Power System Engineering
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Lecture - 24
Sag & Tension Analysis (Contd.)

So, we have just discussed that that sag of the conductor decreases, because it tensions pulls the conductor up right.

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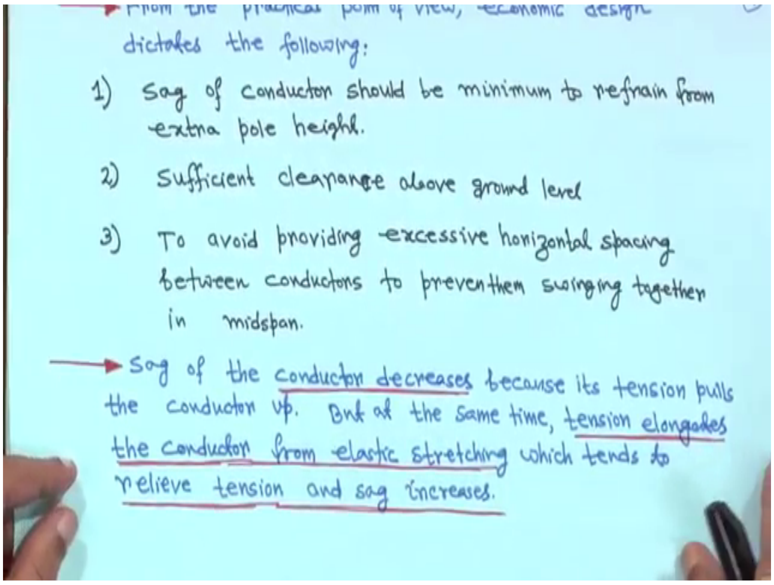

But at the same time tension elongates the conductor from elastic stretching, which tends to relieve tension and sag increases right.

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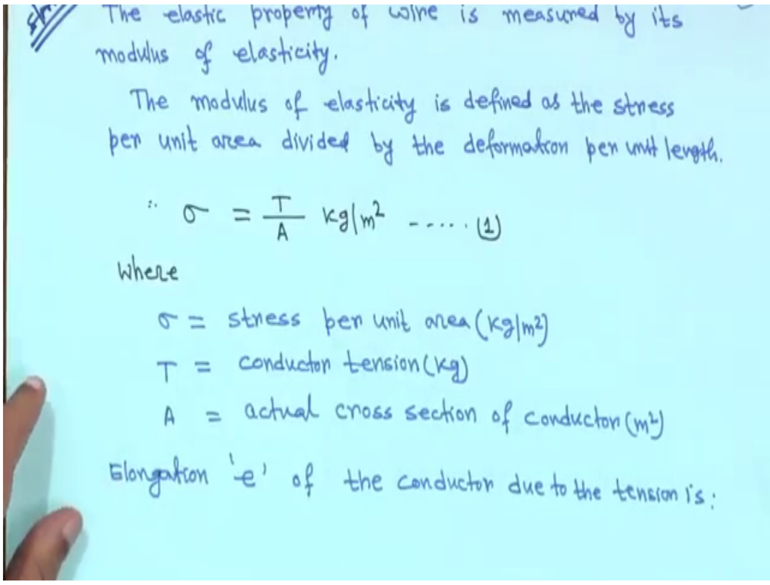

That the elastic properties of wire is measured by, it is modulus of elasticity this you know actually.

The modulus of elasticity is defined as the, stress per unit area divided by the deformation per unit length; that means, sigma is equal to T upon A kg per meter square this is equation one, where sigma is equal to stress per unit area that is kg per meter square, T is equal to conductor tension in kg, and A is equal to actual cross-section of conductor that is in meter square.

So, elongation e of the conductor due to the tension, is that elongation e can be defined as.

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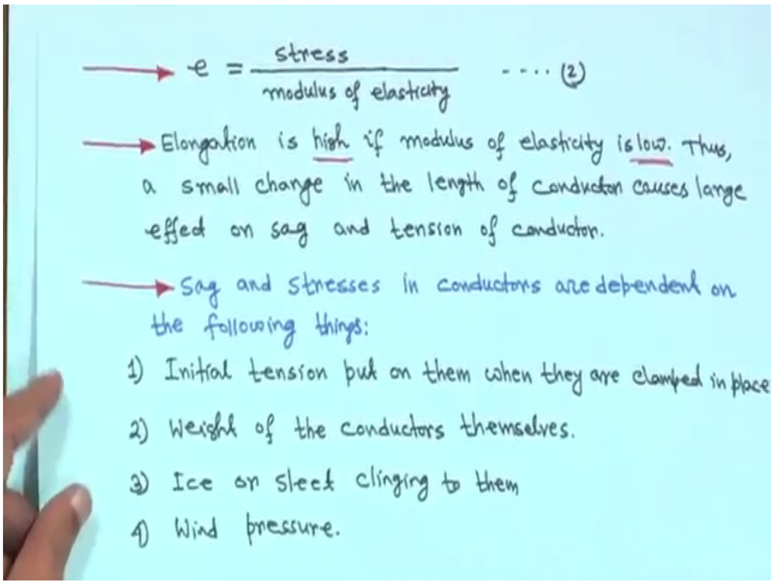

That e is equal to stress divided by modulus of elasticity, this is equation 2. So, elongation is high e is high, that is elongation if modulus of elasticity is low right, thus a small change in the length of conductor causes large effect on sag and tension of conductor.

Now, if you see that sag and stresses in conductor are dependent on the following points, one is initial tension put on them when they are clamped in place right, 2, weight of the conductor themselves ice or sleet clinging on the clinging to them, and then wind pressure right. So, these are the things that sag and stresses actually dependent of this following things.

So, particularly ice or sleet clinging to them it is basically in the mountain area where snow fall is there.

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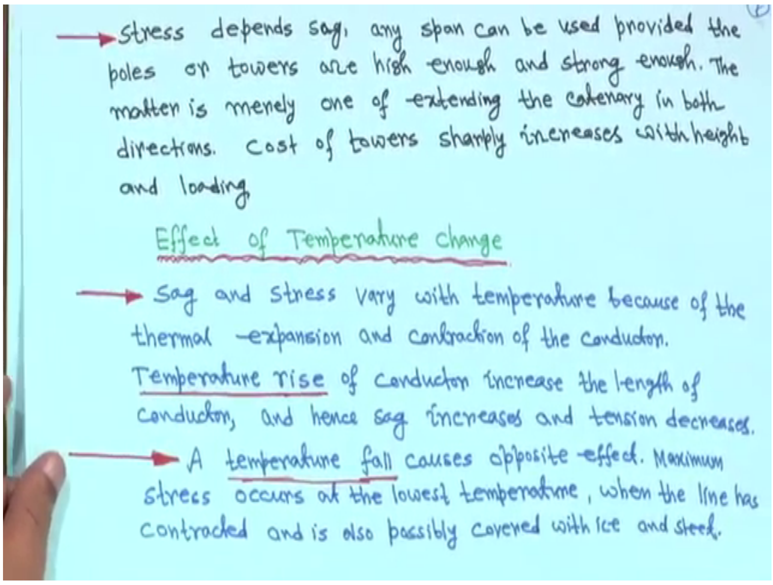

Therefore, that stress actually depend sag any span can be used provided the poles or towers are high enough and strong enough, but if you make poles or towers are high and then naturally that your height of the tower will increase therefore, cost will increase.

The matter is merely are of extending the catenary in both directions. So, cost of towers sharply increases with height and loading right, now another thing is the effect of temperature change, common sense says that in summer that sag will be more and winter it will be less because of the temperature variation.

So, sag and stress vary with temperature because of the thermal expansion and contraction of the conductor, temperature rise of conductor increase the length of conductor hence sag increases and tension decreases and vice versa, and similarly the temperature fall that is in winter say causes just opposite effect right. So, maximum stress occurs at the lowest temperature, when the line has contracted and is also possibly covered with ice and sleet right. So, this this can happen particularly in the winter.

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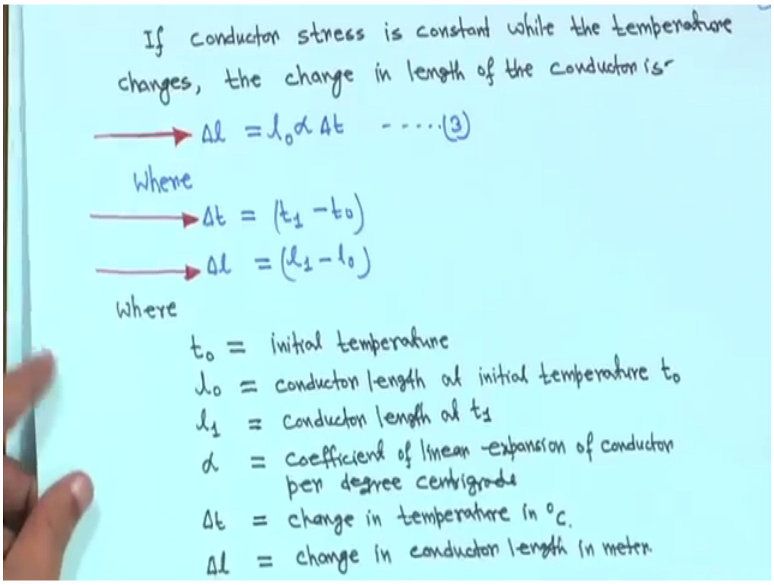

So, in if conductor stress is constant while the temperature changes, the change in the length of the conductor can be given as delta l is equal to l 0 into alpha into delta T this is equation 3, where delta T is equal to T 1 minus T 0 and delta l is equal to l 1 minus l 0, where T 0 is initial temperature l 0 is conductor length at initial temperature say T 0, that is this is T 0 l one is conductor length at temperature T one, and alpha coefficient of linear expansion of conductor per degree centigrade, and delta T that is this one delta T and delta l this delta T is change in temperature in degree centigrade and delta l change in conductor length in meter right.

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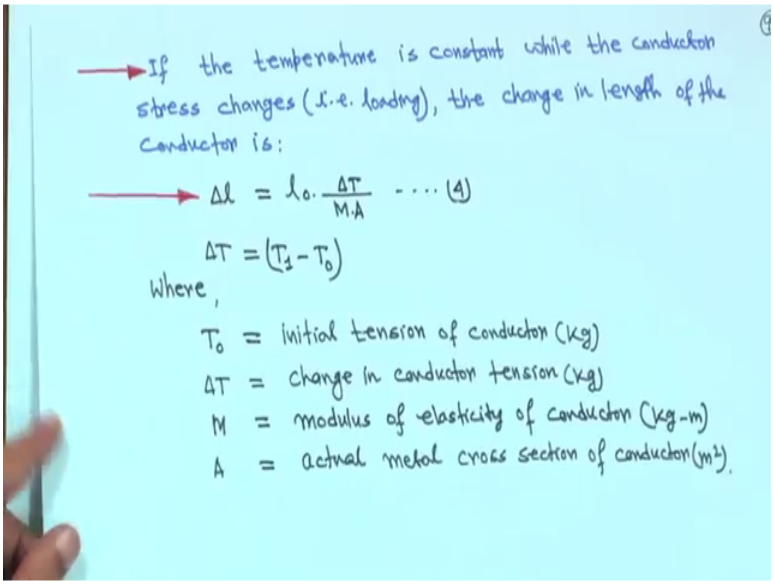

That means that if the temperature is constant, while the conductor stress changes that is loading, the change in lenght of the conductor is it can be given as delta l is equal to l 0 into delta T divided by m into A, where delta T is equal to capital T 1 minus capital T 0, where T 0 is equal to initial tension of conductor that is kg delta T is equal to change in conductor tension, that is in kg m is equal to modulus of elasticity of conductor that kg meter, and A actual metal cross section of the conductor that is in meter square.

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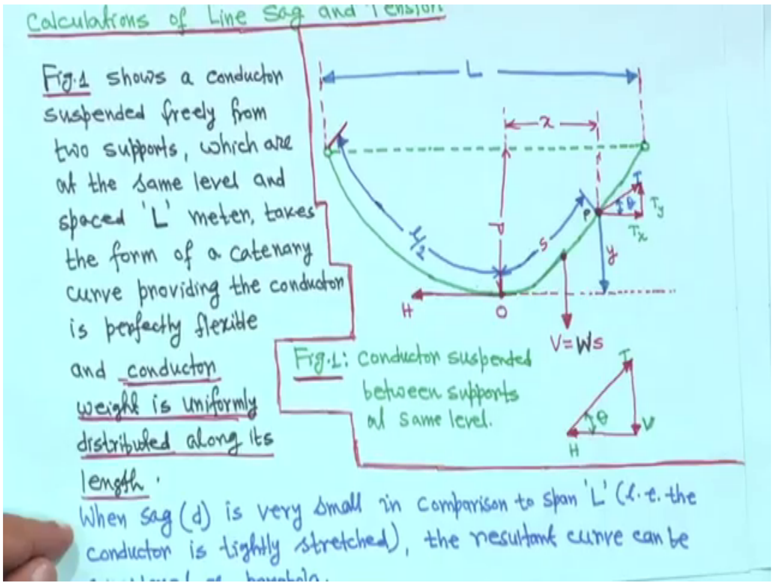

Now, these are some simple thing, now calculation of line sag and tension, actually we have taken a catenary shape these are symmetrical although my handwriting is slightly this way, but this side and this side it is symmetrical right. So, this is your suppose it is your both the side this is tower side this one and this one, they are of equal height right. So, this is the first thing.

So, figure one shows a conductor suspended freely from 2 support, this is one support this is another support a tower side right, which are at the same level that is same height same level, and I mean from this is a horizontal distance right, and if sag and tension if sag happens in conductor. So, basically takes a catenary shape right. So, they are at the same level that is an horizontal distance between them is capital l this is capital l right, takes the form of catenary curve providing the conductor is perfectly flexible.

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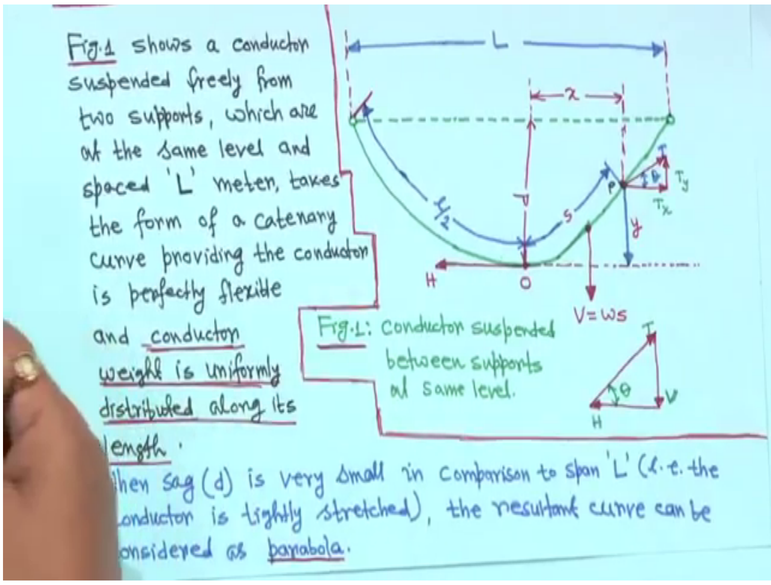

And conductor weight is uniformly distributed along it is line, actually total length of the conductor is small l. So, from here to here we have this is l by 2 this side also total is l by 2 right, but we have taken some point your p. So, in this case that when sag your d is very small when d is very small in computation to span l, that is that conductor is tightly stretched, the resultant curve can be considered as a parabola right.

But in this case parabolas we will see later, but in this case, we have not assume that this is a parabola it is a catenary shape, but if sag is less as compared to the l, then that can be considered as a parabola. So, this is regarding this your point P then V is equal to ws, then horizontal your tension it H then tension T at point P Tx Ty will come little bit later.

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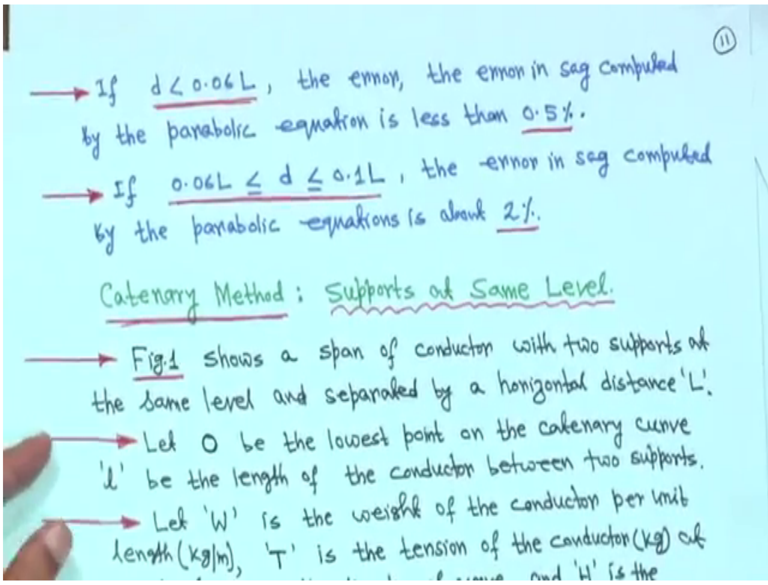

So, if your d less than 0.061, suppose if it is total your distance between the 2 towers is l horizontal distance, and d is your less than 0.061 the error the error in sag right computed by the parabolic equation is less than 0.5 percent; that means, if your I mean if d is less than your what you call that, 0.0611 is the your distance between the your 2 support or 2 tower horizontal distance, then error is very small just point 5 percent, if d lying d is lying in between 0.061 and point one l then the error in sag computed by the parabolic equations is about 2 percent that is also quite less anyway.

So now catenary methods that is supports at the same level; so, this is that both the support at the same level, now figure one this is your figure one this is figure one, it is shows a span of conductor with 2 supports at the same level and supported by a horizontal distance l this is the horizontal distance l right.

Now, this O this is the O is the point is the lowest point on the catenary curve right, this is the lowest point at the catenary curve and this d is the maximum sag d is the maximum sag right, and l is be the length of the conductor between these 2, but it is it has taken a your catenary shape. So, this is the midpoints. So, this length is l by 2 from here to here it is l by 2 right.

So, this is that half half this l is the length of the conductor, and W is the weight of the conductor per unit length, this W actually is the weight of the W this is W right. So, W is the your what you call weight of the conductor per unit length, and if your T is the tension of the conductor that is at point P right, this is the T is the tension at point P and your in direction of curve an H is the tension k kg at origin and this is the horizontal tension.

So, this is this T is work is acting in the direction of the curve, and it is horizontal components is Tx and vertical component is Ty, and this angle is your between T and T x is theta, and distance from this horizontal line from this meaning that is O is that your lowest point of the conductor this is maximum, and here the sag is maximum this distance is y small y and from O to this point P that is op this distance is H O to P that distance is H; that means, here to here and from point O to P we assume the conductor length is small S, this is small S and if the W is the weight per kg meter. So, total vertical force will be active on it V is equal to W into S.

Where W is equal to kg per meter and S in meter. So, this is at some point P you have taken and the weight is uniformly distributed therefore, this point you can my if this point the middle point between O and P this is the middle point between O and P, that V is equal to the vertical Ws say is acting downwards right.

Assuming that weight is and conductors are your round shape this assumption is correct right. So, in that case this tension T it is it can be resolved T in 2, competent one is horizontal component Tx another is vertical component Ty. So, if you see that your force tension balancing equation then Tx will be is equal to H that horizontal tension and this Ty is equal to V is equal to Ws right.

So, if we; that means, this if we make a triangle here in this figure smallest figure that this is a horizontal tension H and this is your Ty that your this this V is acting downwards right. So, this is V Ty is equal to V and Tx is equal to H if you make this triangle and this is tension T and this angle is theta; that means, tan theta will be as actually is equal to V upon h.

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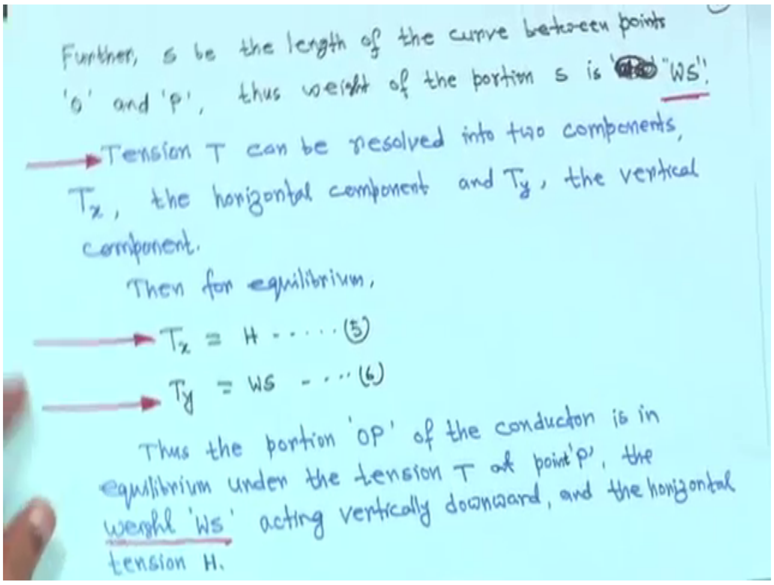

Therefore, just hold on. So, that means Tx is equal to H this is equation 5 I told you, and Ty is equal to W into s this is equation 6, thus the portion op of the conductor is in equilibrium under the under the tension T, that is this portion op is in is in equilibrium under the tension your T right, at point P the weight Ws acting vertically downward at the horizontal tension h. So, this I told you the Ws this is the midpoint between O and P and it your acting vertically and this is the horizontal tension this is h.

Now, suppose if that if the there is a your certain your little bit of increase in the conductor length your say, this is the conductor length if there is a little bit of change in the distance say, it is S suppose there is a little change in ds then then naturally your this is you have taken in catenary shape if you make a triangle then horizontal side will be dx and your vertical side will be dy.

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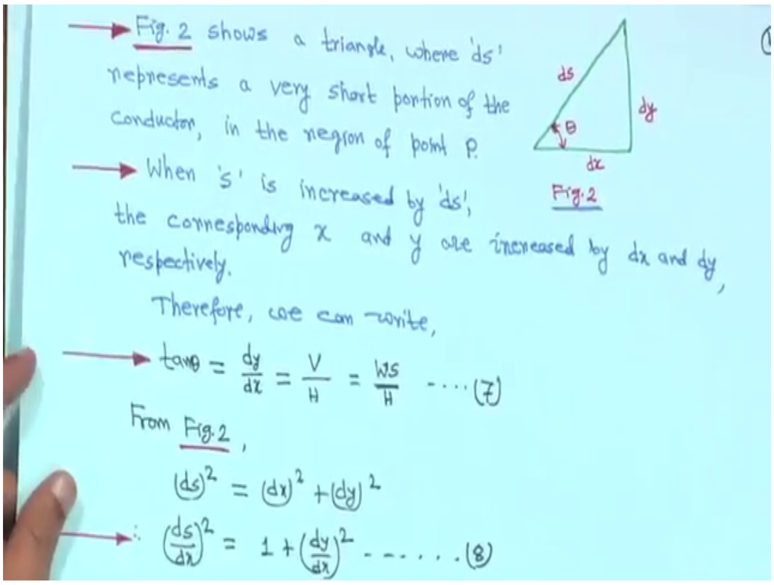

So, that is why now this is your figure 2, so, in figure 2, it is a triangle where ds represents a very short portion of the conductor, you take a very short portion of the conductor in the region of point P, now this is your this is your point P that the, I mean in the vicinity of this one if you consider that your what you call small distance ds.

So, if when S is increases by ds that is why tell you that small increment is corresponding to x and y are increased by dx and dy respectively; that means, just hold on; that means, if you little bit of increase say ds, naturally this distance is y from here to here and this O to P that horizontal one it is P, for your this things suppose I can make this one also P dash. So, op dash actually is equal to x and P dash P is equal to y right. So, if there is take of if this is ds along the conductor. So, horizontal one is ds and vertical one is dy, and this angle is theta; that means, tan theta is equal to dy by dx.

Therefore, we can write tan theta is equal to dy by dx is equal to V by H; that means, this theta and this theta same. So, tan theta here is equal to V upon H, and from here tan theta is equal to dy upon dx. So, tan theta is equal to dy upon dx is equal to V upon H, and V is equal to W into S therefore, we are substituting V is equal to W into S. So, basically tan theta will be W S by H that is dy by dx is equal to Ws by H right and this is equation 7.

Now, from figure 2 from this, figure this is right angle triangle. So, ds square is equal to dx square plus dy square; that means, from figure 2; that means, from this figure ds square is equal to dx square plus dy square; that means, d just divide both side by ds. So, ds by ds whole square is equal to 1 plus dy ds whole square this is equation 8 right.

But just hold on, but we have seen the d your dy by dx is equal to actually Ws by your H right, so; that means, this dy ds is equal to Ws by H you substitute here Ws by h.

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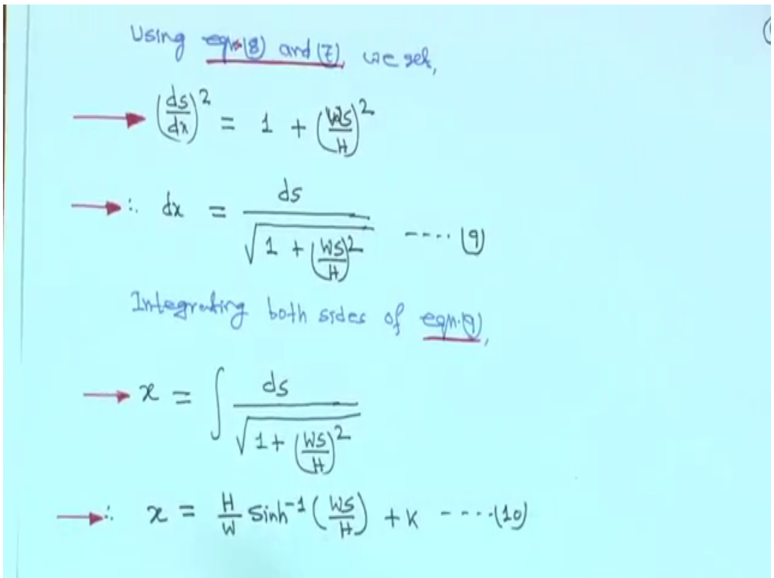

If you do. So, then ds by dx whole square is equal to your 1 plus Ws by H whole square right, or dx is equal to ds upon root over 1 plus Ws by H whole square this is equation 9.

Now, if you integrate both sides, then x will be integration of ds upon root over 1 plus Ws by H whole square, if you integrate this it will be x is equal to H by W this integration, I think you can do it I need not do it right you please do it right very simple thing. So, H upon W sin hyperbolic inverse Ws upon H plus k k is constant. So, this is equation 10.

Now, now if I put the boundary condition to get the value of this k value. So, here when x is equal if you look at this figure, when this is x actually O to P dash x if x is equal to 0, then naturally S also will become 0 because S is measured from this point see if x is equal to 0 then S is equal to 0 if it is. So, if it is. So, then you put the condition that x is equal to 0 then S is equal to 0, then then you will find that k is equal to 0.

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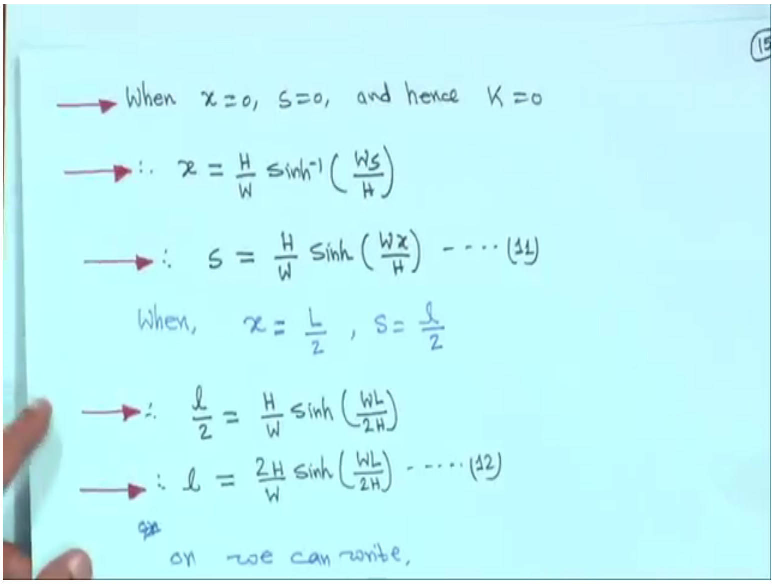

Though in this case your S k will become 0 so, therefore, x is equal to H upon W sin H inverse that is sin hyperbolic inverse Ws by H or S is equal to we can write H by W sin hyperbolic Ws by H, I mean cross multiply Ws by H and then you just you simplify. So, we are writing S is equal to this is equal to H by W sin hyperbolic Ws by H this is equation 11 right.

Now, when x is equal to l by 2 S is equal to l by 2; that means, this figure. So, x is the horizontal distance. So, from here to here it is l by 2 because from this support to support is capital l. So, it is a midpoint. So, when x is equal to l by 2 then this this side also half of the conductor length. So, small S is equal to small l by 2.

Therefore, when x is equal to capital l by 2 S is equal to small l by 2. So, if you substitute here if you substitute here in this equation you will get S is equal to l by 2 is equal to H by W then sin hyperbolic then W l upon 2 H or l is equal to 2 H by W sin hyperbolic W l by 2 H this is equation 12.

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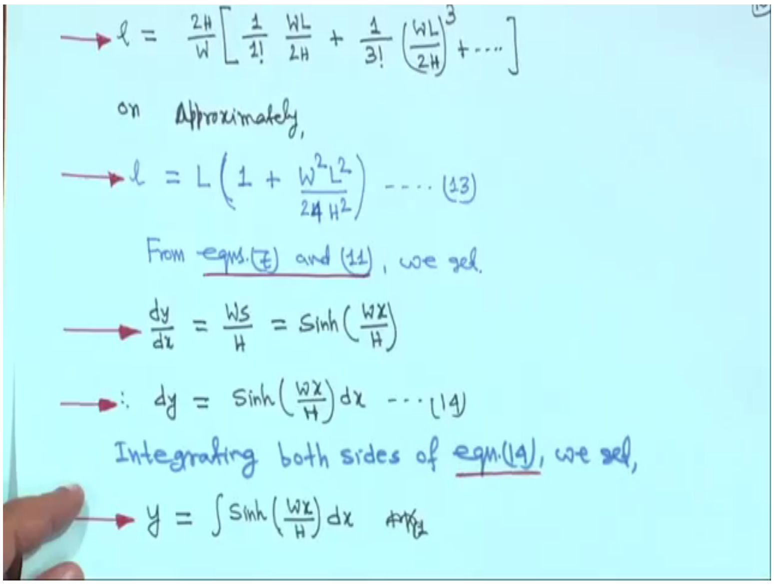

Now, what you can do is you expand this sin hyperbolic double W l by 2 H in series, if you expand it will become l is equal to 2 H by W 1 upon one factorial W l upon 2 H plus 1 upon 3 factorial W WL upon 2 H whole cube plus the higher order terms.

So, and if you take only this first 2 term this term and this term you take W common and H common they will be cancel finally, this expression l will become that is equal to capital l into 1 plus W square l square upon 24 H square, this is equation 13 right, just you take from this that this equation W common and your H common and simplify, and as well as l common you take H common W common and l common you take instead W will be cancel you will find small l is equal to capital l, into 1 plus W square l square upon 24 H square this is equation 13.

Now, from equation 7 and 11 we get, now equation 7 we have seen not showing it again understandable we have seen that dy by dx is equal to Ws by H this is at equation 7 right, and your S is equal to the W is sin hyperbolic W is equal to sin hyperbolic wx by H just let me find out equation 11, I will show you that this is just hold on this is equation 11 right.

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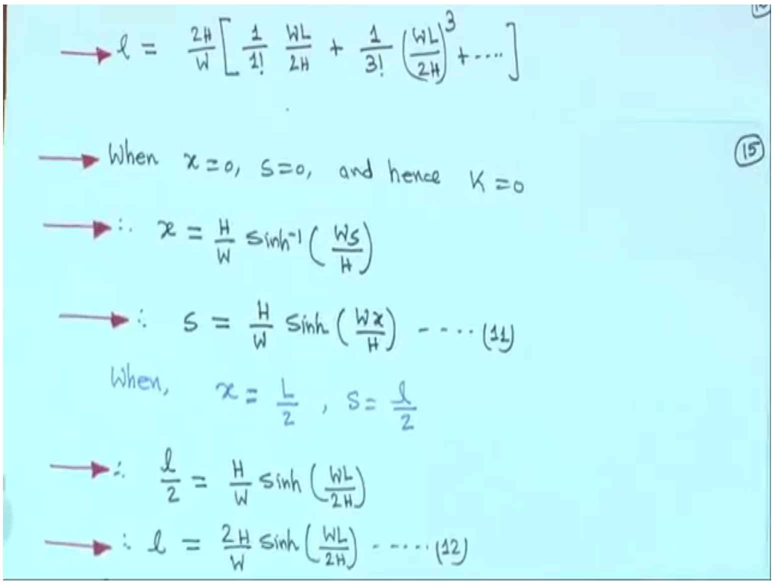

here it is S is equal to H by W sin hyperbolic wx by H, in your equation it is your this Ws by dy by dx is equal to Ws by h.

So, if you cross multiply it will be Ws by H is equal to sin hyperbolic wx by H, that is why you are writing dy by dx is equal to Ws by H, this is known Ws by H actually coming from this equation 11 multiply this cross multiplication Ws by H is equal to sin hyperbolic wx by H right.

So, therefore, dy is equal to your sin hyperbolic wx by H into dx this is equation 14. So, integrating you integrate this equation both side, integrating both sides of equation 14 that y is equal to integration of sin hyperbolic wx upon H in to dx right. So, that integration sin hyperbolic sin function integration is cos hyperbolic right. So, it will be actually.

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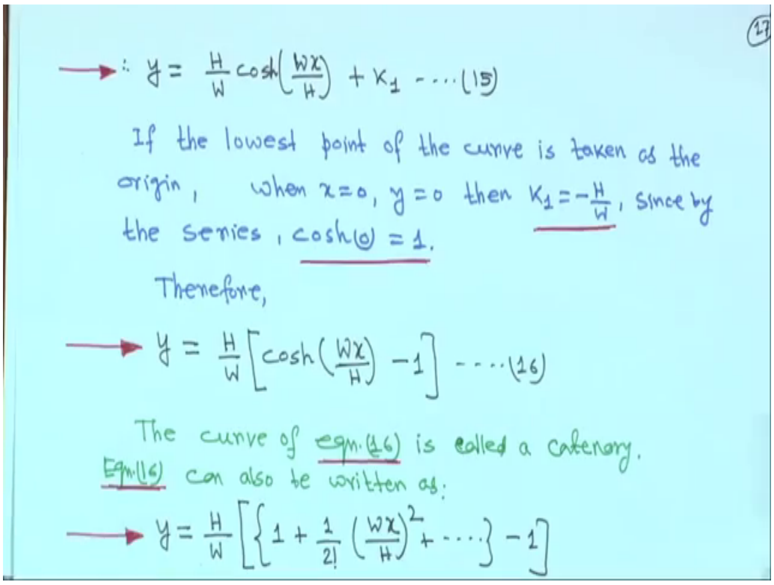

y is equal to H upon W cos hyperbolic wx upon H plus k one this is the equation 15 where k one is the constant.

Now, if the lowest point of the curve is taken as origin that is when x is equal to 0 y is equal to 0, because this is the point is origin, so, at this point x is measured from this point only, so, when x is equal to 0 then y is also is equal to 0 because this is y this is x this is y in general coordinate of this point is xy.

So, when x is equal to 0 y is equal to 0; that means, if you do. So, cos hyperbolic 0 actually is equal to one therefore, small y is equal to H upon W then cos hyperbolic wx upon H minus 1, this is equation 16, because you will get if you put x is equal to 0 and y is equal to 0 in this equation you will get k one is equal to minus H by W right, after that you substitute here k one is equal to minus H by W take H by W common. So, you will get y is equal to H by W cos hyperbolic wx by H minus 1 this is equation 16.

Now, the curve of the equation 16 this one is called a catenary right and equation 16 can also be written as y is equal to small y is equal to H by w, this cos hyperbolic you expand in series I will consider only first 2 term. So, it is 1 plus 1 upon 2 factorial wx by H whole square then higher order terms neglected bracket minus 1 because this minus 1 is here. So, this minus 1 this minus 1 will be cancel if it is. So, then it will become actually that your W because if you this is W square x square in H by w.

So, one H and one W will be cancel, and ultimately it will become W y is equal to wx square upon 2 H, this is equation 17 right, and from figure one I mean from this figure that is T this is right angled triangle. So, T square is equal to a square plus B square here it is T square is equal to a square plus B square that means T is equal to root over a square plus B square if you take H common from this equation. So, T is equal to H in under root 1 plus V upon H whole square right this is equation 18.

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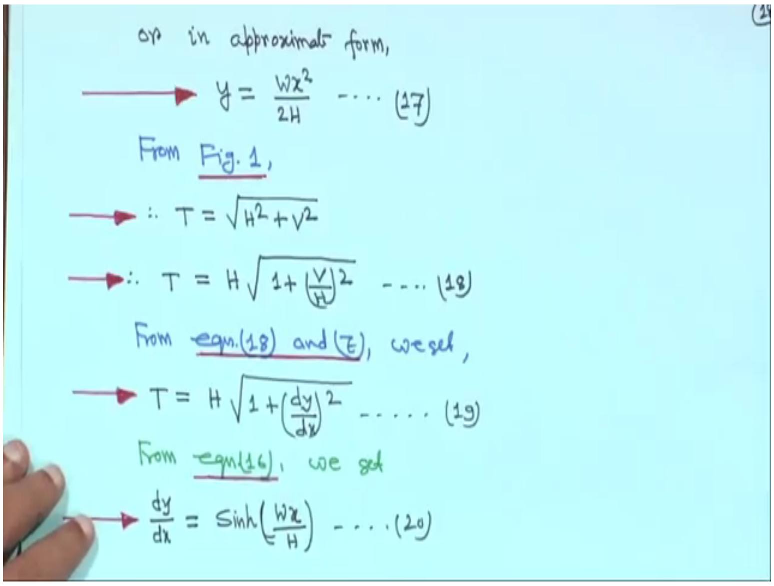

And equation 7 we have seen V upon H actually is equal to dy upon dx. So, here this is equation 7 it is given that it is dy upon dx is equal to V upon h. So, we are putting T is equal to H root over 1 plus dy upon dx whole square this is equation 19. So, from equation 16 we get that dy upon dx is equal to sin hyperbolic wx upon H, that mean this is your equation 16, you take the derivative with respect to x it will be dy upon dx is equal to H by W and it will come out W into W by H then sin hyperbolic x. So, H, H cancelled W, W will be cancel and it will be sin hyperbolic wx by H not showing here because this is a simple thing and understandable to you right.

So; that means, we get the dy by dx actually is equal to sin hyperbolic wx by H, because this one actually it will come this one your what you call this one your d your y is equal to your H by W cos hyperbolic wx by H right. So, if you; that means, your ultimately dy by dx will come that is sin hyperbolic wx by h. So, in this case your what you call that this is from equation 16, that is from equation 6teen we are getting dy by dx is equal to your sin hyperbolic wx just at the derivative of this equation simply this equation right.

Then what you do this dy by dx that is see equation 19 and 20. So, dy by dx if you put here in the in this equation it will be root over 1 plus sin hyperbolic square wx by H right. So, 1 plus sin hyperbolic your Ws it will be cosine what you call that cosine hyperbolic wx by H actually is very simple thing right.

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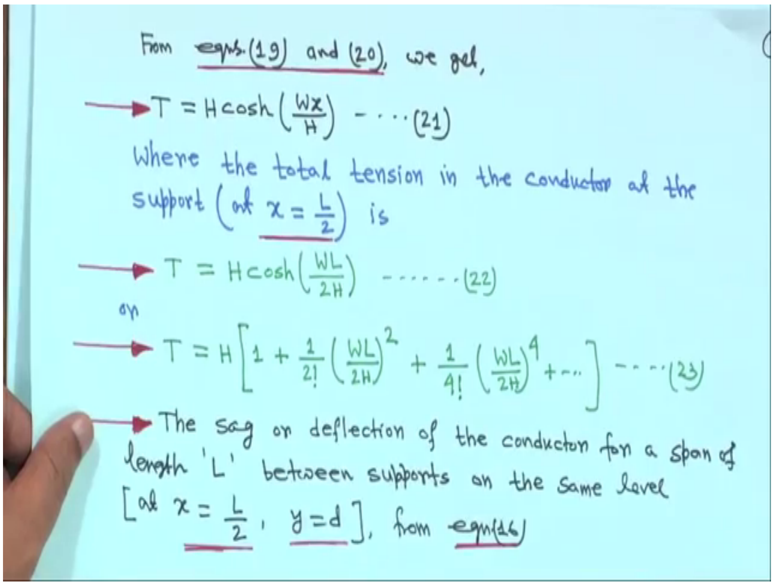

So, your for example, your y is equal to y is equal to H by W then this is cos hyperbolic right your wx divided by H minus 1.

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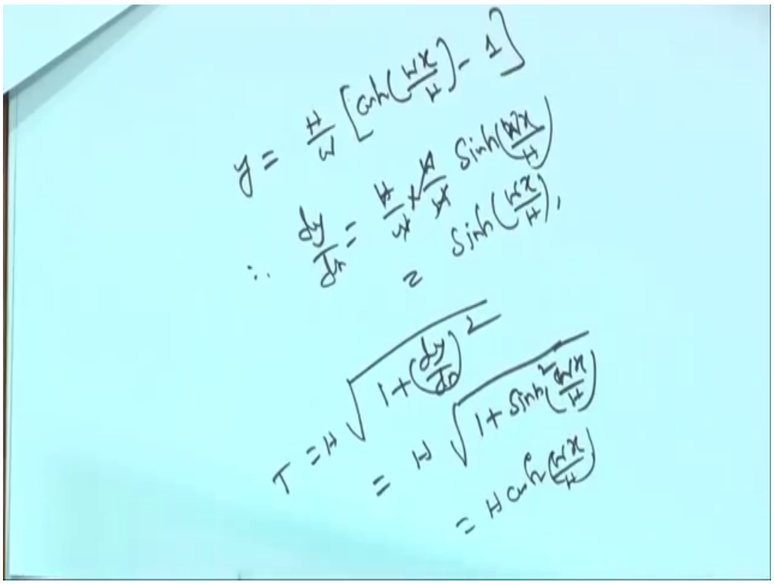

So, if you take the derivative of this one, then dy by dx will be H upon W into W upon H then sin hyperbolic your wx by H right, this one this one will be cancel is equal to sin hyperbolic wx by h.

Now, if you put in that equation that now root over it is given T is equal H, root over 1 plus dy by dx whole square that is equal to H then root over 1 plus sin hyperbolic square wx by H, actually this is nothing but cos square cos hyperbolic square it is cos hyperbolic. So, is equal to H cos hyperbolic wx by H right. So, this is the thing, that is why this equation that is why this equation 20 one from equation 19 and 20, sorry I am writing T is equal to H then cos hyperbolic your wx by H this is equation 21.

So, where the total tension your in the conductor at the support x is equal to l by 2, that is if you look at the figure that is this is total distance is l and this will be l by 2. So, when x is equal to l 2 l by 2, sorry then the total tension in the conductor at the support at x is equal to l by 2 is; that means, T is equal to H cos hyperbolic when x is equal to l by 2 you substitute in question 21 x is equal to l by 2 here it will be W l upon 2 h. So, H cos hyperbolic WL upon 2 H this is equation 22.

Now, if you expand this cos hyperbolic W l by 2 H in series, it will be T is equal to H then 1 plus 1 upon 2 factorial, then W l upon 2 H whole square plus 1 upon 4 factorial, W upon 2 H to the power 4 plus higher order terms and so on this is equation 23.

Now, the sag or deflection of the conductor for a span of length l between supports on the same level, that is at x is equal to l by 2 and your y is equal to your y is equal d. So, from equation 16; that means, this equation actually so many equations are here. So, from equation 16 right from this equation only right. So, and this is the your this is your what you call that diagram that figure one.

So, the sag or deflection of the conductor for a span of length l between supports on the same level that is at x is equal to l by 2 y is equal to d; that means, this is this is your what you call the total. total is your l and half is l by 2. So, at x is equal to your, what you call l by 2 y is equal to d and then equation your in equation 16, you if you put this one that when x is equal to l by 2 y is equal to d.

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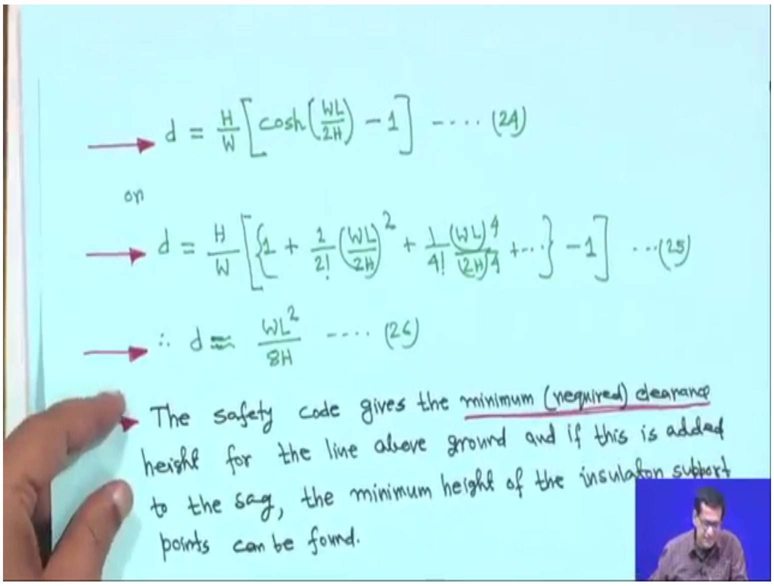

Then what you will get this equation that that if you substitute here then you will get, d is equal to because y is equal to d d is equal to H upon W cos hyperbolic W l upon 2 H because x is equal to l by 2 minus 1 this is equation 24.

If you expand this it will be d is equal to H upon W 1 plus 1 upon 2 factorial WL square upon 2 H plus 1 upon 4 factorial, W l upon 2 H to the power 4. So, WL to the power 4 divided by 2 H, H to the power 4, then higher of the terms then bracket is closed minus 1. So, this one this one will be cancel, and this one approximately you will get d approximately WL square by your 8 H this you this is your what you call this is a simple task you can do right.

So, approximately d is equal to WL square upon 8 H, and this is equation 26 therefore, the safety code gives the minimum clearance your height for the line above ground and if this is added to the sag the minimum height of the insulation support points can be found, actually this sag will be there from the ground the minimum your safety clearances will be there in addition to that you have to add this sag also right such that minimum height of the insulation support points can be found. So, whenever you are designing a line you have to you have to find out the maximum sag what can happen and that can happen when temperature is very high particularly in a summer season.

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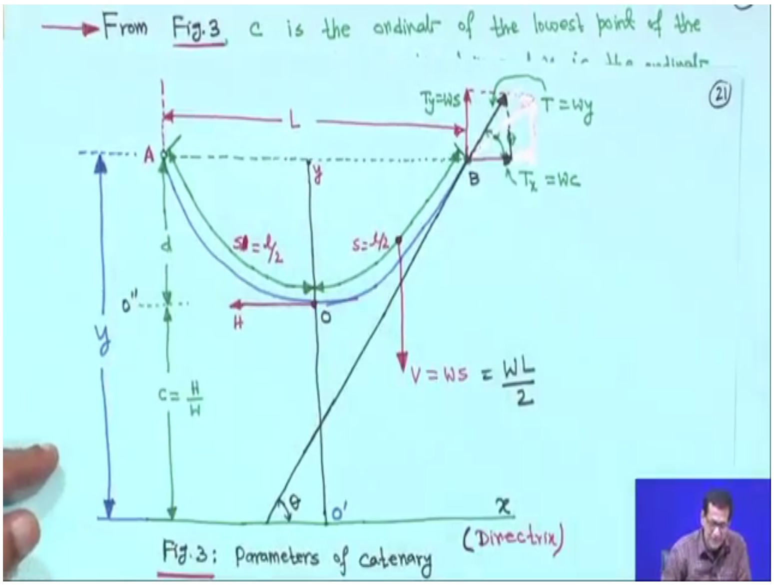

Next what we will do, suppose this is your same thing A and B at the same support right this is parameters of catenary this is your directrix this is say x, and this is the lowest point O this is that horizontal tension H right and this side is l by 2 this side is l by 2, and here that along this along this distance suppose say this this is your between l by 2, that you're at S is equal to l by 2 this weight is acting in the middle of this portion right between O and b, this is the midpoint of the conductor and your horizontal distance between A and B is l.

So, this V is equal to ws and as S is equal to l by 2; that means, this one actually your WL by 2 because S is equal to l by 2 and therefore, this is Ty that vertical tension Ty will be balance by this downwards that vertical conductor weight is acting. So, Ty will be ws.

So, Ty is equal to wx it is written here and Tx horizontal one, say Tx is equal to some c is some length right. So, that that is T x is equal to wc some length and that is equal to h.

So, and this one this is the directrix and this is the horizontal line. So, this angle is theta right, and some correction fluid I have put it. So, do not look into this. So, this angle is theta therefore, this angle is also theta and this one this let us see which H upon w, and this then from here this is this y this is the sag. So, this portion is d. So, we will see later this thing and total from this directrix that total is y, and y is equal to c plus d now this is actually a figure 3, just for the purpose of explanation we have made this diagram.

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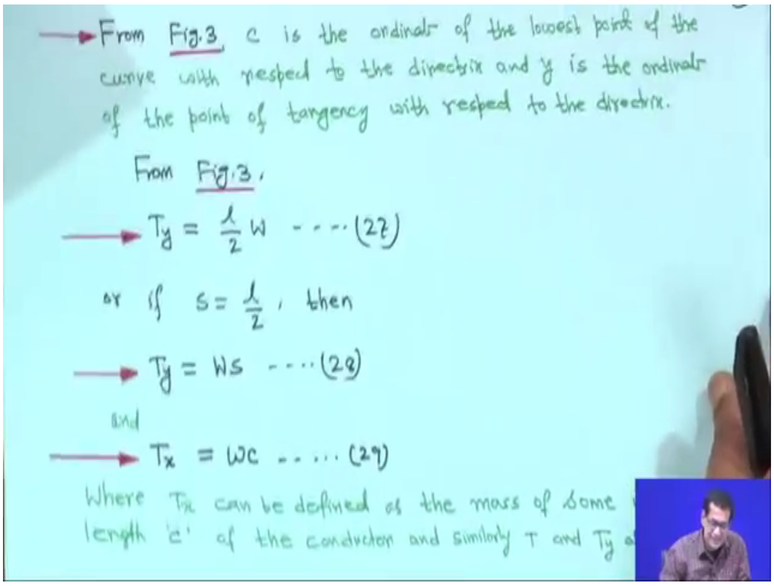

Now, c is the now from figure 3, it is c is the ordinate of the lowest point of the curve with respect to the directrix, and y is the ordinate of the point of tangency with respect to the directrix; that means, this is the c, this is the lowest point O dash is equal to c and this from here actually if you take this this point from this point to this point suppose if it is O this is O dash. So, if I take this is my double O double dash. So, O double dash is equal to d this is basically nothing but the sag, from this height is equal to this height d so.

Thank you very much we will be we will be back again.