Power System Engineering
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Lecture – 16
Transient over voltages and Insulation coordination (Contd.)

Welcome back to this now. Now, we will see the junction of several lines right. So, first you look into this diagram.

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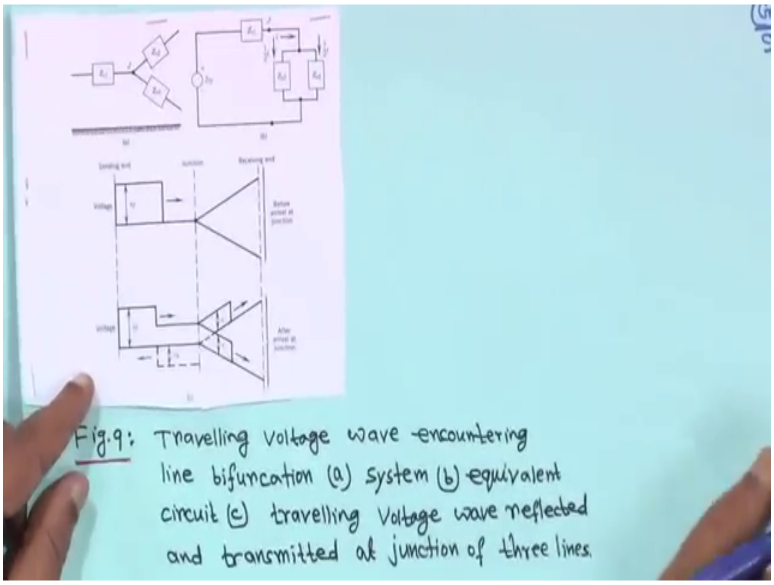

Suppose there is one line surged impedance or characteristic impedance is Z c 1 and line is bifurcated here and this surge impedance of this line this side is Z c 2, this is also Z c 2. These two are equal we have considered for their present study it may be it may be different also right and this is the junction j, J is the junction.

If you draw the equivalent circuit here in voltages here coming 2 v f later we will see why it is 2 v f right, that is your forward wave and this is Z c 1 and current through this at the receiving side is i this is the junction and these 2 are in parallel therefore, it is Z c 2, Z c 2. So, current i, at both are equal half I half I current flowing through this to Z c 2, Z c 2 characteristic impedance right. This one I will come later.

Now, and this figure lines actually this is actually traveling wave only voltage is shown right, only voltage is shown.

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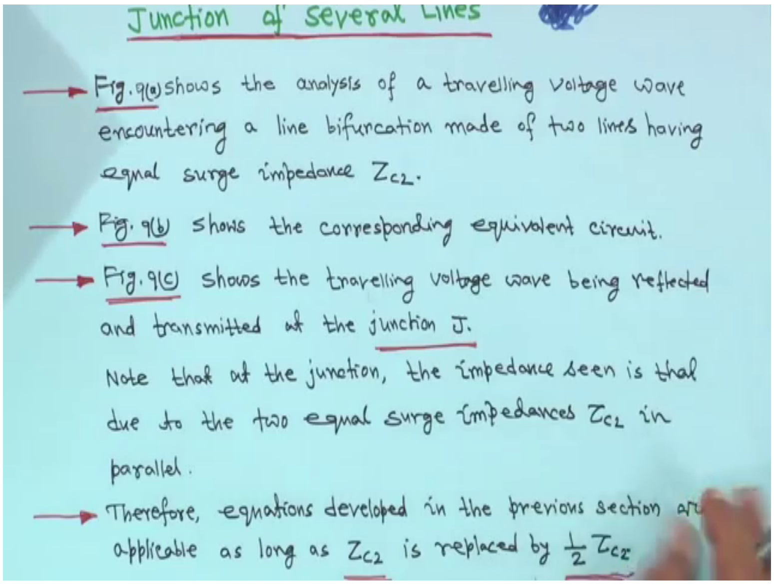

Now, figure nine we nine c actually shows the travelling voltage wave being reflected and transmitted of the junction J right. So, at the at the junction J the impedance seen is that due to the 2 equal surge impedance Z c 2 in parallel right therefore, equation developed in the previous sections are applicable as long as Z c is replaced by half Z c 2. Previously we have seen that you are for this circuit analysis we have seen that line was terminated by only say one is Z c 1 another was only Z c 2. But in this case 2 parallel lines are there therefore, it that those Z c 2 will be replaced by your half of by replaced by half of Z c 2, instead of Z c 2 half of Z c 2 rest will remain same right. That is why the equation developed in the previous section are applicable as long as Z c 2 is replaced by half Z c 2.

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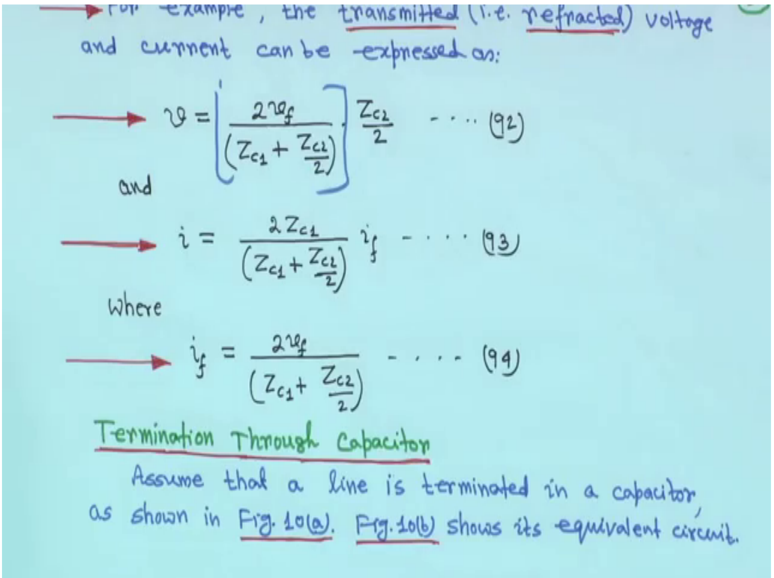

So, if we would do so, then what will happen, that you are for example the transmitted or reflected voltage can current can be expressed. As in previous expression just now we have seen that v was 2 v f and that was your Z c 1 plus Z c 2 into Z c 2 now that was for your line, line your what you call one side is Z c 1 another side was Z c 2. Now, it line is bifurcated parallel line. So, Z c 1 is remain same Z c earlier one, one thing was not there only one 2 junction right to over it line may be connection or one cable one over it connection right the junction, but now this your what you call that line is bifurcated this is that is why Z c 2, Z c 2 in the previous expression just you replace Z instead of Z c 2 you right Z c 2 by 2 therefore, v is equal to 2 v f the Z c 1 plus Z c 2 by 2 into instead of Z c 2 it will be Z c 2 by 2, right.

Similarly, and current also will be 2 Z c 1 divided by Z c 1 plus Z c 2 by 2 into i f therefore, your i f is equal to where i f is equal to 2 here Z c 1 plus Z c 2 by 2. Previously we have seen that when 2 lines are connected with different characteristic impedance or overhead line or cable and overhead line whatever may be we have seen if was previously it was 2 v f upon Z c 1 plus Z c 2 right.

Now, this Z c 2 is replaced by Z c 2 by 2. So, that is why in this diagram it is 2 v f right, i f is equal to 2 v f divided by Z c 1 plus Z c 2 by 2 because these two are in parallel that is why i if is equal to 2 v f upon Z c 1 plus Z c what this be. In terms of Z c 1 and Z c 2 for overhead 2 overhead line connection or the different characteristic impedance or one overhead line another cable connection that these things are derived.

So, in this case also that this is your what you call that forward wave that is voltage wave and this will this is the junction where line is bifurcated this side is receiving and so before arrival at the junction voltage where v f right. Now, at your what you call at the junction while the wave arrive at the junction some will be reflected some will be transmitted right.

So, in that case when your voltage is reflected back, it is your v b that is your minus minus v b. So, here, your direction is from reflected one from right to left this side is minus v f. So, naturally that magnitude actually effective magnitude will go down. So, this is this is your what you call this is your junction point and then v is that you are receiving and the side voltage v. So, this is drawn like this, but this voltage may are your transmitted voltage magnitude right then v then 2 2 will be cancel actual it will be v f into Z c 2 divided by Z c 1 plus Z c 2 right.

So, this way this is shown. One is along this line another along this line as both are both are equal characteristic impedance Z c 2 by 2, a Z c 2 and Z c 2 therefore, this is wise also v this is also v and this is the reflected wave that is your v b. That is why figure 9, that is see the traveling voltage a reflected and transmitted at junction of 3 lines because this is a is incoming line, this is incoming line and these 2 are outgoing line. So, that is why this is after arrival at junction because this v b is the v f plus v b is equal to v. So, it is actually this is the reflected wave. So, that is why and if you want that what will be reflected wave something I have made it for you for this one v f plus v b is equal to v. That means, v b is equal to v minus v f right.

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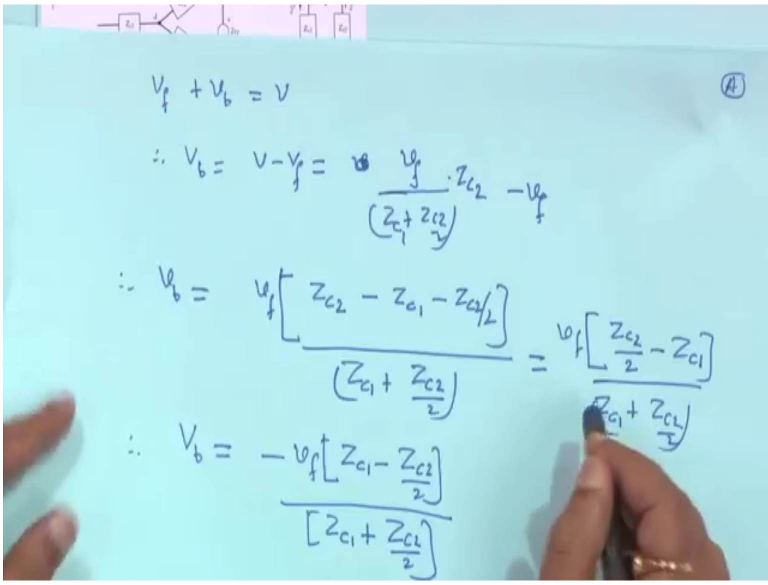

So, your v is equal to this one v is equal to 2 v f upon Z c 1 plus Z c 2 by 2 into Z c 2 by 2. So, 2 2 will be cancel this 2 will be cancel. So, basically it will be v will be v f into Z c 2 divided by Z c 1 plus Z c 2 right minus v f. If you simplify this one you will get v b is equal to minus v f Z c 1 minus Z c 2 by 2 divided by Z c 1 plus Z c 2 by 2. Same as before only thing is that Z c 2 is replace by Z c 2 by 2 and 1 minus sign we can reflected wave right, so negative sign.

So, that is why if you look in this graph that this figure b see that this is that minus v b, because of this minus sign right and this is your v that is your transmitted or refracted wave right. So, this is after arrival at the you are what you call at the junction right.

So, next is, just hold on. Next one is your termination through capacitor right. So, whenever suppose you have a line that is assumed that, assume that you have a line, you have a line right. So, this line is terminated at capacitor c. So, same as before voltage will be 2 v b this is Z c the characteristic impedance line and this is your capacitor c right.

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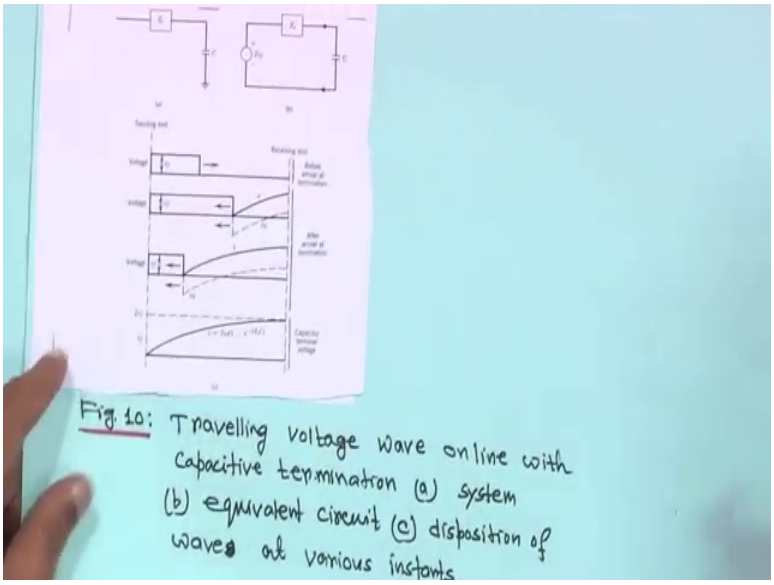

Now, before proceeding further I mean I will come to that when the wave will reach at the end of this your what you call at the capacitor at that time you know that when it reaches capacitor will be where this hot circuit right and when it will reach the steady state it will act as open circuit. So, same philosophy will be applied.

So, now this is the wave traveling before arrival at the junction this is the voltage wave traveling along this direction right. So, from equation your 60.

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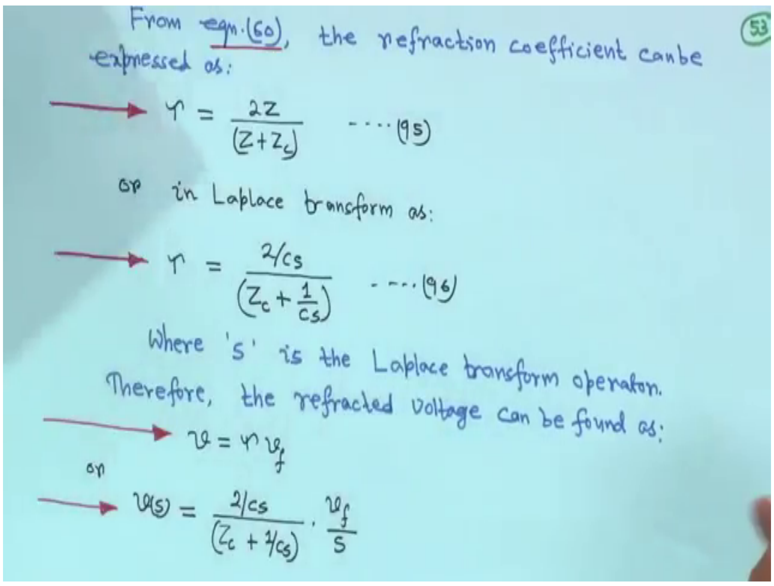

Ah from equation 60 we know the reflection coefficient can be expressed as this we have seen that tau is equal to your 2 Z upon Z plus Z c. So, this is equation 95 right or in if you take the Laplace transform then tau should we it because it is your, it is capacitor it is capacitor right where see therefore, it will be your what you call 2 upon in Z you use one upon CS is the Laplace transform operator right.

So, 2 upon CS divided by Z c plus 1 upon CS because we are trying to represent in Laplace transform. So, because it is a capacitor you know that it is a capacitor. So, Z c will be 1 upon sc right and S is the Laplace transform operator. Therefore, the you are reflected voltage reflected voltage we know this equation v is equal to tau into v f this we know when tau is this much therefore, v s is equal to 2 upon CS divided by Z c plus 1 upon CS and it is a step, it is a step disturbance actually that is why v f is replaced the you know v f upon S because of the step function. Because all the things whatever we are observing it is step function you assumed right.

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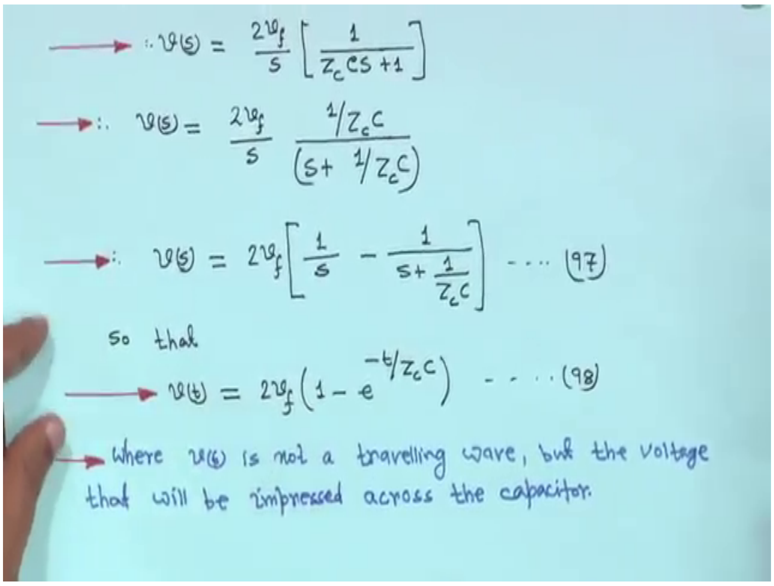

Therefore, if you simplify this one if you simplify this equation this equation you will get that v s is equal to 2 v f upon S in into 1 upon Z c CS plus 1 right.

So, or v s is equal to 2 v f upon S you write a rewrite this equation like this one upon Z c into c divided by S plus 1 upon Z c into c. And if you go for your partial fraction then it will be v s is equal to 2 v f 1 upon S minus 1 upon S plus 1 upon Z c into c this I am not showing I think you know right you know that. You can make it a upon S plus your b upon I mean tau, if it is tau it is given 1 upon S into S plus a you can write a upon S plus b upon S plus a after that you can compute a and b right.

So, that if you take the inverse Laplace transform. So, v 2 will be 2 v f into 1 minus e to the power minus t divided by Z c into c, but this is not the your what you call not a travelling wave rather it is the voltage that will impress across the capacitor; that means, these v t is the voltage that will be impressed across the capacitor. So, it is not a traveling wave. So, we can tell like this that is the voltage that will be impressed across the capacitor.

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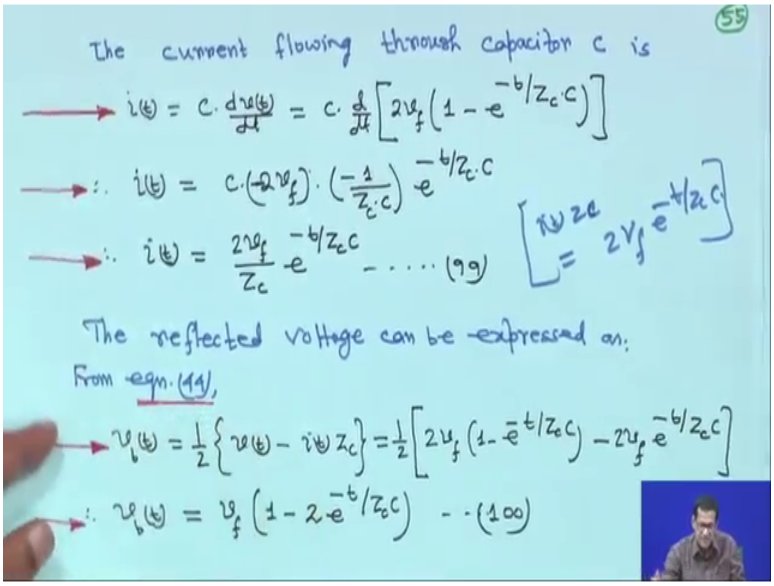

Now, if this is the v t then you have to find out current flowing through the capacitor. So, current flowing through the capacitor is that it is equal to c into d v t by dt that is c, d v t and v t, just now we have seen this one. So, you take the derivative of this. So, you will get that it is equal to 2 v f upon Z c into e to the power minus t upon Z c into c this is equation 99 right. Therefore, and if you go for cross multiplication this is required. So, your i t into Z c, i t Z into c is equal is equal to your what you call that this is it into your Z c this Z c is the suffix here right because it is the characteristic impedance of the line right. So, it is Z c, C is the suffix here.

So, these 2 v f upon Z c e to the power minus t upon Z c into C; that means, Z c into it, so Z c into it is equal to 2 v f e to the power minus t upon Z c into C. That means, from equation 44 the reflected voltage can be expressed these we have these we have seen in equation 44, not showing again and again because this we have seen. So, directly we were writing that if your v b t is equal to half that there is from equation 44 right that is your v t minus it into Z c.

Now, is equal to half that v t is equal to your just now just now we have seen that v t is equal to your v t is equal to this2 v f into 1 minus e to the power minus t upon Z c into C.

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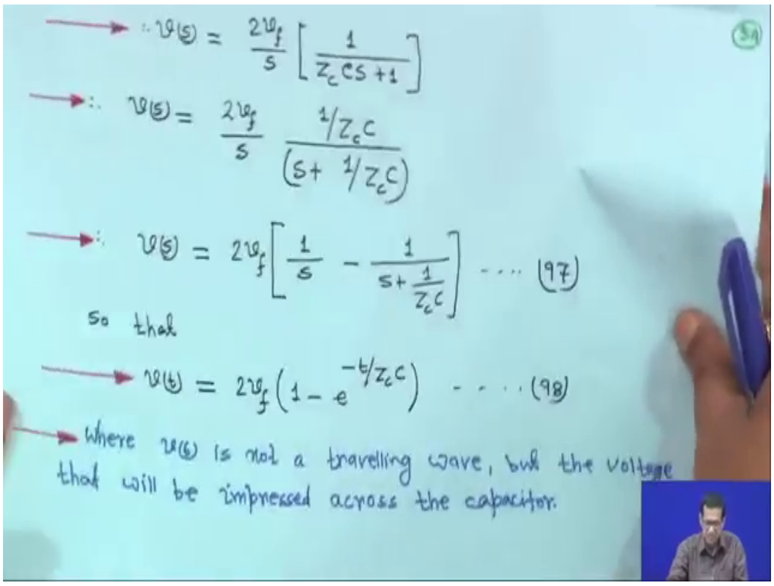

So, this one you substitute here v t is equal to this one and then it into Z c this one actually minus 2 here e to the power minus t divided by Z c into C; that means, you are v b t a proper simplification v b t is equal to v f 1 minus 2 e to the power minus t Z c into C this is equation 100 right. Therefore, if you look into this I mean suppose although here it is a it cannot be your what to call explain your interpret that for example, for t is equal to any value say any value if we can interpret that suppose for example, suppose t is equal to 0, suppose t is equal to 0, therefore, what will happen? t is equal to 0 means it will be 1 minus 2, so minus. So, v b t is equal to minus your v f.

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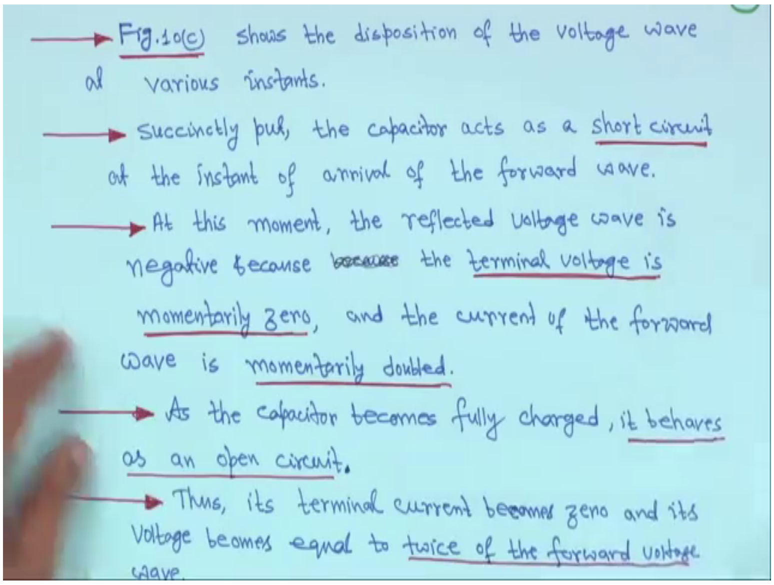

So, in this case what will happen that just little bit of understanding little bit of understanding. So, in this case what will happen that this is that, this position of the voltage wave at various instant 2 instant are there this is a voltage this is voltages. Look at the one first one only more than enough look at this one right. So, put the capacitor like I mean if you if you logically if you try to put the capacitor acts as a short circuit at the instant of arrival of the forward wave. That means, when it will when it will arrive that while we are discussing about know, you are traveling wave that so many sound capacitors are there inductor immediately your capacitor and instantly it will be short circuited inductor will be open circuited. So, it takes time to reach to the receiving end, but as soon as when it will reach to the receiving end say. So, capacitor will behave like a short circuit.

So, if capacitor act at that instant of arrival of the forward wave, so at this moment the reflected voltage wave is negative because the terminal voltage is momentarily 0; that means, when your traveling wave is arriving here traveling wave that capacitor is short circuited; that means, your; that means, your this thing, this one.

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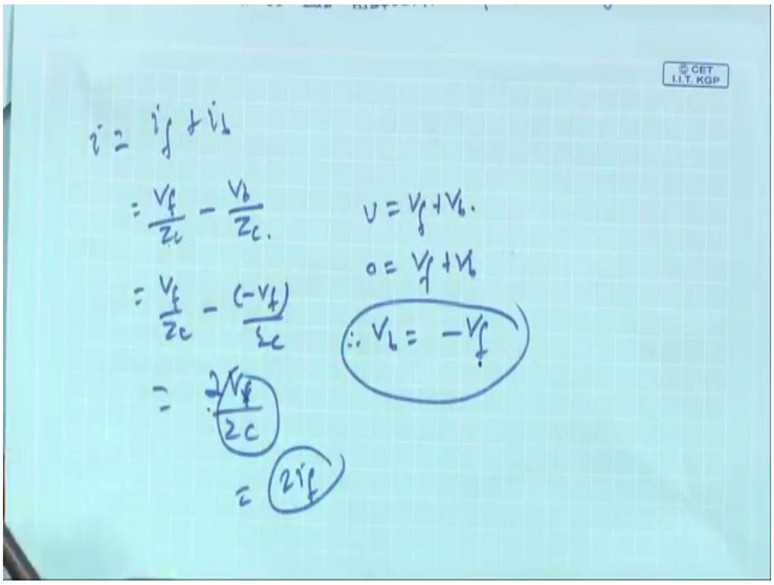

Suppose we know that v is equal to v f plus v b, but when traveling will be arriving here the capacitor give the short circuit. So, momentarily this voltage will be 0. If it is 0 means that is 0 is equal to v f plus v b; that means, v b is equal to minus v f at that moment at that moment right. But after that capacitor will be slowly your charge and when it is when it will restore the steady state that this will behave as an open circuit.

So, in that case in that case what will happen that you are just hold on and, so the terminal voltage is momentarily 0 and the current of the forward wave right it momentarily double. So, I mean this one, this one you can, of your own you can verify that that you are what you call that current. Current is equal to actually current is equal to your i is equal to your i f plus i b right that i b is equal to v f upon your say an i v is equal to minus v b upon your Z c right, but question is that v b is equal to minus v f therefore, v f upon Z c and minus minus v f upon Z c. So, it will be 2 v f upon Z c right therefore, it v f upon Z c if is equal to forward wave current if is equal to v f upon Z c, but at that moment it will be 2 into v f upon Z c right.

So, momentarily that is why it is that is why we are writing here that momentarily the voltage is 0 or at the current of the forward wave is momentarily that is your what you call that doubled. So, in that case your what you call that; that means, this is actually v f upon Z c; that means, we can write 2 into i f right, because v f upon Z c is equal to i f.

So, that is your 2 i f. So, in that case, or as the capacitor become fully charged then it behaves as an open circuit.

So, the when it is fully charged now capacitor will act as open circuit that is known to us. So, in that thus the terminal current becomes 0 and its voltage becomes equal to twice the forward voltage wave. So, in that case if you look that, that is this one that is this one right. So, if you look into that at any instant when your, what you call that v b is equal to minus v f it is showing like this and, but after that capacitor will be slowly and slowly it will be getting charged. So, v should be is equal to v f plus v b. So, after that with that wave from this one will added and this v will be your arrival after arrival at the terminal the termination that v will move will be characteristic or the v will be like this, this is at a particular instant.

Similarly, this voltage another instant also it will be the same philosophy, but one is sufficient for understandable understanding right. And the capacitor one if you look at the capacitor one that v is equal to this, this expression we have got it v is equal to 2 v f one minus e to the power minus t upon Z c into C. So, in t is equal to 0 the right when t is equal to 0, v is equal to 0. And when it capacitor terminal voltage that is after having this, finally, is gradually it is increasing and it is your what you call final it is equal to the v is equal to 2 v f right.

So; that means, that this is actually the capacitor terminal voltage where this whatever you have derived right and this is your, that your this position of wave and various instant. So, this is that capacitor terminal voltage variation and this one either this one or this one this is the your what you call that is that v is equal to your v f plus v b right which is arriving at the your receiving end. So, this is for the capacitor.

So, how we can decide that this one or this one you can make it. Similarly this an exercise for you instead of voltage you just plot the your what to call that current right because here all voltage wave searched on. So, I request you this one and this one this is current, current waveform also that you should draw right. So, this is your what you call the termination at capacitor. These are special cases right these are special cases and in the reality is something else, in reality it may not happen and termination through inductor it is just dual of capacitor.

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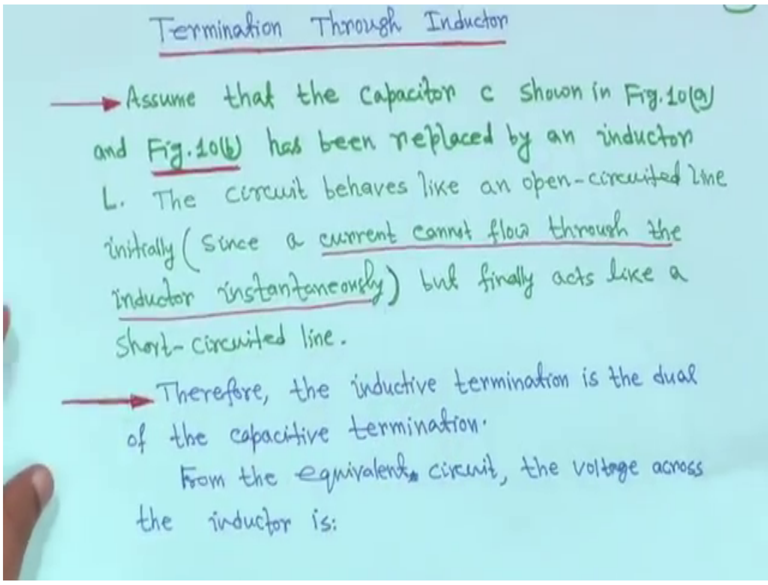

So, in this case capacitor instead of capacitors c you can you can replace this one by an inductor is the by an inductor. So, it will just opposite it behavior will be just opposite to the capacitor right and this is the characteristic impedance. So, in that case the circuit behaves like an open circuit line initially right and in the case of capacitor it was first at it in this case it is initially open circuit since a current cannot flow to the inductor instantaneously this we know, but finally, acts like a short circuited line.

So, therefore, the inductive termination is the dual of the capacitive termination. It will be just what is what is happening for your current for the capacitor it will happen for the voltage I mean just dual right. So, from the equivalent circuit the voltage across the inductor is.

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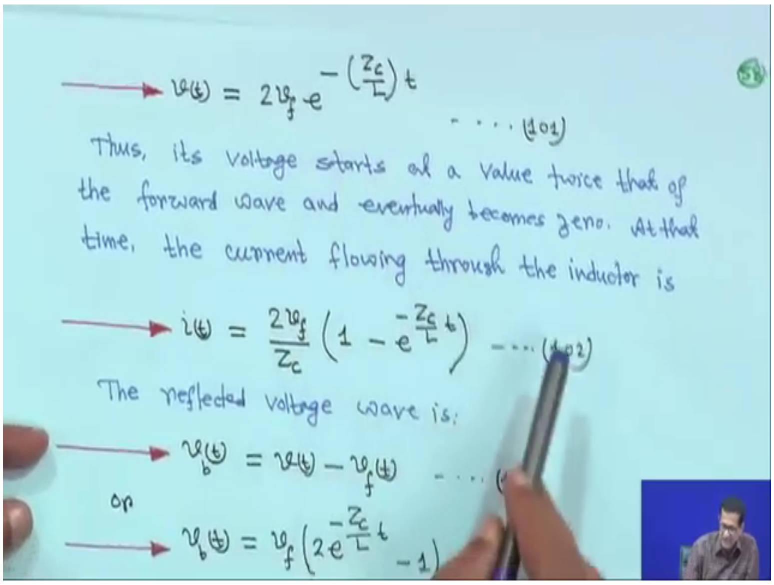

So, you can, we can write this one we can write your this thing that v t is equal to your, v t is equal to 2 v f into e to the power minus Z c upon L t this is v or v t. And similarly the current expression of whatever we are getting for voltages this terminology that i t is equal to 2 v f upon Z c 1 minus e to the power minus Z c upon L into t.

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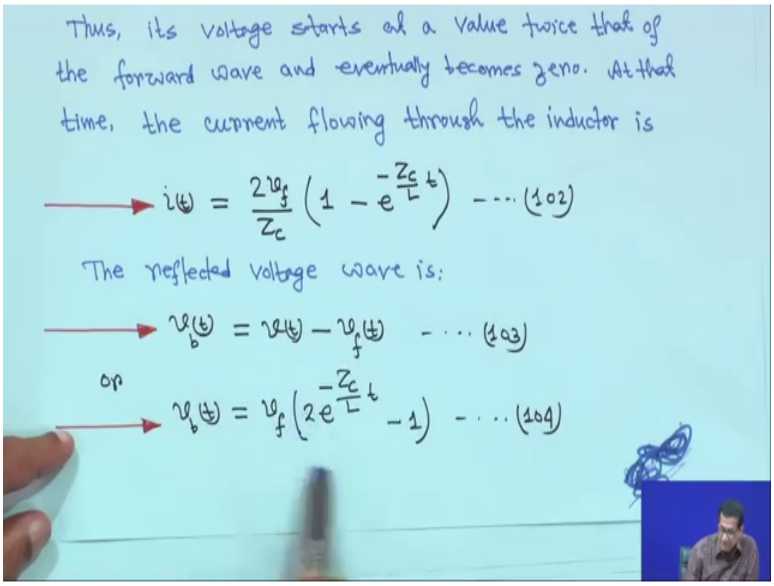

So, the then for the reflected voltage wave is that v b t is equal to v t minus may we know v or v b plus v f is equal to v t therefore, v b t is equal to v t minus v f t. So, this v t expression is known in terms of v f. If you substitute then reflected wave that v b t you will get v f into 2 e to the power minus Z c upon L t minus 1 this is equation 104.

So, this is actually your what you call this is your inductor if line is terminated your what you call your inductive only pure inductive your circuit right. But in reality it does not happen that only x should not be there that resistance also will be there or in general Z, but this is a special case for this one I will request you that you will draw the voltage wave forward v f then your v and showing also the v b right this will be an exercise for you.

Next is your, up to this, this traveling wave more or less things are right. I mean there is not much mathematics there only little bit of understanding. Numericals are also quite easy, which you can easy, which you can easily solve right.

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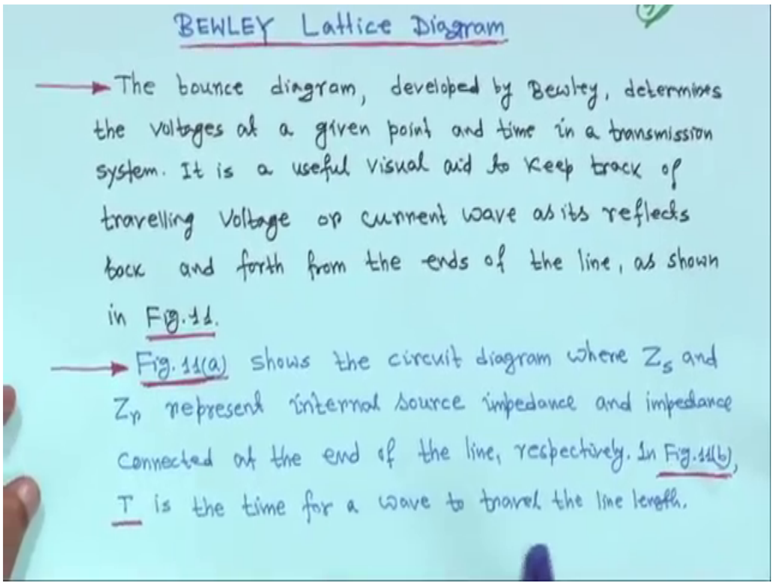

Now, next one is that your Bewley Lattice diagram, a bounce diagram. I think it was given in 1951, more nearly 66 years back all right.

So, it actually gives some ideas right at suppose it will give voltage at given point and time in a transmission system. So, it is a useful visual aid to keep track of traveling voltage or current wave as it reflects back and forth from the end of the line because it will be you know sending and receiving in it will be reflected back and forth. So, that question is that how to understand this Bewley are diagram right.

First thing is that before showing the diagram that figure 11 a. So, I will show that that circuit diagram where Z s and Z r represent internal source impedance and impedance connected to the end of the line. So, Z s is the source impedance or internal impedance and Z r is the or impedance connected at the end of the line and figure 11 b, t is the time for a wave to travel the line length.

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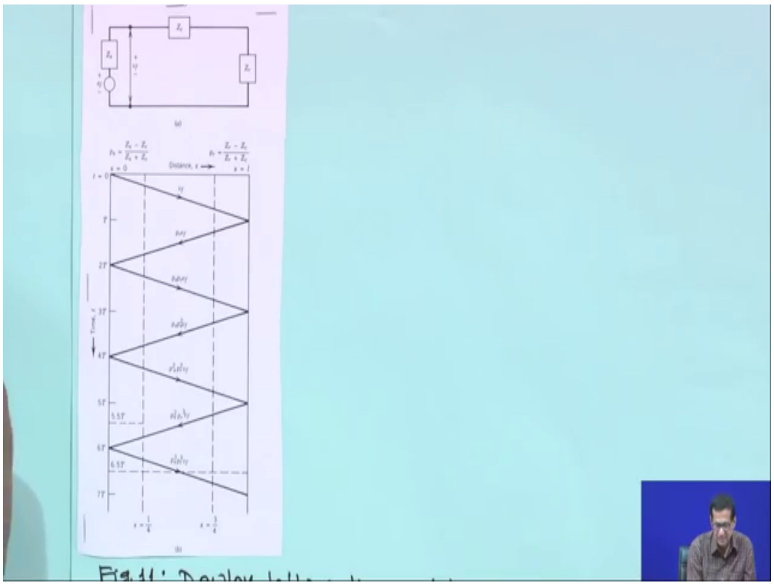

That means, it shows suppose you have, suppose you have this is the diagram this is voltage this across this v f this is actually internal impedance Z s and it is small v f given, but across this the voltage is v f and this characteristic impedance is Z c right. And receiving inside it is terminal Z. Now, first if you look into this diagram first you see how things are lattice diagram is drawn.

So, this side if you look this side is time T is equal to 0, capital T, capital 2 T, capital 3 T, 4 T and so on this is time and this side your distance this is actually distance. Now, suppose at any instant I can we can find out what will be the voltage. So, what will happen? That this is v f this is; that means, what this is at T is equal to 0 and this is at x is your distance is x, it is T is equal to 0 x is equal to 0 and when it is traveling that line length is x is equal to l and the 2 reflection coefficient your os and or both have been, both have been given.

So, that is your what you call that expression we have derived earlier os is given Z s minus Z c upon Z s plus Z c this is actually sending inside sending inside and rho r is equal to Z r minus Z c Z r plus Z c this is are the receiving inside. These 2 coefficients are given. We have found out all your what you call that reflection refraction all these things we have made it right.

So, in this case what happened that traveling wave travels, so it is starting from here right when is traveling it is v f. Now, receiving inside the reflects and factor is rho r; that means, this reflection multiplied by rho r, rho r into v f right. And this printing is little bit small, but understandable when you will when you see this one this is understandable right.

Now, from the sending and when again it will be reflected back sending inside reflection rho s, with this rho s will be multiplied. So, rho s rho r into v f right and this time this side when again when it will be reflected back receiving inside it will be look it will be rho s and this way actually is printing error it will be rho s rho r square it I am writing here rho s rho r square v f here, right.

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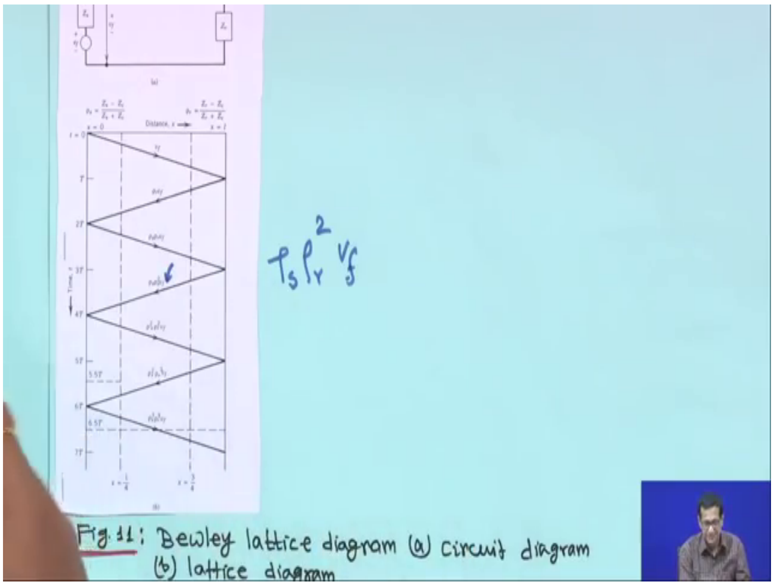

So, it will be because it is reflecting from the disturbing inside the reflection coefficient is rho r. So, it will be rho s the rho r square not rho s square rho s into rho r square into your v f right into v f now when again it is reflecting from sending inside sending inside is rho s. So, it will be multiplied into rho s; that means, will be rho s square rho r square into v f rho s were rho s where square v f is already there and again it will be reflected should we rho s square rho r square v f.

Now, again when it will come here right when it will be when it will coming here again it will be reflected back from the receiving a therefore, this side again it will be multiplied by rho r. So, already it was rho s square rho r square. So, it will multiply rho r it will be rho s square rho r q into v f and when from this side coming at 60 when it will multiplied by rho s it will be rho s cube rho r cube into v f and so on right. So, that way these things will be, what you call this, this lattice diagram can be constructed.

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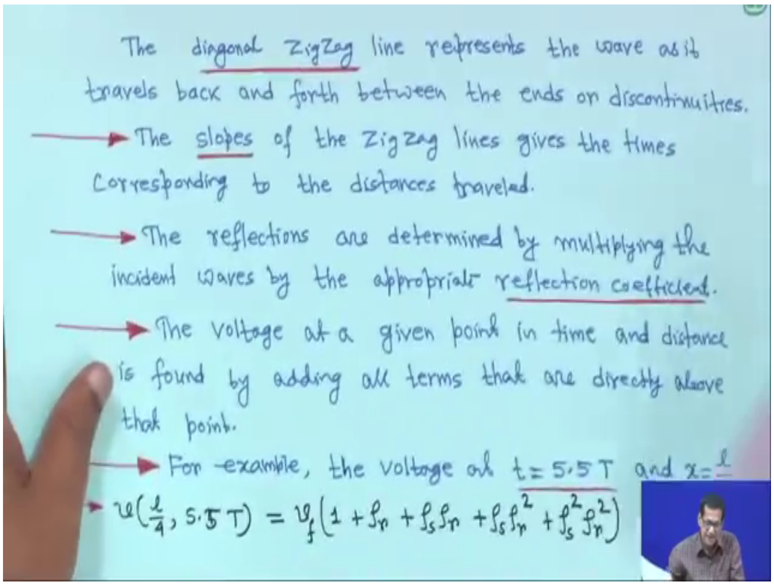

So, this side is time, it is time T it is given here, this is time. That means, that the diagonal zigzag line represent the wave now I am explain everything, but for you it is right up is here. The diagonal zigzag line represent the wave as it travels back and forth between the ends or discontinuities. Now corresponding to the distance travel, the reflections are you are what you call determine by multiplying the incident wave by the appropriate reflection coefficient. This is sending and reflection factorial coefficient and this is receiving an inflection factor or coefficient right. So, the voltage at a given point in time and distance is found by adding all terms that are directly above that point. How to find out the voltage? I will come, will be coming, we will come back to soon.

Thank you.