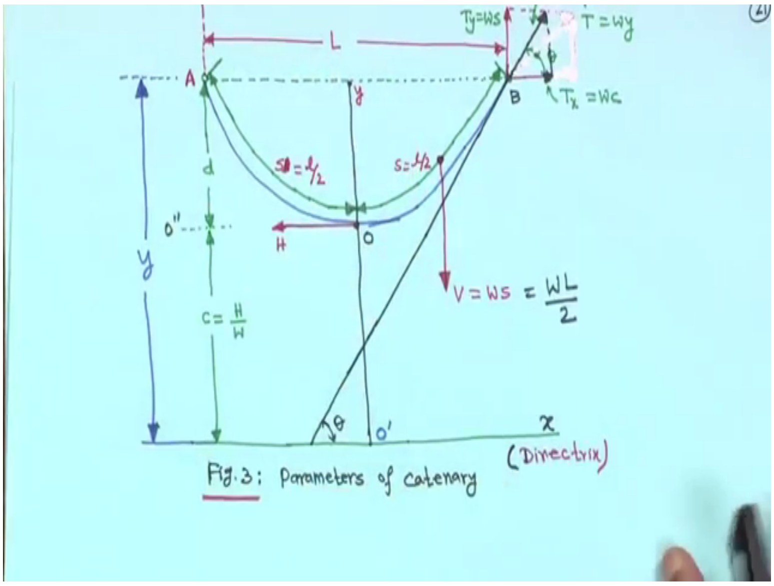
Power System Engineering
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Sag & Tension Analysis (Contd.)

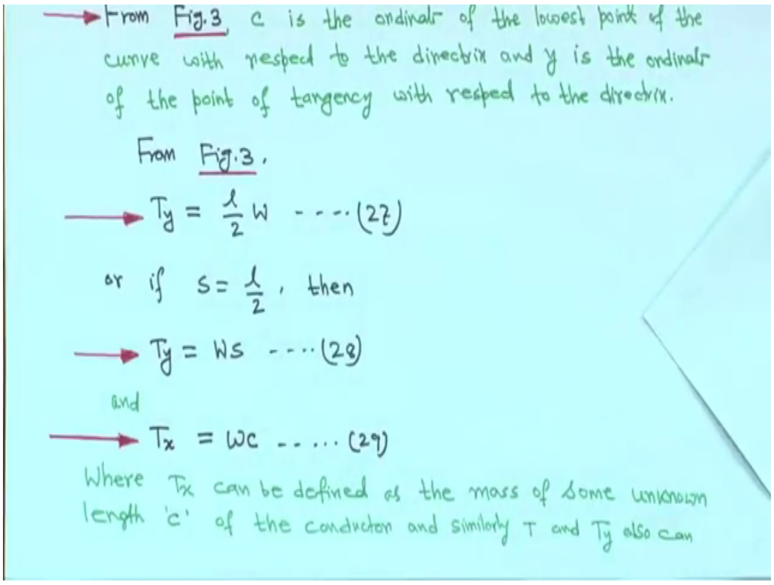
So, this is actually this is the sag list and this is D.

(Refer Slide Time: 00:17)



Therefore from figure – 3.

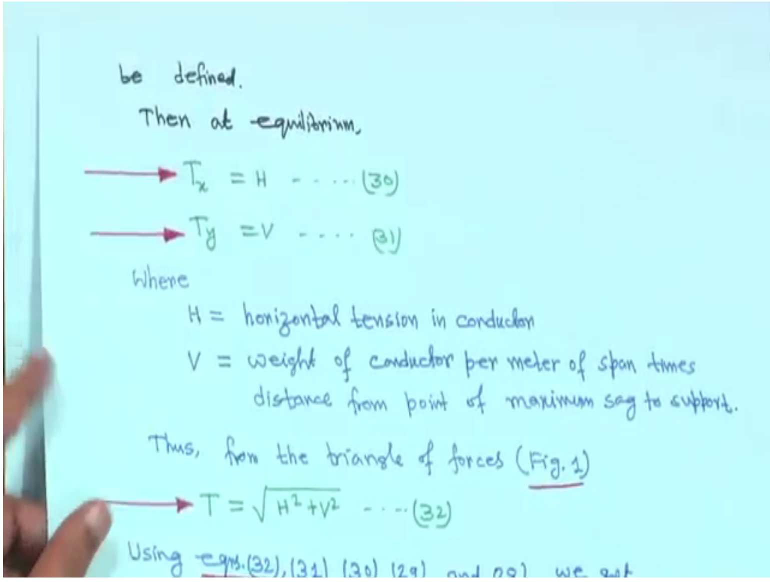
(Refer Slide Time: 00:23)



You can write, I just told you that Ty is equal to WS actually basically it will be WL by 2. So, that is why Ty is equal to W into your sorry this is small l because this is L by 2 sorry this is small l right. So, this will be your WL by 2. So, and if s is equal to l by 2 small s is equal to l by 2 then Ty is equal to WS; that means, this whole thing from here to here is if S is equal to small l by 2 then only Ty is equal to WS means W into small l by 2. So, that is Ty is equal to WS that is why we are writing this is the vertical one and this is balancing this is conductor weight. So, Ty is equal to WS.

Now, another thing is Tx that horizontal component say it is W into C where Tx can be defined as the mass of some unknown length c this is W is known kg per meter square conductor weight you know and C some unknown length of the conductor and similarly T and Ty also your what you call also can be your can be written as defined as that Tx if you look into that the Tx is equal to the horizontal or tension H and Ty is equal to WS that is your vertical one V, that means, basically Ty is equal to WS is equal to V is equal to W small l by 2.

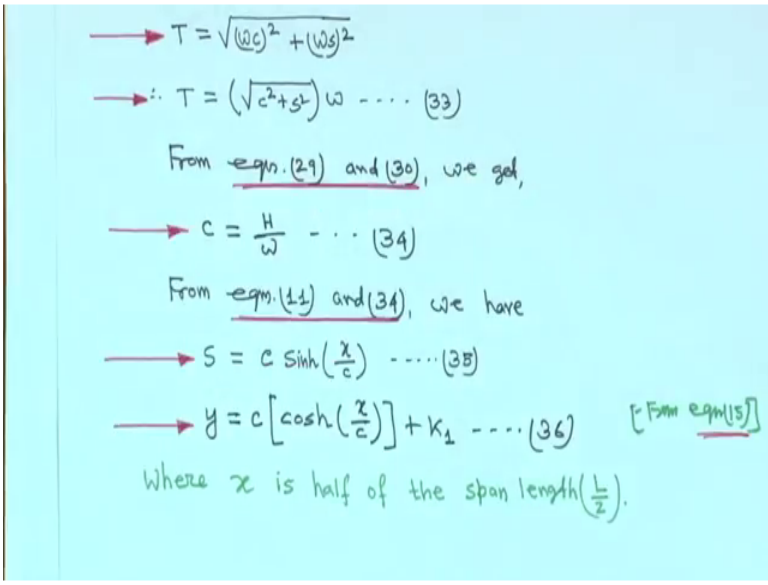
(Refer Slide Time: 02:01)



So, that means, the Tx is equal to WS that is equation 3 and Ty is equal to V this is equation – 31, where H is equal to horizontal tension in conductor and V is equal to weight of conductor per meter of span time right. So, distance from point of maximum sag to support. So, thus for the triangle of forces figure 1 we have seen earlier that t is equal to root over H square plus V square this we have seen equation 32 therefore, from equation 32, 31, 30, 29 and 28 we get you will know one is your what you call that Tx is equal to H horizontal one that is equal to your W into C and Ty is equal to V that is equal to WS.

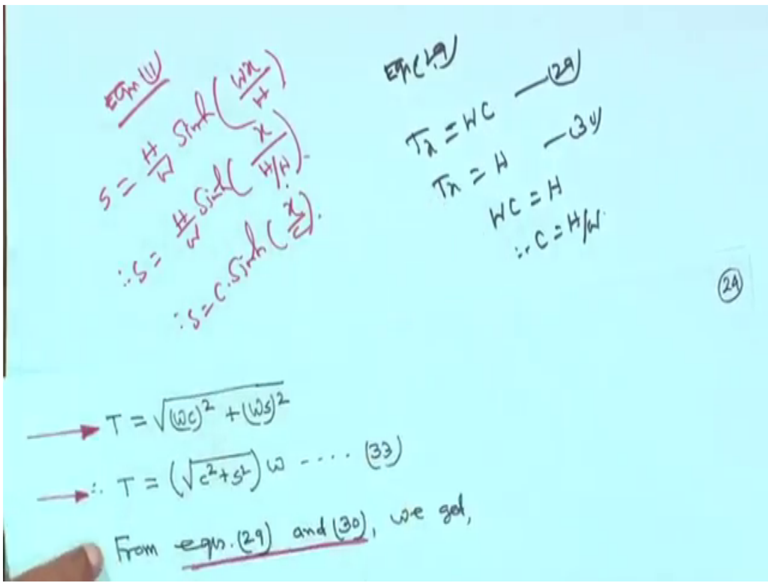
So, basically T is equal to root over WS C square and plus W your S square.

(Refer Slide Time: 02:54)



So, that is why, it is coming T is equal to root over WC square plus WS square. So, W you take common to T is equal to root over C square plus S square into W actually conductor weight this is equation 33. Now, from equation 29 and 30 you will get that is c is equal to H by what you call W. So, if you go back to equation just hold on equation 29, this is in 29 equation this one is equal to your 29 and 30, I making it here for you equation 29.

(Refer Slide Time: 03:28)



Equation 29, so, 29 actually T x is equal to W into C and 30 actually T x is equal to H this is equation 29 and this is 30; that means, WC is equal to H; that means, C is equal to H by W, that is why you are writing this in this equation that in this equation that C is equal to H by W.

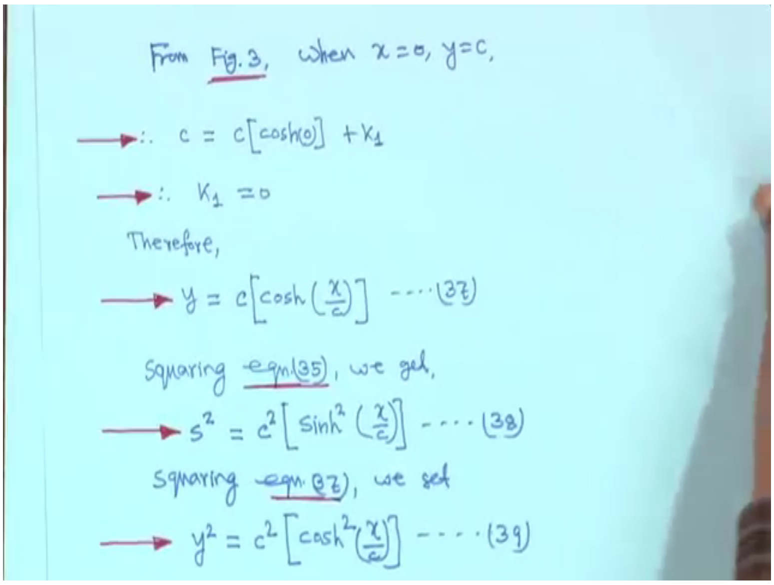
So, if I go back to this figure that is why this C we have written it is H by W. Look at this figure it is written C is equal to H by W. So, now, again from equation and 11 and 34 we have small s is equal to C sin hyperbolic x by C because that equation 11, if you see equation 11 actually it was small s it was H by W then sin hyperbolic then it is Wx by H this was equation 11.

Now, this one can be written as this one sin hyperbolic then x upon H by W and H by W is equal to C that is why you are writing s is equal to C then sin hyperbolic then x upon C, what we are making from equation 11 and 34 x is equal to C sin hyperbolic x by C, whatever I have done here right. So, this is equation your 35.

Therefore, your from equation 15 this similar way equation 15 we can write y is equal to C cos hyperbolic then x by C plus K 1 this is equation 36. This is actually from equation 15 we are just making that C is equal to H by W. So, you can easily make it. So, no need to tell this is one a simple thing, where x is half of the span length l by 2.

Now, from figure 3 when x is equal to 0, y is equal to C; that means, this figure if you look into these that just hold on this figure if you look into these that this is figure 3 when x is equal to 0, I mean then y is equal to C because this is the height C this is the origin say. So, when x is equal to 0, y is equal to C. Therefore, this equation you put this boundary condition in this equation then you will get it is equation 36.

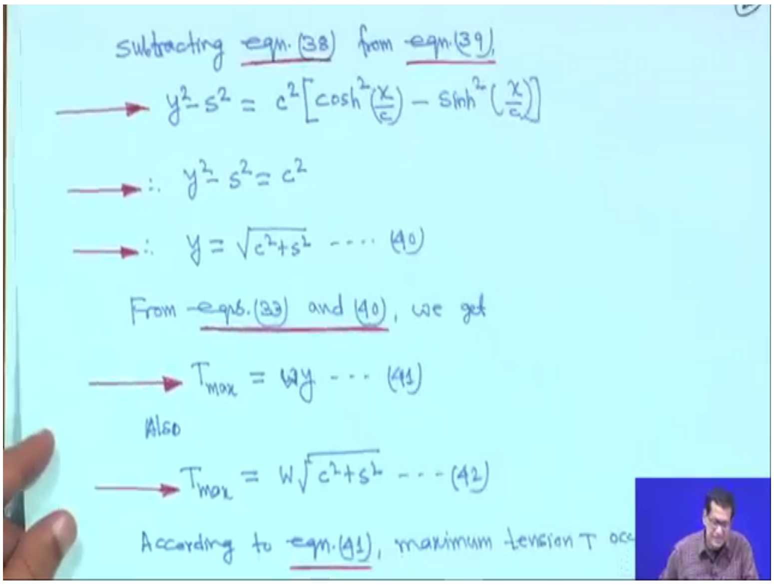
(Refer Slide Time: 06:18)



Then you will get C is equal to your C in to cos hyperbolic 0 plus K 1, hence K is equal to 0 because cos hyperbolic 0 is 1 therefore, K 1 is equal to 0 because C is equal to C plus K 1. So, K 1 is zero. Therefore, this y is equal to C cos hyperbolic x by C; that means, this equation 36 this K 1 you put here 0, therefore, y is equal to your C cos hyperbolic x this is equation 37.

Now, squaring equation 35; that means, this equation in square this equation you square this equation S square is equal to C square sin hyperbolic square x upon C. So, if you do, so S square is equal to C square sin hyperbolic square x by C and next is your square equation 37 if you do. So, it is y square is equal to C square cos hyperbolic square x upon C.

(Refer Slide Time: 07:25)



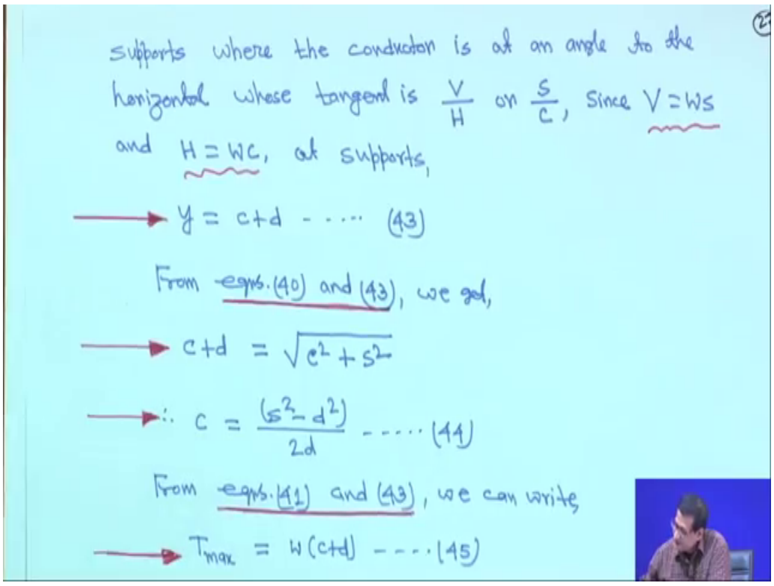
Now, what you do you subtract equation 38 from equation 39 you just subtract equation 38 from equation 39 that means, subtracting equation 38 if you do so it will be y square minus S square is equal to C square cos hyperbolic square x upon C minus sin hyperbolic square x upon C. So, this is 1 it is hyperbolic function. So, cos square hyperbolic minus sin square hyperbolic it is 1. So, therefore, y square minus S square is equal to C square; that means, y is equal to root over C square plus S square this is equation 40.

Therefore, from equation 33 and 40 we get, equation 33 again we have to go back to you just hold on I have to search those, equation 33. So, this is equation 33 T is equal to root over C square plus S square into W this one so; that means, and it is root over C square plus y square is equal to y root over that that you have got the root over C square plus y square is equal to y.

That means, this equation can be written T is equal to y into W that is why you are writing that that is the maximum tension from equation 33 we get T max is equal to W into y because y is the maximum one, y is equal to C plus d actually from this figure that figure we have seen again and again so; that means, T max we can write W into y this is equation 41 and y is equal to root over C square plus S square; that means, T max is equal to then we can write again W into root over C square plus square this is equation 42.

Now, according to equation 41 this one maximum tension T occurs at the support as per this equation maximum tension will occur at the support.

(Refer Slide Time: 09:14)

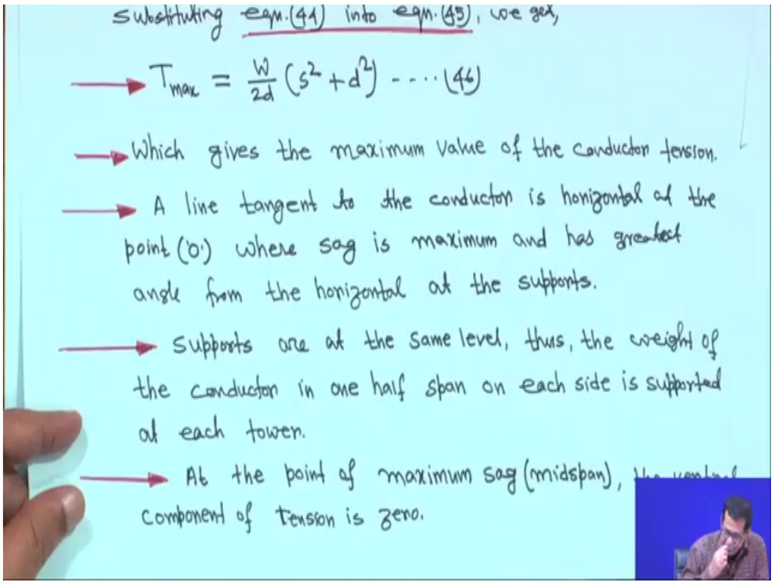


That means both side of the tower; that means, where the conductor is at an angle to the horizontal whose tangent is V by H that we have seen or S by C since V is equal to omega WS, W into S the vertical weight of the mid from the through the midpoint acting downwards that conductor weight. So, V is equal to WS and H is equal to W into C because this is horizontal one. So, at support therefore, that we know y is equal to C plus d just let me C if I can get back that figure again otherwise we have seen that figure again and again anyway.

So, from that figure 3 only that y is equal to C plus d. So, this is equation 43; that means, we have seen just now that y is equal to root over C square plus S square; that means, y is equal to C plus d. So, C plus d is equal to root over C square plus S square; that means, C is equal to S square minus d square upon 2d just both side you square it and simplify you will get C is equal to S square minus d square upon 2d this is equation 44.

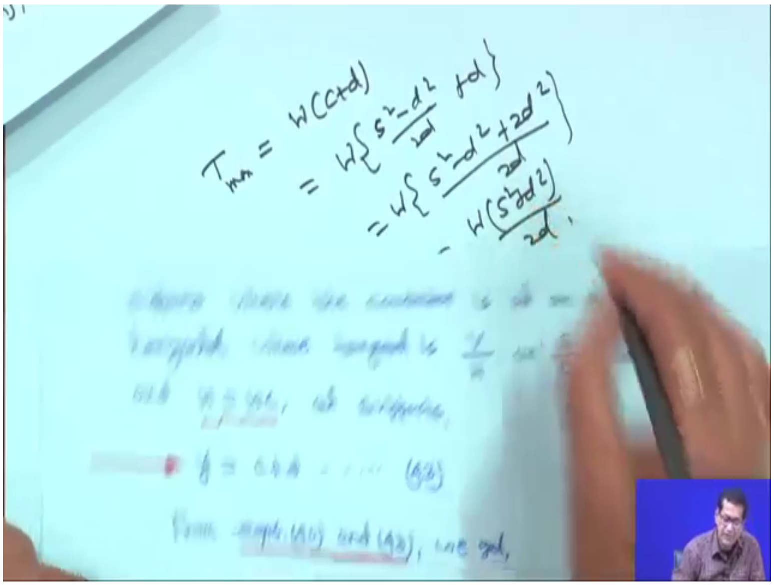
So, from equation 41 and 43 we can write we have seen T max is equal to W into y, but y is equal to C plus d. So, t max is equal to W into C plus d this is equation 44; that means, substituting equation 44; that means, this one into equation 45 if you substitute and this your this one and simplify.

(Refer Slide Time: 10:49)



You will get T max is equal to W upon 2d S square plus d square, I mean it is coming this way.

(Refer Slide Time: 10:58)

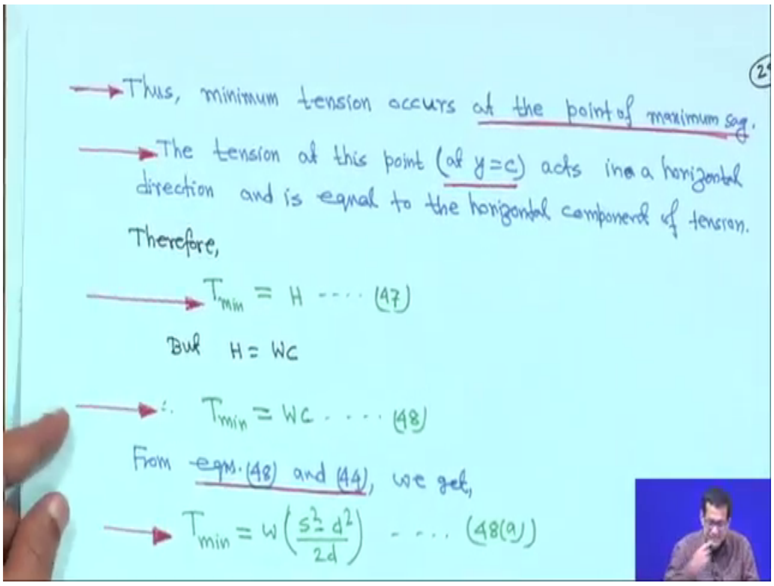


Your T max is equal to is equal to your W into C plus d is equal to W then your C is equal to your S square minus d square upon 2d plus d that is W into S square minus d square plus 2d square upon 2d. So, that is W into S square plus d square divided 2d. So, that is why we get that thing the T max is equal to your W upon 2d S square plus d square equation – 46, this, that means, which gives the maximum value of the conductor tension, this is the maximum tension.

A line tangent to the conductor is horizontal at the point where sag is maximum and has greatest angle from the horizontal at the support and the supports are at the same level thus the weight of the conductor in one half span on each side is your supported at each tower because tower height is same.

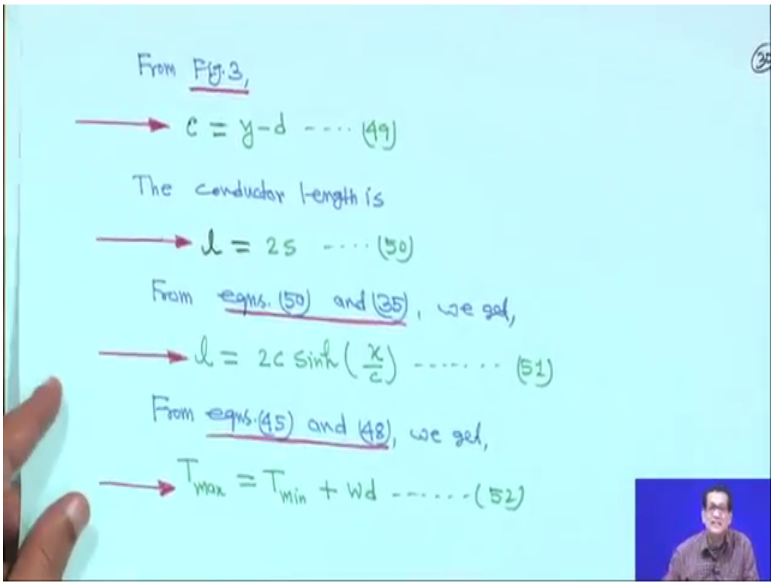
So, naturally your what you call that whole weight of the conductor your at the midpoint of the your conductor at the point of maximum sag that is the mid span that is the midpoint of the conductor the vertical component of the tension is 0, because there at the midpoint you will find only H exists, but vertical component your of the tension will be 0.

(Refer Slide Time: 12:35)



So, in this case the maximum thus maximum tension occurs at the point of maximum sag sorry minimum tension will occur at the point of maximum sag therefore, the tension at this point that is at y is equal to C if you think that graph that acts in a horizontal direction and is equal to the horizontal component of tension therefore, T minimum should be H because it is at the midspan. So, t minimum should be that we know H is equal to W into C. So, T minimum will be W into C and we have seen that C is equal to root over sorry S square minus d square upon 2d. So, what you do this T minimum C you substitute is equal to W into S square minus d square upon 2d say this is equation 48. a right.

(Refer Slide Time: 13:31)

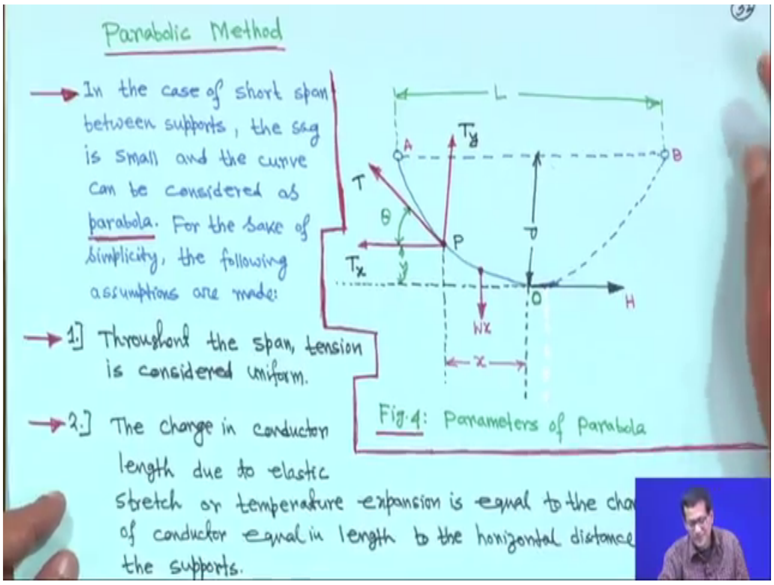


So, from figure – 3, again and again not showing the figure – 3, it is will we have seen know that C plus d is equal to y therefore, from figure 3 only C is equal to y minus d this is equation 49 therefore, the conductor length that is we know that l by 2 is equal S. So, l is equal to 2s, s small 2 into small s this is equation 50.

Some equation 50 and 35 we get l is equal to 2 and from 35 s is equal to C sin hyperbolic x by C you substitute here from equation 35 s is equal to. So, small l is equal to 2C sin hyperbolic x by C this is equation 51 and from therefore, from equation 45 and 48; that means, your this is your 48 and this is just hold on this is your equation 45, 45 T max is equal to WC plus wd actually and; that means, your T max and T min is equal to you know the WC just now here it is T min is equal to WC.

Therefore, from equation 45 and 48 actually T max is equal to T min plus Wd because here T max is equal to WC plus WC and WC is equal to T min therefore, T max is equal to T min plus Wd that is why we are making it the T max is equal to T min plus Wd, this is equation 52 right. So, this is your using all catenary shape and these are all relationship when you will take the numerical at the time we will see the usefulness of this formula.

(Refer Slide Time: 15:23)

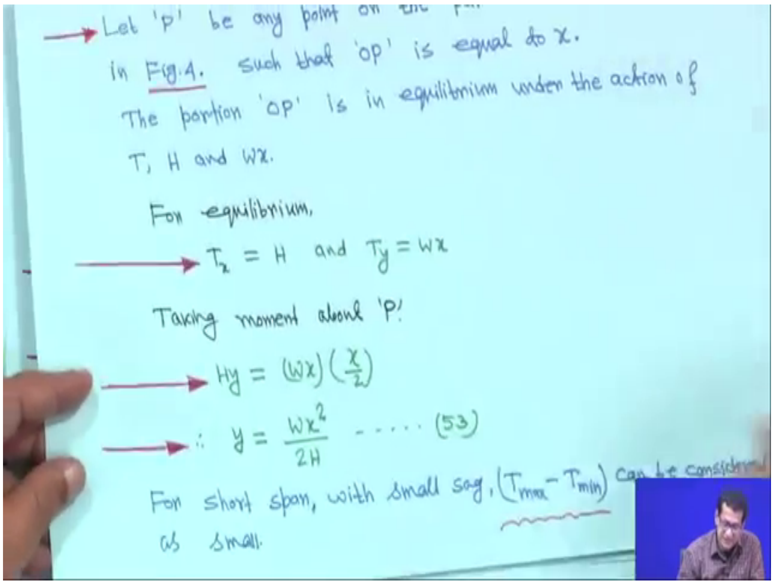


Now, suppose if it is a parabola now whatever we have seen that is basically general case now if it is a parabola. So, equal y your shapes are this is the same level distance is L and this is the maximum sag d some point P we have taken. So, tension T y acting along this direction upward and here it is your T x is the horizontal tension angle between this is the tension T tangential it is working.

This angle is theta and this is the your mid span that is O, this is the point of maximum sag and from this distance is y some distance is y and this is some point P and this angle is theta this is parameters of parabola and this is the horizontal tension H and in between O and P this is the midpoint something. So, say some distance x, W into x is you are working downwards this is the conductor weight say if you consider the point P and between O and P this is the midpoint between O and P this is Wx and this is figure – 4 and this is the distance between the 2 support.

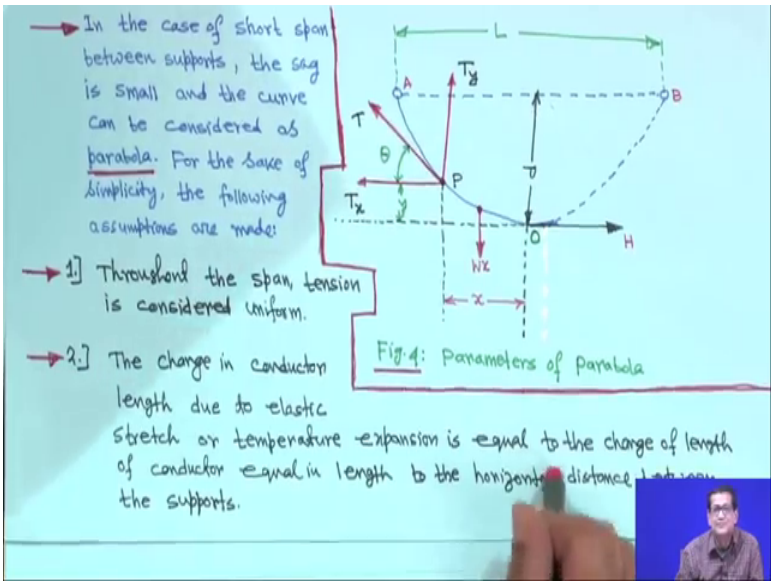
So, for the sake of simplicity the following assumptions are made throughout the span tension is considered uniform this is one assumption second is the change in conductor length due to elastic stretch or temperature expansion is equal to the change of length of conductor equal in length to the horizontal distance between the support, these two assumptions we are making.

(Refer Slide Time: 17:02)



And, if you assume let this is the point P. So, let P be any point on the parabolic curve as shown in figure four. So, P is any point any point such that OP is equal to x. So, this distance this is your OP the distance is your OP is equal to actually this is if you take your S, it will be whatever we are writing that OP is equal to x because of this assumption because we have to just consider the two assumptions.

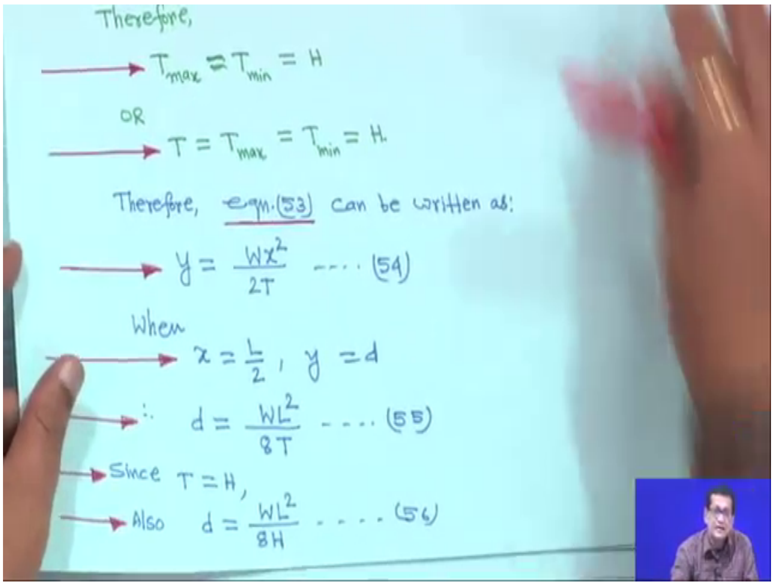
(Refer Slide Time: 17:36)



This is because of the second assumption. So, that is why OP is equal to x and the portion OP is in equilibrium under the action of T, H and W for equilibrium, T x is equal to H same thing this is the horizontal one T x is equal to H and T y is equal to Wx right. So, T x is equal to H and T y is equal to Wx. So, taking if you take moment about P, this about this point P then it will become that is your H this is the H and this distance is y. So, H into y and this is Wx, W into x and it is midpoint, Wx into x by 2. So, your taking moment about P. So, Hy is equal to Wx into x by 2 because this is the midpoint x by 2. So, and this is x by 2 is coming because of our second assumption.

So, for short span small sag T max minus T min can be considered as small; that means, T max minus T min we have seen is equal to your W into d. So, for small sag the T max minus T min can be considered as small. So, in that case if it so, we can write t max is equal to T min approximately is equal to H.

(Refer Slide Time: 18:55)

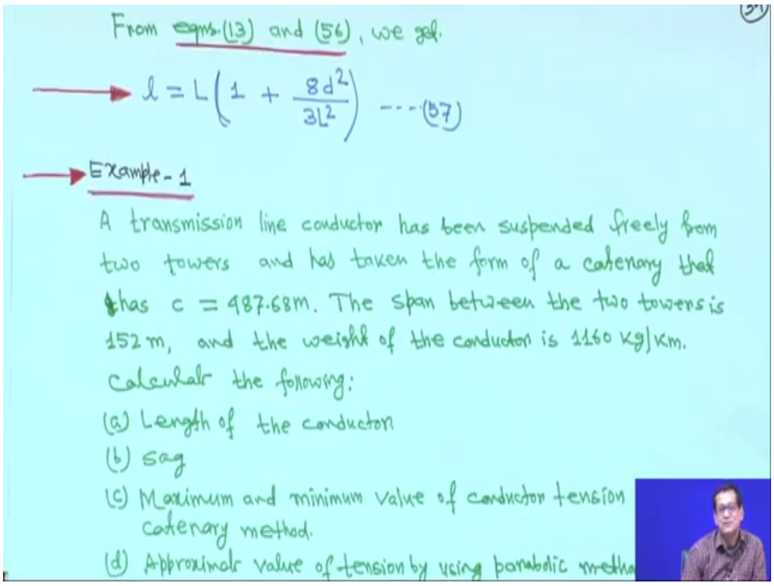


So, t max is equal to T min is H; that means, that we can write that one also T. So, T is equal to T max is equal to T min is equal to H therefore, equation 53; that means, this equation y is equal to Wx square by 2H, we can write this equation that as T is equal to ultimately it is coming T is equal to H therefore, we can write y is equal to Wx square upon 2H this is equation 53. So, we can write this equation y is equal to Wx square upon 2T is because H is equal to T this is equation 54.

When x is equal to capital L by 2 then y is equal to d; that means, this is the midpoint the total length is L. So, in x will be L by 2 then sag will be d therefore, x by when x is equal to capital L by 2 then y small y is equal to d.

Therefore, d is equal to WL square by 8T this is equation 55, because here you substitute your y is equal to d and x is equal to L by 2 you will get d is equal to W square by 8T since T is equal to H because from this equation T is equal to H also, this equation it also can be written as the d is equal to WL square by 8H. So, this is equation 56. So, from equation – 13 and 56 you know that d is equal to WL square by 8H. So, what you do, you substitute that thing.

(Refer Slide Time: 20:35)



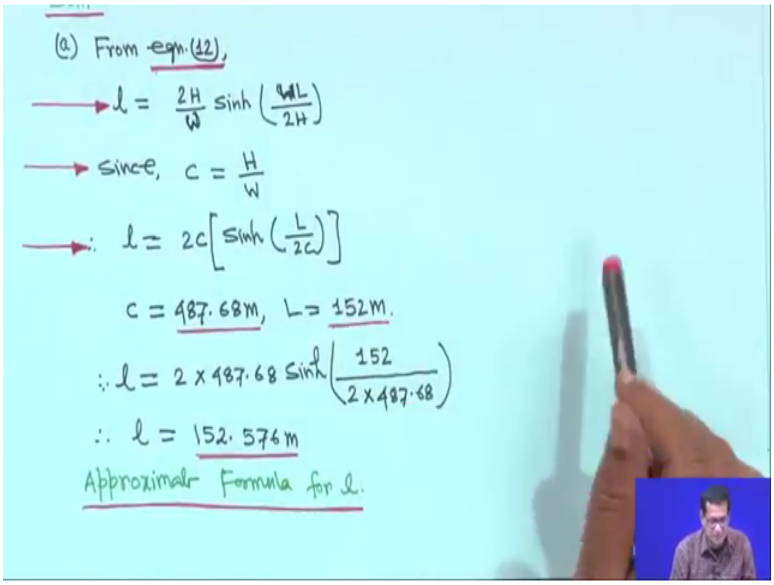
And, in equation – 13, you will get small is equal to capital L in bracket 1 plus 8d square divided by 3L square this is equation 57. So, with these whatever derivation was there for equal supports one is catenary shape, another is parabolic shape these are all the mathematical expression up to equation 57.

Now, that we will take an example after that we will consider that your wind pressure, then ice your size formation on the conductor those things will be considered. So, first you take an example, say example – 1: a transmission line conductor has been suspended freely from two towers and has taken the form of a catenary that has C is equal to 487.68 meter this is given.

The span between the two towers is 152 meter and the weight of the conductor is 1160 kg per kilometre, this is per kilometre is given you have to calculate the following 1, length of the conductor b. the sag C maximum and minimum value of conductor tension that is T max and T min using catenary method approximate value of tension by using your parabolic method this is the numerical.

So, how to solve it? So, whatever problem formula we have derived mathematical derivation. So, just you have to stick to with that right.

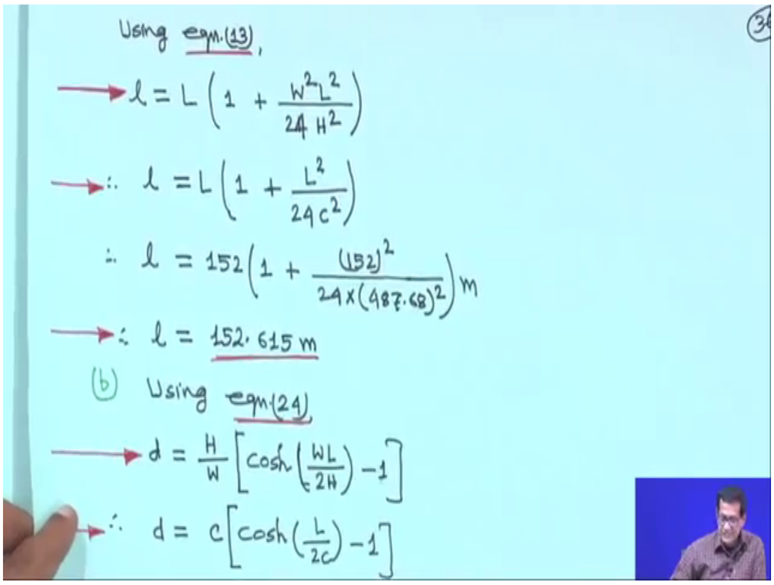
(Refer Slide Time: 22:13)



Problems here are quite simple if you can call those formula from equation – 12, we know l is equal to 2H upon W sin hyperbolic WL upon 2H. Since we know C is equal to H by W, this we have seen; therefore, l is equal to we can write 2C because H by W upon 2C sin hyperbolic capital L upon 2C because C is equal to H by W. So, it is W by H denominator your if you just divide by W. So, that is why you can write sin hyperbolic l by capital L by 2C.

Now, it is given C is equal to 487.68 meter and given L is equal to 152 meter. So, substitute all these value. So, small l will be is equal to 2 into 487.68 sin hyperbolic 152 upon 2 into 487.68; that means, l is equal to 152.576 your meter. So, if you look into that that horizontal distance between two towers is given 152 meter and if you look the length of conductor it is now 152.576 meter. So, as it has taken a catenary shape. So, length has increased, now approximate formula for l.

(Refer Slide Time: 23:45)

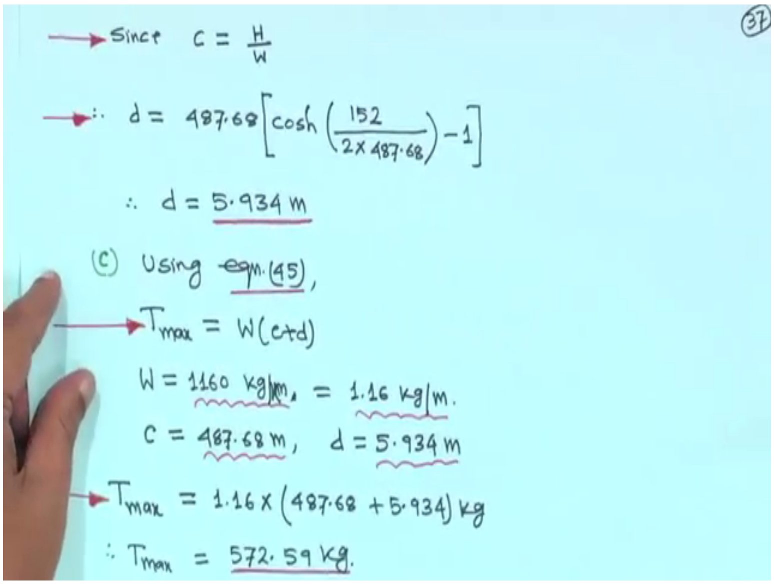


Now, we have derived that in equation – 13, the small l is equal to it is equation – 13 only 1 plus W square L square upon 24 H square. So, or l is equal to L 1 plus that capital l square upon 24 C square because C is equal to H by W. So, here you just replace that H by W by C. So, L square upon 24 C square.

So, capital L is given L is equal to horizontal distance 152 and it is 1 plus 152 square divided by 24 into 487.68 whole square meter. So, if you see l is equal to 152.615. So, this is one formula approximate formula and this one we got that your 152.576 this is exact one using sin hyperbolic and this is approximate formula for l is, 152.615. So, it is not much difference approximate formula. Initially, I told that if error maximum error will be 2 percent.

Next, is using equation 24 because next part is a part – b. So, in the part – b, just hold on part – b that you have to find out that sag part – b is the sag. So, in that case that using equation 24 d is equal to H upon W cos hyperbolic WL upon 2H minus 1. So, d is equal to H upon WC. So, just ratio is by W replaced by C it is cos hyperbolic capital L by 2C minus 1.

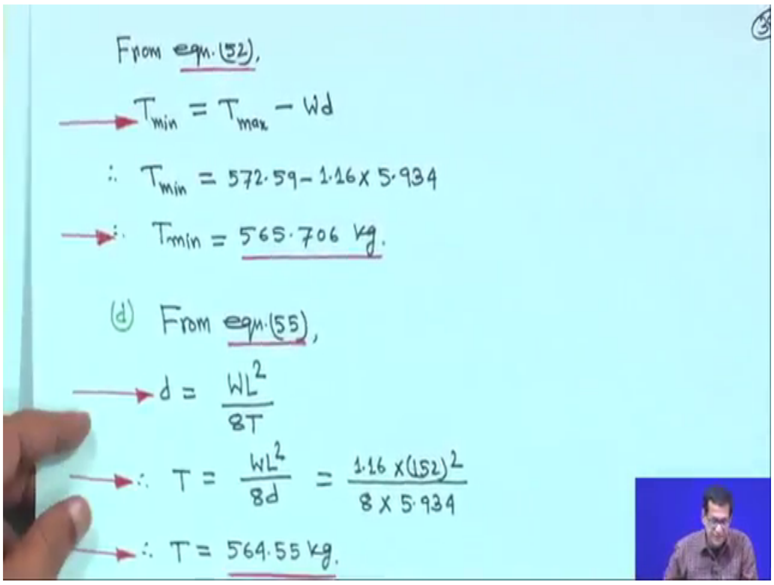
(Refer Slide Time: 25:34)



That means, if you substitute all that data, if you substitute since C is equal to H by W, I have written here once again d is equal to 487.68. So, cos hyperbolic divided by 1 which hyperbolic 152 divided by 2 into 487.68 minus 1, so, that means, d is equal to sag is 5.934 meter.

Now, C we have to find out this one maximum and minimum value of conductor tension using catenary method. So, using equation 45 we have seen just T max is equal to W into C plus d, W is given 1160 kg per kilometre that is actually 1.16 kg per meter and C is given 487.68 meter and d we have calculated here 5.934 meter. So, d is equal to 5.93 meter. So, T max is equal to 1.16 into 487.68 plus d 5.934 kg. So, 572.59 kg, this is your T max.

(Refer Slide Time: 26:55)



Next, one is that you have to find out the minimum one. We know that T max plus T min is equal to Wd that we have seen using. Equation 52 where it has been shown then T max is equal to T min plus Wd that mean T min is equal to T max minus Wd; that means, T min is equal to T max just we have calculated 572.59 minus, W is 1.16 and d also sag we have calculated 5.934. Therefore, T min is equal to 565.706 kg, that means, between T max and T min you will find there is not much difference.

Now, last one is that approximate value of tension by using parabolic method. If you use parabolic method then first you have to calculate approximate method using. So, equation 55 d is equal to WL square by 8T this we have seen W is equal to 1.16 and your T; that means, this equation we can write T is equal to WL square upon 8d; d is coming here, T is going here. T is equal to WL square by 8d.

So, W you know 1.16 kg per meter L is 152 meter. So, 152 square divided by 8 into d we have calculated at the sag 5.934 you will get T is equal to 564.55 kg. This is the using your approximate method using that parabolic method. So, numerical are very simple, but only those catenary methods and parabolic one, little bit practice is necessary, a little bit understanding particularly the boundary condition and little bit of integration, with that I think you can easily do it with that.

Thank you very much.