

Biomedical Signal Processing
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Lecture - 10
Artifact Removal (Contd.)

(Refer Slide Time: 00:20)

Time Domain Filtering contd.

Moving Average Filters

$$y(n) = \sum_{k=0}^N b_k x(n-k)$$
$$H(z) = Y(z)/X(z) = \sum_{k=0}^N b_k z^{-k}$$

The block diagram illustrates a discrete-time filter. The input signal $x(n]$ enters from the left. It is split into two paths: one goes directly to a multiplier b_0 , and the other goes through a chain of delay elements z^{-1} . Each delay element is followed by a multiplier $b_1, b_2, \dots, b_k, \dots, b_N$. The outputs of all these multipliers are summed at a summation node Σ to produce the output signal $y(n]$.

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66

So, the next, in the time domain filtering, we take the technique called moving averaging filters. Now, moving averaging filter actually, helps to get rid of the problems we faced with the synchronous averaging. In case of moving averaging filter, it takes care of the fact that it can work with the single input of the signal. It does not need multiple realizations and thereby it is much more suited for real time operation.

Here, we show a simple form that the output, what is y_n , that is made out of actually addition of the present signal and past few samples of the signal and we have used an weighted average. Below, we are showing that the block diagram that the input is fed to a actually that chain of registers which is acting as a delay line and there that at each step it is tapped and multiplied with some rate and the all the products of this multiplier or the output of the multiplier they are added together and that gives us the output y_n .

So, here if we look at the transfer function, $H(z) = Y(z)/X(z)$. So, we get that. That is given here. So, it is also expressed in terms of the b_k 's.

(Refer Slide Time: 02:24)

Time Domain Filtering contd.

Moving Average Filters

- Finite number of terms $h(k) = b_k, k=0,1,2 \dots N$ b₀ = 1
- An FIR, realized non-recursively with no feedback
- Output depends on present and past few samples
- Tapped delay filter
- Transfer function has no poles except at $z=0$
- Linear phase if series of tap weights are symmetric or anti-symmetric

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So, now let us try to look at the form that what are the things we need. First of all we need finite number of terms, that is, if we call them as a impulse response h_k that is nothing, but this b_k 's, starting from 0 to N. So, we have taken N terms and to make it unique usually that b_0 is kept as 1; that b_0 is kept at usually 1, because if we scale it change it then all the things can be scaled and that it will be just simple scaling of the output. So, usually b_0 is kept as 1 and this filter is a FIR filter or Finite Impulse Response filter because there is no recursion or feedback here and how that helps because it is an FIR filter there is we need not have to worry about the stability of that filter. Any filter we choose that is inherently stable.

So, filter design is much more easy and in this case the output depends on the present and past few values. So, it is a causal actually filter, it does not depend on the future, that way that we get good response that we need not have to that the filter is causal and it is represented by that delay filter or tapped delay filter that we have shown in the previous page, that the architecture what we have shown here, that this is the architecture of tapped delay filter below filter. So, this is the architecture of the tapped delay filter. So, this tapped delay filter is used that is gives a easy way to implement it.


(Refer Slide Time: 04:42)

Time Domain Filtering contd.

Moving Average Filters

- Finite number of terms $h(k) = b_k, k=0,1,2 \dots N$
- An FIR, realized non-recursively with no feedback
- Output depends on present and past few samples
- Tapped delay filter
- Transfer function has no poles except at $z=0$
- Linear phase if series of tap weights are symmetric or anti-symmetric

$$\sum_{k=0}^N b_k z^{-k}$$



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And the transfer function what we got transfer function in these case we got in the form of summation of $b_k z^{-k}$ where k is equal to 0 to N , that is what the transfer function you got.


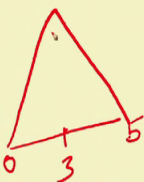
It does not have any pole, except for at 0. So, we need not have to worry about the stability of this signal and if we can choose the weights properly; that means, if the weights are symmetric or anti symmetric, then in that case that we can have linear phase.

(Refer Slide Time: 05:45)

Time Domain Filtering contd.

Moving Average Filters

- Finite number of terms $h(k) = b_k, k=0,1,2 \dots N$
- An FIR, realized non-recursively with no feedback
- Output depends on present and past few samples
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So, that means, say if we have a 5 weights starting from say 0 to 5 and if we choose the weights in this way that where 3 is the midpoint, it is we are getting it actually symmetric over that point, in that case we get the linear phase. So, we can also have anti symmetric; that means, it could be drawn like this. So, in both the case we will get linear phase. We will go through some example that will help us to realize that what we mean and the important actually the importance of this linear phase is that if we look at any signal it consists of multiple actually mono component signals. For example, if we take the Fourier series to the present that signal, then we find that or if you do the Fourier transform we see the signal consists of multiple sinusoids. Now, for each of these frequency that there is a phase associated with it along with a amplitude that the phase or the initial that the position of that signal that determines that the shape of the signal very well.


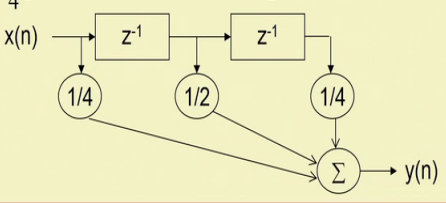
Now, in our body some of the sensors they are very much sensitive to the that this phase and some of the sensor on the other hand they are not sensitive to it; for example, when we listen our ear is not particularly sensitive to the phase of the signal, but if we talk about that our vision, if the signal phase is changed, so what we see that in a image if we artificially create and just the change the phase, but the spectrum or the spectral energy remains the same. We will see completely another picture. So, our eyes are actually very much sensitive to the phase. So, if it is we are interested in the shape of the signal it is better that we should try to preserve the shape and one way to do that using the linear phase signal or linear phase filter.

(Refer Slide Time: 08:29)

Time Domain Filtering contd.

Moving Average Filters

Hanning Filter $y(n) = \frac{1}{4} [x(n) + 2x(n-1) + x(n-2)]$

$$h(n) = \frac{1}{4} [\delta(n) + 2\delta(n-1) + \delta(n-2)]$$



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68

So, now, here we take an example of the moving average filter. Here, in particular we have taken a Hanning filter. Hanning filter is nothing, but a simple triangular filter. The weights are like a triangle. So, this is a base is a time and we get a triangle and in these particular case that the filter weights what is chosen we have chosen them as integers. So, we can also call these filters as an integer filter.

(Refer Slide Time: 09:16)

Time Domain Filtering contd.

Moving Average Filters

Hanning Filter $y(n) = \frac{1}{4} [x(n) + 2x(n-1) + x(n-2)]$

$h(n) = \frac{1}{4} [\delta(n) + 2\delta(n-1) + \delta(n-2)]$

Integer filter

$2^n - 2$

The diagram shows a block diagram of the Hanning filter. The input $x(n]$ is fed into a chain of two delay elements, each labeled z^{-1} . There are three taps: the first tap is at the input with a weight of $1/4$; the second tap is after the first delay element with a weight of $1/2$; and the third tap is after the second delay element with a weight of $1/4$. All three weighted signals are fed into a summing junction labeled Σ , which produces the output $y(n)$.

Now, what is the benefit of these integer filters? Actually one may argue that how we are calling this as an integer filter, because we have a scaling function here that is one fourth.

So, we are having actually that fractions, but we can represent that function with a help of that the integers and there is a easy way to implement that thing and that comes as an attraction of the integer filter, that if you see that there is a scaling, that scaling is all the time it is 2 to the power n in this form. Now, what is the benefit of making it 2 to the power n? As we had the binary representation of the data, we can have the scaling that by actually shifting of that beat in the register.

So, if you have to divide it by 4, which can be represented by 2 to the power minus 2, what we can simply do? We can have 2 right shift, that can give actually that scaling and same way if we choose more carefully that the multiplication that coefficients what we have done they are also power of 2. So, for that first 2 cases that we need not have to that first the x_n and x_{n-2} multiplication of with 1, we need not have to do any multiplication the that the middle one that x_{n-1} that multiplication with 2 we can actually do it with the help of

that again shifting and shifting is a much easier operation or faster operation in microprocessor. In fact, when you face the signal at the same time you can do that job and get it without spending any extra machine cycle.

So, here we get in the next line that we get the impulse response of it that h_n is a impulse response. So, we get that delayed 3 deltas here and below we show the form of it that how the tabular filter with the help of that we are getting y_n , but if you implement it as a in a computer then it is better to do it in that way that using the shifting and if required that if you have odd values then you may have to use the multiplication. But, the integer multiplication is much faster and in case of integer filter that is the advantage we take to provide the result in real time and in a cheaper hardware; that means, we can use a say DSP which is integer DSP to do this job.

(Refer Slide Time: 12:51)

Time Domain Filtering contd.



Moving Average Filters

Hanning Filter

$$H(z) = \frac{1}{4} [1 + 2z^{-1} + z^{-2}]$$

$$H(\omega) = \frac{1}{4} [1 + 2\cos\omega] e^{-j\omega} \text{ for } z = e^{-j\omega T} \text{ and } T = 1$$

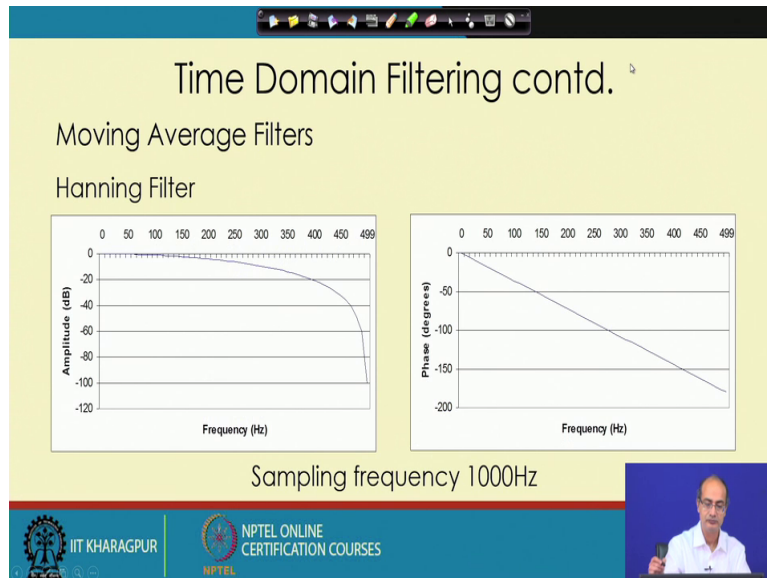
$$|H(\omega)| = \frac{1}{4} [1 + 2\cos\omega]; \angle H(\omega) = -\omega$$



69

So, now let us move forward. We see that here is that from the transfer function we have represented H_z in the z domain and corresponding to that we compute that what would be the that if you are interested the spectrum of it then we can look at the value on the unit circle which is represented by H_ω ; that means, we are taking z equal to e to the power $j\omega$; that means, we are just looking the value of the transfer function on the unit circle and we get it is the that for T equal to 1, this is the value. So, we get here the 2 parts of it; one is the amplitude response and we get the phase response. And as the Hanning window the

weights are symmetric we get the phase is linear. Phase is varying from that 0 to minus omega. So, that is the way it is moving.

(Refer Slide Time: 14:16)




So, now, here we have plotted the output of the Hanning filter, we see that this filter that the phase where it is linear the drooping characteristics is not very fast. So, if you look at that 20 dB, reduction in the amplitude we have to go up to 400 hertz. Here, the sampling frequency is 500. So, almost 80 percent of that we need to take that, I think that you know sampling is that 1000 hertz and we are getting after going up to 400 we get that minus 20 dB reduction, for 3 dB, we may have to go lower, but then we have to go in between some 200 to 250 hertz, that is very slow kind of reduction we get. Only very high frequency terms they would be eliminated by this filter.

So, that is the characteristics of this signal. However, what we can do? We can change the number of taps we can increase the number of taps and that can give us a better kind of averaging and we can get a better drooping characteristics.

(Refer Slide Time: 16:00)

Time Domain Filtering contd.

Moving Average Filter Example

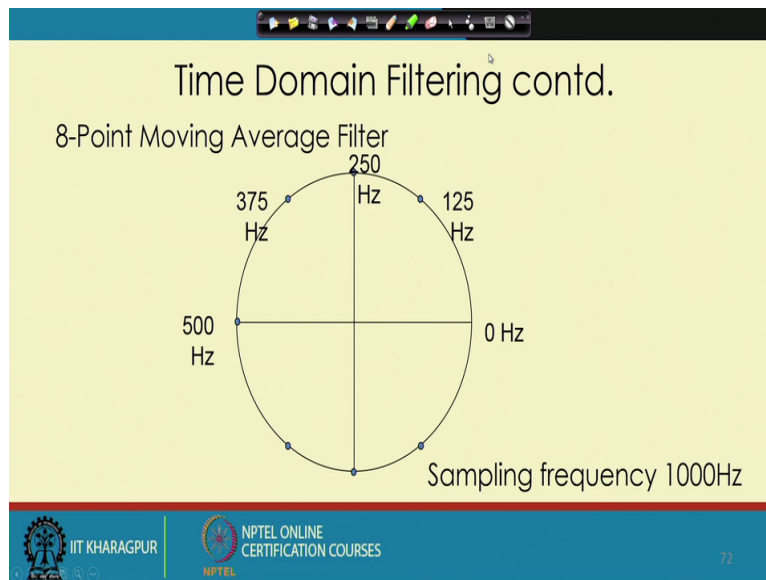
$$y(n) = 1/8 \sum_{k=0}^7 x(n-k)$$
$$H(z) = 1/8 \sum_{k=0}^7 z^{-k}$$
$$H(\omega) = 1/8 \sum_{k=0}^7 e^{-j\omega k} \text{ for } z = e^{j\omega T} \text{ and } T = 1$$
$$= 1/8 \left[1 + e^{-j4\omega} \times \{1 + 2 \cos(\omega) + 2 \cos(2\omega) + 2 \cos(3\omega)\} \right]$$


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So, next we look one such filter, that is, taking actually 8 taps and here we have taken a special care that again it is an integer filter, but it is uniformly weighted filter. So, if you look at the tap weights it would be just like a rectangle. It is uniform weighted all the points and that to take care of a scaling there is a term 1 by 8 and 1 by 8 can be represented by 2 to the power minus 3; that means, these division can be implemented with the help of 3 shifts in the right side.

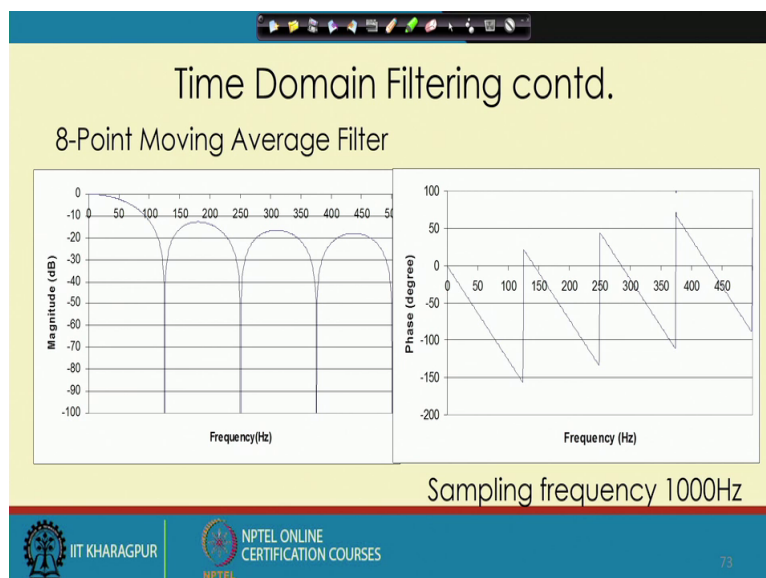
So, for that filter this or that H z in this case, we can look at the frequency domain characteristics by taking z equal to e power minus j omega T and T equal to 1. So, we get the H omega here and then we can simplify it further that replacing the e with cosine and sine term and we can get it consists of only the cosine terms; that means, it would be symmetric and the maximum will occur at 0.

(Refer Slide Time: 17:30)



So, here first look at the pole 0 diagram. What we get, the zeros are located like this at different frequency; that means, that we have, if there is a component of signal at 125 hertz, that will come down to actually 0. If there is a component at 250 hertz that will come down to 0, 375 hertz also will come down to 0, so it is coming in that way.

(Refer Slide Time: 18:07)



And, now we look at the spectrum. So, the characteristics of the zeros are coming here and as you see that it has to go to 0 at 125 hertz, very quickly we are getting that minus 20 dB

reduction just beyond that 100. It is becoming minus 20 dB reduction and if you are interested to say 3 dB cut off, then it is coming at 50 hertz kind of thing.

Now, if you look at the phase plot, the phase plot is becoming actually again linear. But it is a piecewise linear, that means, it has some discontinuity and those location of the discontinuities are coinciding with actually that the location of the 0. Wherever there is a 0 there would be a discontinuity and again it actually flows in a linear way. So, it is not as good as they completely linear phase, but it is better than a non-linear phase.

(Refer Slide Time: 19:29)

Time Domain Filtering contd.

Moving Average Filter to Integration $y(t) = \int_{t_1}^{t_2} x(t) dt$

More general form of Integration $y(t) = \int_{-\infty}^{t_2} x(t) dt$

For causal signal, the form of Integration $y(t) = \int_0^{t_2} x(t) dt$

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Next, we look for the moving average filter to integration, the relation between these 2. The integration here in the right hand side that we are showing that input is x. So, we can integrate from t 1 to t 2 at some interval and we get the output y t. In a more general form if the signal is starting from minus infinity we get it the integration from minus infinity to the present instant may be t 2 and that could be giving us the signal that y t. For a causal signal, however, when the signal is starting at 0, that integration should be done from the 0.

(Refer Slide Time: 20:37)

Time Domain Filtering contd.

FT of the integral relationship

$$Y(\omega) = \frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$$

Frequency response $H(\omega) = \frac{1}{j\omega}$

Magnitude and phase responses

$$|H(\omega)| = \left| \frac{1}{\omega} \right| \text{ and } \angle H(\omega) = -\frac{\pi}{2}$$

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So, we can see that there is a relationship between the signal and that moving average filter. Now, here FT is the integer, here what we are showing by Y_n , if we take the integration of the Fourier transform if we take what we will get, it will be 1 by $j\omega$, $X(\omega)$ plus there would be a delta function and 0 frequency. So, that's the form we get. So, immediately what we can get that the frequency response would be 1 by $j\omega$ and the magnitude of it would be 1 by ω .

So, this one gives us one clear indication that the value would be maximum at 0 and as ω is increasing it will actually reduce, so, that means, it is a low pass filter and because of the presence of that 1 by j that we get a constant phase lack of minus π by 2 .

(Refer Slide Time: 22:10)

Time Domain Filtering contd.

Accumulation of a discrete time signal till present $H(z) = \frac{1}{1-z^{-1}}$

8-point MA in different form $y(n) = y(n-1) + \frac{1}{8}x(n) - \frac{1}{8}x(n-8)$

Transfer function and Frequency response

$$H(z) = \frac{1}{8} \left[\frac{1-z^{-8}}{1-z^{-1}} \right] \quad \text{and} \quad H(\omega) = \frac{1}{8} \left[\frac{1-e^{-j8\omega}}{1-e^{-j\omega}} \right] = \frac{1}{8} e^{-j\frac{7}{2}\omega} \left[\frac{\sin(4\omega)}{\sin(\frac{\omega}{2})} \right]$$

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So, the same thing can be implemented by actually accumulation filter in the discrete domain. So, the form would be that we can take this form that $H(z)$ equal to $1/(1-z^{-1})$. However; that means, it has a infinite memory kind of thing. So, instead of implementing it in this way we can implement it in a little different way. So, that we can implement it that 8-point MA that we can take the last output of the filter plus the latest value and if you take the as we have taken the 8-point that MA filter that what is just coming outside the window that we need to subtract. So, we are taking the 8-point MA filter it is actually equivalent to that. So, using that filter the only change what we have done here that this is no longer a FIR filter, because in case of FIR filter we have the only the; that input signal and past few inputs, but here we are using the output of the signal also to generate the present output.

So, here we get the corresponding that transfer function and from there we can compute that what would be the frequency response here using e to the power z equal to the power $j\omega$. So, that way we can get the response and it comes like a sine function, this one it is coming in the form of a sine function.

(Refer Slide Time: 24:41)

Time Domain Filtering contd.

Derivative-based approach for low-frequency artifacts

$$y(n) = \frac{1}{T} [x(n) - x(n-1)]$$

Transfer function $H(z) = \frac{1}{T} (1 - z^{-1})$

Transfer function and Frequency response

$$H(\omega) = \frac{1}{T} [1 - e^{-j\omega}] = \frac{1}{T} e^{-j\omega/2} [2j \sin \frac{\omega}{2}] \text{ leads to}$$
$$|H(\omega)| = \frac{2}{T} |\sin \frac{\omega}{2}| \text{ and } \angle H(\omega) = \frac{\pi}{2} - \frac{\omega}{2}$$

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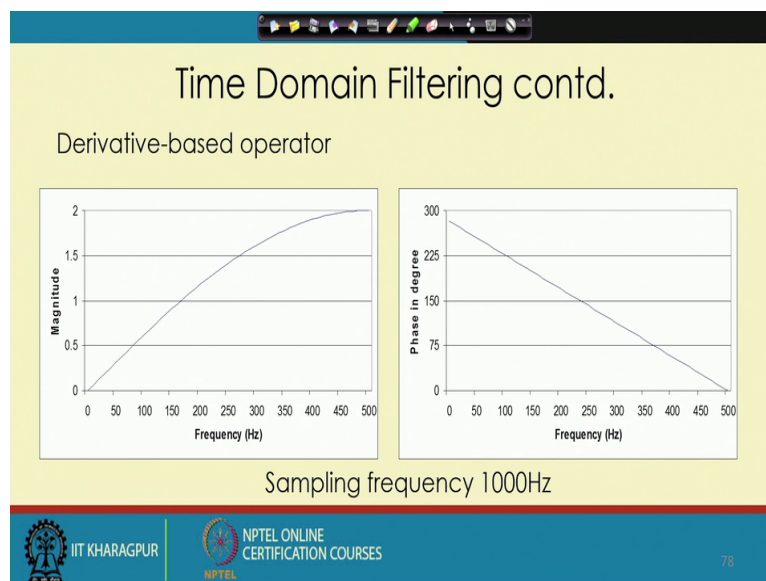
Derivative-based then we can look at the other kind of signals like we can look at that the low frequency artifacts; for example, we have seen that the baseline filter that baseline wandering. So, for the filters what we have seen that is moving average filter, synchronous averaging they are primarily used or good for removing the high frequency noise.

Now, when we are going for actually that when we are going for low frequency noise that is, baseline wandering then we need to go for a little different kind of filter and we need to go for a derivative filter. Again, it is a time domain filter and again it is in the form of a m a filter, but some of the coefficients need to be negative in this case.

So, here we have taken the simplest form of it that we are taking the difference between the present and the past input value and the corresponding transfer function is given here is H z. So, using that we can taking that if we take z equal e to the power j omega T, then with the help of that we can take this actually form and with that we can actually compute the value H omega it comes with a sine and the angle that we get pi by 2 minus omega by 2.

So, what we are getting that is actually a decreasing function that sine means at 0 values, the value would be the omega equal to 0 the value would be 0, then it will slowly raise. So that means, it will be a high pass filter and we get that it would have also have a linear phase.

(Refer Slide Time: 27:22)



So, here we show the magnitude response here and the corresponding phase response is also shown here. So, from that we can get, it is a high frequency filter and having a linear phase. So, we stop here for the session.

Thank you.