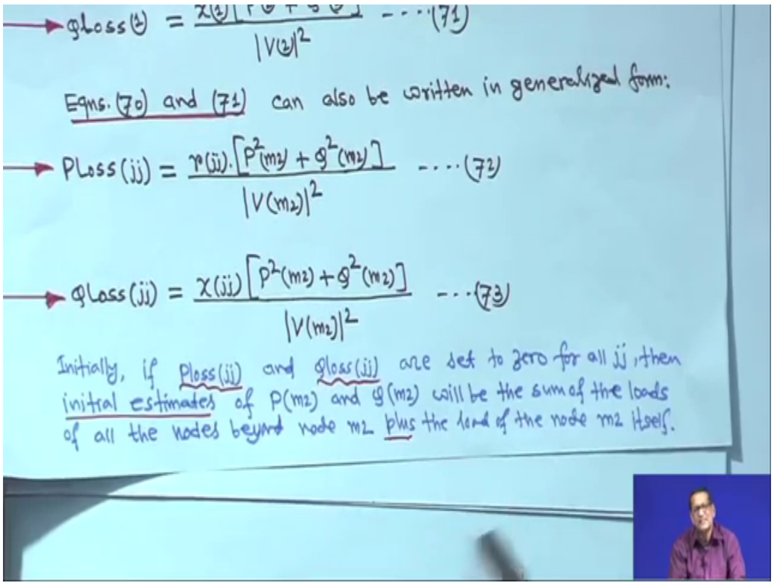

So, initially what you have to do is for this kind of load flow that we have to your what you call we have to assume that initial P loss and Q loss of all the branches are 0 right, because I have initially you have to start the algorithm. So, initially P loss jj and Q loss set to 0 or for all jj then initial estimate of Pm 2 and Qm 2 will be the some of the loads of all the nodes beyond node m 2 plus the load of the node m 2 itself.

So, this we will see this this we will see when we will take the example right. So, if initially you have to start the algorithm and you have to assume that initial your loss will be 0, but in that case, there is no question of flag voltage start only substation voltage V 1 will assume based on that your V 2 V 3 magnitude all will be computed. So, no question of flag voltage start for the second method right. So, this is your what you call for second method.

So, 2 load flow studies right from this 2-load flow studies method 1 and method 2.

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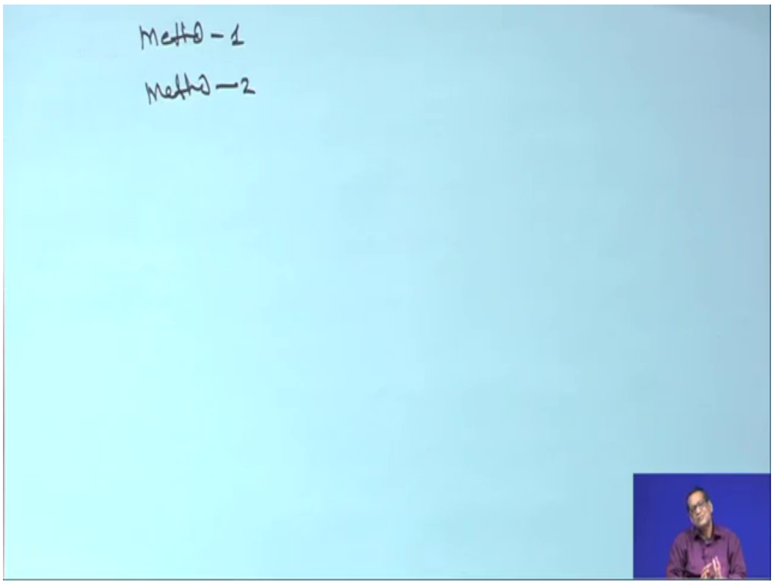

Method 1 right method 1 and method 2, actually it is sometimes we can call it is a backward forward sleeping method right, but 2 methods are given. So, first method whatever first method we have studied right, so in that in that case what you are doing first that we are trying to find out initial initially we are going for a flag voltage start and based on the flag voltage start we are trying to find out the load current, once all the load currents are known we are computing the branch current and when branch currents are computed we are trying to find out almost recursive relationship that both sending and receiving end relationship right.

So, V m 2 is equal to V m 1 minus your ijj into zjj that your first method we are doing it. So, accordingly we are going iteration after iteration numericals we will see later right. And in the second method case what you are doing is actually we are assuming the losses of all branches are 0. So, initially you have to find out the what are the total loads real and reactive power load fed through each node and based on based on that we will try to find out, the substation voltage substation is a slag bus.

So, substation voltage is known and from that we will try to find out the voltage of node V 2 magnitude of course, when voltage V 2 magnitude is known then again from this relationship we will try to find out voltage V 3 and so on.

So, in the second method no question of flag voltage start only initial loss has to be taken 0 and, but in the case of first volt first method that we have to compute your what you call that we need flag voltage start. First method we need complex numbers are involved because that load current computation branch current voltage all complex number are involve.

So, first method easily you can get the voltage angle although angle variation is very less second method also that from those from those mathematical derivation angle expression also can be obtained some recursive relationship can be obtained, but second method that voltage angles for distribution system is not significantly varying from the substation to the tailed end of the figure it may be hardly 2 3 degrees if you take a realistic problem right. So, in that case second voltage magnitudes of concern and the power losses. So, can easily be obtained.

Thank you very much next after this we will come to the algorithm.