Power System Engineering
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Lecture – 13
Transient over voltages and Insulation coordination (Contd.)

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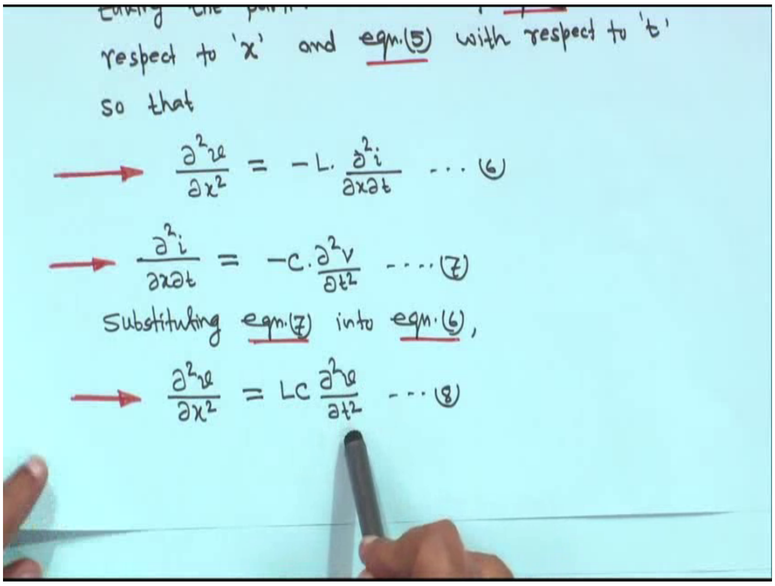

Welcome back so this del square v del x square is equal to LC del square v del t square this is equation 8, right.

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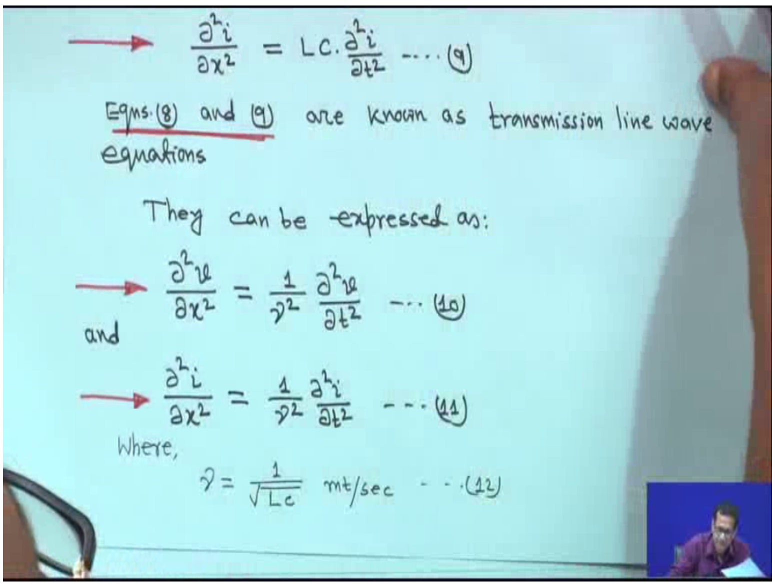

So, similarly this is you will do it, I will not do it for you will do it, I have done it for you here only. Similarly you can show that delta square i del x square is equal to LC delta square I delta t square equal, I mean other 2 equations are there also you take similar way you take the derivative and you try establish this one. So, this one is del square v del x square is equal to LC del v del square v del t square, here also del square i del x square is equal to LC del square i del t square. Other 2 equations are there similar way you take the derivative and put it right. So, equation 8 and 9 you known as transmission line wave equation; that means, this is the voltage, this is your for the voltage and this is for the equation 9 is for the currents right.

Therefore, this they can be expressed as equation 8, can be expressed as del square v del x square is equal to 1 upon gamma square del square v delta t square and this equation can be written as del square i del x square is equal to 1 upon gamma square del square i del t square this is equation 11, then your gamma is equal to 1 upon root over LC meter per second right.

So, gamma we have defined that 1 upon root over LC. So, this we can write this equation right.

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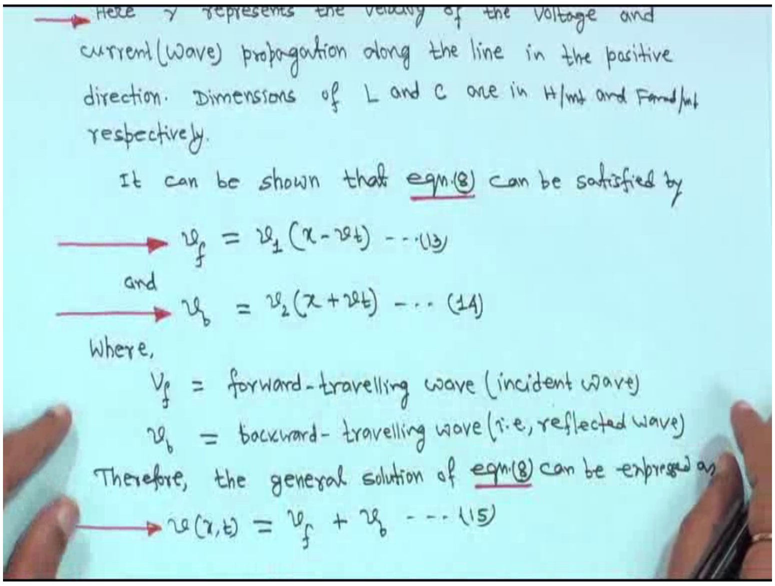

Now here gamma actually represent the velocity of the voltage and current wave, that is propagation velocity of voltage and current propagation along the line this is the gamma right; in the positive direction. Dimension of L and C same as the Henry per meter and farad per meter respectively right. So, it can be shown that equation 8 can be satisfied by v f is equal to that means your equation 8 look this one, this equation 8, it can be shown that I am not going to the solution of this one, but you will make it in different way for understandable right.

So, can be shown that one is v f is equal to v 1 it is x minus v t that is your 13, another is v b is equal to v t x plus v t that is 14 and v f is equal to your forward your travelling wave that is your incident wave you know that forward travelling wave and v b is backward travelling wave that is reflected wave.

So, forward travelling wave is incident wave and backward travelling wave is after reflection it will come back, so it is called backward travelling wave right and therefore the general solution of equation 8, I mean this equation general solution of equation 8 right, can be expressed as v x t is equal to v f plus v b right. This way we define one is your forward incident wave, forward travelling wave, plus your; what we call the backward travelling wave right.

So, this thing so this way you define. So, v x t it has the solution as one is v f, another is plus v b. So, this is equation say 15 right; that means, v f is equal to your v 1 x minus v t and v b is equal to v 2 x plus v t so substitute here.

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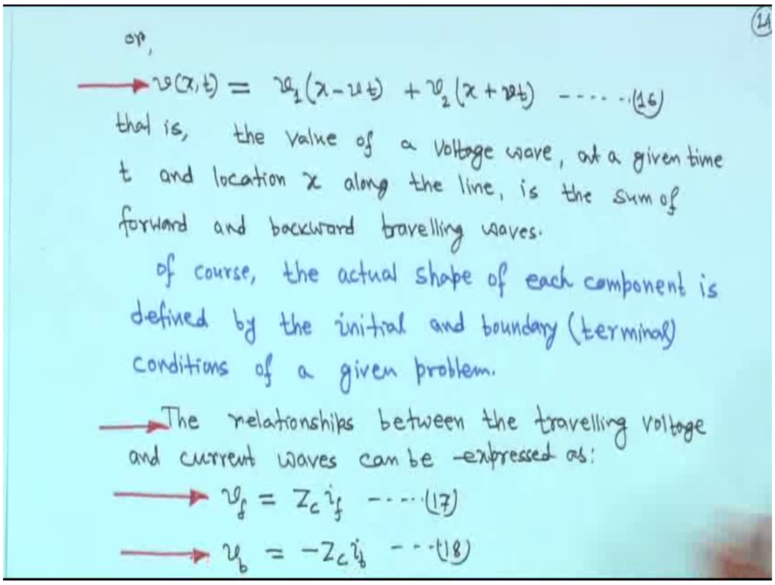

If you do so it is v x t is equal to v 1 x minus v t plus v 2 x plus v t this is equation 16. Now that is the value of a voltage wave at a given time t and location x along the line is the sum of forward and backward travelling waves; if you know this to this expression of these 2 you can easily find it out. Of course, the actual shape of each component is defined by the initial and boundary conditions of a given problem. You have to know the initial and boundary conditions that is the terminal conditions sending end and receiving end right.

For example the relationship between the travelling voltage and current waves can be expressed as that forward wave right, v f can be written as Z c into i p you know Z c is the characteristic of surge impedance, v f should be is equal to Z c into i f and v b the backward ones should be minus your Z c into i b, because it is reflected back right. So, it will be v b will be minus Z c into i b this is equation 18 right; where Z c is the surge or characteristic impedance of the line.

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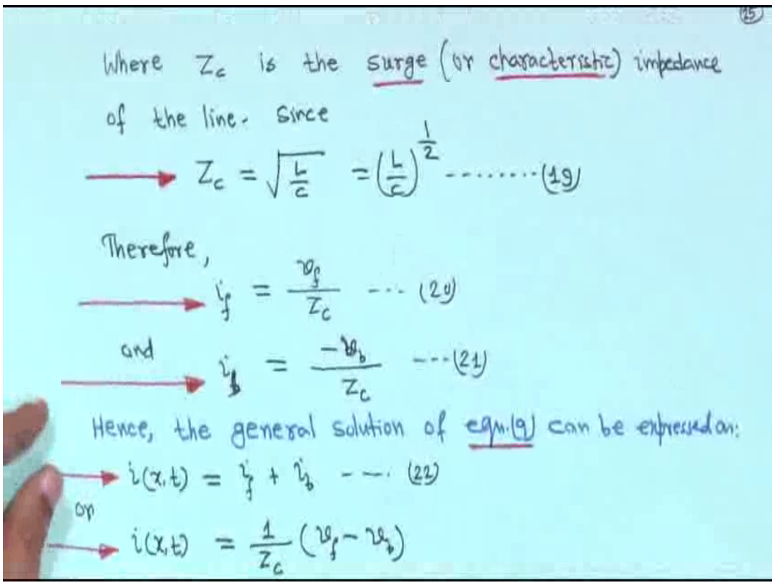

Next is that means, you know that Z c is equal to root over L by c that is L c to the power half this is equation 19, therefore v f is equal to I f into z c; therefore, i f is equal to v f upon Z c this is equation 20 and this one i b this is actually I am writing on a separate page.

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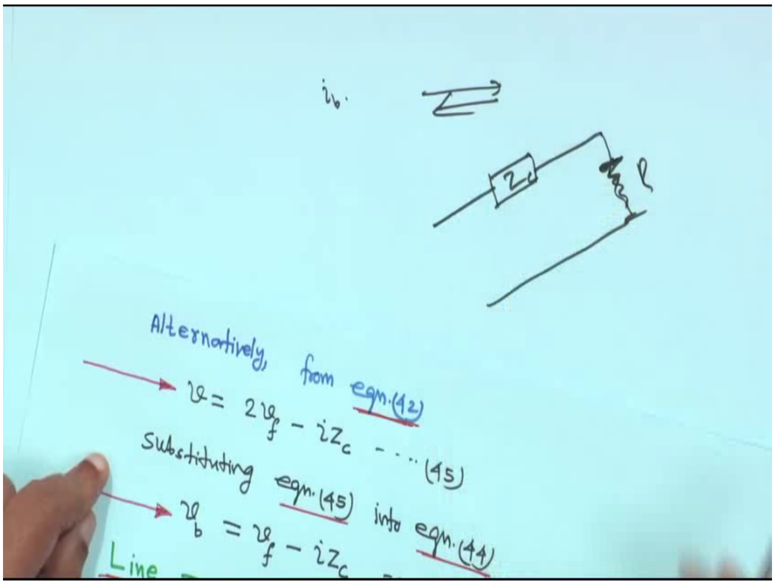

This is actually i b right i b will be minus v b upon Z c because, v b is equal to minus i b Z c. So, i b is equal to minus v b by Z c this is equation 21. Hence the general solution of equation 9 same as before; the equation 9 we can write i x t is equal to i f plus i b, I mean this equation just hold on this equation 9 right.

So, we can write same as before we are writing no for v is equal to i plus i b, similarly I we can write i x t is equal to i plus i b. So, if we substitute this i f and i b here it will be 1 upon Z c v f minus v b right. So, this is the expression of i.

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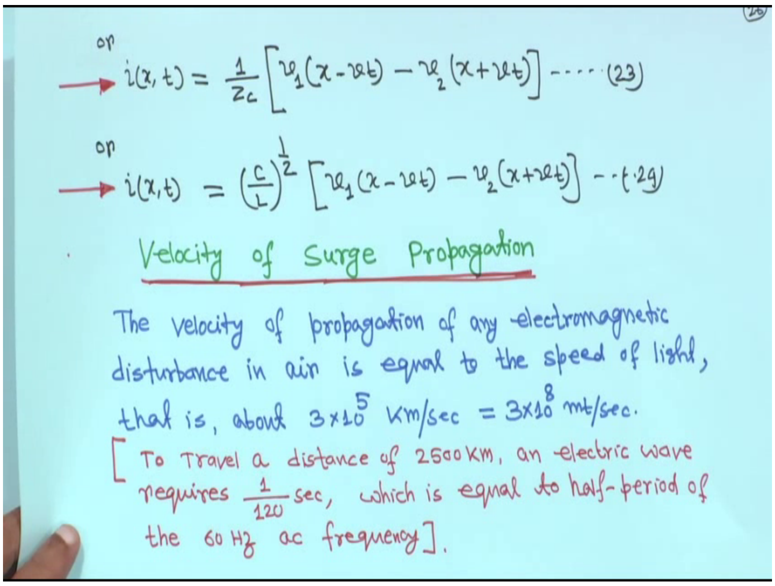

Therefore i x t will be 1 upon Z c so v f is v 1 x minus your v t z minus v t and your v b is equal to your x 2 plus sorry v t x plus v t this is equation 23 or Z c you know root over L by c. So, it is 1 upon Z c, therefore I x t will be c upon L to the power half v 1 x minus v t minus v 2 x plus v t this is equation 24 right. Now take something the velocity of propagation of any electromagnetic disturbance in air is equal to the speed of the light, this you know that is about 3 into 10 to the power of 5 kilometre per second or 3 into 10 to the power 8 meter per second; that means, to travel a distance of 2500 kilometre say an electric wave requires 1 upon 120 second, which is equal to the half period of the 60 hertz a c frequency.

So, you can imagine how fast it is moving with your velocity of propagation of any electromagnetic disturbance in air is equal to the speed of the light; that means, if for example, a line 2500 milometer long it will take just 14 upon 120 second right, which is equal to the half period of the 60 hertz ac frequency right. So, you can imagine how fast it is right.

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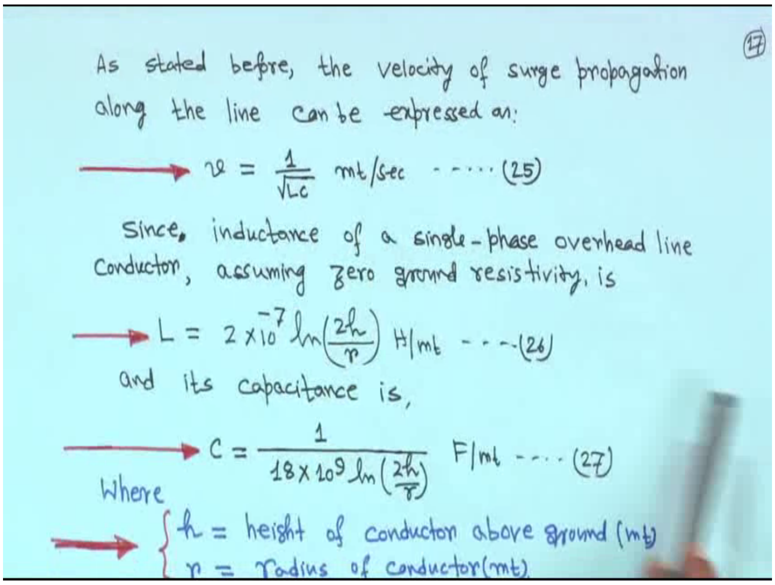

So, as stated before the velocity of surge propagation along the line can be v is equal to 1 upon root over LC that is you meter per second. Since inductance of a single phase overhead line this velocity v and voltage v should not be confused right. So, there should be v also I am using, but should there should not be any confusion right.

So, v is equal to 1 upon root over LC meter per second, so since inductance of a single phase overhead conductor assuming 0 ground resistivity, you know this expression from your inductance chapter l is equal to 22 into 10 to the power minus 7 ln 2 h upon r Henry per meter; this is 26 and similarly the capacitor c is equal to 1 upon 18 into 10 to the power 9, I mean all this epsilon 0 after consideration of the epsilon 0 approximately 18 into 10 to the power 9, 1 upon 18 into 10 to the power 9 ln 2 h upon r farad per meter, this is equation. So, h is the height of the conductor above ground in meter and r is the radius of the conductor in meter right.

Therefore this surge velocity in a single phase overhead line, it is v is equal to root over velocity 1 upon root over LC.

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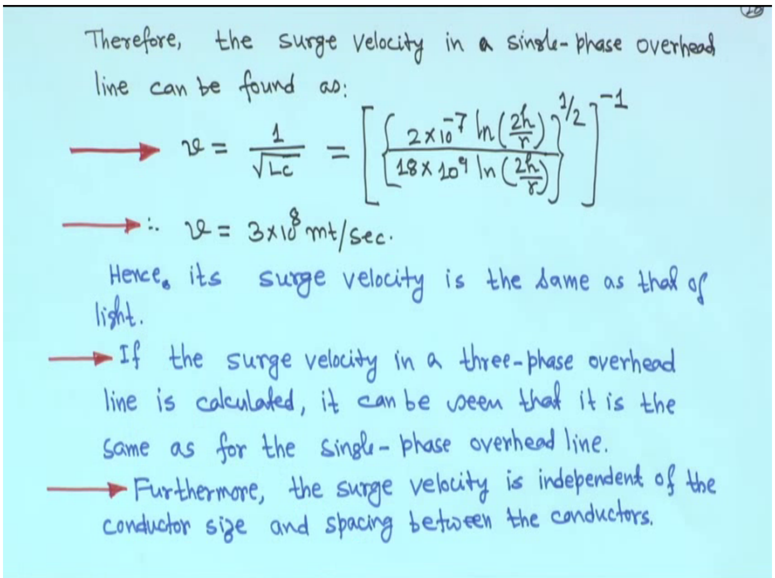

So, substitute this values L and c values right and it is reciprocal everything I have taken into to the power half and then overall to the power minus 1 because it is reciprocal right; so it coming around 3 into 10 to the power 8 meter per second. So, therefore surge velocity is same that of the light right; that means, you can imagine that travel that travelling waves velocity.

So, if the surge velocity in a 3 phase overhead line is calculated, if you try to calculate for 3 phase line you will find it is same as this value right. So, furthermore the surge velocity is independent of the conductor size and spacing between the conductors because, here this is a direct calculation 3 into 10 to the power 8 meter per second. So, it is neither the function of conductor size and spacing. So, it is the independent of the conductor size and spacing between the conductors right.

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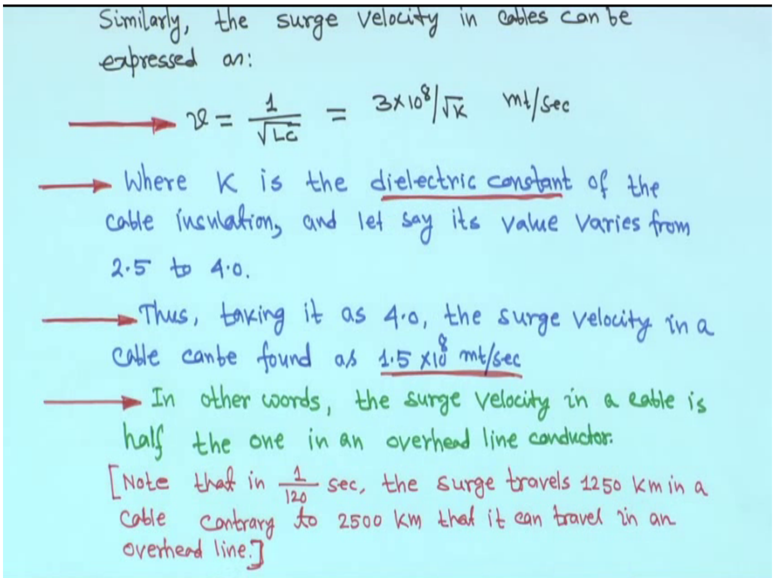

So, similarly the surge velocity in cables can be expressed as, I mean if it is a cable it will be v upon v is equal to 1 upon root over LC, only thing is that it will 3 into 10 to the power 8 root over k meter per second, because the dielectric constant for cable will come here right. So, in this case k is the dielectric constant f the cable insulation this you can try also n you put the expression of L and expression of c of cable and root it that one extra term will come that is root k. So, that is it varies between 2.5 to 4 for example, if you take it is 4 k is equal to 4.

So, root k will be 2 right the surge velocity in a cable can be found 1.5 into 10 to the power 8 meter per second, because, it is 3 by 2 is 1.5. In other words the surge velocity in cable is half the 1 in an overhead line conductors, if you the value of k is 4. So, in this case also in 1 by 120 second the surge travels 25 kilometres in a cable as compared to a overhead transmission line it was 2500 kilometre I showed you, but in the case of cable it will travel 25 kilometres right this is due to dielectric surge power input to an energy storage.

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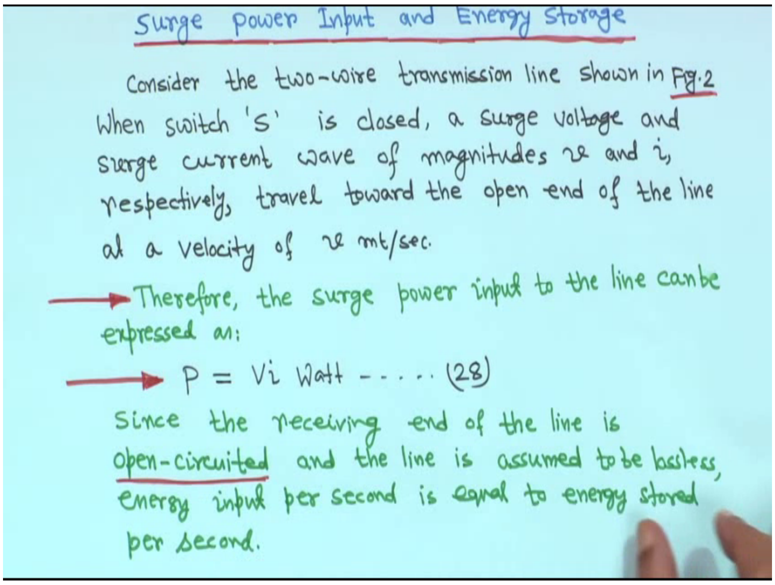

So, you consider the same 12 lines shown in figure 2 I have already showed you when switch s is closed surge voltage and surge current wave of magnitudes v and i respectively this you know travel toward the open end of the line at a velocity of, I mean this receiving end of the side for the time being as it is open end right. So, from sending end to receiving end it is moving with a velocity v meter per second, therefore the surge power input to the line can be expressed as P is equal to v into i watts this is equation 28.

Now since the revving end of the line is open circuited, so line is open circuit and you have assumed the line is lossless right. So, energy input per second is equal to the energy stored per second right as the lines will lossless. So, whatever input it is going so it will be energy stored per second right. If it is the energy stored in turn equal to the sum of the electrostatic and electromagnetic energies stored right.

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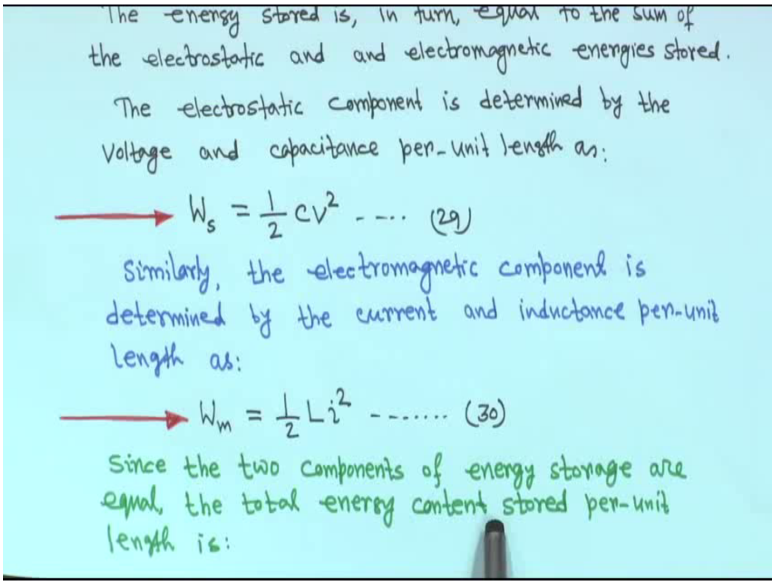

So; that means, the electrostatic component is determined by the voltage and capacitance per unit length is you know that it is w s is equal to half c v square this is equation 29 this is electrostatic one and for electromagnetic component is it can be determined by the current and inductance per unit length we know that w is equal to half Li square.

Since the 2 components of energy storage are equal right, both are equal the total energy content stored per unit length will be w should be is equal to w s plus wm because both are equal right and we are considering it is a lossless right. So, that means, if both are equal so w is equal to w s plus wm. So, we can write either w is equal to 2 w s or is equal to 2 w m this is equation 32.

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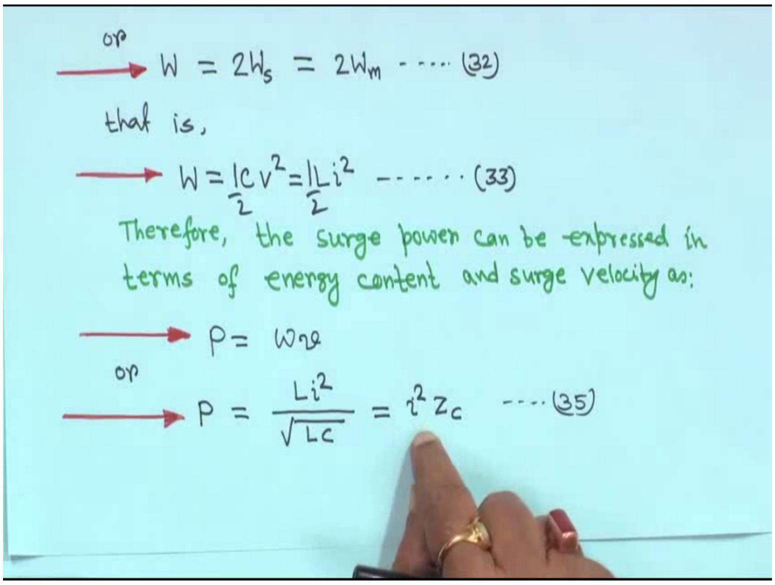

Now, w is equal to if both are equal it is half c v square is equal to half Li square; that means, we can write that w is equal to your half c v square and half Li square right if it is so, you can find out that your surge impedance right, therefore usage power can be expressed in terms of energy content and surge velocity; that means, surge power P is equal to w into v that is P is equal to your w is P is equal to that is your what is called Li square, then your by root over LC is equal to i square Z c right.

I mean from here also you can easily find out right P is equal to w v finally; it will become Li square upon root over Lc. So, it is actually i square Z c otherwise from i square Z c you can easy from i square Z c you can calculate this one right you can just put this 1 and this expression automatically will get it. If you want you start P is equal to i square Z c right and Z c component you know and i square also you know substitute this thing i square Z c, if you substitute here all i and Z in terms of this thing and you will get P is equal to w into v.

So, it is actually i square Z c, so this is what you call your surge power, P is called the surge power right or another thing is other way you can write this is v square by zc, because this one i is equal to v by zc. So, it will be v square upon Z c square one Z c in the denominator we cancel therefore, you can get that P is equal to v square by Z c so this is equation 36 right.

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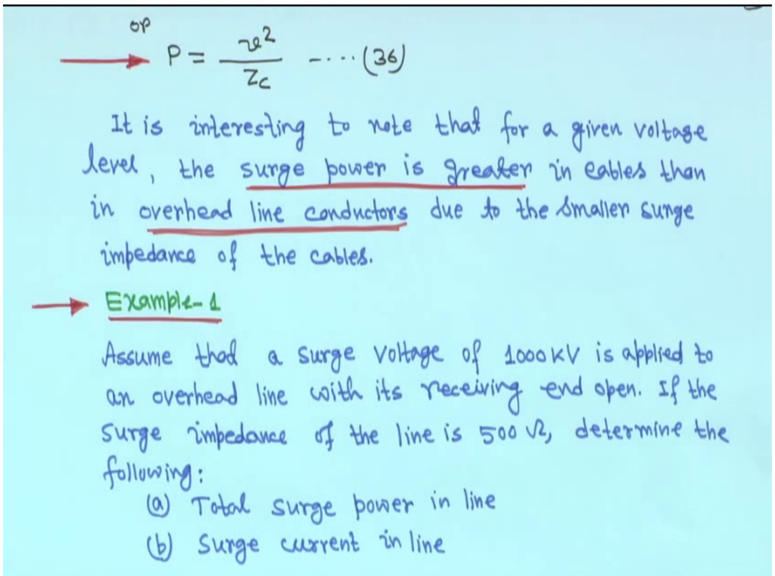

So, now it is interesting to note that for a given voltage level, the surge power is greater in cables than in overhead line conductors due to the smaller surge impedance of the cable, in the cable the surge impedance is small right. So, that is why the given voltage level the surge power is greater in cables because, your denominator is less right than in overhead line conductors and due to the smaller surge impedance of the cable right, slowly we will go into the deep of it.

But first you take a small example right 1 or 2 small example suppose assume this is example one, assume that a surge voltage of 1000 k v is applied top an overhead line with it is receiving end open; that means, surge voltage 1000 k v applied, but receiving end of the line is open circuited if the surge impedance of the line is 500 ohm, determine the following total surge power in the line and we need the surge current in the line right.

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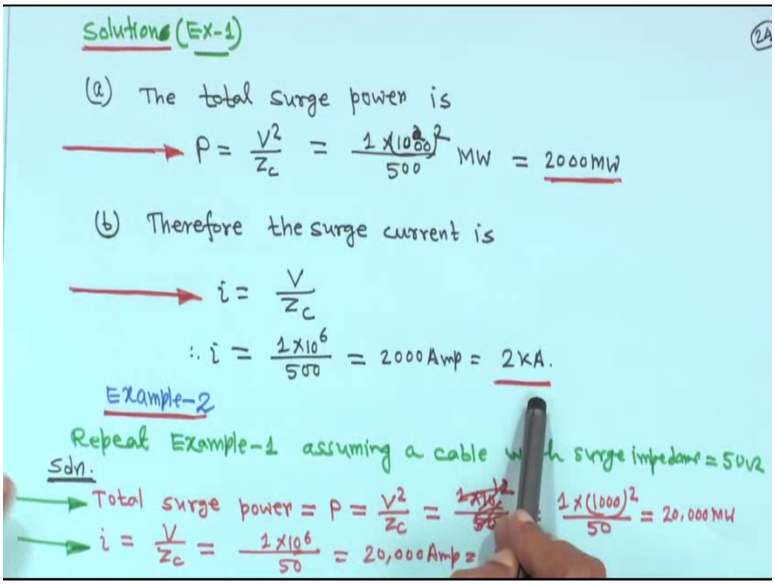

So, this is the simple thing this is a simple thing this solution. So, total surge power is P is equal to v square by Zc. So, 1 into 1000 square by 500 is megawatts, so k v square by ohm so megawatt. So, it is 2000 megawatt and b therefore surge current is i is equal to v upon Zc. So, v is given your 1000 k v, so it is converted to the volt 10 to the power 6 volt right 1000 into 1000 divided by 500, 2000 ampere that is 2 kilo ampere right.

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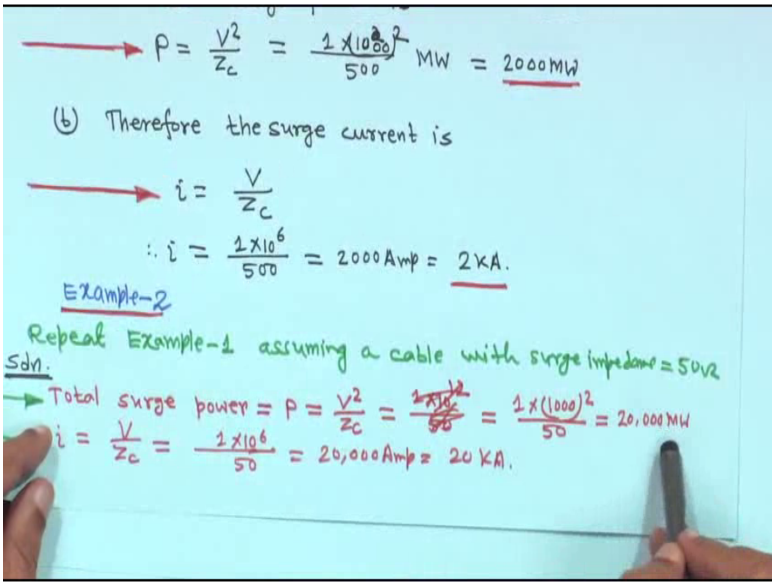

Now so repeat example, assume a cable with surge impedance 50 ohm, suppose if a cable having 50 ohm then total surge power will be v square upon Zc, it will become actually here it was 2000 megawatt there it will become 10 times more 20000 megawatt right. Similarly I is equal to V by Z c it is 1 into 10 to the power 6 by 50 that is 20000 ampere, it is 20 kilo ampere here also it is 10 times more here it is 2 kilo ampere here it is 20 kilo ampere right.

So, this one couple of 1 or 2 examples which we took, next we will n go the your what you call that super position of forward and backward travelling waves just hold on. Super position of forward and backward travelling waves, so first step before explaining all this first I will show you the look at the figure right.

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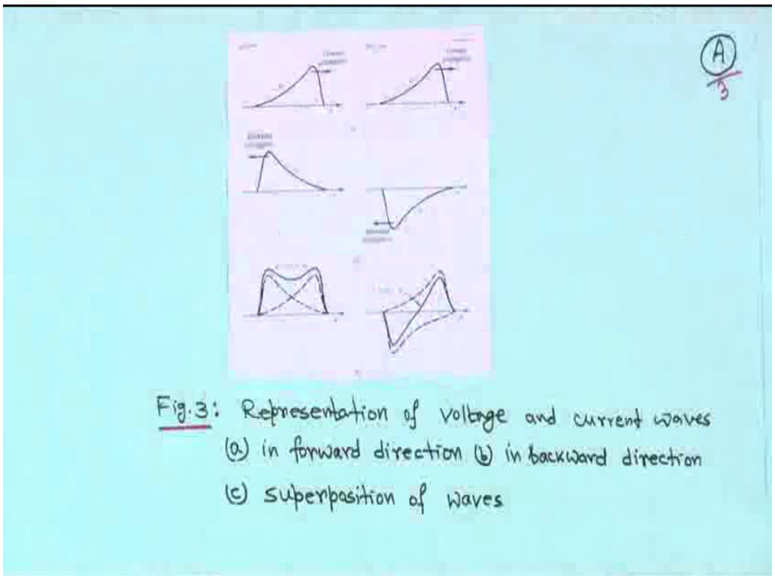

So, this here this one is the forward propagation this figure. So, I have taken from book right. So, because then see figures are symmetrical it will be easy for you to understand. So, this is a forward propagation this is voltage here this is x is increasing for this side right and this is forward propagation this is i f the current right shape banged by your dimension different, but shape is remaining same values may be different shape is remain same.

Now for backward propagation it is moving this way, so backward propagation this is v b right. So, in the case of current backward propagation current is negative right so it is ib. So, it is backward propagation, but it is current this is v b and this is ib right. So now if you add both v f plus v b right, so in that case this continuous line is vf plus vb and that line is your this is your backward and other 1 is forward if you add this it will be v f plus v b.

Similarly, in the case of current this is your this continuous line it is If plus ib, but this is your the dash line this is backward propagation and this is forward propagation. So, this is your backward propagation voltage vb is positive, but back backward propagation current it is your what you call it as negative. Now this is the super position right now if it is.

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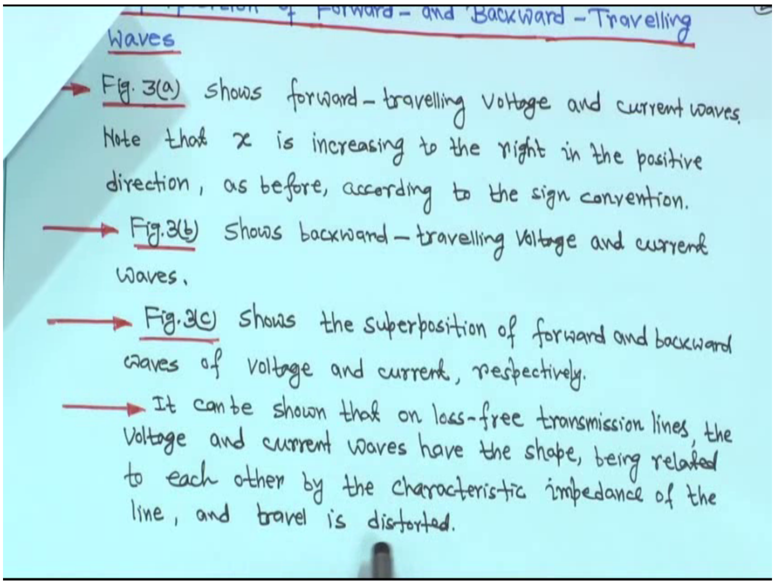

So, now the super position and figure 3 is shows forward travelling voltage and current wave this is your figure a b and figure c right. So, in this case the travelling wave current note that x is increasing to the right in the positive direction I told you this way x is increasing right and as before according to the sign convention; figure 3 b shows backward travelling voltage and current wave this is backward your propagation voltage wave and this is current wave right.

So, figure 3 shows the super position of both right this is showed you.

So, forward and backward waves of voltage and current respectively, it can be shown that on the loss free transmission lines the voltage and the current waves are the same shapes being related to each other by the characteristic impedance of the line and travel is distorted right. So, that we will see that we will see later that how things are happening right. So, further when 2 waves meet this is very interesting they do not affect each other, suppose in the same conductor right or cable when they there is when is going this way another is going opposite direction.

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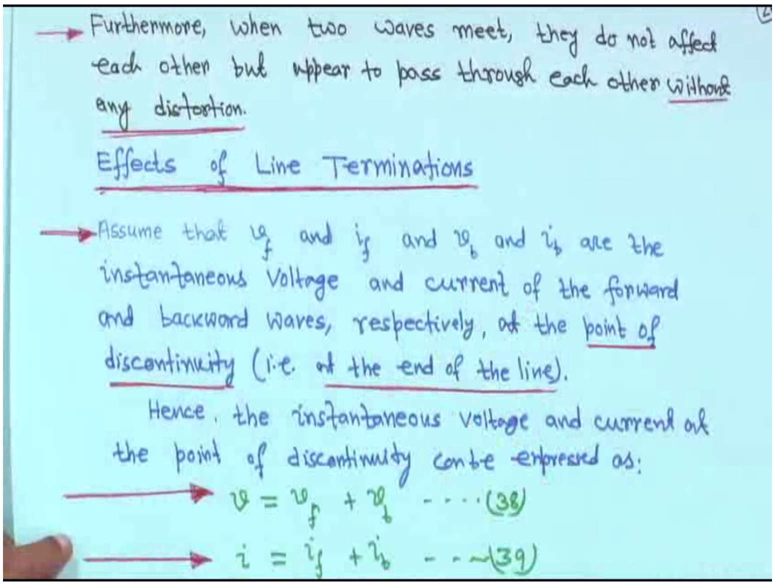

If they both the waves meet they do not affect each other, but appear to pass through each other without any distortion, I mean when both are moving I mean one is if 1 is moving this way another wave is moving this way there will be no distortion actually, without any distortion they pass you what you call through each other without any distortion this is very interesting right.

Now is the affect of line termination now you see assume that v f and i f and v b and i b are the instantaneous voltage and current of forward and backward waves, respectively at the point of discontinuity mean at the end of the line; it is this is sometimes we call at the point of discontinuity at the end of the line. Now hence the instantaneous voltage and the current and the point of discontinuity that is at the end of the line v is equal v f plus v b and i is equal i f plus i b, this we know actually previously we have seen, so this is equation 38 and this is 39. So, now we know that substitution of equation 20 and 21 right. So, we know that from equation 20 we know i f is equal to v f by Z c and i b is equal to minus v b by Z c right.

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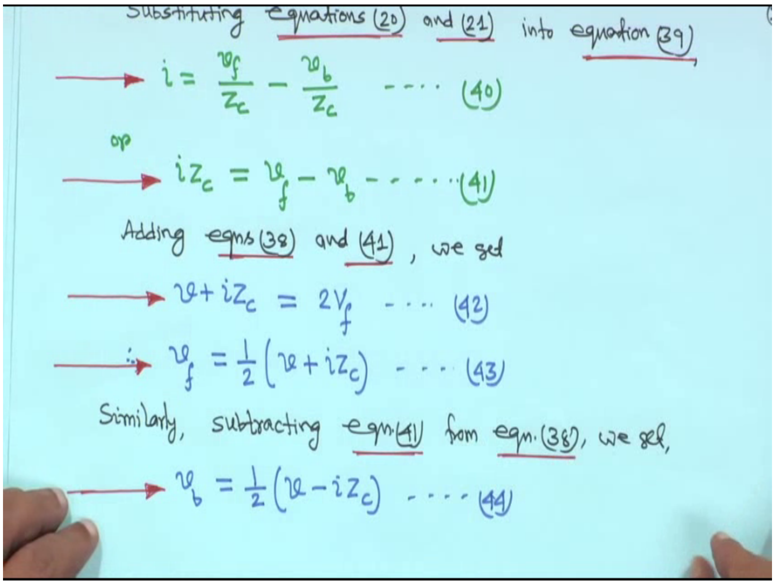

Therefore this thing that equation 20 and 21 our put here in 39; that means this equation here you put i f is equal to v f upon Z c from equation 20 and here you put i b is equal to your minus your v b by Z c right in this equation 39 you put it, I am not showing again equation 20 and 21 no need understandable.

If you do so you will get I is equal to v f upon Z c minus v b upon Z c this is equation 40; that means, I into Z c is equal to v f minus v b next is you add equation 38 and 41; that means, this equation 38 this is your v is equal to v f plus v b and this equation 41 this 1 you add, if you do so you will get v plus I Z c is equal to 2 v f therefore, v f will be half v plus I Z c this is equation 43 right. Similarly your subtract equation 41 from equation 38 you will get this 1, I am not doing for you it is understandable you just from equation 38 you subtract equation 41 and just simplify you will get v b will be half of v minus I into Z c this is equation 44 right. Then another thing alter alternatively from equation 42.

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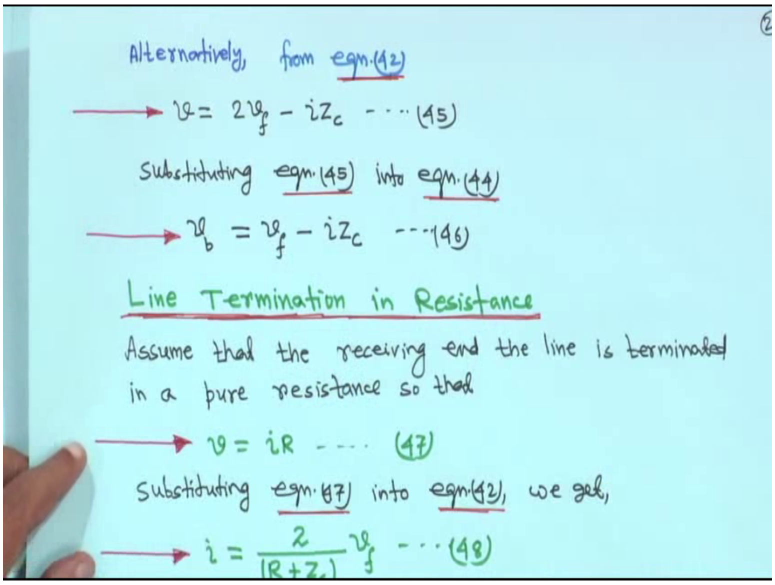

I mean this equation 42 we can write v is equal to 2 v f minus i Z c; that means this equation you can write that v is equal to 2 v f minus i into Z c this is equation 45; now you substituting equation 45 into equation 44; that means, this v is equal to your 2 v f my see this 1 this v value 2 v f minus I Z c you put in this equation 44 you put it here, if you do so then you will get v b is equal to v f minus i into your Z c this is equation your 46 right.

Now, next is line termination in resistance, I mean it is not open circuited. Now, as soon as the receiving end line is terminated in a pure resistance right; I mean you have a receiving end line I mean it is something like this you have a sending end, I am just putting like this down way long transmission line you make right; now at this thing and here it is terminated by a resistance R right is termination by resistance by your sending end side everything is there source and other things.

So, assume that the receiving end line is terminated in a pure resistance. So, that v is equal to IR this is equation 47. Now substitute equation 47 into equation 42 and here you have to consider case by case right. So, substitute this one into equation 42 I mean in this equation you substitute v is equal to IR here you substitute because, line is terminated in the receiving end and resistance r. So, v is equal to IR you substitute. If you substitute then you will get i is equal to 2 upon r plus Z c into v f this is equation 48 right similarly and from equation 47 this I you got 2 upon r plus Z c. So, this I value you put it here in 47. So, I is equal to 2 upon r plus Z c into v f you put it here.

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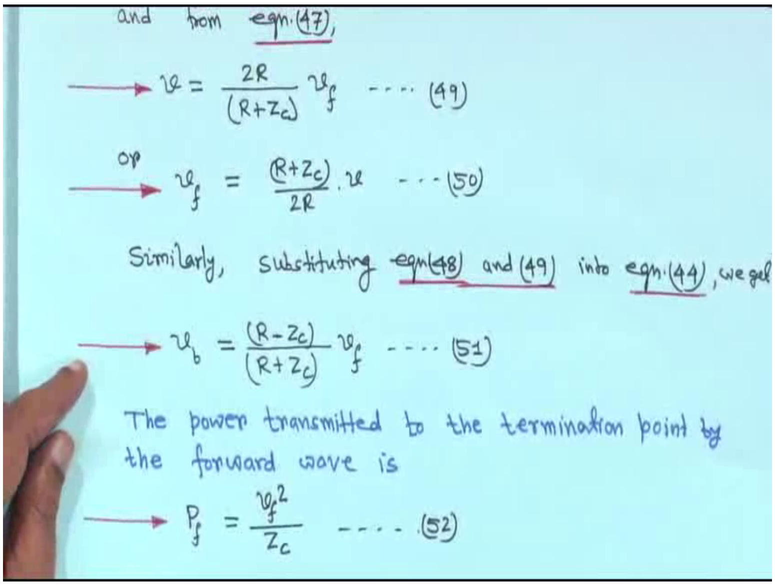

So, if you do so you will get v is equal to 2 R upon R plus Z c into v f this is equation 49 or from this equation only v f is equal to R plus Z c upon 2 r into v this is equation 50.

Similarly, for v b you will do it for the all these substituting equation 48 and 49 into equitation 44, you will get v b is equal to R minus Z c by R plus Z c into v f this is equation 51 this is the expression for v b right. So, just you do it yourself I have done all this thing is showed, so just I have written also just you put it you will get it there right. So, the power transmitted to the termination point by the forward wave will be v f square Zc. Zc is the characteristic impedance of the line. So, it will be v f square by Z c this is equation 52 right.

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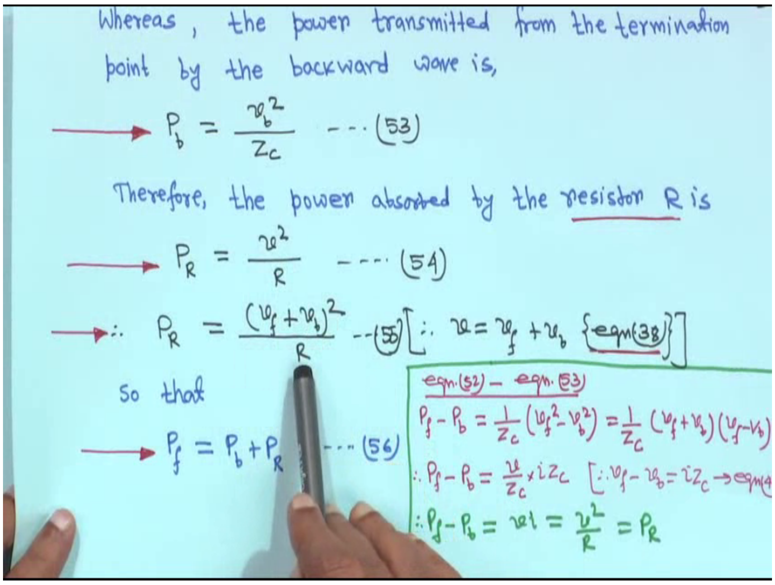

So, next is that whereas the power transmitted from the termination point by the backward wave it will be P b will be v b square upon Z c, this is understandable right. So, it is 53 therefore, the power absorbed by the resistor R is it is v square by R because across the resistance voltage is v. So, P R will be v square upon R this and it is a lossless line right. So, P R will be I mean is along this thing line resistance not considered.

So, P R will be v square by R so line is terminated at resistive value resistance r right, so this is equation 54; that means, P R is equal to v f v is equal to v f plus v b. So, v f plus v b whole square upon R this is equation 55. Now so that P f is equal to it is actually P f will be is equal to P b plus P R or should not be confused that P f forward wave will be this thing do not write P f plus P v, suddenly no P f will be p b plus P R because P f is the from the common sense you can make out from your intuition that P f actually is the forward wave, something will be reflected back and some will go to the resistor.

So, P f will be P b plus P R, how we got it equation 52 minus you this thing equation 52 you subtract equation 53 from equation 52, then you will get P f minus p b is equal to 1 upon Z c v b v f square minus v b square is equal to 1 upon Z c it is a plus b into a minus b formula. So, we can write v f plus v b into v f minus v b right. Now v f minus v b we have from equation 41, we have seen that v f minus v b is equal to i into Z c right. So, v f minus v f you put i into Z c and you know that v is equal to v f plus v b. So, P f minus p b will be v by Z c into i into z c.

So, Z c cancel it will be v into i, so P f minus P b will be v into i that is nothing, but v square by r that is equal to PR, that is why I am writing so that P f is equal to p b plus P R common sense is that wave forward wave is moving and after that at the receiving end something will be transmitted to r, because it is not open circuited and some will be reflected back. So, basically P f will be p b plus PR. So, how things have been done it is shown. So, thank you very much, welcome.