Biomedical Signal Processing
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Lecture - 63
Tutorial – V

Now we will go through the question number 5.1.

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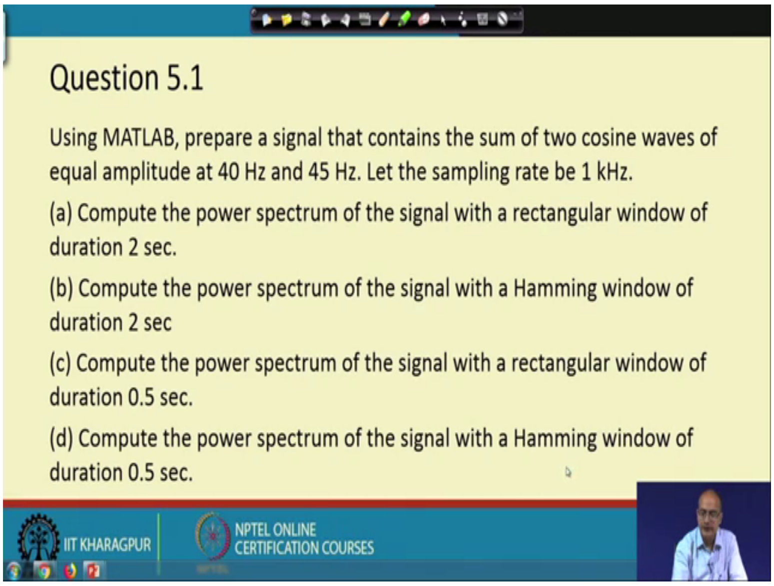

So, the task here that we have taken two cosine waves, which are closely spaced of equal amplitude. They are of frequency 40 and 50 Hertz. That means, they are closely spaced and they are sampled with 1 kilohertz sampling frequency. That means, at a much higher sampling rate they are sampled.

And now the task is that we have to compute the power spectrum of the signal with a rectangular window first and signal duration within the window with 2 seconds ok. So, we have signal. We can assume that it is starting from 0 to perpetual that waves and we have taken 2 seconds using a rectangular window and we have to compute the power spectrum.

Now, you repeat the same thing, where we will take a Hamming window of duration 2 seconds. So, changing the window what is the difference it creates we would like to look at. Next, that again will take the rectangular window, but this time the duration would be a 0.5 second. That is one-fourth of the previous duration, and we will repeat the same experiment of computing the power spectra using a Hamming window of duration 0.5 seconds.

So, we have two windows; rectangular window and Hamming window and two durations, 2 seconds and 0.5 seconds and with that, what we get that, we here four choices a, b, c, d. So, we have to compute the power spectra.

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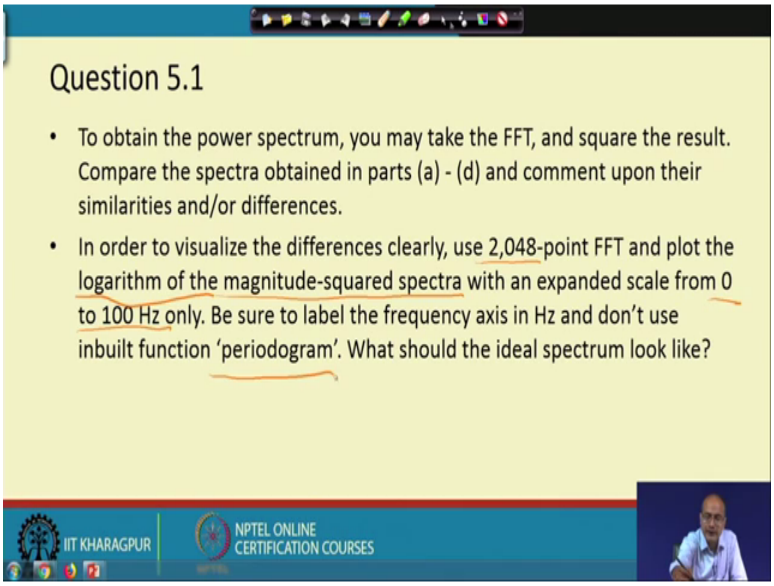

Now, to compute the power spectra, we are asked to use FFT, and take the square to compute the result and we have to do that for all the four cases and for the FFT we are told that we should take 2048-point FFT.

Now, how it matters? The more number of points we will take, we would get we are supposed to get better frequency resolution. So, that is the reason that a high number is given to compute the FFT. And we should actually, plot that the result in the log scale; logarithm of the magnitude squared spectrum ok. So, we need to plot that in the log scale and we should concentrate only on the frequency 0 to 100 Hertz.

In this case, as you recall that sampling frequency is 1 kilohertz. So, we have a quite a big range compared to the, that the frequency of the signals. The two frequencies they are 40 and 45. So, as we have two signals 40 and 45 Hertz, there is no point in look into the whole range. So, we should concentrate into the part that 0 to 100 Hertz only ok.

And, we are asked not to use the periodogram function which is inbuilt in the MATLAB and we should also get that how the ideal spectrum would look like and compared to that what we are getting ok. So, that is a task we have.

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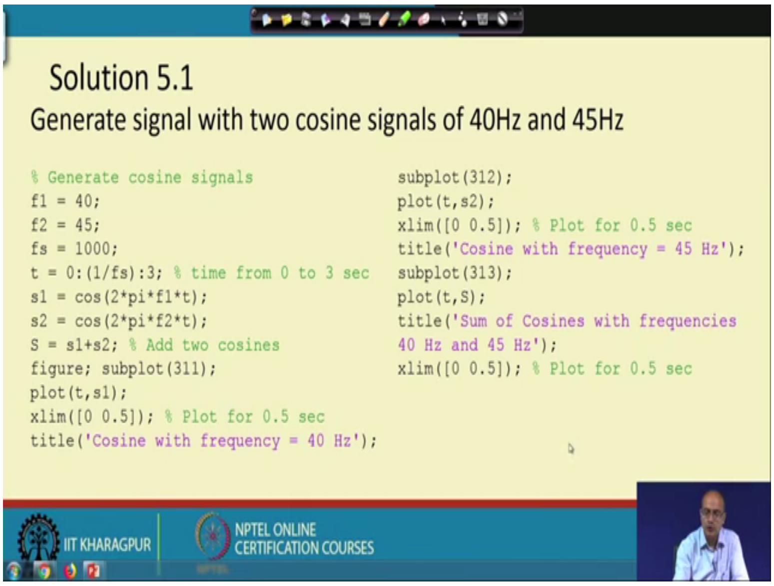

So, here so first we have to create the two cosine wave; one with frequency 40 Hertz another with 45 Hertz. So, we have assigned the two frequencies f1 and f2 and sampling frequency we have taken 1 kilohertz.

Now, first we have taken created the signal for 3 seconds, because what we have to do that our maximum duration of the signal required is 2 seconds. So, 2 seconds or anything more than that would actually serve the purpose in this case. So, here we have taken 3 second signal and we have taken two cosines; s1 and s2 we have assigned that. And here, please look into the structure here that the time axis, the time axis is 0 to 3 seconds and increment is 1 by fs; 1 by fs means, it is that time period; the sampling time period. So, we would increment by the sampling time period.

So, that way the frequency time axis is actually taken. The samples would come and with that vector the t, we have created the that signal s1 and s2. And we have combined them and stored in a variable S here. And after that we divide the space of the plot into three parts. The top part; top one-third, that is subplot 311; we are plotting s1 and we are taking the second one, 312 that is for s2 and 313, the lowest part; we are giving the we are plotting the combined signal S.

Now, to see that plot clearly we have reduced the, the time up to which we would like to see. So, in the plot we are limiting it within the time window 0 to 0.5 second. So, instead of going for though the signal is there for 3 seconds, we are looking into first half seconds. And for each of the subplot we have given the, the title. The first one is, for the 40 Hertz second one is 45 Hertz and third one is the, the combined one ok. So, this is the way we would like to plot that and see that how the, the signal looks like.

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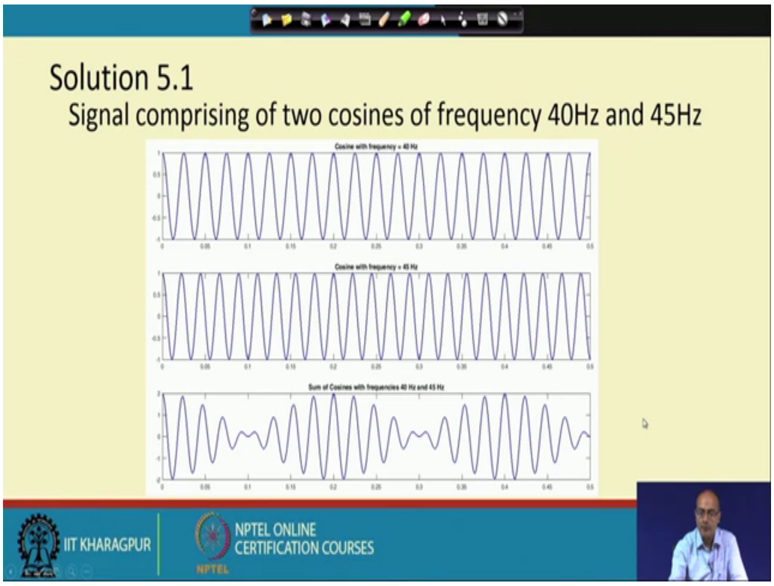

So, here we get the, the three waves. The top one is 40 Hertz, this is 45 Hertz, as the two signal frequencies are very close the two signals they look also very close to each other. And just if we look at the time domain it is difficult to even differentiate them quickly. But, when they we add them up what is happening, that at times they are actually combining with each other and we have that both the magnitudes they are adding them giving a high magnitude.

So, initially both value are one so, adding up to two. And after that as the frequency is different slowly they are going out of phase and the combined signal we see the magnitude is getting diminished. And coming here, at this position we see that they are actually the two signals they are almost in opposite phase. So, if we tell that at the beginning we had, constructive interference here we can call it is destructive interference. They are cancelling each other and again slowly the signal is building up here, again it is getting the high magnitude touching plus 2 and minus 2. Again, it is waning it is coming near to 0, again it is growing.

So, this oscillatory kind of signal, we get as a resultant ok. So, looking at this signal, it looks more or less like a frequency modulated or rather amplitude modulated signal rather than it is a combination of two closed frequencies. Now, our task is to find out, that the spectrum and from there to identify that what are these two frequencies ok. That is the reason we go for the spectrum analysis.

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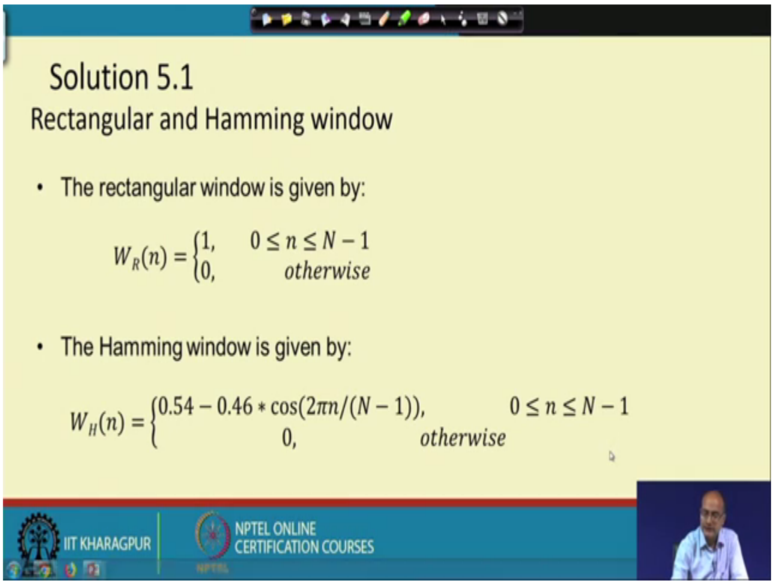

So, here first let us look at that the two windows, the rectangular window and the Hamming window. The rectangular window, it is a very simple window that it raises to 1 and again goes to 0. So, in this case that the rectangular window it is starting at sample value zero and it is having magnitude 1 up to sample N minus 1. So, for N samples it is having the amplitude 1 beyond that again it is becoming 0. On the other hand, Hamming window it is little different we will see that Hamming window how it looks like. It, it will have a actually hump kind of structure ok, both the side it will be 0. Within 0 to N minus 1 sample it will peak and again it will come down, and the equation is given here ok.

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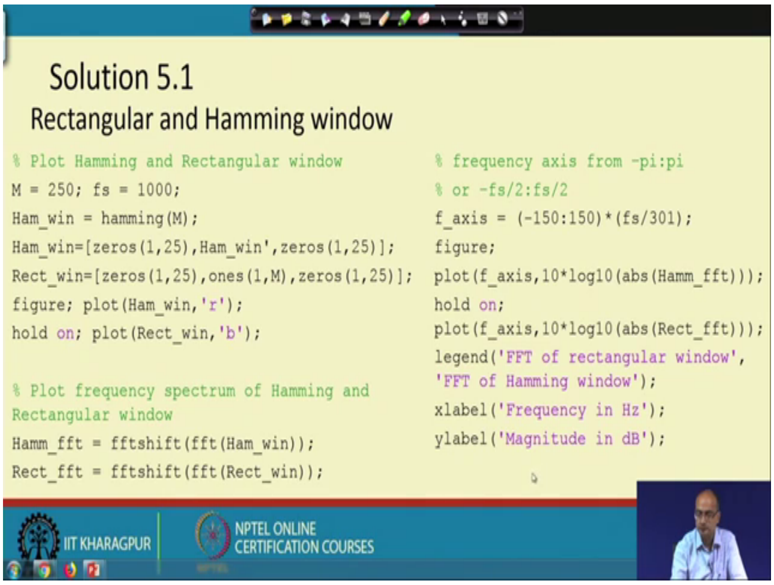

So, let us see how the, the signal looks like or these windows first look like. So, for that what we have taken we have taken the, the signal for say 250 samples we have taken M equal to 250; for that we have taken a Hamming window of M. And here, what we have done we have taken the Hamming window, using the inbuilt MATLAB function, hamming M, M is the number of samples and the variable, Ham underscore win it will give us a column vector.

So, we have transposed it here and made it a row vector and we have padded zeros in both the sides, to appreciate that how it would look like if we go beyond the window. The same way, the triangular window also is created we have padded both the side with 25 zeros and in between we have 250 ones. So, first thing is to plot actually the Hamming window using the color red and hold that plot. So, on the same plot we have given the rectangular window. Plot frequency spectrum of Hamming and the Rectangular window is. So, along with the plot of the, that signal we would like to see that in the frequency domain how they look like.

So, for that we have taken the fft of the Hamming window as well as the Rectangular window and we have used the f shift function in both the case. What it will do, that it will say when we take the, the Fourier transform it shows the results 0 to pi. Now, when we use the f shift function it will show us the result from minus pi to pi ok. So, you can actually instead of pi you can say it is fs by 2 ok.

So, first we have used that f shift and in this case, that as we have that 300 samples in the signal and we have taken the fft of that. So, the number of samples in the frequency axis taking care of the sign it is starting from minus 150 to plus 150 ok. And, so many samples we have to divide multiply it with the corresponding frequency. So, that frequency is the sampling frequency divided by the total number of samples what we have. So, that gives us the frequency axis.

And, for the new plot we have first given the command figure to create a pen. And then, we plot the x axis is fx f axis and our y axis is the 10 multiplied with log to the base 10 absolute value of the, that magnitude of the that window. First we have taken the Hamming window followed by the rectangular window. And then we use the labels to label the x axis and y axis and we have given the legend also for the, that the part that FFT of the rectangular window or FFT of the Hamming window. So, now with that with this code we are ready to show the, the two windows in the time domain as well as in the frequency domain. So, let us see how it looks.

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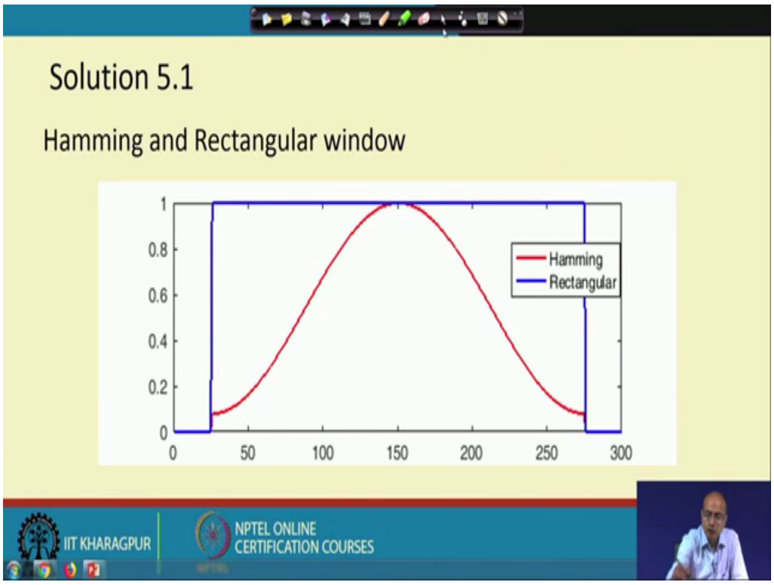

First we show, that the time domain that the window that here in the red we are having the Hamming window in the blue we are showing the Rectangular window. Actually we could make it smaller start from here and in here, but in that case we would not get that idea that how the values are beyond this window. But, for all practical purpose this is the useful actually range of the window over which we will get some sample values outside it would be zero.

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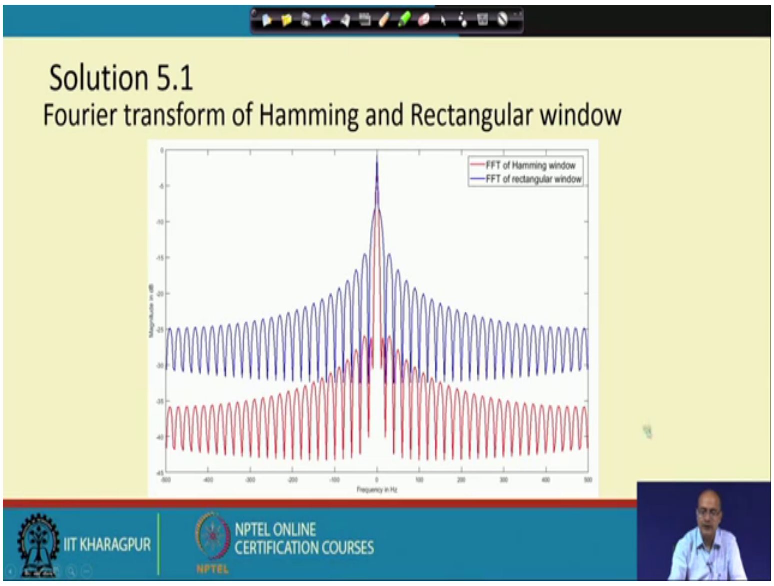

So, next we look at that the FFT of the Hamming window and FFT of the Rectangular window. Now, the first thing that comes clear that here is the peak of the Rectangular window in blue and here is the peak of the, that Hamming window. There is a different small difference compared to that, please look at that the difference between the, the nearest side lobes ok.

So, we see that in case of the Rectangular window the side lobe heights are much more compared to the, that our Hamming window. In fact, that is the one of the, the big advantage of going for the Hamming window that side lobes are suppressed to a great extent ok. So, this is we observe and now let us get back to our work that we have to apply these Hamming window and the Rectangular window one by one on the signal. So, let us go back to the, that task.

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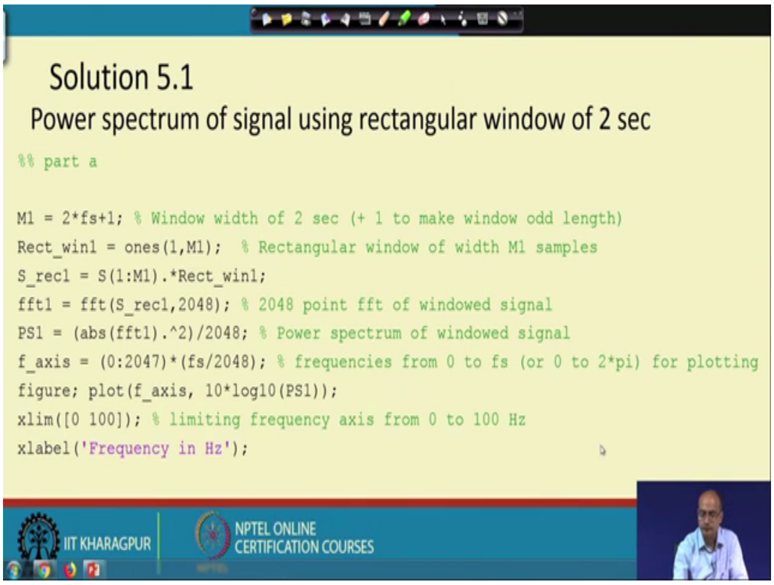

So, first we have taken the, that rectangular window of 2 seconds. To do that, that we have taken the, that M1 number of samples, how do we get that M1 in 2 seconds? We have 2 into fs number of samples, that is 2 into 1000 that means, 2000 as it is a even number and we prefer to make the window odd. So, that the symmetry is maintained we add 1 to it. So, that is our size of the window and, so our Rectangular window here in the next line that it consists of ones from 1 to M1.

And, what we do for the signal we take the same number of samples of the signal from the beginning and we take the dot product with the window ok. This sign that dot star it is sample by sample multiplication. So, this is giving us the dot product between the two vectors. And after that we perform 2048 point fft, so the command is fft our now the windows signal is S underscore rec1. And, we give the number of points for which we need to compute the fft; 2048 point fft.

And, once the fft is computed what we do? We take the amplitude square for the fft so, that we can get the power for each frequency. So, the command is here that we have taken absolute value of fft so we get a vector and dot this hat 2 it means that, for each of this sample we have to take square. And, here is a scaling with the number of that fft coefficients we have that 2048. So, after that we compute the frequency axis we have 0 to that 2047 samples or sample numbers; that means, total 2048 points we have. So, we have to multiply it with the sampling frequency divided by number of points that is the way the increment has happened.

So, we are scaling it by that to get the exact frequency and then, we take the plot first we created the figure to create the pen and we plot f axis is providing the x axis and 10 log to the base 10 that PS1 that is the power spectrum. So, we are getting the spectrum in the, that dB scale in this way ok. And again, we limit our view using this command xlim for the x axis we are limiting it from 0 to 100 Hertz ok. Because, we know our signals of interest are within that it should be at 40 and 45. And, here we are showing that how to put the levels that we already know.

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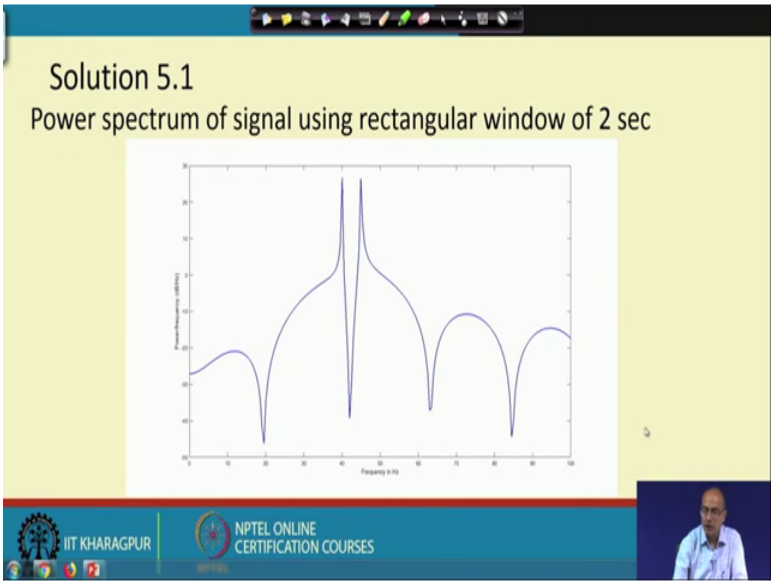

So, now here is how the signal looks like, it is a magnified portion we get the two peaks here one peak here another peak here. So, if you look at one peak is around 40 another is here 45. And though, that these parts it seems to be that a lot of other peaks looks like. We should look at the scale that if they are near 30 here it is below minus 20. So, that means about 50 dB down these magnitudes are. So, if you have not taken log scale these two would look like two spikes, and rest of the thing are actually it will clamp to 0.

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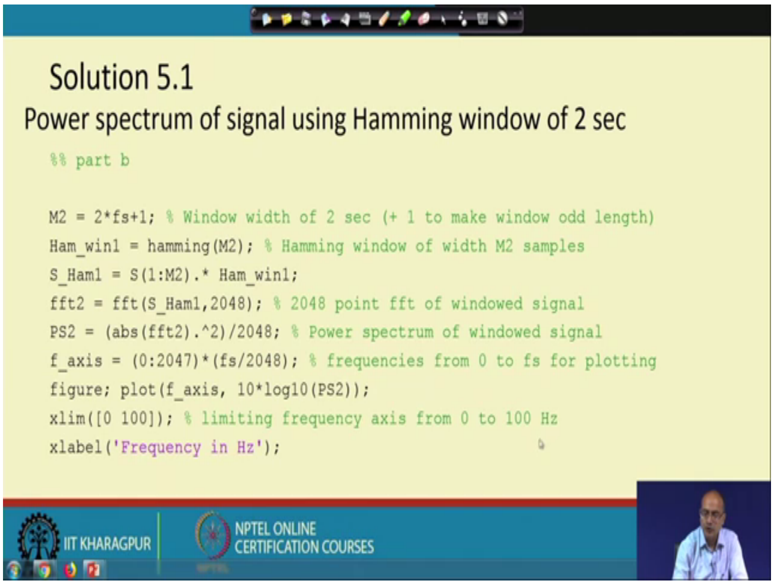

And, here we go forward we do the same for the Hamming window. For the Hamming window we take the number of samples in the same way the only thing for defining the window instead of once we take the, that inbuilt function hamming and we multiply with the signal and correspondingly take the fft, then we take the, the square of the amplitude of the, that Fourier transform of the signal.

And then we again generate the frequency axis and plot the, the log value of the, that spectrum multiplied by 10. So, so that we can tell that it is in dB scale, and we limit our view again within 0 to 100 Hertz ok. So, we are repeating the same thing as we did for the rectangular window for the Hamming window.

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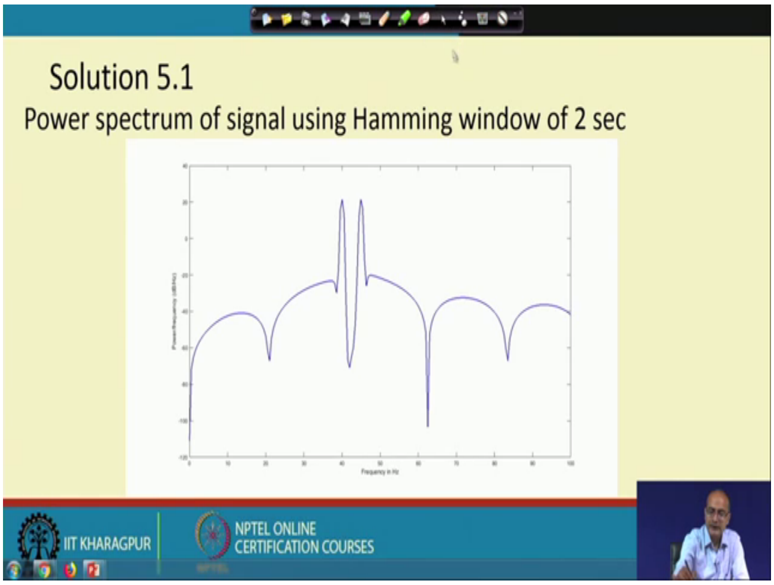

And, here is the result we get here, we get the peaks again at around 40 and 45 and corresponding to the peak we get the sub banks or the side lobes. We have good reduction here, even better if we had previous case, less than 50 dB here it is from here to here it is more than 60 dB. So, more suppression of the side lobes have happened.

But, if we look at the, these peaks these peaks are little more wide ok. So, we can appreciate that better if we go back here and again come back we can appreciate it better that we have more separation of the side lobes, but have a little wider peaks.

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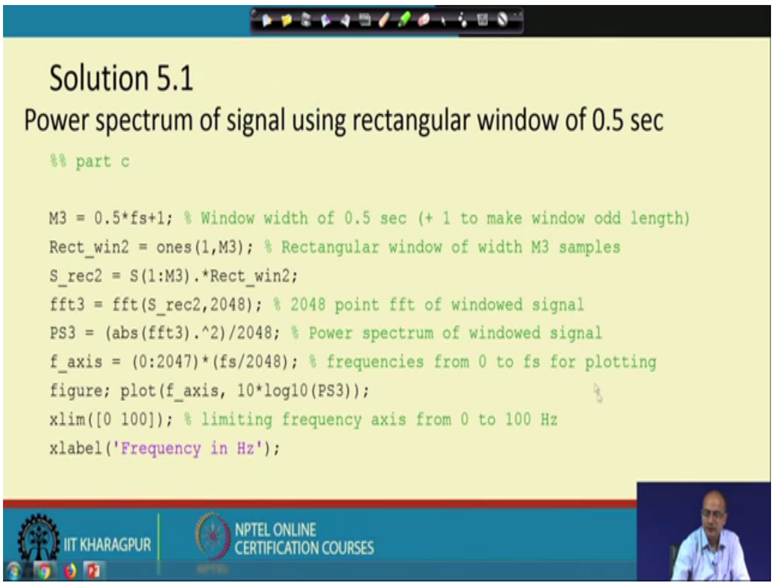

Now, we have to repeat the same experiment with rectangular window with window size of 0.5 second. So, essentially what is change is the number of samples we take that as M3 that is 0.5 into s fs that means, we will get 1000 plus 1 to make it or that is 1000 one samples we will take and for the rectangular window we repeat what we have done for the, the 2 seconds. Rest of the thing remains to be the same and here we get the output.

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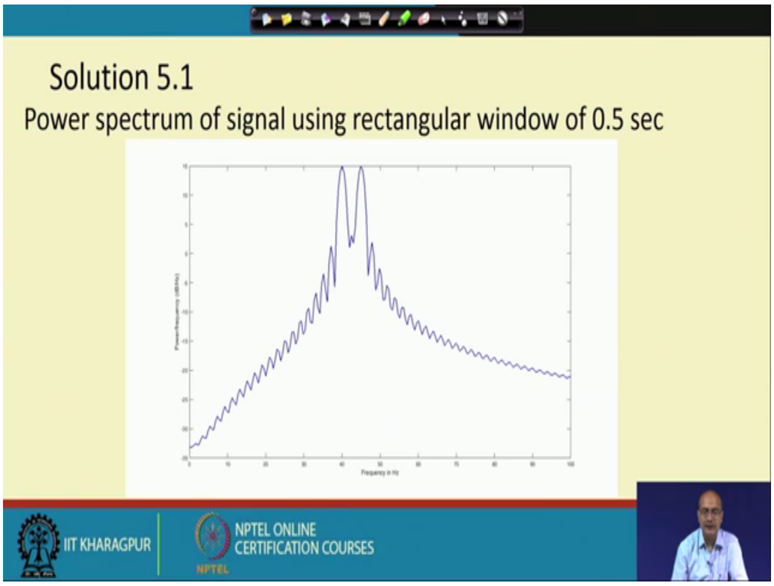

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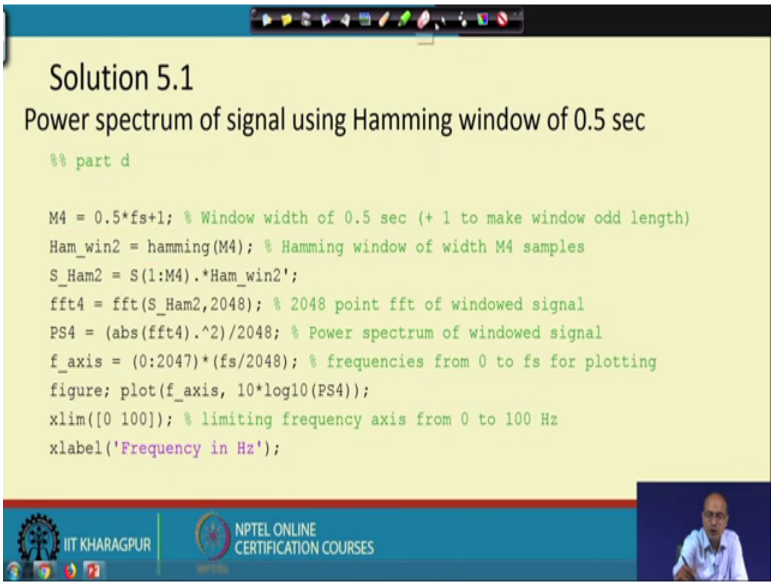

We see that, the two peaks there in the same location, but it has become wide, that is a the change we get. Next we, test that Hamming window for duration 0.5 second. Again, our number of samples is 1000 and 1 and we have taken that the window that is hamming window for 1000 1 samples and rest of the thing remains same and we have the plot of the, that signal.

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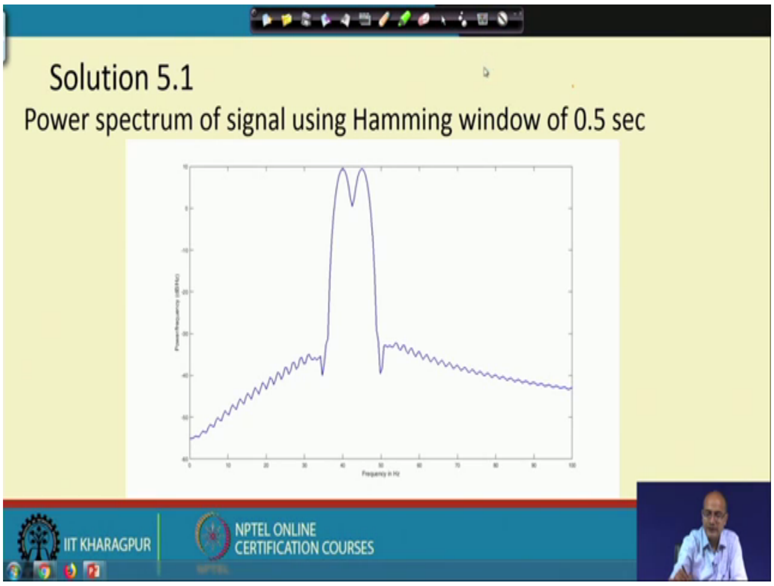

We get it looks little different and the two peaks if we look at carefully, the two peaks here and here they are almost getting merged. Because, the frequencies are close they are almost getting merged fortunately there is a kink we can separate them. So, we can go back two steps to appreciate it better, this was the result for the rectangular window and this is for the Hamming window ok. So, all the four cases we have seen.

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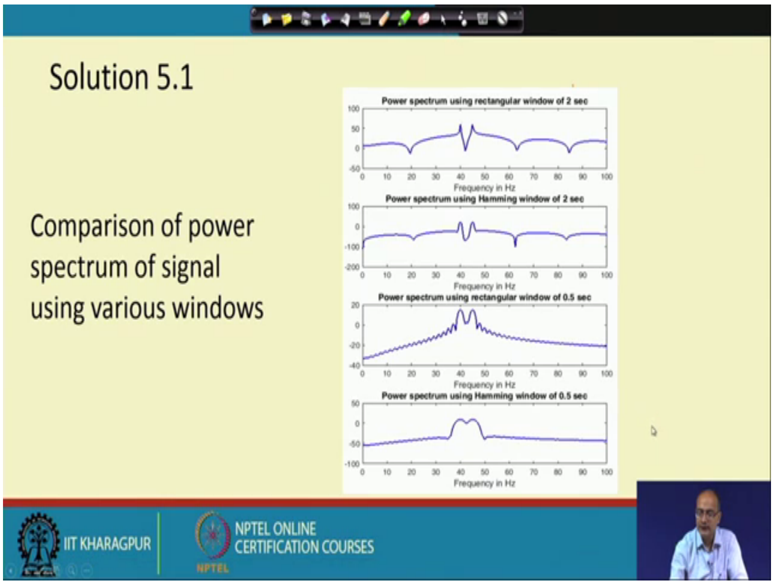

Now, we would like to see them side by side or one after another so, that we can appreciate the difference better ok. So, what we get that for the rectangular window, the spikes are sharp compared to the Hamming window ok. And, we have already seen that Hamming window provides better suppression of the side lobe ok.

And, once we are reducing the duration of the window what we see, that we are losing the frequency resolution. And, because of that see irrespective of the window, the spikes have become wider ok. So, the spikes have become wider and as a result in these case for the Hamming window with 0.5 seconds they have almost merged had the two frequencies where closer we would have look a single spike inside up to different frequencies.

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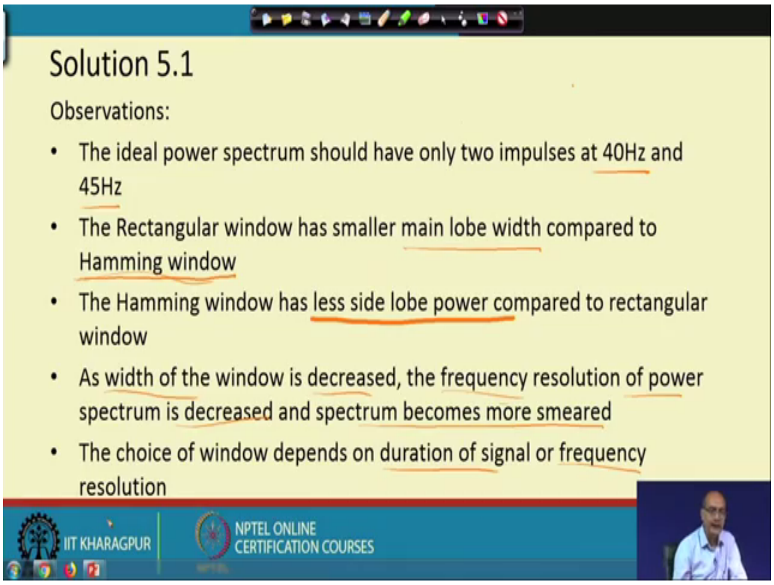

So, here is our observation that, the ideal power spectrum should have been two impulses, at the two chosen frequencies 40 and the 45 Hertz. That would have been the ideal case and for the Rectangular window, we have smaller main lobe width compared to the Hamming window. How we can get that? We have seen, that when we are using the rectangular window then what is happening that the window spectrum or in the frequency domain when we are windowing the signal in the frequency domain that multiplication becomes convolution. So, Fourier transform of the window is getting convolved with the Fourier transform of the signal.

So, signal Fourier transform are actually the two impulses if we take for any of them any of the signal. So, when we convolve it with the rectangular window we see that the width is smaller, that means the rectangular window that main lobe width is smaller compared to the Hamming window ok. On the other hand Hamming window has a side lobe compared to the rectangular window. So, there is a trade off both are desirable quality, that main lobe width should be small and why that should be small? If the frequencies are close, it may happen if the main lobe widths are bigger than the difference of the frequency these two signals may get merged.

On the other hand, that more side lobes means the smearing would be more so, side lobe should be lower. So, we get a situation that we have a trade off of these two in this case. Next, what we note that as the width of the window is decreased frequency resolution of the power spectrum is decreased; the spectrum becomes more smeared ok. The choice of window depends on the duration of the signal or the frequency resolution ok. Now, which window would be better in what case that depends on what is the duration of the signal and how closely spaced these frequencies are ok. So, that is the thing we learn from the first experiment.

Thank you.